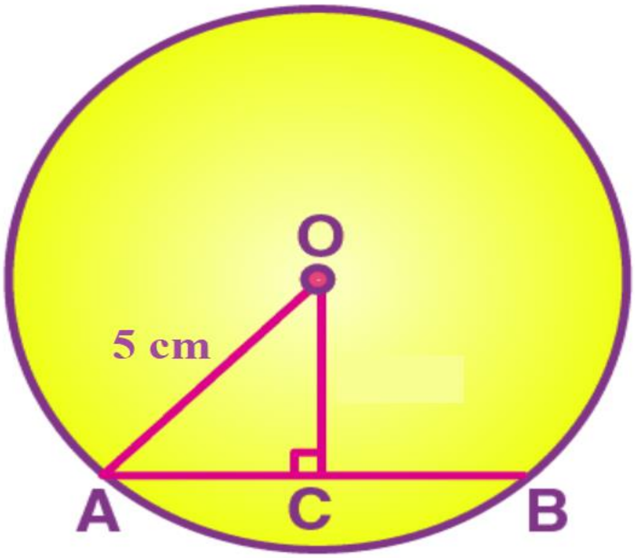
EXERCISE 17A PAGE: 210

1. A chord of length 6 cm is drawn in a circle of radius 5 cm. Calculate its distance from the centre of the circle.

Solution:

Consider AB as the chord and O as the centre of the circle.

Take OC as the perpendicular drawn from the centre O to AB.



Here, the perpendicular to a chord, from the centre of a circle, bisects the chord.

So, AC = CB = 3 cm

In △ OCA,

$OA^2 = OC^2 + AC^2$ [Using Pythagoras Theorem]

Substituting the values

$OC^2 = 5^2 - 3^2$

$OC^2 = 16$

So we get

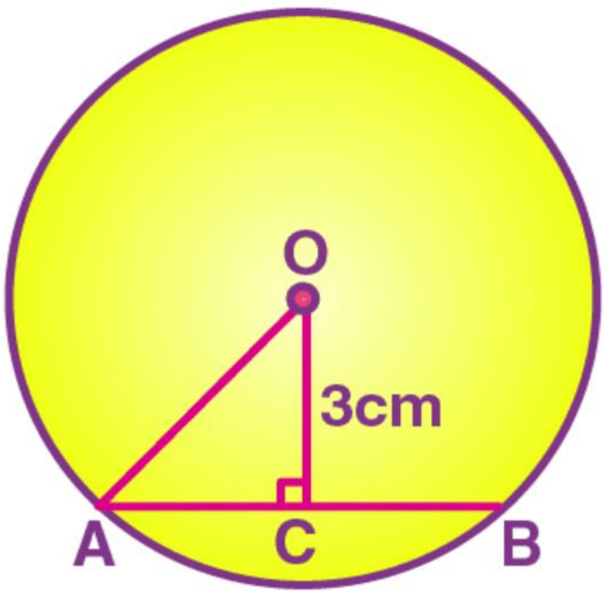
OC = 4 cm

2. A chord of length 8 cm is drawn at a distance of 3 cm from the centre of a circle. Calculate the radius of a circle.

Solution:

Consider AB as the chord and O as the centre of the circle.

Take OC as the perpendicular drawn from the centre O to AB.

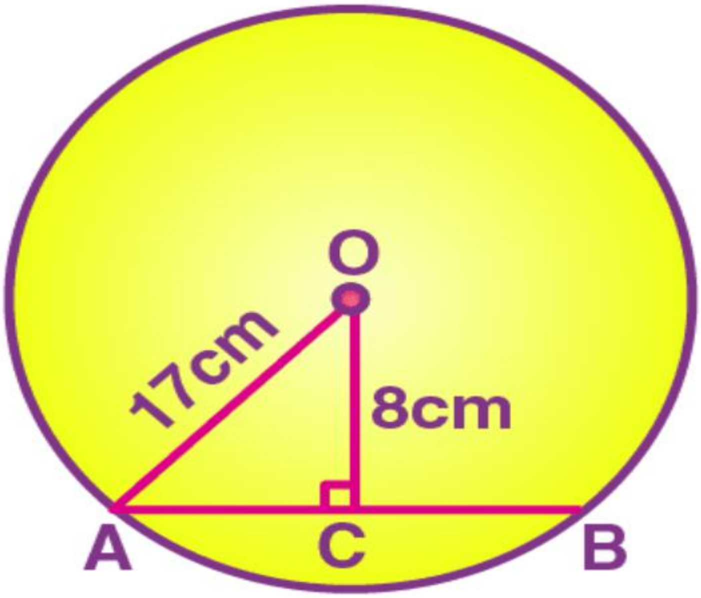


Here, the perpendicular to a chord, from the centre of a circle, bisects the chord.
So, AB = 8 cm
We know that
AC = CB = AB/2
Substituting the value of AB
AC = CB = 8/2
AC = CB = 4 cm

In △ OCA,
$OA^2 = OC^2 + AC^2$ [Using Pythagoras Theorem]
Substituting the values
$OA^2 = 4^2 + 3^2$
OA = 25
So we get
OA = 5 cm

Therefore, radius of the circle is 5 cm.

3. The radius of a circle is 17.0 cm and the length of perpendicular is drawn from its center to a chord is 8.0 cm. Calculate the length of the chord.
Solution:
Consider AB as the chord and O as the centre of the circle.
Take OC as the perpendicular drawn from the centre O to AB.



Here, the perpendicular to a chord, from the centre of a circle, bisects the chord.
So, AC = CB

In △ OCA,
$OA^2 = OC^2 + AC^2$ [Using Pythagoras Theorem]
Substituting the values
$AC^2 = 17^2 – 8^2$
AC = 225
So we get
AC = 15 cm

AB = 2 AC = 2 × 15 = 30 cm

4. A chord of length 24 cm is at a distance of 5 cm from the centre of the circle. Find the length of the chord of the same circle which is at a distance of 12 cm from the centre.
Solution:

Consider AB as the chord of length 24 cm and O as the centre of the circle.
Take OC as the perpendicular drawn from the centre O to AB.

Here, the perpendicular to a chord, from the centre of a circle, bisects the chord.
So, AC = CB = 12 cm

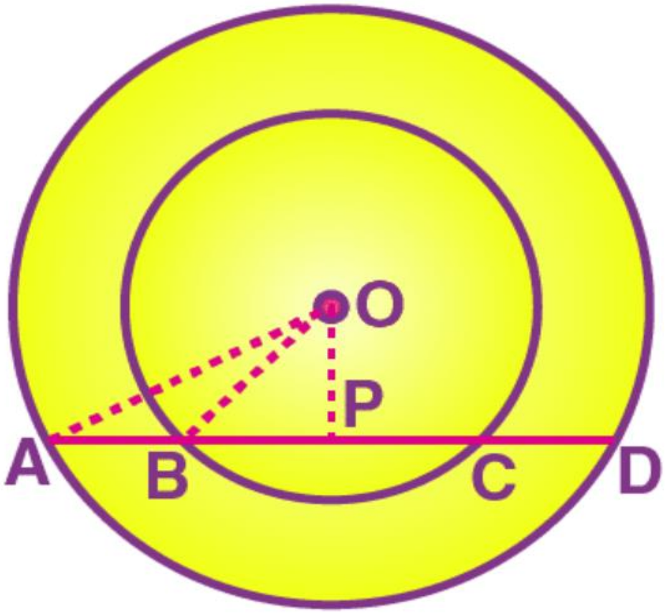
In △ OCA,
$OA^2 = OC^2 + AC^2$ [Using Pythagoras Theorem]
Substituting the values
$OA^2 = 5^2 + 12^2$
OA = 169

So we get
OA = 13 cm
Therefore, radius of the circle is 13 cm.

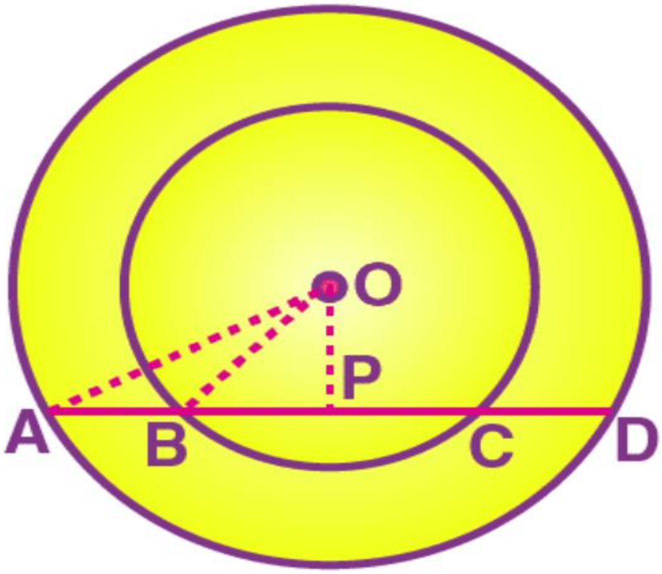
Consider A'B' as the new chord at a distance of 12 cm from the centre.
$(OA')^2 = (OC')^2 + (A'C')^2$
Substituting the values
$(A'C')^2 = 13^2 - 12^2$
$(A'C')^2 = 25$
A'C' = 5 cm

Length of the new chord = 2 × 5 = 10 cm

5. In the following figure, AD is a straight line. OP ⊥ AD and O is the centre of both circles. If OA = 34 cm, OB = 20 cm and OP = 16 cm; find the length of AB.



Solution:



In the inner circle, BC is the chord and OP ⊥ BC
Here, the perpendicular to a chord, from the centre of a circle, bisects the chord.
So, BP = PC

In △ OBP,
$OB^2 = OP^2 + BP^2$ [Using Pythagoras Theorem]
Substituting the values
$BP^2 = 20^2 – 16^2$
$BP^2 = 144$
So we get
BP = 12 cm

In the outer circle, AD is the chord and OP ⊥ AD
Here, the perpendicular to a chord, from the centre of a circle, bisects the chord.
So, AP = PD

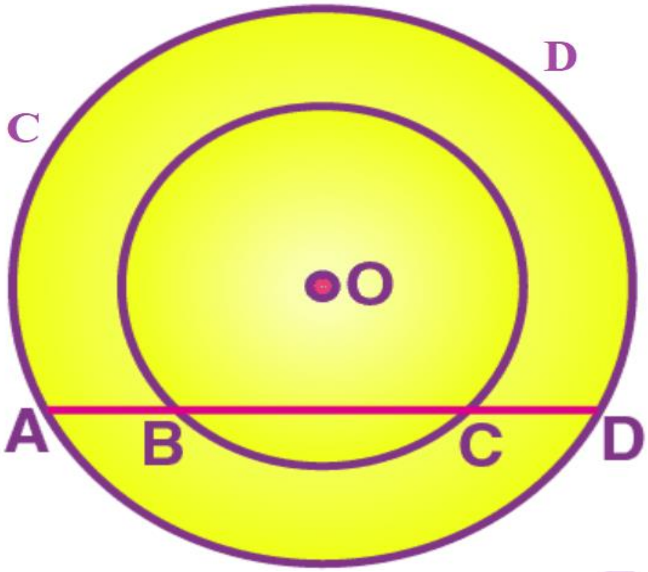
In △ OAP,
$OA^2 = OP^2 + AP^2$ [Using Pythagoras Theorem]
Substituting the values
$AP^2 = 34^2 – 16^2$
$AP^2 = 900$
So we get
AP = 30 cm

Here, AB = AP – BP = 30 – 12 = 18 cm.

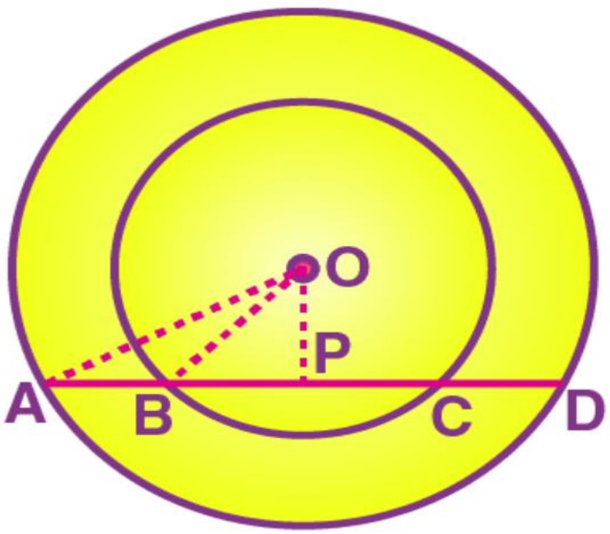
EXERCISE 17B

PAGE: 217

1. The figure shows two concentric circles and AD is a chord of larger circle. Prove that: AB = CD.



Solution:

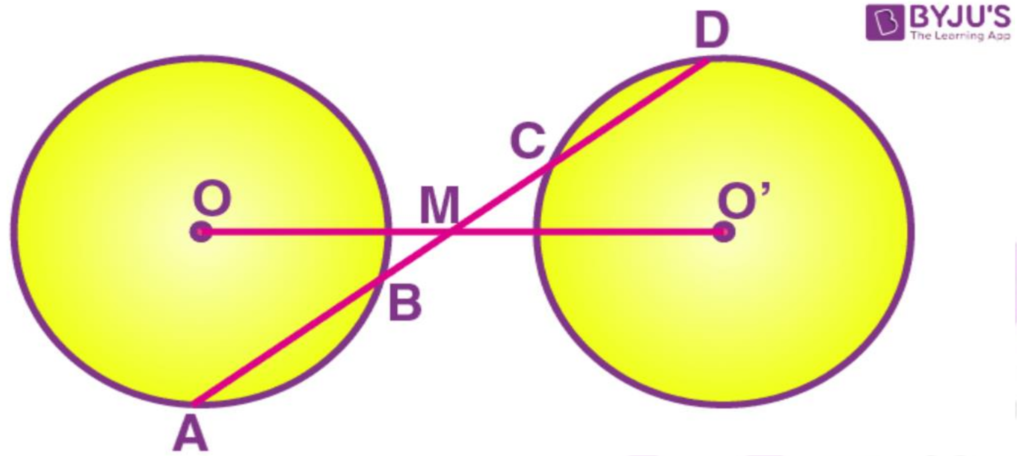


Draw OP ⊥ AD
So OP bisects AD
[Perpendicular drawn from the centre of a circle to a chord bisects it.]
AP = PD ……… (i)

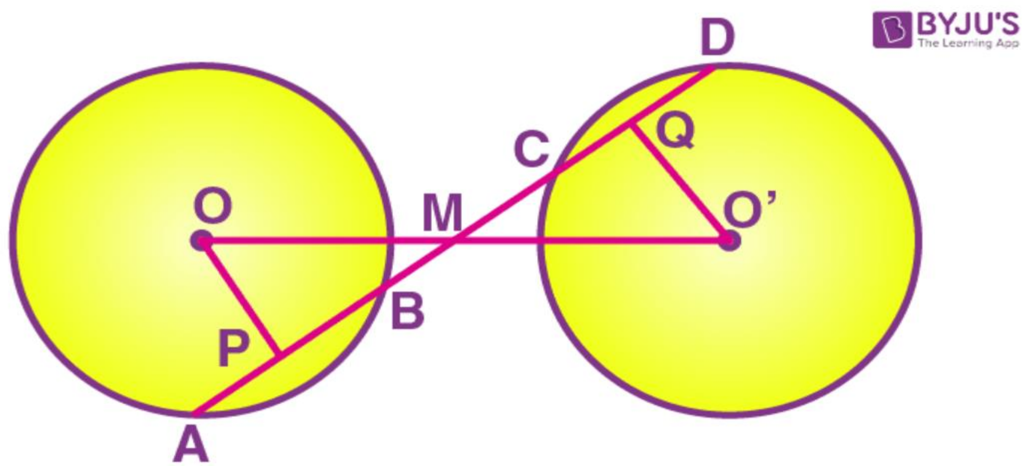
BC is a chord for the inner circle and OP ⊥ BC
So OP bisects BC
[Perpendicular drawn from the centre of a circle to a chord bisects it.]
BP = PC ………. (ii)

By subtracting equation (ii) from (i),
AP – BP = PD – PC
AB = CD

2. A straight line is drawn cutting two equal circles and passing through the midpoint M of the line joining their centres O and O'.
Prove that chords AB and CD, which are intercepted by the two circles are equal.



Solution:



Given –
A straight line AD intersects two circles of equal radii at A, B, C and D.
Line joining the centres OO' intersect AD at M
M is the midpoint of OO'

To prove – AB = CD.

Construction – From the centre O, draw OP ⊥ AB and from O' draw O'Q ⊥ CD.

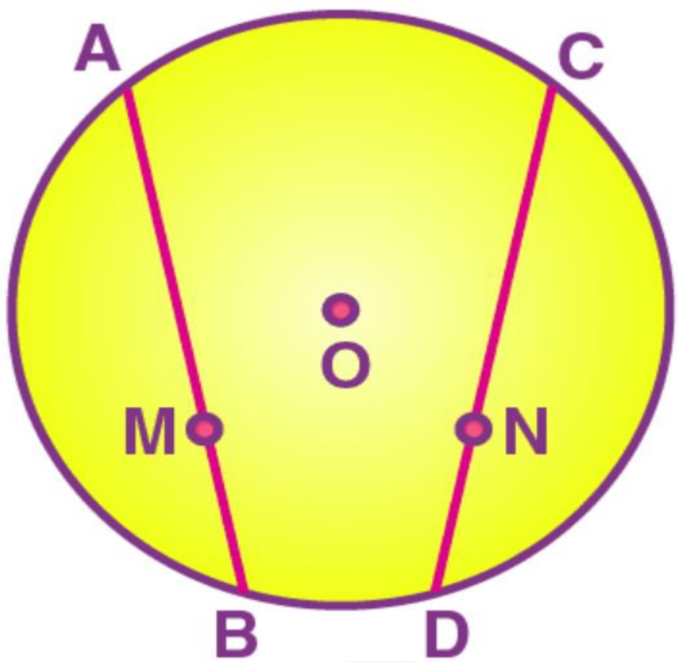
Proof –
In △ OMP and △ O'MQ,
∠ OMP = ∠ O'MQ [vertically opposite angles]
∠ OPM = ∠ O'QM [each = 90^0]

OM = O'M [given]

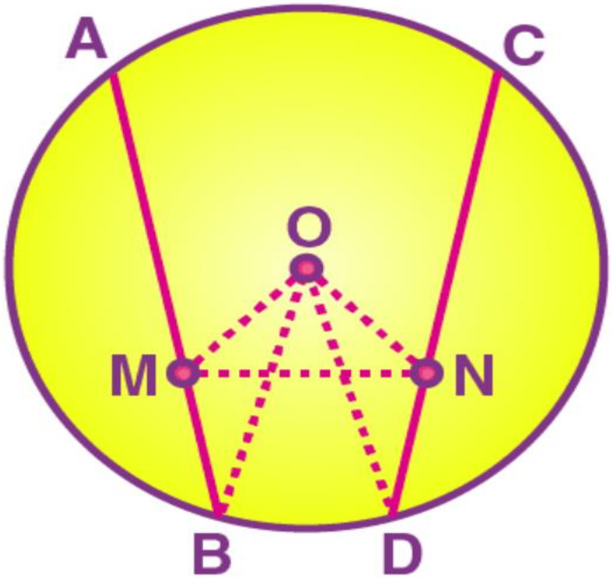
By AAS criterion of congruence,
△ OMP ≅ △ O'MP
OP = O'Q [c.p.c.t]

Here, two chords of a circle or equal circles which are equidistant from the centre are equal.
AB = CD.

3. M and N are the mid-points of two equal chords AB and CD respectively of a circle with centre O. Prove that:
(i) ∠ BMN = ∠ DNM,
(ii) ∠ AMN = ∠ CNM.



Solution:



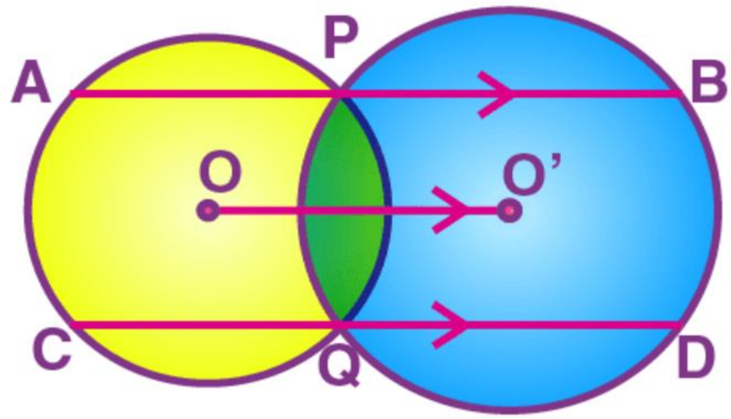
Draw OM ⊥ AB and ON ⊥ CD
So OM bisects AB and ON bisects CD
[Perpendicular drawn from the centre of a circle to a chord bisects it.]
BM = ½ AB = ½ CD = DN ……..(1)

In △ OMB,
$OM^2 = OB^2 + BM^2$ [Using Pythagoras Theorem]
We can write it as
$OM^2 = OD^2 – DN^2$ [using equation (1)]
$OM^2 = ON^2$
OM = ON
So we get
∠ OMN = ∠ ONM ……. (2) [Angles opposite to the equal sides are equal]

(i) ∠ OMB = ∠ OND [both 90^0]
By subtracting (2) from above
∠ BMN = ∠ DNM

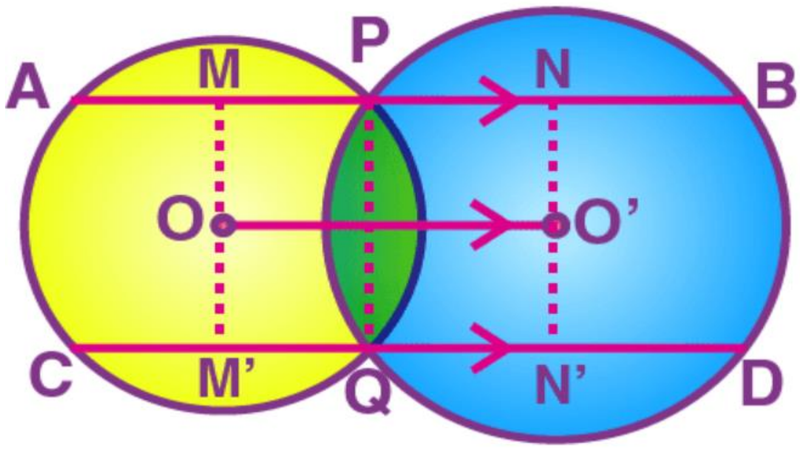
(ii) ∠ OMA = ∠ ONC [both 90^0]
By adding (2) to above
∠ AMN = ∠ CNM

4. In the following figure: P and Q are the points of intersection of two circles with centres O and O’. If straight lines APB and CQD are parallel to OO’. Prove that
(i) OO’ = ½ AB
(ii) AB = CD



Solution:

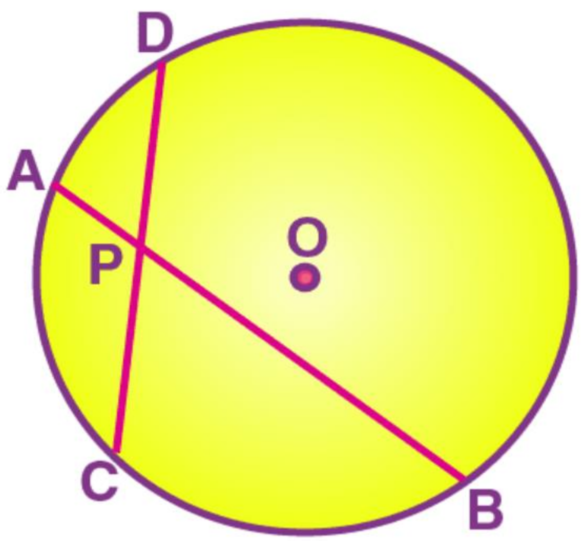
Draw OM and ON perpendicular on AB and OM' and O'N' perpendicular on CD.



So OM, O'N, OM' and O'N' bisect AP, PB, CQ and QD respectively
[Perpendicular drawn from the centre of a circle to a chord bisects it.]
MP = ½ AP, PN = ½ BP, M'Q = ½ CQ, QN' = ½ QD

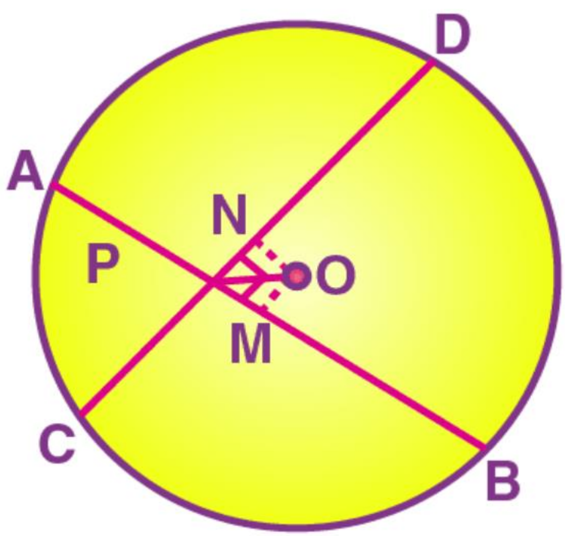
We know that
OO' = MN = MP + PN = ½ (AP + BP) = ½ AB ……. (i)
OO' = M'N' = M'Q + QN' = ½ (CQ + QD) = ½ CD …… (ii)
Equating (i) and (ii)
AB = CD

5. Two equal chords AB and CD of a circle with centre O, intersect each other at a point P inside the circle. Prove that:
(i) AP = CP
(ii) BP = DP



Solution:

Draw OM and ON perpendicular on AB and CD.
Join OP, OB and OD.



So OM and ON bisect AB and CD respectively.
[Perpendicular drawn from the centre of a circle to a chord bisects it.]
MB = ½ AB = ½ CD = ND ……. (i)

In right triangle △ OMB,
$OM^2 = OB^2 - MB^2$ ……. (ii)
In right triangle △ OND,
$ON^2 = OD^2 - ND^2$ ……. (iii)
From equation (i), (ii) and (iii)
OM = ON

In △ OPM and △ OPN,
∠ OMP = ∠ ONP [both 90^0]
OP = OP [Common]
OM = ON [Proved]

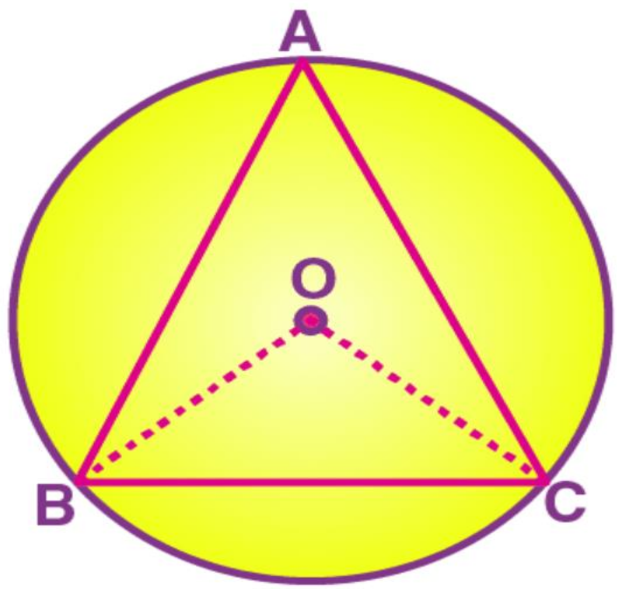
Using RHS criterion of congruence,
△ OPM ≅ △ OPN
PM = PN [c.p.c.t]

By adding (i) both sides
MB + PM = ND + PN
BP = DP

We know that
AB = CD
AB – BP = CD – DP [BP = DP]
AP = CP

EXERCISE 17C PAGE: 220

1. In the given figure, an equilateral triangle ABC is inscribed in a circle with centre O. Find:
(i) ∠ BOC
(ii) ∠ OBC



Solution:

From the given figure, △ ABC is an equilateral triangle.
So all the three angles of the triangle will be 60^0.
∠ A = ∠ B = ∠ C = 60^0

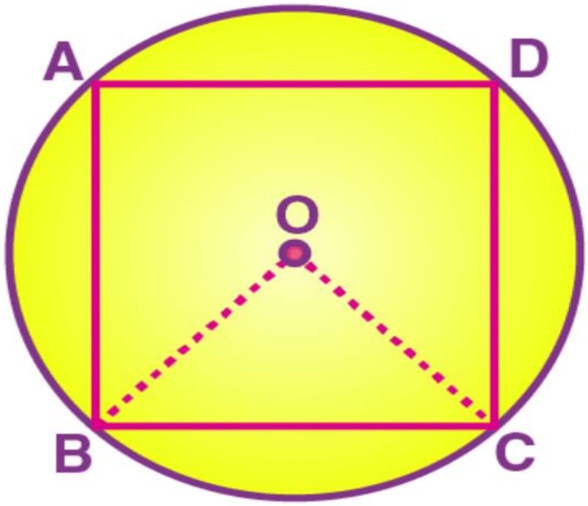
As the triangle is equilateral, BO and CO will be the angle of bisectors of ∠ B and ∠ C respectively.
∠ OBC = ∠ ABC/2 = 30^0

From the given figure,
OB and OC are the radii of the given circle and are of equal length.
△ OBC is isosceles triangle with OB = OC.

In △ OBC,
∠ OBC = ∠ OCB as they are angles opposite to the two equal sides of an isosceles triangle.
∠ OBC = 30^0 and ∠ OCB = 30^0
As the sum of all the angles of a triangle is 180^0
In △ OBC,
∠ OCB + ∠ OBC + ∠ BOC = 180^0
Substituting the values
30^0 + 30^0 + ∠ BOC = 180^0
60^0 + ∠ BOC = 180^0
So we get
∠ BOC = 180^0 – 60^0 = 120^0

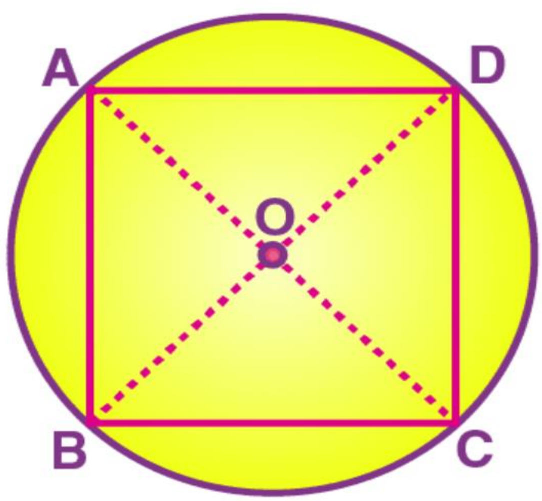
Therefore, ∠ BOC = 120^0 and ∠ OBC = 30^0.

2. In the given figure, a square is inscribed in a circle with centre O. Find:
(i) ∠ BOC
(ii) ∠ OCB
(iii) ∠ COD
(iv) ∠ BOD
Is BD a diameter of the circle?



Solution:

From the figure, extend a straight-line OB to BD and CO to CA.
We get the diagonals of the square which intersect each other at 90^0 by the property of square.



From the above mentioned statement, we know that
∠ COD = 90^0
Here the sum of the angle ∠ BOC and ∠ OCD is 180^0 as BD is a straight line.
∠ BOC + ∠ OCD = ∠ BOD = 180^0

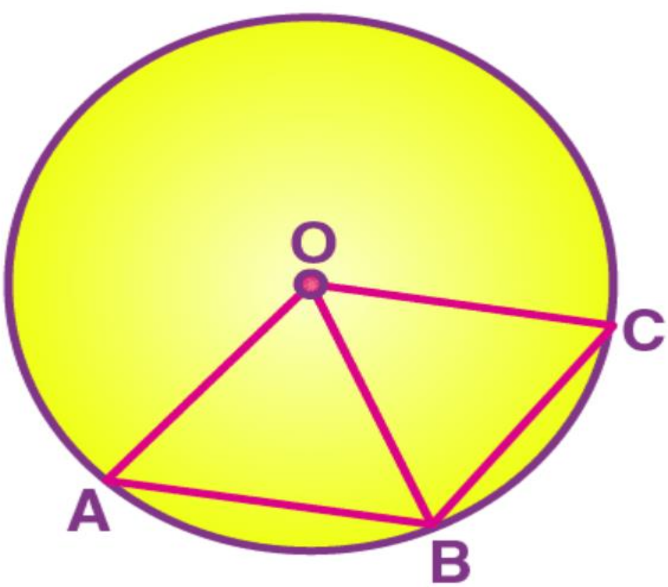
It can be written as
$\angle BOC + 90^0 = 180^0$
$\angle BOC = 180^0 - 90^0$
$\angle BOC = 90^0$

Therefore, triangle OCB is an isosceles triangle with sides OB and OC of equal length as they are the radii of the same circle.

In $\triangle$ OCB,
$\angle OBC = \angle OCB$ [Opposite angles to the two equal sides of an isosceles triangle]
Here sum of all the angles of a triangle is 180^0
$\angle OBC + \angle OCB + \angle BOC = 180^0$
It can be written as
$\angle OBC + \angle OBC + 90^0 = 180^0$ [$\angle OBC = \angle OCB$]
So we get
$2 \angle OBC = 180^0 - 90^0$
$2 \angle OBC = 90^0$
$\angle OBC = 45^0$

Here, $\angle OBC = \angle OCB = 45^0$
Yes, BD is the diameter of the circle.

3. In the given figure, AB is a side of regular pentagon and BC is a side of regular hexagon.
(i) $\angle$ AOB
(ii) $\angle$ BOC
(iii) $\angle$ AOC
(iv) $\angle$ OBA
(v) $\angle$ OBC
(vi) $\angle$ ABC



Solution:

Given –
AB is the side of a pentagon where the angle subtended by each arm of the pentagon at the centre of the circle = $360^0/5 = 72^0$
Hence, $\angle AOB = 72^0$

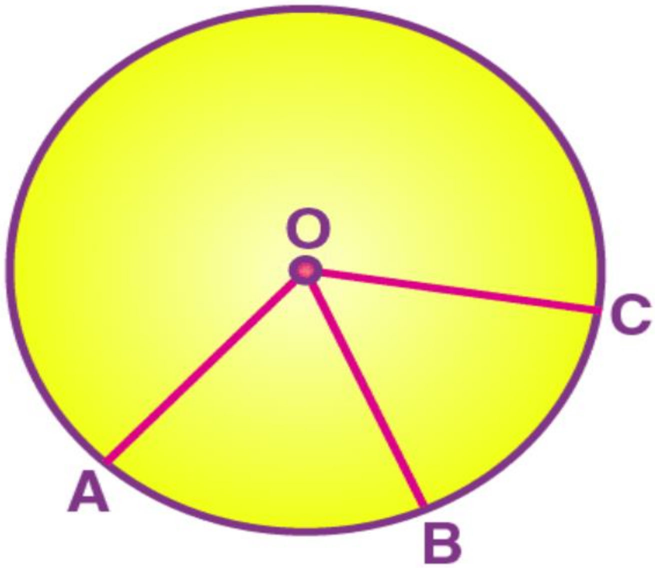
BC is the side of a hexagon where the angle subtended by BC at the centre = $360^0/6 = 60^0$
Hence, $\angle BOC = 60^0$

$\angle AOC = \angle AOB + \angle BOC$
$\angle AOC = 72^0 + 60^0 = 132^0$
The triangle formed i.e., $\triangle AOB$ is an isosceles triangle with OA = OB as they are radii of the same circle.
$\angle OBA = \angle BAO$ [opposite angles of equal sides of an isosceles triangle]
We know that the sum of all the angles of a triangle is 180^0
$\angle AOB + \angle OBA + \angle BAO = 180^0$
$2\angle OBA + 72^0 = 180^0$ [$\angle OBA = \angle BAO$]
So we get
$2 \angle OBA = 180^0 – 72^0$
$2 \angle OBA = 108^0$
$\angle OBA = 54^0$
Here $\angle OBA = \angle BAO = 54^0$

So the triangle formed, $\triangle BOC$ is an isosceles triangle with OB = OC as they are radii of the same circle.
$\angle OBC = \angle OCB$ [opposite angles of equal sides of an isosceles triangle]

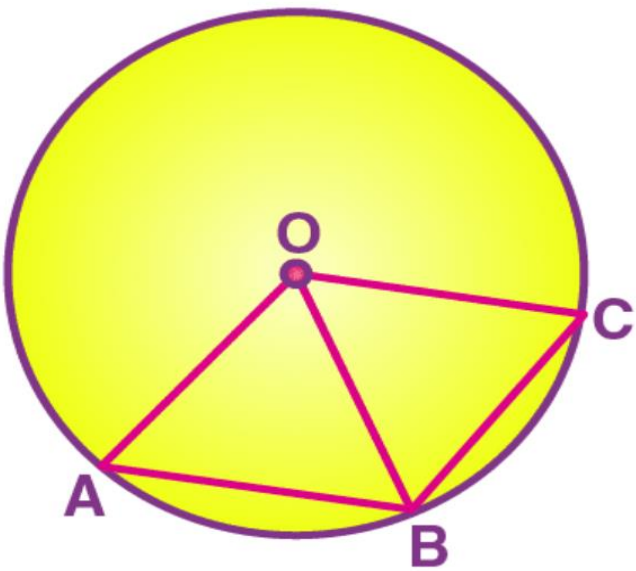
We know that the sum of all the angles of a triangle is 180^0
$\angle BOC + \angle OBC + \angle OCB = 180^0$
Substituting the values
$2 \angle OBC + 60^0 = 180^0$ [$\angle OBC = \angle OCB$]
$2 \angle OBC = 180^0 – 60^0$
$2 \angle OBC = 120^0$
$\angle OBC = 60^0$
Here $\angle OBC = \angle OCB = 60^0$
So $\angle ABC = \angle OBA + \angle OBC = 54^0 + 60^0 = 114^0$

4. In the given figure, arc AB and arc BC are equal in length. If $\angle AOB = 48^0$, find:
(i) $\angle BOC$
(ii) $\angle OBC$
(iii) $\angle AOC$
(iv) $\angle OAC$



Solution:

The arc of equal lengths subtends equal angles at the centre.



$\angle$ AOB = $\angle$ BOC = 48^0
$\angle$ AOC = $\angle$ AOB + $\angle$ BOC = $48^0 + 48^0 = 96^0$

So the triangle formed $\triangle$ BOC is an isosceles triangle with OB = OC as they are radii of the same circle.
$\angle$ OBC = $\angle$ OCB [opposite angles of equal sides of an isosceles triangle]

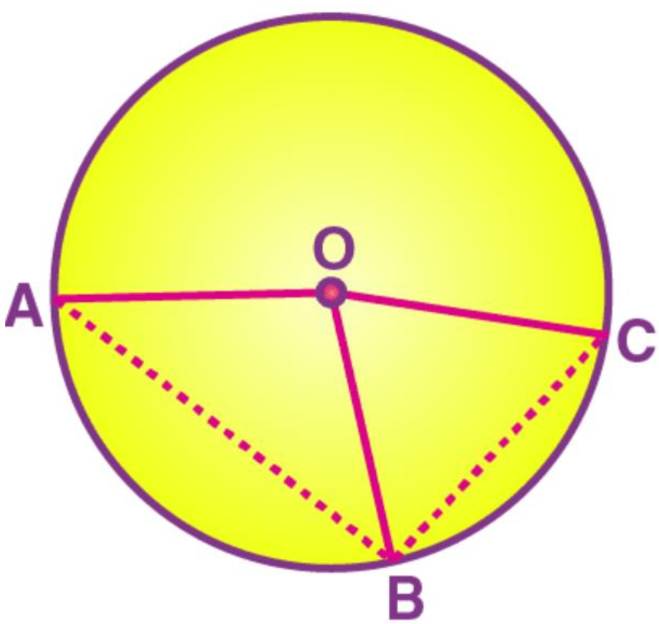
We know that the sum of all the angles of a triangle is 180^0
$\angle$ BOC + $\angle$ OBC + $\angle$ OCB = 180^0
2 $\angle$ OBC + 48^0 = 180^0 [$\angle$ OBC = $\angle$ OCB]

$2 \angle OBC = 180^0 – 48^0$
$2 \angle OBC = 132^0$
$\angle OBC = 66^0$
Here $\angle OBC = \angle OCB = 66^0$

So the triangle formed $\triangle AOC$ is an isosceles triangle with OA = OC as they are radii of the same circle
$\angle OAC = \angle OCA$ [opposite angles of equal sides of an isosceles triangle]

We know that the sum of all the angles of a triangle is 180^0
$\angle COA + \angle OAC + \angle OCA = 180^0$
Substituting the values
$2 \angle OAC + 96^0 = 180^0$ [$\angle OAC = \angle OCA$]
$2 \angle OAC = 180^0 - 96^0$
$2 \angle OAC = 84^0$
$\angle OAC = 42^0$
Here $\angle OCA = \angle OAC = 42^0$

5. In the given figure, the lengths of arcs AB and BC are in the ratio 3:2. If $\angle AOB = 96^0$, find:
(i) $\angle BOC$
(ii) $\angle ABC$



Solution:

The two arcs are in the ratio 3:2
$\angle AOB : \angle BOC = 3: 2$
$\angle AOC = 96^0$
So $3x = 96$
$x = 32$
Hence, $\angle BOC = 2 \times 32 = 64^0$

So the triangle formed, △ AOB is an isosceles triangle with OA = OB as they are radii of the same circle.
∠ OBA = ∠ BAO [opposite angles of equal sides of an isosceles triangle]

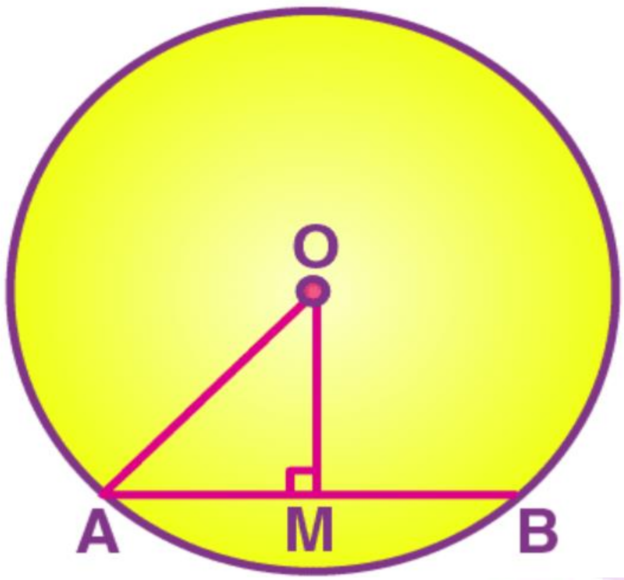
We know that the sum of all the angles of a triangle is 180^0
∠ AOB + ∠ OBA + ∠ BAO = 180^0
2 ∠ OBA + 96^0 = 180^0 [∠ OBA = ∠ BAO]
2 ∠ OBA = $180^0 - 96^0$
2 ∠ OBA = 84^0
2 ∠ OBA = 42^0
Here ∠ OBA = ∠ BAO = 42^0

So the triangle formed, △ BOC is an isosceles triangle with OB = OC as they are radii of the same circle.
∠ OBC = ∠ OCB [opposite angles of equal sides of an isosceles triangle]

We know that the sum of all the angles of a triangle is 180^0
∠ BOC + ∠ OBC + ∠ OCB = 180^0
2 ∠ OBC + 64^0 = 180^0 [∠ OBC = ∠ OCB]
2 ∠ OBC = $180^0 - 64^0$
2 ∠ OBC = 116^0
∠ OBC = 58^0
Here ∠ OBC = ∠ OCB = 58^0
∠ ABC = ∠ BOA + ∠ OBC = $42^0 + 58^0 = 100^0$

EXERCISE 17D PAGE: 221

1. The radius of a circle is 13 cm and the length of one of its chords is 24 cm. Find the distance of the chord from the centres.
Solution:

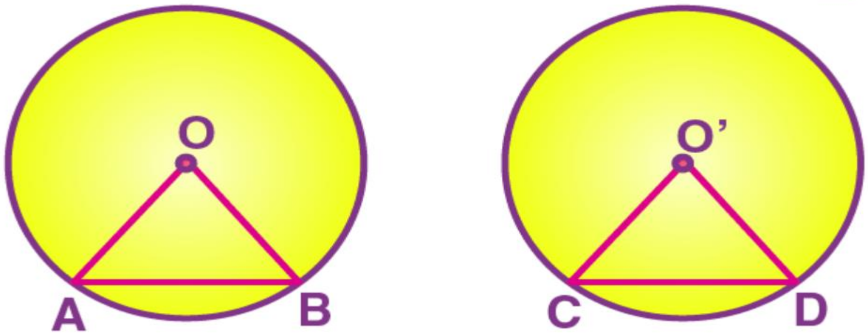


To find – OM
Given – AB = 24 cm
As OM ⊥ AB
OM bisects AB
AM = 12 cm

In right △ OMA,
$OA^2 = OM^2 + AM^2$
$OM^2 = OA^2 – AM^2$
Substituting the values
$OM^2 = 13^2 – 12^2$
$OM^2 = 25$
OM = 5 cm

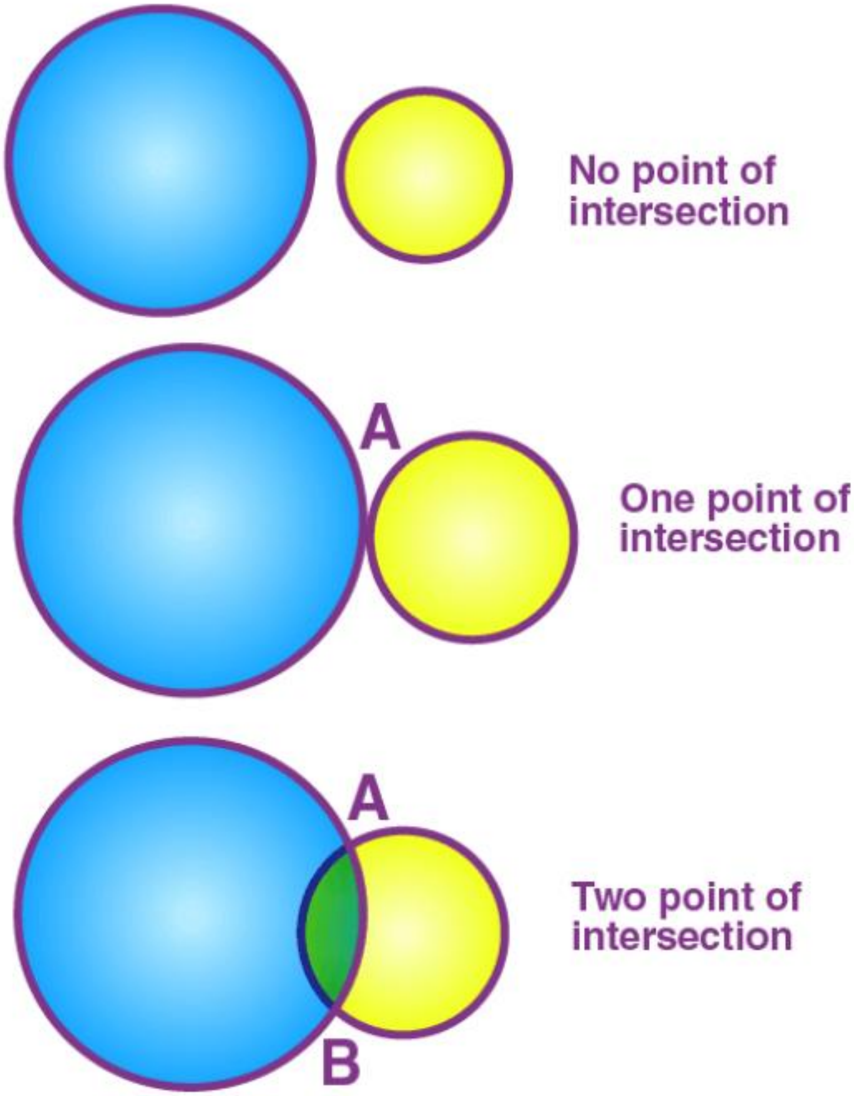
Therefore, the distance of the chord from the centre is 5 cm.

2. Prove that equal chords of congruent circles subtend equal angles at their centre.
Solution:



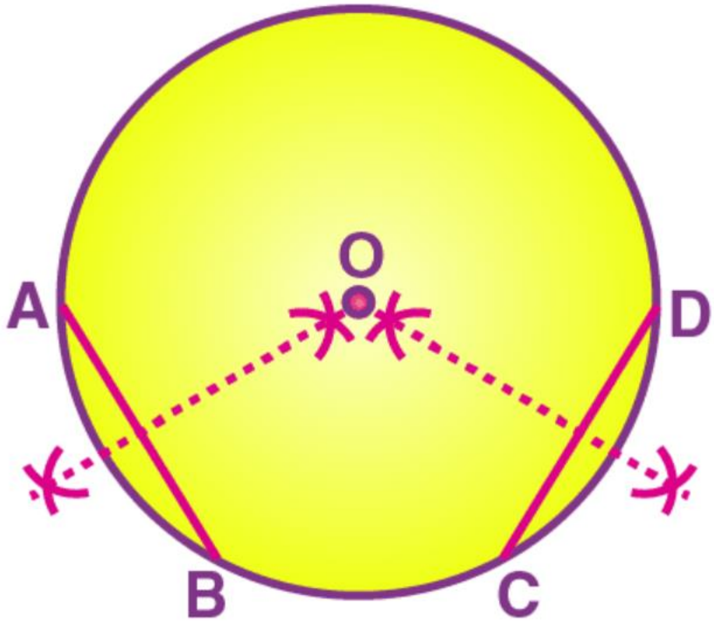
Given – AB and CD are two equal chords of congruent circles with centres O and O' respectively.
To prove -
∠ AOB = ∠ CO'D
Proof – In △ OAB and △ O'CD
OA = O'C (radii of congruent circles)
OB = O'D (radii of congruent circles)
AB = CD (Given)
△ OAB ≅ △ O'CD [By SSS congruence criterion]
∠ AOB = ∠ CO'D [c.p.c.t]

3. Draw two circles of different radii. How many points these circles can have in common? What is the maximum number of common points?
Solution:



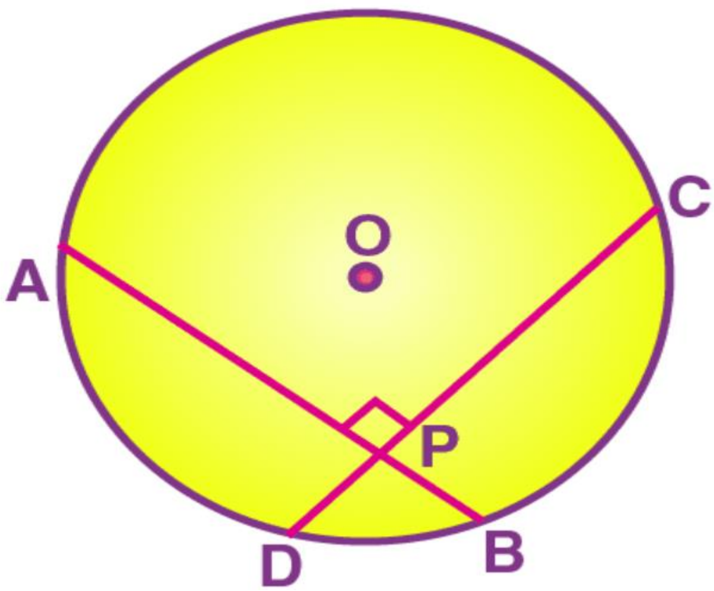
The circle can have 0, 1 or 2 points in common.
The maximum number of common points is 2.

4. Suppose you are given a circle. Describe a method by which you can find the centre of this circle.
Solution:

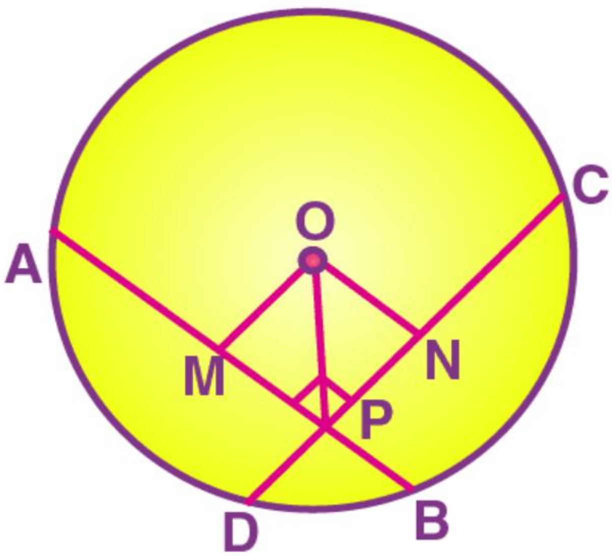


In order to draw the centre of a given circle:
1. Construct the circle.
2. Taking any two different chords AB and CD of this circle, construct perpendicular bisectors of these chords.
3. Now let the perpendicular bisectors meet at point O.
Hence, O is the centre of the given circle.

5. Given two equal chords AB and CD of a circle, with centre O, intersecting each other at point P. Prove that:
(i) AP = CP
(ii) BP = DP



Solution:



In △ OMP and △ ONP,
OP = OP (common side)
∠ OMP = ∠ ONP [Both are right angles]
OM = OM [side both the chords are equal, so the distance of the chords from the centre are also equal]
△ OMP ≅ △ ONP [RHS congruence criterion]
MP = PN [cpct] …… (a)

(i) AB = CD [given]
AM = CN [Perpendicular drawn from the centre to the chord bisects the chord]
AM + MP = CN + NP [from (a)]
AP = CP …… (b)

(ii) AB = CD
AP + BP = CP + DP
BP = DP [from (b)]
Therefore, proved.