EXERCISE 19A

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1. Find the mean of 43, 51, 50, 57 and 54.
Solution:

Numbers given are 43, 51, 50, 57 and 54.
Mean of given numbers

$$= \frac{43 + 51 + 50 + 57 + 54}{5}$$

$$= \frac{255}{5}$$

= 51

2. Find the mean of first six natural numbers.
Solution:

First six natural numbers are 1, 2, 3, 4, 5, 6.
Mean of first six natural numbers

$$= \frac{1 + 2 + 3 + 4 + 5 + 6}{6}$$

$$= \frac{21}{6}$$

= 3.5

3. Find the mean of first ten odd natural number.
Solution:

First ten odd natural numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19
Mean of first ten odd numbers

$$= \frac{1 + 3 + 5 + 7 + 9 + 11 + 15 + 17 + 19}{10}$$

$$= \frac{100}{10}$$

4. Find the mean of all factors of 10.
Solution:

All factors of 10 are 1, 2, 5, 10
Mean of all factors of 10

$$= \frac{1+2+5+10}{4}$$

$$= \frac{18}{4}$$

= 4.5

5. Find the mean of x + 3, x + 5, x + 7, x + 9 and x + 11.
Solution:

Values given are x + 3, x + 5, x + 7, x + 9 and x + 11
Mean of the values

$$= \frac{x+3+x+5+x+7+x+9+x+11}{5}$$

$$= \frac{5x+35}{5}$$

$$= \frac{5(x+7)}{5}$$

= x + 7

6. If the different values of variable x are 9.8, 5.4, 3.7, 1.7, 1.8, 2.6, 2.8, 8.6, 10.5 and 11.1; find
(i) the mean $\overline{x}$

(ii) the value of $\sum_{i=1}^{10}(x_i - \overline{x})$

Solution:

(i) Numbers given are 9.8, 5.4, 3.7, 1.7, 1.8, 2.6, 2.8, 8.6, 10.5 and 11.1

$$\overline{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + ... + x_n}{n}$$

$$= \frac{9.8+5.4+3.7+1.7+1.8+2.6+2.8+8.6+10.5+11.1}{10}$$

= 5.8

(ii)

$$The\ value\ of \sum_{i=1}^{10}(x_i - \overline{x})$$

$$\sum_{i=1}^{n}(x_i - \overline{x}) = (x_1 - \overline{x}) + (x_2 - \overline{x}) ..... + (x_n - \overline{x}) = 0$$

$\overline{x} = 5.8$

$\sum_{i=1}^{10}(x_i - \overline{x})$

= (9.8 – 5.8) + (5.4 – 5.8) + (3.7 – 5.8) + (1.7 – 5.8) + (1.8 – 5.8) + (2.6 – 5.8) + (8.6 – 5.8) + (10.5 – 5.8) + (11.1 – 5.8)
= 4 – 4 – 2.1 – 4.1 – 4 – 3.2 – 3 + 2.8 + 4.7 + 5.3
= 0

7. The mean of 15 observations is 32. Find the resulting mean, if each observation is:
(i) Increased by 3
(ii) Decreased by 7
(iii) Multiplied by 2
(iv) Divided by 0.5
(v) Increased by 60%
(vi) Decreased by 20%
Solution:

It is given that
Mean of 15 observations is 32

(i) Resulting mean if each observation is increased by 3 = 32 + 3 = 35

(ii) Resulting mean if each observation is decreased by 7 = 32 – 7 = 25

(iii) Resulting mean if each observation is multiplied by 2 = 32 × 2 = 64

(iv) Resulting mean if each observation is divided by 0.5 = 32/0.5 = 64

(v) Resulting mean if each observation is increased by 60% = 32 + 60/100 × 32
= 32 + 19.2
= 51.2

(vi) Resulting mean if each observation is decreased by 20% = 32 – 20/100 × 32
= 32 – 6.4
= 25.6

8. The mean of 5 numbers is 18. If one number is excluded, the mean of remaining number becomes 16. Find the excluded number.
Solution:

It is given that
Mean of 5 numbers is 18
Total sum of 5 numbers = 18 × 5 = 90

Excluding an observation, the mean of the remaining 4 number becomes 16 = 16 × 4 = 64
Sum of remaining 4 observations = Total of 5 observations – Total of 4 observations
= 90 – 64
= 26

9. If the mean of observations x, x + 2, x + 4, x + 6 and x + 8 is 11, find:
(i) The value of x;
(ii) The mean of first three observations.
Solution:

(i) It is given that
Mean of observations x, x + 2, x + 4, x + 6 and x + 8 is 11
We know that

$$Mean = \frac{Observations}{n}$$

$$11 = \frac{x + x + 2 + x + 4 + x + 6 + x + 8}{5}$$

$$11 = \frac{5x + 20}{5}$$

By taking out 5 as common
11 = [5 (x + 4)]/5
11 = x + 4
By transposing we get
x = 11 – 4
x = 7

(ii) Mean of first three observations

$$= \frac{x + x + 2 + x + 4}{3}$$

$$= \frac{3x + 6}{3}$$

$$= \frac{3(7) + 6}{3} [As\ x = 7]$$

$$= \frac{21 + 6}{3}$$

= 9

10. The mean of 100 observations is 40. It is found that an observation 53 was misread as 83. Find the correct mean.
Solution:

It is given that

Mean of 100 observations is 40

$$\frac{\sum x}{n} = \overline{x}$$

$$\frac{\sum x}{n} = 40$$

x = 40 × 100 = 4000

Here the incorrect value of x = 4000
So the correct value of x = Incorrect value of x – Incorrect observation + Correct observation
Substituting the values
= 4000 – 83 + 53
= 3970

We know that

$$Correct\ mean = \frac{Correct\ value\ of \sum x}{n}$$

= 3970/100
= 39.7

EXERCISE 19B

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1. Find the median of:
(i) 25, 16, 26, 16, 32, 31, 19, 28 and 35
(ii) 241, 243, 347, 350, 327, 299, 261, 292, 271, 258 and 257
(iii) 63, 17, 50, 9, 25, 43, 21, 50, 14 and 34
(iv) 233, 173, 189, 208, 194, 194, 185, 200 and 220.
Solution:

(i) Arrange the numbers in ascending order
16, 16, 19, 25, 26, 28, 31, 32, 35
As n = 9 (odd)

$$Median = Value\ of\ (\frac{n+1}{2})^{th}\ term$$

$$= (\frac{9+1}{2})^{th} term$$

= 5^{th} term
= 26

Hence, the median is 26.

(ii) Arrange the numbers in ascending order
241, 243, 257, 258, 261, 271, 292, 299, 327, 347, 350
As n = 11 (odd)

$$Median = Value\ of\ (\frac{n+1}{2})^{th}\ term$$

$$= (\frac{11+1}{2})^{th} term$$

= 6^{th} term
= 271

Hence, the median is 271.

(iii) Arrange the numbers in ascending order
9, 14, 17, 21, 25, 34, 43, 50, 50, 63
As n = 10 (even)
Median = ½ [value of $(n/2)^{th}$ term + value of $(n/2 + 1)^{th}$ term]
= ½ [value of $(10/2)^{th}$ term + value of $(10/2 + 1)^{th}$ term]
= ½ [25 + 34]
= ½ [59]
= 29.5

Hence, the median is 29.5.

(iv) Arrange the numbers in ascending order

173, 185, 189, 194, 194, 200, 204, 208, 220, 223
As n = 10 (even)
Median = ½ [value of $(n/2)^{th}$ term + value of $(n/2 + 1)^{th}$ term]
= ½ [value of $(10/2)^{th}$ term + value of $(10/2 + 1)^{th}$ term]
= ½ [200 + 194]
= ½ [394]
= 197

Hence, the median is 197.

2. The following data have been arranged in ascending order. If their median is 63, find the value of x.
34, 37, 53, 55, x, x + 2, 77, 83, 89 and 100.
Solution:

Numbers given are 34, 37, 53, 55, x, x + 2, 77, 83, 89 and 100
As n = 10 (even)
Median = ½ [value of $(n/2)^{th}$ term + value of $(n/2 + 1)^{th}$ term]
= ½ [value of $(10/2)^{th}$ term + value of $(10/2 + 1)^{th}$ term]
= ½ [value of 5^{th} term + value of $(5 + 1)^{th}$ term]
= ½ [value of 5^{th} term + value of 6^{th} term]
Substituting the values
63 = ½ [x + x + 2]
[2 + 2x]/2 = 63
Taking 2 as common
x + 1 = 63
x = 62

3. In 10 numbers, arranged in increasing order, the 7^{th} number is increased by 8, how much will the median be changed?
Solution:

We know that for any given set of data, the median is the value of its middle term
Total observations n = 10 (even)
Median = ½ [value of $(n/2)^{th}$ term + value of $(n/2 + 1)^{th}$ term]
= ½ [value of $(10/2)^{th}$ term + value of $(10/2 + 1)^{th}$ term]
= ½ [value of 5^{th} term + value of 6^{th} term]

Therefore, if the 7^{th} number is diminished by 8, there will be no change in the median value.

4. Out of 10 students, who appeared in a test, three secured less than 30 marks and 3 secured more than 75 marks. The marks secured by the remaining 4 students are 35, 48, 66 and 40. Find the median score of the whole group.
Solution:

Total observations n = 10 (even)

Median = = ½ [value of $(10/2)^{th}$ term + value of $(10/2 + 1)^{th}$ term]
= ½ [value of 5^{th} term + value of 6^{th} term]
Substituting the values
Median = ½ [40 + 48]
= 88/2
= 44

Hence, the median score of the whole group is 44.

5. The median of observations 10, 11, 13, 17, x + 5, 20, 22, 24 and 53 (arranged in ascending order) is 18; find the value of x.
Solution:

Total observations n = 9 (odd)
As n is odd

$$Median = (\frac{n+1}{2})^{th} term$$

$$Median = (\frac{9+1}{2})^{th} term$$

= 5^{th} term
= x + 5

It is given that, Median = 18
x + 5 = 18
x = 13

EXERCISE 19C

1. Find the mean of 8, 12, 16, 22, 10 and 4. Find the resulting mean, if each of the observations, given above be:
(i) Multiplied by 3.
(ii) Divided by 2.
(iii) Multiplied by 3 and then divided by 2.
(iv) Increased by 25%.
(v) Decreased by 40%.
Solution:

Mean of the given data

$$= \frac{8 + 12 + 16 + 22 + 10 + 4}{6}$$

$$= \frac{72}{6}$$

= 12

(i) Multiplied by 3
If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots\ldots x_n$,
Mean of $ax_1, ax_2, ax_3, \ldots\ldots, ax_n$ is $a\bar{x}$
When each of the given data is multiplied by 3, then mean is also multiplied by 3
Mean of the original data = 12
Therefore, the new mean = 12 × 3 = 36

(ii) Divided by 2
If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots\ldots x_n$,
Mean of $x_1/a, x_2/a, x_3/a, \ldots\ldots x_n/a$ is $\bar{x}/a$
When each of the given data is divided by 2, the mean is also divided by 2.
Mean of the original data = 12
Therefore, the new mean = 12/2 = 6

(iii) Multiplied by 3 and then divided by 2
If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots\ldots x_n$,
Mean of $a/b\ x_1, a/b\ x_2, a/b\ x_3, \ldots..\ a/b\ x_n$ is $a/b\ \bar{x}$
When each of the given data is multiplied by 3/2, the mean is also multiplied by 3/2.
Mean of original data = 12
Therefore, the new mean = 3/2 × 12 = 36/2 = 18

(iv) Increased by 25%
We know that
New mean = Original mean + 25% of original mean
New mean = 12 + 25% of 12
New mean = 12 + 25/100 × 12
So we get

New mean = 12 + ¼ × 12
New mean = 12 + 3
New mean = 15

(v) Decreased by 40%
We know that
New mean = 12 – 40% of 12
New mean = 12 – 40/100 × 12
So we get
New mean = 12 – 2/5 × 12
New mean = 12 – 0.4 × 12
New mean = 12 – 4.8
New mean = 7.2

2. The mean of 18, 24, 15, 2x + 1 and 12 is 21. Find the value of x.
Solution:

We know that
Mean of given data

$$= \frac{18 + 24 + 15 + 2x + 1 + 12}{5}$$

$$21 = \frac{70 + 2x}{5}$$

By cross multiplication
5 × 21 = 70 + 2x
105 = 70 + 2x
On further calculation
2x = 105 – 70
2x = 35
x = 35/2 = 17.5

3. The mean of 6 numbers is 42. If one number is excluded, the mean of remaining number is 45. Find the excluded number.
Solution:

If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots\ldots x_n$

$$Mean\ of\ given\ data = \frac{x_1 + x_2 + x_3 + \ldots.. + x_n}{n}$$

It is given that mean of 6 numbers is 42

$$\frac{x_1 + x_2 + x_3 + \ldots.. + x_6}{6} = 42$$

$x_1 + x_2 + x_3 + \ldots\ldots + x_6 = 42 \times 6$
$x_1 + x_2 + x_3 + x_4 + x_5 = 252 - x_6 \ldots\ldots (1)$

It is also given that the mean of 5 numbers is 45

$$\frac{x_1 + x_2 + x_3 + x_5}{5} = 45$$

$x_1 + x_2 + x_3 + x_4 + x_5 = 45 \times 5$

$x_1 + x_2 + x_3 + x_4 + x_5 = 225$ …… (2)

From equating both the equations

$252 - x_6 = 225$

$x_6 = 252 - 225 = 27$

4. The mean of 10 numbers is 24. If one more number is included, the new mean is 25. Find the included number.

Solution:

If $\bar{x}$ is the mean of n number of observations $x_1, x_2, x_3, \ldots\ldots x_n$

$$Mean\ of\ given\ data = \frac{x_1 + x_2 + x_3 + \ldots.. + x_n}{n}$$

It is given that the mean of 10 numbers is 24

$$\frac{x_1 + x_2 + x_3 + \ldots.. + x_{10}}{10} = 24$$

$x_1 + x_2 + x_3 + \ldots\ldots + x_{10} = 24 \times 10$

$x_1 + x_2 + x_3 + x_4 + \ldots\ldots + x_{10} = 240$

$x_1 + x_2 + x_3 + x_4 + \ldots\ldots + x_{10} = 240 + x_{11}$ …… (1)

It is also given that the mean of 11 numbers is 25.

$$\frac{x_1 + x_2 + x_3 + \ldots.. + x_{10} + x_{11}}{11} = 25$$

$x_1 + x_2 + x_3 + x_4 + \ldots\ldots + x_{10} + x_{11} = 11 \times 25$

$x_1 + x_2 + x_3 + x_4 + \ldots\ldots + x_{10} + x_{11} = 275$ ….. (2)

From equating both the equations

$240 + x_{11} = 275$

$x_{11} = 275 - 240 = 35$

5. The following observations have been arranged in ascending order. If the median of the data is 78, find the value of x.

44, 47, 63, 65, x + 13, 87, 93, 99, 110.

Solution:

The data given is

44, 47, 63, 65, x + 13, 87, 93, 99, 110

Total number of observations (n) = 9 which is odd

$$Median = (\frac{n+1}{2})^{th} term$$

$$Median = (\frac{9+1}{2})^{th} term$$

= 5^{th} term
= x + 13

It is given that the median is 78
x + 13 = 78
x = 78 – 13 = 65