EXERCISE 21A PAGE: 269

1. The length, breadth and height of a rectangular solid are in the ratio 5: 4: 2. If the total surface area is 1216 cm^2, find the length, the breadth and the height of the solid.
Solution:

Consider the angle as 5x, 4x and 2x.
Total Surface Area = 2 (lb + bh + hl) = 1216
Substituting the values
$2(20x^2 + 8x^2 + 10x^2) = 1216$
$76x^2 = 1216$
$x^2 = 1216/76$
$x^2 = 16$
x = 4 cm

So we get
5x = 5 × 4 = 20 cm
4x = 4 × 4 = 16 cm
2x = 2 × 4 = 8 cm
Hence, the dimensions are 20 cm, 16 cm and 8 cm.

Here
Volume = lbh
= 20 × 16 × 8
= 2560 cm^3

2. The volume of a cube is 729 cm^3. Find its total surface area.
Solution:

Consider one edge of a cube = a
We know that
Volume = a^3
$729 = a^3$
$9^3 = a^3$
9 = a
a = 9 cm

So we get
Total surface area = $6a^2 = 6 \times 9^2 = 486$ cm^2

3. The dimensions of a Cinema Hall are 100 m, 60 m and 15 m. How many persons can sit in the hall, if each requires 150 m^3 of air?
Solution:

We know that
Volume of cinema hall = 100 × 6 × 15 = 90000 m^3

It is given that
$150\ m^3 = 1$ person
$90000\ m^3 = (1/150) \times 90000 = 600$ persons

Hence, 600 persons can sit in the hall.

4. 75 persons can sleep in a room 25 m by 9.6 m. If each person requires 16 m^3 of air; find the height of the room.
Solution:

Consider the height of the room = h
1 person = $16\ m^3$
75 person = $75 \times 16 = 1200\ m^3$

We know that
Volume of room is $1200\ m^3$
$1200 = 25 \times 9.6 \times h$
By further calculation
$h = 1200/(25 \times 9.6)$
$h = 5$ m

5. The edges of three cubes of metal are 3 cm, 4 cm and 5 cm. They are melted and formed into a single cube. Find the edge of the new cube.
Solution:

We know that
Volume of melted single cube = $3^3 + 4^3 + 5^3$
$= 27 + 64 + 125$
$= 216\ cm^3$

Consider the edge of the new cube as a
Volume = $216\ cm^3$
$a^3 = 216$
$a^3 = 6^3$
$a = 6$ cm

Hence, 6 cm is the edge of cube.

6. Three cubes, whose edges are x cm, 8 cm and 10 cm respectively, are melted and recasted into a single cube of edge 12 cm. Find 'x'.
Solution:

We know that
Volume of a melted single cube = $x^3 + 8^3 + 10^3\ cm^3$
$= x^3 + 512 + 1000\ cm^3$
$= x^3 + 1512\ cm^3$

From the question, 12 cm is the edge of the single cube.
$12^3 = x^3 + 1512 \text{ cm}^3$
It can be written as
$x^3 = 12^3 – 1512$
$x^3 = 1728 – 1512$
$x^3 = 216$
So we get
$x^3 = 6^3$
x = 6 cm

7. Three equal cubes are placed adjacently in a row. Find the ratio of the total surfaced area of the resulting cuboid to that of the sum of the total surface areas of the three cubes.
Solution:

Consider the side of a cube = a units
Total surface area of one cube = $6a^2$
Total surface area of 3 cubes = $3 \times 6a^2 = 18a^2$
So after joining 3 cubes in a row,
Length of cuboid becomes 3a
Breadth and height of cuboid = a
Here the total surface area of cuboid = $2 (3a^2 + a^2 + 3a^2) = 14a^2$
We know that
Ratio of total surface area of cuboid to the total surface area of 3 cubes = $14a^2/18a^2 = 7/9$

8. The cost of papering the four walls of a room at 75 paisa per square meter is Rs. 240. The height of the room is 5 metres. Find the length and the breadth of the room, if they are in the ratio 5: 3.
Solution:

Consider the length and breadth as 5x and 3x respectively.
From the question,
The cost of papering the four walls of a room at 75 paisa per square meter is Rs. 240.
240 = Area × 0.75
Area = 240/0.75
Area = 24000/75
Area = 320 m

We know that
Area = 2 × Height (Length + Breadth)
Substituting the values
320 = 2 × 5 (5x + 3x)
320 = 10 × 8x
32 = 8x
x = 4

So we get
Length = 5x = 5 × 4 = 20 m
Breadth = 3x = 3 × 4 = 12 m

9. The area of a playground is 3650 m^2. Find the cost of covering it with gravel 1.2 cm deep, if the gravel costs Rs. 6.40 per cubic metre.
Solution:

Area of the playground = 3650 m^2
Gravels are 1.2 cm deep
So the total volume to be covered = 3650 × 0.012 = 43.8 m^3
As the cost of per cubic metre is Rs. 6.40
Total cost = 43.8 × Rs. 6.40 = Rs. 280.32

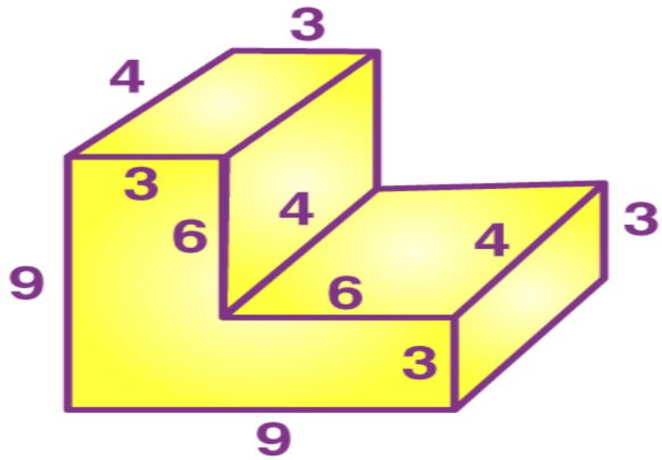
10. A square plat of side 'x' cm is 8 mm thick. If its volume is 2880 cm^3; find the value of x.
Solution:

We know that
1 mm = 1/10 cm
8 mm = 8/10 cm

Here
Volume = Base area × Height
Substituting the values
2880 cm^3 = x × x × 8/10
So we get
2880 × 10/8 = x^2
x^2 = 3600
x = 60 cm

EXERCISE 21B PAGE: 273

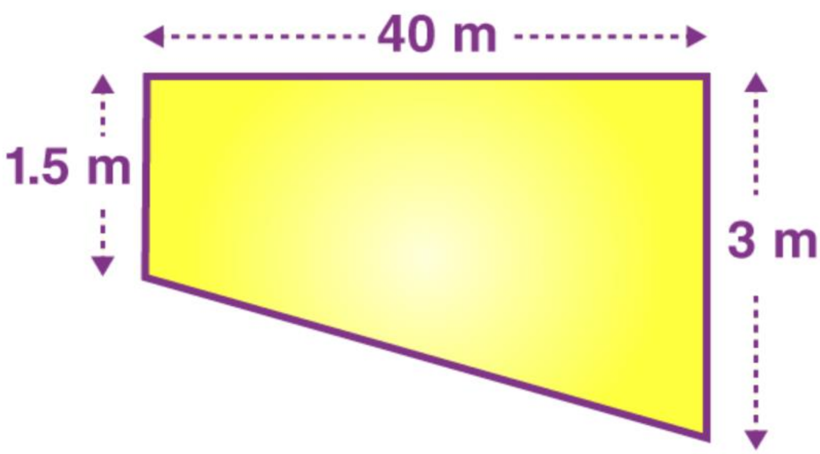
1. The following figure shows a solid uniform cross-section. Find the volume of the solid. All measurements are in centimetres. Assume that all angles in the figures are right angles.



Solution:

We can divide the figure into two cuboids of dimensions 6 cm, 4 cm, 3 cm and 9 cm respectively.
So the volume of solid = $9 \times 4 \times 3 + 6 \times 4 \times 3$
= 108 + 72
= 180 cm^3

2. A swimming pool is 40 m long and 15 m wide. Its shallow and deep ends are 1.5 m and 3 m deep respectively. If the bottom of the pool slopes uniformly, find the amount of water in litres required to fill the pool.



Solution:

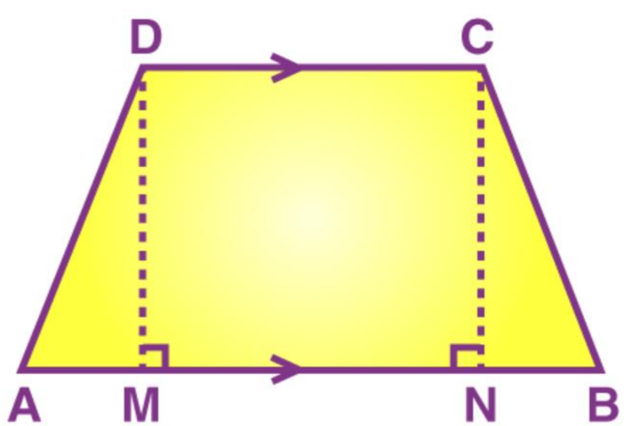
We know that
Area of cross section of the solid = ½ (1.5 + 3) × 40 cm^2
By further calculation
= ½ × 4.5 × 40 cm^2

= 90 cm^2

Here
Volume of solid = Area of cross section × length
= 90 × 15 cm^3
= 1350 cm^3

We know that,
1 cm^3 = 1000 lt
So 1350 cm^3 = 1350000 litres

3. The cross-section of a tunnel perpendicular to its length is a trapezium ABCD as show in the following figure; also given that:
AM = BN; AB = 7 m; CD = 5 m. The height of the tunnel is 2.4 m. The tunnel is 40 m long. Calculate:



(i) The cost of painting the internal surface of the tunnel (excluding the floor) at the rate of Rs. 5 per m^2 (sq. metre).
(ii) The cost of paving the floor at the rate of Rs. 18 per m^2.
Solution:

It is given that
AB = 7 m, CD = 5 m and AM = BN
Height = 2.4 m
Length = 40 m

(i) We know that
AM = BN = (7 – 5)/2 = 2/2 = 1m

In △ ADM,
Using Pythagoras Theorem
$AD^2 = AM^2 + DM^2$
Substituting the values
$AD^2 = 1^2 + 2.4^2$
$AD^2 = 1 + 5.76$

$AD^2 = 6.76$
$AD^2 = 2.6^2$
AD = 2.6 m

Here
Perimeter of the cross section of the tunnel = 7 + 2.6 + 2.6 + 5 = 17.2 m
Length = 40 m

So the internal surface area of the tunnel (except floor) = 17.2 × 40 – 40 × 7
By further calculation
= 688 – 280
= 408 m^2
Rate of paining = Rs. 5 per m^2
So the total cost of painting = 5 × 408 = Rs. 2040

(ii) We know that
Area of floor of tunnel = l × b
= 40 × 7
= 280 m^2
Rate of cost of paving = Rs. 18 per m^2
So the total cost of painting = 280 × 18 = Rs. 5040

4. Water is discharged from a pipe of cross-section area 3.2 cm^2 at the speed of 5 m/s. Calculate the volume of water discharged:
(i) In cm^3 per sec.
(ii) In litres per minute.
Solution:

(i) Rate of speed = 5 m/s = 500 cm/s
Volume of water flowing per sec = 3.2 × 500 = 1600 cm^3

(ii) Volume of water flowing per min = 1600 × 60 = 96000 cm^3
We know that 1000 cm^3 = 1 litre
So the volume of water flowing per min = 96000/1000 = 96 litres

5. A hose-pipe of cross-section area 2 cm^2 delivers 1500 litres of water in 5 minutes. What is the speed of water in m/s through the pipe?
Solution:
Volume of water flowing in 1 sec = (1500 × 1000)/ (5 × 60) = 5000 cm^3

We know that
Volume of water flowing = Area of cross section × speed of water
5000 cm^3/s = 2 cm^2 × speed of water
So we get
Speed of water = 5000/2 cm/s
= 2500 cm/s

= 25 m/s

EXERCISE 21C PAGE: 274

1. Each face of a cube has perimeter equal to 32 cm. Find its surface area and its volume.
Solution:

Perimeter of a cube is
Perimeter = 4a where a is the length
It is given that perimeter of the face of the cube = 32 cm
4a = 32 cm
a = 32/4
a = 8 cm

Surface area of a cube with side 'a' = $6a^2$
So the surface area = $6 \times 8^2 = 6 \times 64 = 384$ cm^2

Volume of a cube with side 'a' = a^3
So the volume = $8^3 = 512$ cm^3

2. A school auditorium is 40 m long, 30 m broad and 12 m high. If each student requires 1.2 m^2 of the floor area; find the maximum number of students that can be accommodated in this auditorium. Also, find the volume of air available in the auditorium, for each student.
Solution:

It is given that
Dimensions of the auditorium = 40 m × 30 m × 12 m
Area of the floor = 40 × 30
It is given that each student requires 1.2 m^2 of the floor area
Maximum number of students = (40 × 30)/ 1.2 = 1000

Volume of the auditorium
= 40 × 30 × 12 m^3
= Volume of air available for 1000 students

Air available for each student = (40 × 30 × 12)/ 10000 = 14.4 m^3

3. The internal dimensions of a rectangular box are 12 cm × x cm × 9 cm. If the length of the longest rod that can be placed in this box is 17 cm; find x.
Solution:

We know that
Length of longest rod = Length of the diagonal of the box

$$17 = \sqrt{12^2 + x^2 + 9^2}$$

By squaring on both sides
$17^2 = 12^2 + x^2 + 9^2$
$x^2 = 17^2 - 12^2 - 9^2$

$x^2 = 289 - 144 - 81$
$x^2 = 64$
x = 8 cm

4. The internal length, breadth and height of a box are 30 cm, 24 cm and 15 cm. Find the largest number of cubes which can be placed inside this box if the edge of each cube is
(i) 3 cm
(ii) 4 cm
(iii) 5 cm
Solution:

(i) Number of cubes which can be placed along length = 30/3 = 10
Number of cubes along the breadth = 24/3 = 8
Number of cubes along the height = 15/3 = 5
So the total number of cubes placed = 10 × 8 × 5 = 400

(ii) Number of cubes which can be placed along length = 30/4 = 7.5 (Take 7)
Number of cubes along the width = 24/4 = 6
Number of cubes along the height = 15/4 = 3.75 (Take 3)
So the total number of cubes placed = 7 × 6 × 3 = 126

(iii) Number of cubes which can be placed along length = 30/5 = 6
Number of cubes along the width = 24/5 = 4.5 (Take 4)
Number of cubes along the height = 15/5 = 3
So the total number of cubes placed = 6 × 4 × 3 = 72

5. A rectangular field is 112 m long and 62 m broad. A cubical tank of edge 6 m is dug at each of the four corners of the field and the earth so removed is evenly spread on the remaining field. Find the rise in level.
Solution:

It is given that
Volume of the tank = Volume of the earth spread
4×6^3 m^3 = $(112 \times 62 - 4 \times 6^2)$ m^2 × Rise in level
We know that

$$Rise\ in\ level = \frac{4 \times 6^3}{112 \times 62 - 4 \times 6^2}$$

$$= \frac{864}{6800}$$

= 0.127 m
= 12.7 cm