Exercise 5(A)

Factorise by taking out the common factors:

1. 2(2x - 5y) (3x + 4y) - 6(2x - 5y) (x - y)

Solution:

Identifying and taking (2x - 5y) common from both the terms, we have

= (2x - 5y) [2(3x + 4y) - 6(x - y)]

= (2x - 5y) (6x + 8y - 6x + 6y)

= (2x - 5y) (8y + 6y)

= (2x - 5y) (14y)

= (2x - 5y)14y

2. $xy(3x^2 - 2y^2) - yz(2y^2 - 3x^2) + zx(15x^2 - 10y^2)$

Solution:

We have, $xy(3x^2 - 2y^2) - yz(2y^2 - 3x^2) + zx(15x^2 - 10y^2)$

Changing signs to arrive at a common term

So,

$= xy(3x^2 - 2y^2) + yz(3x^2 - 2y^2) + zx(15x^2 - 10y^2)$

$= xy(3x^2 - 2y^2) + yz(3x^2 - 2y^2) + 5zx(3x^2 - 2y^2)$

$= (3x^2 - 2y^2) (xy + yz + 5zx)$

3. $ab(a^2 + b^2 - c^2) - bc(c^2 - a^2 - b^2) + ca(a^2 + b^2 - c^2)$

Solution:

We have, $ab(a^2 + b^2 - c^2) - bc(c^2 - a^2 - b^2) + ca(a^2 + b^2 - c^2)$

Changing signs to arrive at a common term

So,

$= ab(a^2 + b^2 - c^2) + bc(a^2 + b^2 - c^2) + ca(a^2 + b^2 - c^2)$

$= (a^2 + b^2 - c^2) (ab + bc + ca)$

4. 2x(a - b) + 3y(5a - 5b) + 4z(2b - 2a)

Solution:

We have, 2x(a - b) + 3y(5a - 5b) + 4z(2b - 2a)

Taking common factors, we get

= 2x(a - b) + 15y(a - b) - 8z(a - b)

= (a - b) (2x + 15y - 8z)

Factorize by the grouping method:

5. $a^3 + a - 3a^2 - 3$

Solution:

We have, $a^3 + a - 3a^2 – 3$

Grouping to arrive at a common term

$= a (a^2 + 1) - 3(a^2 + 1)$

Taking common, we get
= $(a^2 + 1)(a - 3)$

6. $16(a + b)^2 - 4a - 4b$
Solution:

We have, $16(a + b)^2 - 4a - 4b$
Grouping to arrive at a common term
= $16(a + b)^2 - 4(a + b)$
Taking common, we get
= $4(a + b)[4(a + b) - 1]$
= $4(a + b)(4a + 4b - 1)$

7. Factorize by the grouping method:
$a^4 - 2a^3 - 4a + 8$
Solution:

We have, $a^4 - 2a^3 - 4a + 8$
Grouping to arrive at a common term
= $a^3(a - 2) - 4(a - 2)$
Taking common, we get
= $(a^3 - 4)(a - 2)$

8. $ab - 2b + a^2 - 2a$
Solution:

We have, $ab - 2b + a^2 - 2a$
Grouping to arrive at a common term
= $b(a - 2) + a(a - 2)$
Taking common, we get
= $(b + a)(a - 2)$

9. $ab(x^2 + 1) + x(a^2 + b^2)$
Solution:

We have, $ab(x^2 + 1) + x(a^2 + b^2)$
On expanding,
= $abx^2 + ab + a^2x + b^2x$
Now, grouping to arrive at a common term
= $abx^2 + a^2x + b^2x + ab$
= $ax(bx + a) + b(bx + a)$
Taking common, we get
= $(ax + b)(bx + a)$

10. $a^2 + b - ab - a$
Solution:

We have, $a^2 + b - ab - a$
Grouping to arrive at a common term
$= a^2 – a + b – ab$
$= a(a - 1) – b(-1 + a)$
$= a(a - 1) – b(a - 1)$
Taking common, we get
$= (a – b) (a – 1)$

11. $(ax + by)^2 + (bx - ay)^2$
Solution:

We have, $(ax + by)^2 + (bx - ay)^2$
On expanding,
$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 – 2abxy$
$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$
Rearranging terms, we get
$= a^2x^2 + b^2x^2 + a^2y^2 + b^2y^2$
Taking common, we get
$= x^2(a^2 + b^2) + y^2(a^2 + b^2)$
$= (x^2 + y^2) (a^2 + b^2)$

12. $a^2x^2 + (ax^2 + 1)x + a$
Solution:

We have, $a^2x^2 + (ax^2 + 1)x + a$
Regrouping the terms, we have
$= a^2x^2 + a + (ax^2 + 1)x$
$= a(ax^2 + 1) + x(ax^2 + 1)$
Taking common, we get
$= (ax^2 + 1) (a + x)$

13. $(2a - b)^2 - 10a + 5b$
Solution:

We have, $(2a - b)^2 - 10a + 5b$
Taking common,
$= (2a - b)^2 – 5(2a - b)$
Now,
$= (2a - b) [(2a - b) – 5]$
$= (2a – b) (2a – b – 5)$

14. $a(a – 4) – a + 4$
Solution:

We have, $a(a – 4) – a + 4$

By grouping, we get
= a(a - 4) -1(a - 4)
Now, taking the common term
= (a - 4) (a - 1)

15. $y^2 - (a + b) y + ab$
Solution:

We have, $y^2 - (a + b) y + ab$
On expanding,
= $y^2 – ay – by + ab$
= $(y^2 – ay) – by + ab$
Taking 'y' and 'b' common from the group, we get
= y(y - a) – b(y - a)
= (y – a) (y – b)

16. $a^2 + 1/a^2 – 2 – 3a + 3/a$
Solution:

We have, $a^2 + 1/a^2 – 2 – 3a + 3/a$
On grouping terms, we get
= $(a^2 – 2 + 1/a^2) – 3a + 3/a$
= $[a^2 – (2 \times a \times 1/a) + 1/a^2] – 3(a – 1/a)$
= $(a – 1/a)^2 – 3(a – 1/a)$ {Since, $(x - y)^2 = x^2 – 2xy + y^2$}
Taking (a – 1/a) as common, we get
= (a – 1/a) [(a – 1/a) - 3]
= (a – 1/a) (a – 1/a - 3)

17. $x^2 + y^2 + x + y + 2xy$
Solution:

We have, $x^2 + y^2 + x + y + 2xy$
On rearranging terms, we get
= $(x^2 + y^2 + 2xy) + (x + y)$ {Since, $(x + y)^2 = x^2 + 2xy + y^2$}
Now,
= $(x + y)^2 + (x + y)$
= (x + y)(x + y + 1)

18. $a^2 + 4b^2 - 3a + 6b - 4ab$
Solution:

We have, $a^2 + 4b^2 - 3a + 6b - 4ab$
On rearranging terms, we get
= $a^2 + 4b^2 - 4ab - 3a + 6b$
Now,
= $a^2 + (2b)^2 - 2 \times a \times (2b) - 3(a - 2b)$ {Since, $(a - b)^2 = a^2 - 2ab + b^2$}

= $(a - 2b)^2 - 3(a - 2b)$
= $(a - 2b) [(a - 2b) - 3]$
= $(a - 2b) (a - 2b - 3)$

19. $m (x - 3y)^2 + n (3y - x) + 5x - 15y$
Solution:

We have, $m (x - 3y)^2 + n (3y - x) + 5x - 15y$
Now,
Taking (x - 3y) common from all the three terms, we get
= $m(x - 3y)^2 - n(x - 3y) + 5(x - 3y)$
= $(x - 3y) [m(x - 3y) - n + 5]$
= $(x - 3y) (mx - 3my - n + 5)$

20. $x(6x - 5y) - 4(6x - 5y)^2$
Solution:

We have, $x(6x - 5y) - 4(6x - 5y)^2$
Now,
Taking (6x - 5y) common from the three terms, we get
= $(6x - 5y) [x - 4(6x - 5y)]$
= $(6x - 5y) (x - 24x + 20y)$
= $(6x - 5y) (-23x + 20y)$
= $(6x - 5y) (20y - 23x)$

Exercise 5(B)

Factorize:

1. $a^2 + 10a + 24$

Solution:

We have, $a^2 + 10a + 24$
By splitting the middle term, we get
$= a^2 + 6a + 4a + 24$
$= a(a + 6) + 4(a + 6)$
$= (a + 4)(a + 6)$

2. $a^2 - 3a – 40$

Solution:

We have, $a^2 - 3a – 40$
By splitting the middle term, we get
$= a^2 - 8a + 5a – 40$
$= a(a - 8) + 5(a - 8)$
$= (a + 5)(a – 8)$

3. $1 - 2a - 3a^2$

Solution:

We have, $1 - 2a - 3a^2$
By splitting the middle term, we get
$= 1 - 3a + a - 3a^2$
$= 1(1 – 3a) + a(1 – 3a)$
$= (1 + a)(1 – 3a)$

4. $x^2 - 3ax - 88a^2$

Solution:

We have, $x^2 - 3ax - 88a^2$
By splitting the middle term, we get
$= x^2 – 11ax + 8ax - 88a^2$
$= x(x – 11a) + 8a(x – 11a)$
$= (x + 8a)(x – 11a)$

5. $6a^2 – a – 15$

Solution:

We have, $6a^2 – a – 15$
By splitting the middle term, we get
$= 6a^2 + 9a – 10a – 15$
$= 3a(2a + 3) – 5(2a + 3)$
$= (3a - 5)(2a + 3)$

6. $24a^3 + 37a^2 - 5a$
Solution:

We have, $24a^3 + 37a^2 - 5a$
Taking 'a' common from all
$= a(24a^2 + 37a - 5)$
$= a(24a^2 + 40a - 3a - 5)$ {By splitting the middle term}
$= a[8a(3a + 5) - 1(3a + 5)]$
$= a[(8a - 1)(3a + 5)]$
$= a(8a - 1)(3a + 5)$

7. a(3a - 2) - 1
Solution:

We have, $a(3a - 2) - 1$
On expanding,
$= 3a^2 - 2a - 1$
By splitting the middle term, we get
$= 3a^2 - 3a + a - 1$
$= 3a(a - 1) + 1(a - 1)$
$= (3a + 1)(a - 1)$

8. $a^2b^2 + 8ab - 9$
Solution:

We have, $a^2b^2 + 8ab - 9$
By splitting the middle term, we get
$= a^2b^2 + 9ab - ab - 9$
$= ab(ab + 9) - 1(ab + 9)$
$= (ab - 1)(ab + 9)$

9. 3 - a (4 + 7a)
Solution:

We have, $3 - a(4 + 7a)$
On expanding,
$= 3 - 4a - 7a^2$
By splitting the middle term, we get
$= 3 + 3a - 7a - 7a^2$
$= 3(1 + a) - 7a(1 + a)$
$= (1 + a)(3 - 7a)$

10. $(2a + b)^2 - 6a - 3b - 4$
Solution:

We have, $(2a + b)^2 - 6a - 3b – 4$
$= (2a + b)^2 – 3(2a + b) – 4$
Let's assume that $(2a + b) = x$
So, the expression becomes
$= x^2 – 3x – 4$
By splitting the middle term, we get
$= x^2 – 4x + x – 4$
$= x(x - 4) + 1(x - 4)$
$= (x - 4) (x + 1)$
Resubstituting the value of x, we get
$= (2a + b - 4) (2a + b + 1)$

11. $1 - 2 (a+ b) - 3 (a + b)^2$
Solution:

We have, $1 - 2 (a+ b) - 3 (a + b)^2$
Let's assume $(a + b) = x$
Then, the expression becomes
$= 1 – 2x – 3x^2$
By splitting the middle term, we get
$= 1 – 3x + x – 3x^2$
$= 1(1 – 3x) + x(1 – 3x)$
$= (1 – 3x) (1 + x)$
Resubstituting the value of x, we get
$= [1 – 3(a + b)] [1 + (a + b)]$
$= (1 – 3a – 3b) (1 + a + b)$

12. $3a^2 - 1 - 2a$
Solution:

We have, $3a^2 - 1 - 2a$
Rearranging,
$= 3a^2 – 2a – 1$
By splitting the middle term, we get
$= 3a^2 – 3a + a – 1$
$= 3a(a - 1) + 1(a - 1)$
$= (3a + 1) (a - 1)$

13. $x^2 + 3x + 2 + ax + 2a$
Solution:

We have, $x^2 + 3x + 2 + ax + 2a$
By splitting the middle term, we get
$= (x^2 + 2x + x + 2) + ax + 2a$
$= x(x + 2) + 1(x + 2) + a(x + 2)$

= (x + 2) (x + a + 1)

14. $(3x - 2y)^2 + 3(3x - 2y) - 10$
Solution:

We know, $(3x - 2y)^2 + 3(3x - 2y) – 10$
Let's assume that (3x – 2y) = a
So, the expression becomes
= $a^2 + 3a – 10$
By splitting the middle term, we get
= $a^2 + 5a – 2a – 10$
= a(a + 5) – 2(a + 5)
= (a – 2) (a + 5)
= (3x – 2y + 5) (3x – 2y - 2)

15. $5 - (3a^2 - 2a)(6 - 3a^2 + 2a)$
Solution:

Given, $5 - (3a^2 - 2a)(6 - 3a^2 + 2a)$
= $5 – (3a^2 – 2a)[6 – (3a^2 – 2a)]$
Let's substitute $(3a^2 - 2a) = x$
And, the expression becomes,
= 5 – x(6 - x)
= $5 – 6x – x^2$
= $5 – 5x – x – x^2$
= 5(1 - x) – x(1 - x)
= (1 - x) (5 - x)
= (x - 1) (x - 5)
= $(3a^2 - 2a – 1)(3a^2 - 2a - 5)$
Now,
= $(3a^2 - 3a + a – 1)(3a^2 + 3a - 5a – 5)$ {By splitting the middle term}
= [3a(a - 1) + 1(a - 1)] [3a(a + 1) - 5(a + 1)]
= [(3a + 1) (a - 1)] [(3a - 5) (a + 1)]
= (3a + 1) (3a - 5) (a + 1)(a - 1)

16. $1/35 + 12a/35 + a^2$
Solution:

We have, $1/35 + 12a/35 + a^2$
Taking common,
= $1/35 (1 + 12a + 35a^2)$
= $1/35 (35a^2 + 12a + 1)$
= $1/35 (35a^2 + 7a + 5a + 1)$ {By splitting the middle term}
= 1/35 [7a(5a + 1) + 1(5a + 1)]
= 1/35 [(7a + 1) (5a + 1)]

= [(7a + 1) (5a + 1)]/ 35

17. $(x^2 - 3x)(x^2 - 3x - 1) - 20$.
Solution:

We have, $(x^2 - 3x)(x^2 - 3x - 1) - 20$
$= (x^2 - 3x)[(x^2 - 3x) - 1] – 20$
Let's
= a[a - 1] - 20 ….(Taking $x^2 - 3x = a$)
$= a^2 - a - 20$
$= a^2 - 5a + 4a - 20$
= a(a - 5) + 4(a - 5)
= (a - 5)(a + 4)
$= (x^2 - 3x - 5)(x^2 - 3x + 4)$

18. Find each trinomial (quadratic expression), given below, find whether it is factorisable or not. Factorise, if possible.
(i) $x^2 - 3x - 54$
(ii) $2x^2 - 7x - 15$
(iii) $2x^2 + 2x - 75$
(iv) $3x^2 + 4x - 10$
(v) x(2x - 1) - 1
Solution:

(i) Given, $x^2 - 3x - 54$
On comparing with the general form $ax^2 + bx + c$, we get
a = 1, b = -3 and c = -54
So, $b^2 – 4ac = (-3)^2 – 4(1)(-54) = 9 + 216 = 225$
225 is a perfect square
Thus, $x^2 - 3x – 54$ is factorisable
Now,
$x^2 - 3x – 54 = x^2 - 9x + 6x – 54$
= x(x - 9) + 6(x – 9)
= (x + 6) (x - 9)

(ii) Given, $2x^2 - 7x - 15$
On comparing with the general form $ax^2 + bx + c$, we get
a = 2, b = -7 and c = -15
So, $b^2 – 4ac = (-7)^2 – 4(2)(-15) = 49 + 120 = 169$
169 is a perfect square
Thus, $2x^2 - 7x - 15$ is factorisable
Now,
$2x^2 - 7x - 15 = 2x^2 - 10x + 3x – 15$
= 2x(x - 5) + 3(x - 5)
= (2x + 3) (x - 5)

(iii) Given, $2x^2 + 2x - 75$
On comparing with the general form $ax^2 + bx + c$, we get
a = 2, b = 2 and c = -75
So, $b^2 - 4ac = (2)^2 - 4(2)(-75) = 4 + 600 = 604$
604 is not a perfect square
Thus, $2x^2 + 2x - 75$ is not factorizable

(iv) Given, $3x^2 + 4x - 10$
On comparing with the general form $ax^2 + bx + c$, we get
a = 3, b = 4 and c = -10
So, $b^2 - 4ac = (4)^2 - 4(3)(-10) = 16 + 120 = 136$
136 is a not perfect square
Thus, $3x^2 + 4x - 10$ is not factorizable

(v) Given, $x(2x - 1) - 1$
$= 2x^2 - x - 1$
On comparing with the general form $ax^2 + bx + c$, we get
a = 2, b = -1 and c = -1
So, $b^2 - 4ac = (-1)^2 - 4(2)(-1) = 1 + 8 = 9$
9 is a perfect square
Thus, $x(2x - 1) - 1$ is factorisable
Now,
$x(2x - 1) - 1 = 2x^2 - x - 1$
$= 2x^2 - 2x + x - 1$
$= 2x(x - 1) + 1(x - 1)$
$= (2x + 1)(x - 1)$

19. Factorise:
(i) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$
(ii) $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$
Solution:

(i) We have, $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$
By splitting the middle term, we get
$= 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$
$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$
$= (4x - \sqrt{3})(\sqrt{3}x + 2)$

(ii) We have, $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$
By splitting the middle term, we get
$= 7\sqrt{2}x^2 - 14x + 4x - 4\sqrt{2}$
$= 7\sqrt{2}x(x - \sqrt{2}) + 4(x - \sqrt{2})$
$= (7\sqrt{2}x + 4)(x - \sqrt{2})$

20. Give possible expressions for the length and the breadth of the rectangle whose area is $12x^2 - 35x + 25$.

Solution:

We have, $12x^2 - 35x + 25$
By splitting the middle term, we get
$= 12x^2 - 20x - 15x + 25$
$= 4x(3x - 5) - 5(3x - 5)$
$= (3x - 5)(4x - 5)$

Hence,
Length = $(3x - 5)$ and breadth = $(4x - 5)$ or,
Length = $(4x - 5)$ and breadth = $(3x - 5)$

Exercise 5(C)

Factorize:

1. $25a^2 - 9b^2$

Solution:

We have, $25a^2 - 9b^2$

$= (5a)^2 – (3b)^2$

$= (5a + 3b) (5a – 3b)$ [As $x^2 – y^2 = (x + y)(x - y)$]

2. $a^2 - (2a + 3b)^2$

Solution:

We have, $a^2 - (2a + 3b)^2$

$= [a - (2a + 3b)] [a + (2a + 3b)]$ [As $x^2 – y^2 = (x + y)(x - y)$]

$= (a – 2a – 3b) (a + 2a + 3b)$

$= (-a – 3b) (3a + 3b)$

$= -3(a + 3b) (a + b)$

3. $a^2 - 81(b-c)^2$

Solution:

We have, $a^2 - 81(b-c)^2$

$= a^2 – [9(b - c)]^2$

$= [a – 9(b - c)] [a + 9(b - c)]]$ [As $x^2 – y^2 = (x + y)(x - y)$]

$= (a – 9b + 9c) (a + 9b – 9c)$

4. $25(2a - b)^2 - 81b^2$

Solution:

We have, $25(2a - b)^2 - 81b^2$

$= [5(2a - b)]^2 – (9b)^2$

$= [5(2a - b) – 9b] [5(2a - b) + 9b]$ [As $x^2 – y^2 = (x + y)(x - y)$]

$= (10a – 5b – 9b) (10a – 5b + 9b)$

$= (10a – 14b) (10a + 4b)$

$= 2(5a – 7b). 2(5a + 2b)$

$= 2(5a – 7b) (5a + 2b)$

5. $50a^3 - 2a$

Solution:

We have, $50a^3 - 2a$

$= 2a(25a^2 - 1)$

$= 2a[(5a)^2 - 1^2]$

$= 2a(5a - 1)(5a + 1)$ [As $x^2 – y^2 = (x + y)(x - y)$]

6. $4a^2b - 9b^3$
Solution:

We have, $4a^2b - 9b^3$
$= b(4a^2 - 9b^2)$
$= b[(2a)^2 – (3b)^2]$
$= b[(2a + 3b) (2a – 3b)]$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= b(2a + 3b)(2a – 3b)$

7. $3a^5 - 108a^3$
Solution:

We have, $3a^5 - 108a^3$
$= 3a^3(a^2 – 36)$
$= 3a^3(a^2 - 6^2)$
$= 3a^3(a – 6) (a + 6)$ [As $x^2 – y^2 = (x + y)(x - y)$]

8. $9(a - 2)^2 - 16(a + 2)^2$
Solution:

We have, $9(a - 2)^2 - 16(a + 2)^2$
$= [3(a - 2)]^2 – [4(a + 2)]^2$
$= [3(a - 2) – 4(a + 2)] [3(a - 2) + 4(a + 2)]$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= [3a – 6 – 4a - 8] [3a – 6 + 4a + 8]$
$= [-a - 14] [7a + 2]$
$= -(a + 14) (7a + 2)$

9. $a^4 – 1$
Solution:

We have, $a^4 – 1$
$= (a^2)^2 – 1^2$
$= (a^2 - 1) (a^2 + 1)$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= [(a - 1)(a + 1)] (a^2 + 1)$
$= (a - 1)(a + 1)(a^2 + 1)$

10. $a^3 + 2a^2 – a - 2$
Solution:

We have, $a^3 + 2a^2 – a – 2$
$= a^2(a + 2) – 1(a + 2)$
$= (a^2 - 1) (a + 2)$
$= (a - 1) (a + 1) (a + 2)$ [As $x^2 – y^2 = (x + y)(x - y)$]

11. $(a + b)^3 - a - b$
Solution:

We have, $(a + b)^3 - a - b$
$= (a + b)^3 - (a + b)$
$= (a + b) [(a + b)^2 - 1]$
$= (a + b) [(a + b - 1) (a + b + 1)]$ [As $x^2 - y^2 = (x + y)(x - y)$]
$= (a + b) (a + b - 1) (a + b + 1)$

12. $a(a - 1) - b(b - 1)$
Solution:

We have, $a(a - 1) - b(b - 1)$
$= a^2 - a - b^2 + b$
$= (a^2 - b^2) - (a - b)$
$= (a + b) (a - b) - (a - b)$ [As $x^2 - y^2 = (x + y)(x - y)$]
$= (a - b) [(a + b) - 1]$
$= (a - b) (a + b - 1)$

13. $4a^2 - (4b^2 + 4bc + c^2)$
Solution:

We know, $4a^2 - (4b^2 + 4bc + c^2)$
$= (2a)^2 - [(2b)^2 + 2(2b)(c) + c^2]$
$= (2a)^2 - (2b + c)^2$
$= (2a - 2b - c) (2a + 2b + c)$ [As $x^2 - y^2 = (x + y)(x - y)$]

14. $4a^2 - 49b^2 + 2a - 7b$
Solution:

We know, $4a^2 - 49b^2 + 2a - 7b$
$= (2a)^2 - (7b)^2 + (2a - 7b)$
$= [(2a - 7b) (2a + 7b)] + (2a - 7b)$ [As $x^2 - y^2 = (x + y)(x - y)$]
$= (2a - 7b) [(2a + 7b) + 1]$
$= (2a - 7b) (2a + 7b + 1)$

15. $9a^2 + 3a - 8b - 64b^2$
Solution:

We have, $9a^2 + 3a - 8b - 64b^2$
$= 9a^2 - 64b^2 + 3a - 8b$
$= (3a)^2 - (8b)^2 + (3a - 8b)$
$= [(3a - 8b) (3a + 8b)] + (3a - 8b)$ [As $x^2 - y^2 = (x + y)(x - y)$]
$= (3a - 8b) [(3a + 8b) + 1]$

= (3a – 8b) (3a + 8b + 1)

16. $4a^2 - 12a + 9 - 49b^2$
Solution:

We have, $4a^2 - 12a + 9 - 49b^2$
$= [(2a)^2 - 2(2a)(3) + 3^2] - (7b)^2$
$= (2a - 3)^2 - (7b)^2$
$= (2a - 7b - 3)(2a + 7b - 3)$ [As $x^2 - y^2 = (x + y)(x - y)$]

17. $4xy - x^2 - 4y^2 + z^2$
Solution:

We have, $4xy - x^2 - 4y^2 + z^2$
On rearranging,
$= z^2 - x^2 - 4y^2 + 4xy$
$= z^2 - (x^2 + 4y^2 - 4xy)$
$= z^2 - (x - 2y)^2$
$= (z - x + 2y)(z + x - 2y)$ [As $x^2 - y^2 = (x + y)(x - y)$]

18. $a^2 + b^2 - c^2 - d^2 + 2ab - 2cd$
Solution:

We have, $a^2 + b^2 - c^2 - d^2 + 2ab - 2cd$
On rearranging,
$= a^2 + 2ab + b^2 - c^2 - d^2 - 2cd$
$= (a^2 + 2ab + b^2) - (c^2 + d^2 + 2cd)$
$= (a + b)^2 - (c + d)^2$
$= (a + b + c + d)(a + b - c - d)$ [As $x^2 - y^2 = (x + y)(x - y)$]

19. $4x^2 - 12ax - y^2 - z^2 - 2yz + 9a^2$
Solution:

We have, $4x^2 - 12ax - y^2 - z^2 - 2yz + 9a^2$
On rearranging,
$= 4x^2 - 12ax + 9a^2 - y^2 - z^2 - 2yz$
$= (4x^2 - 12ax + 9a^2) - (y^2 + z^2 + 2yz)$
$= (2x - 3a)^2 - (y + z)^2$
$= (2x - 3a + y + z)(2x - 3a - y - z)$ [As $x^2 - y^2 = (x + y)(x - y)$]

20. $(a^2 - 1)(b^2 - 1) + 4ab$
Solution:

We have, $(a^2 - 1)(b^2 - 1) + 4ab$
By cross multiplying and expanding, we get
$= (1 - a^2 - b^2 + a^2b^2) + 4ab$

On manipulating,
$= (a^2b^2 + 1 + 2ab) - (a^2 + b^2 – 2ab)$
Now,
$= (ab + 1)^2 – (a - b)^2$
$= [(ab + 1) – (a – b)] [(ab + 1) + (a – b)]$
$= (ab + 1 – a + b) (ab + 1 + a - b)$ [As $x^2 – y^2 = (x + y)(x - y)$]

21. $x^4 + x^2 + 1$
Solution:

We have, $x^4 + x^2 + 1$
$= x^4 + 2x^2 + 1 – x^2$
$= (x^2 + 1)^2 – x^2$ [As $a^2 – b^2 = (a + b)(a - b)$]
$= (x^2 + 1 - x) (x^2 + 1 + x)$

22. $(a^2 + b^2 - 4c^2)^2 - 4a^2b^2$
Solution:

We have, $(a^2 + b^2 - 4c^2)^2 - 4a^2b^2$
$= (a^2 + b^2 - 4c^2)^2 – (2ab)^2$
$= [(a^2 + b^2 - 4c^2) + (2ab)] [(a^2 + b^2 - 4c^2) - (2ab)]$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= [(a^2 + b^2 + 2ab) - 4c^2] [(a^2 + b^2 - 2ab) - 4c^2]$
$= [(a + b)^2 – (2c)^2] [(a - b)^2 – (2c)^2]$
$= [(a + b – 2c) (a + b + 2c)] [(a - b – 2c) (a - b + 2c)]$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= (a + b – 2c) (a + b + 2c) (a - b – 2c) (a - b + 2c)$

23. $(x^2 + 4y^2 - 9z^2)^2 - 16x^2y^2$
Solution:

We have, $(x^2 + 4y^2 - 9z^2)^2 - 16x^2y^2$
$= (x^2 + 4y^2 - 9z^2)^2 – (4xy)^2$
$= (x^2 + 4y^2 - 9z^2 – 4xy) (x^2 + 4y^2 - 9z^2 + 4xy)$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= [(x^2 – 4xy + 4y^2) - 9z^2] [(x^2 + 4xy + 4y^2) - 9z^2]$
$= [(x – 2y)^2 – (3z)^2] [(x + 2y)^2 – (3z)^2]$
$= [(x – 2y + 3z) (x – 2y - 3z)] [(x +2y + 3z) (x + 2y - 3z)]$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= (x – 2y + 3z) (x – 2y - 3z)] [(x +2y + 3z) (x + 2y - 3z)$

24. $(a + b)^2 - a^2 + b^2$
Solution:

We have, $(a + b)^2 - a^2 + b^2$
On expanding,
$= (a^2 + 2ab + b^2) – a^2 + b^2$
$= 2b^2 + 2ab$
$= 2b (b + a)$

25. $a^2 - b^2 - (a + b)^2$
Solution:

We have, $a^2 - b^2 - (a + b)$
On expanding,
$= a^2 - b^2 - (a^2 + b^2 + 2ab)$
$= a^2 - b^2 - a^2 - b^2 - 2ab$
$= -2b^2 – 2ab$
$= -2b(b + a)$

26. $9a^2 - (a^2 - 4)^2$
Solution:

We have, $9a^2 - (a^2 - 4)^2$
$= (3a)^2 - (a^2 - 4)^2$
$= [3a – (a^2 - 4)] [3a + (a^2 - 4)]$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= [3a – (a^2 - 2^2)] [3a + (a^2 - 2^2)]$
$= (3a – a^2 + 4) (3a + a^2 – 4)$
$= (-a^2 + 3a + 4) (a^2 + 3a – 4)$
$= (-a^2 + 4a – a + 4) (a^2 + 4a – a – 4)$ [By splitting the middle term]
$= [a(-a + 4) + 1(-a + 4)] [a(a + 4) – 1(a + 4)]$
$= [(-a + 4) (a + 1)] [(a - 1) (a + 4)]$
$= (4 - a) (a + 1) (a - 1) (a + 4)$

27. $x^2 + 1/x^2 - 11$
Solution:

We have, $x^2 + 1/x^2 - 11$
$= x^2 + 1/x^2 – 2 – 9$
$= (x^2 + 1/x^2 – 2 \times x \times 1/x) - 9$
$= (x - 1/x)^2 - 3^2$
$= (x - 1/x + 3) (x - 1/x - 3)$ [As $a^2 – b^2 = (a + b)(a - b)$]

28. $4x^2 + 1/4x^2 + 1$
Solution:

We have, $4x^2 + 1/4x^2 + 1$
$= 4x^2 + 1/4x^2 + 2 – 1$
$= [(2x)^2 + (1/2x)^2 + 2 \times 2x \times 1/2x)] - 1^2$
$= (2x + 1/2x)^2 - 1^2$
$= (2x + 1/2x + 1) (2x – 1/2x - 1)$ [As $x^2 – y^2 = (x + y)(x - y)$]

29. $4x^4 - x^2 - 12x - 36$
Solution:

We know, $4x^4 - x^2 - 12x – 36$
$= 4x^4 – (x^2 + 12x + 36)$
$= (2x^2)^2 – [x^2 + 2(x)(6) + 6^2]$
$= (2x^2)^2 – (x + 6)^2$
$= (2x^2 + x + 6) (2x^2 – x – 6)$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= (2x^2 + x + 6) (2x^2 – 4x + 3x – 6)$ [By splitting the middle term]
$= (2x^2 + x + 6) [2x(x - 2) + 3(x – 2)]$
$= (2x^2 + x + 6) [(2x + 3) (x – 2)]$
$= (2x^2 + x + 6) (2x + 3) (x – 2)$

30. $a^2(b + c) - (b + c)^3$
Solution:

We have, $a^2(b + c) - (b + c)^3$
$= (b + c) [a^2 – (b + c)^2]$
$= (b + c) [(a – b – c) (a + b + c)]$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= (b + c) (a – b - c) (a + b + c)$

Exercise 5(D)

Factorize:

1. $a^3 - 27$

Solution:

We have, $a^3 – 27$

$= a^3 - 3^3$

$= (a - 3) [a^2 + (a \times 3) + 3^2]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]

$= (a - 3) (a^2 + 3a + 9)$

2. $1 - 8a^3$

Solution:

We have, $1 - 8a^3$

$= 1^3 – (2a)^3$

$= (1 – 2a) [1^2 + (1 \times 2a) + (2a)^2]$

$= (1 – 2a) (1 + 2a + 4a^2)$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]

3. $64 - a^3b^3$

Solution:

We have, $64 - a^3b^3$

$= 4^3 – (ab)^3$

$= (4 – ab) [4^2 + (4 \times ab) + (ab)^2]$

$= (4 – ab) (16 + 4ab + a^2b^2)$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]

4. $a^6 + 27b^3$

Solution:

We have, $a^6 + 27b^3$

$= (a^2)^3 + (3b)^3$

$= (a^2 + 3b) [(a^2)^2 - (a^2 \times 3b) + (3b)^2]$

$= (a^2 + 3b) (a^4 - 3a^2b + 9b^2)$ [As, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$]

5. $3x^7y - 81x^4y^4$

Solution:

We have, $3x^7y - 81x^4y^4$

$= 3xy (x^6 – 27x^3y^3)$

$= 3xy [(x^2)^3 – (3xy)^3]$

$= 3xy (x^2 – 3xy) [(x^2)^2 + (x^2 \times 3xy) + (3xy)^2]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]

$= 3xy (x^2 – 3xy) (x^4 + 3x^3y + 9x^2y^2)$

$= 3xy.\ x(x – 3y).\ x^2(x^2 + 3xy + 9y^2)$ [Taking common from terms]

$= 3x^4y (x – 3y) (x^2 + 3xy + 9y^2)$

6. $a^3 – 27/a^3$
Solution:

We have, $a^3 – 27/a^3$
$= a^3 – (3/a)^3$
$= (a – 3/a) [a^2 + a \times 3/a + (3/a)^2]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
$= (a – 3/a) (a^2 + 3 + 9/a^2)$

7. $a^3 + 0.064$
Solution:

We have, $a^3 + 0.064$
$= a^3 + (0.4)^3$
$= (a + 0.4) [a^2 – (a \times 0.4) + 0.4^2]$
$= (a + 0.4) (a^2 – 0.4a + 0.16)$ [As, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$]

8. $a^4 - 343a$
Solution:

We have, $a^4 - 343a$
$= a (a^3 - 343)$
$= a (a^3 - 7^3)$
$= a (a - 7) [a^2 + (a \times 7) + 7^2]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
$= a (a - 7) (a^2 + 7a + 49)$

9. $(x - y)^3 - 8x^3$
Solution:

We have, $(x - y)^3 - 8x^3$
$= (x - y)^3 - (2x)^3$
$= (x - y - 2x) [(x - y)^2 + 2x(x - y) + (2x)^2]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
$= (-x – y) [x^2 + y^2 - 2xy + 2x^2 - 2xy + 4x^2]$
$= -(x + y) [7x^2 - 4xy + y^2]$

10. $8a^3/27 – b^3/8$
Solution:

We have, $8a^3/27 – b^3/8$
$= (2a/3)^3 – (b/2)^3$
$= (2a/3 – b/2) [(2a/3)^2 + (2a/3 \times b/2) + (b/2)^2]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
$= (2a/3 – b/2) (4a^2/9 + ab/3 + b^2/4)$

11. $a^6 - b^6$
Solution:

We have, $a^6 - b^6$

$= (a^3)^2 – (b^3)^2$
$= (a^3 + b^3) (a^3 - b^3)$ [As $x^2 – y^2 = (x + y) (x - y)$]
Now,
$= [(a + b) (a^2 – ab + b^2)] [(a – b) (a^2 + ab + b^2)]$ [Using identities]
$= (a + b) (a - b) (a^2 – ab + b^2) (a^2 + ab + b^2)$

12. $a^6 - 7a^3 - 8$
Solution:

We have, $a^6 - 7a^3 – 8$
By splitting the middle term,
$= a^6 - 8a^3 + a^3 – 8$
$= a^3(a^3 - 8) + 1(a^3 - 8)$
$= (a^3 + 1) (a^3 - 8)$
We know that,
$a^3 – b^3 = (a - b) (a^2 + ab + b^2)$ … (1)
$a^3 + b^3 = (a + b) (a^2 - ab + b^2)$ … (2)
Now,
$(a^3 + 1) (a^3 - 8)$
$= [(a + 1) (a^2 – a + 1)] [(a - 2)(a^2 + 2a + 4)]$ … [Using (1) and (2)]
$= (a + 1) (a - 2) (a^2 + 2a + 4) (a^2 – a + 1)$

13. $a^3 - 27b^3 + 2a^2b - 6ab^2$
Solution:

We have, $a^3 - 27b^3 + 2a^2b - 6ab^2$
$= [a^3 – (3b)^3] + 2ab(a – 3b)$
$= (a – 3b) (a^2 + 3ab + 9b^2) + 2ab(a – 3b)$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
Now, taking (a – 3b) as common
$= (a – 3b) [(a^2 + 3ab + 9b^2) + 2ab]$
$= (a – 3b) (a^2 + 5ab + 9b^2)$

14. $8a^3 - b^3 - 4ax + 2bx$
Solution:

We have, $8a^3 - b^3 - 4ax + 2bx$
$= (2a)^3 - b^3 – 2x(2a - b)$
$= (2a - b) [(2a)^2 – 2ab + b^2] – 2x(2a - b)$
Taking (2a - b) as common,
$= (2a - b) [(4a^2 + 2ab + b^2) – 2x]$
$= (2a – b) (4a^2 + 2ab + b^2 – 2x)$

15. $a - b - a^3 + b^3$
Solution:

We have, $a - b - a^3 + b^3$
$= (a - b) – (a^3 – b^3)$
$= (a - b) – [(a - b) (a^2 + ab + b^2)]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
Now, taking (a - b) as common
$= (a - b) [1 - (a^2 + ab + b^2)]$
$= (a - b) (1 - a^2 - ab - b^2)$

16. $2x^3 + 54y^3 - 4x - 12y$
Solution:

We have, $2x^3 + 54y^3 - 4x - 12y$
$= 2(x^3 + 27y^3 - 2x - 6y)$
Now,
$= 2 \{[(x)^3 + (3y)^3] - 2(x + 3y)\}$
$= 2 \{[(x + 3y) (x^2 - 3xy + 9y^2)] - 2(x + 3y)\}$ [As, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$]
$= 2 (x + 3y) (x^2 - 3xy + 9y^2 - 2)$

17. $1029 - 3x^3$
Solution:

We have, $1029 - 3x^3$
$= 3(343 - x^3)$
$= 3(7^3 - x^3)$
$= 3(7 - x) (7^2 + 7x + x^2)$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
$= 3(7 - x) (49 + 7x + x^2)$

18. Show that:
(i) $13^3 - 5^3$ is divisible by 8
(ii)$35^3 + 27^3$ is divisible by 62
Solution:

(i) We have, $(13^3 - 5^3)$
Now, using identity $(a^3 - b^3) = (a - b) (a^2 + ab + b^2)$
$= (13 - 5) (13^2 + 13 \times 5 + 5^2)$
$= 8 \times (169 + 65 + 25)$

Hence, the number is divisible by 8.

(ii) $(35^3 + 27^3)$
Now, using identity $(a^3 + b^3) = (a + b) (a^2 - ab + b^2)$
$= (35 + 27) (35^2 + 35\times 27 + 27^2)$
$= 62 \times (35^2 + 35 \times 27 + 27^2)$

Hence, the number is divisible by 62.

19. Evaluate:

$$\frac{5.67 \times 5.67 \times 5.67 + 4.33 \times 4.33 \times 4.33}{5.67 \times 5.67 - 5.67 \times 4.33 + 4.33 \times 4.33}$$

Solution:

Let a = 5.67 and b = 4.33

Then,

$$\frac{5.67 \times 5.67 \times 5.67 + 4.33 \times 4.33 \times 4.33}{5.67 \times 5.67 - 5.67 \times 4.33 + 4.33 \times 4.33}$$

$$= \frac{a \times a \times a + b \times b \times b}{a \times a - a \times b + b \times b}$$

$$= \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$= \frac{(a + b)(a^2 - ab + b^2)}{a^2 - ab + b^2}$$

= a + b

= 5.67 + 4.33

= 10

Exercise 5(E)

Factorize:

1. $x^2 + 1/4x^2 + 1 – 7x – 7/2x$

Solution:

We have, $x^2 + \frac{1}{4x^2} + 1 - 7x - \frac{7}{2x}$

= $[x^2 + 1/(2x)^2 + 2 \times x \times 1/(2x)] – 7 [x + 1/(2x)]$

= $(x + 1/2x)^2 – 7(x + 1/x)$

Taking out (x + 1/2x) as common,

= (x + 1/2x) (x + 1/2x - 7)

2. $9a^2 + 1/9a^2 – 2 – 12a + 4/3a$

Solution:

We have, $(9a)^2 + \frac{1}{(9a)^2} - 2 - 12a + \frac{4}{3a}$

= $(3a)^2 + \frac{1}{(3a)^2} - 2 \times 3a \times \frac{1}{3a} - 4\left(3a - \frac{1}{3a}\right)$

= $\left(3a - \frac{1}{3a}\right)^2 - 4\left(3a - \frac{1}{3a}\right)$

Taking (3a - 1/3a) as common,

= $\left(3a - \frac{1}{3a}\right)\left[\left(3a - \frac{1}{3a}\right) - 4\right]$

= $\left(3a - \frac{1}{3a}\right)\left(3a - 4 - \frac{1}{3a}\right)$

3. $x^2 + (a^2 + 1) x/a + 1$

Solution:

We have, $x^2 + \frac{a^2+1}{a}x + 1$

$= x^2 + ax + \frac{1}{a}x + 1$

$= x(x+a) + \frac{1}{a}(x+a)$

$= (x+a)\left(x + \frac{1}{a}\right)$

4. $x^4 + y^4 - 27x^2y^2$
Solution:

We have, $x^4 + y^4 - 27x^2y^2$
$= x^4 + y^4 - 2x^2y^2 - 25x^2y^2$
$= [(x^2) + (y^2) - 2x^2y^2] - 25x^2y^2$
$= (x^2 - y^2) - (5xy)^2$
$= (x^2 - y^2 - 5xy)(x^2 - y^2 + 5xy)$ [As $x^2 - y^2 = (x + y)(x - y)$]

5. $4x^4 + 9y^4 + 11x^2y^2$
Solution:

We have, $4x^4 + 9y^4 + 11x^2y^2$
$= 4x^4 + 9y^4 + 12x^2y^2 - x^2y^2$
$= (2x^2)^2 + (3y^2)^2 + 2(2x^2)(3y^2) - (xy)^2$
$= (2x^2 + 3y^2)^2 - (xy)^2$
$= (2x^2 + 3y^2 - xy)(2x^2 + 3y^2 + xy)$ [As $x^2 - y^2 = (x + y)(x - y)$]

6. $x^2 + 1/x^2 - 3$
Solution:

We have, $x^2 + 1/x^2 - 3$
$= x^2 + 1/x^2 - 2 - 1$
$= [x^2 + 1/x^2 - (2 \times x \times 1/x)] - 1^2$
$= (x - 1/x)^2 - 1^2$
$= (x - 1/x - 1)(x - 1/x + 1)$ [As $x^2 - y^2 = (x + y)(x - y)$]

7. $a - b - 4a^2 + 4b^2$
Solution:

We have, $a - b - 4a^2 + 4b^2$
$= (a - b) - 4(a^2 - b^2)$

= (a - b) – 4(a - b)(a + b) [As $x^2 - y^2 = (x + y)(x - y)$]
Taking (a - b) common,
= (a - b) [1 – 4(a + b)]
= (a - b) [1 – 4a – 4b]

8. $(2a - 3)^2 - 2(2a - 3)(a - 1) + (a - 1)^2$
Solution:

We have, $(2a - 3)^2 - 2(2a - 3)(a - 1) + (a - 1)^2$
Comparing with the identity, $(a - b)^2 = a^2 - 2ab + b^2$
= $[(2a - 3) - (a - 1)]^2$
= $(2a - a - 3 + 1)^2$
= $(a - 2)^2$

9. $(a^2 - 3a)(a^2 - 3a + 7) + 10$
Solution:

Let's substitute $(a^2 - 3a) = x$
Then the given expression becomes,
= x(x + 7) + 10
= $x^2 + 7x + 10$
= $x^2 + 5x + 2x + 10$ [By splitting the middle term]
= x(x + 5) + 2(x + 5)
= (x + 2) (x + 5)
Resubstituting the value of x,
= $(a^2 - 3a + 2)(a^2 - 3a + 5)$
= $(a^2 - 3a + 5)(a^2 - 3a + 2)$
= $(a^2 - 3a + 5)[a^2 - 2a - a + 2]$ [By splitting the middle term]
= $(a^2 - 3a + 5)[a(a - 2) - 1(a - 5)]$
= $(a^2 - 3a + 5)[(a - 1)(a - 2)]$
= $(a^2 - 3a + 5)(a - 1)(a - 2)$

10. $(a^2 - a)(4a^2 - 4a - 5) - 6$
Solution:

Let's $a^2 - a = x$
Then the expression becomes,
= x(4x - 5) – 6
= $4x^2 - 5x - 6$
= $4x^2 - 8x + 3x - 6$
= 4x(x - 2) + 3(x - 2)
= (4x + 3) (x - 2)
Resubstituting the value of x,
= $(4a^2 - 4a + 3)(a^2 - a - 2)$
= $(4a^2 - 4a + 3)(a^2 - 2a + a - 2)$
= $(4a^2 - 4a + 3)[a(a - 2) + 1(a - 2)]$

= $(4a^2 – 4a + 3) [(a + 1) (a – 2)]$
= $(4a^2 – 4a + 3) (a + 1) (a – 2)$

11. $x^4 + y^4 - 3x^2y^2$
Solution:

We have, $x^4 + y^4 - 3x^2y^2$
= $(x^4 + y^4 - 2x^2y^2) - x^2y^2$
= $(x^2 - y^2) – (xy)^2$ [As $x^2 – y^2 = (x + y)(x - y)$]
= $(x^2 – y^2 – xy) (x^2 – y^2 + xy)$

12. $5a^2 - b^2 - 4ab + 7a - 7b$
Solution:

We have, $5a^2 - b^2 - 4ab + 7a - 7b$
= $4a^2 + a^2 - b^2 - 4ab + 7a - 7b$
= $a^2 - b^2 + 4a^2 - 4ab + 7a - 7b$
= $(a^2 - b^2) + 4a(a – b) + 7(a – b)$
= $(a + b)(a - b) + 4a(a - b) + 7(a - b)$ [As $x^2 – y^2 = (x + y)(x - y)$]
= $(a - b) [(a + b) + 4a + 7]$
= $(a - b) (5a + b + 7)$

13. $12(3x - 2y)^2 - 3x + 2y - 1$
Solution:

We have, $12(3x - 2y)^2 - 3x + 2y – 1$
= $12(3x - 2y)^2 – (3x - 2y) – 1$
Let's substitute $(3x – 2y) = a$
Then, the expression becomes
= $12a^2 – a – 1$
= $12a^2 – 4a + 3a – 1$
= $4a (3a - 1) + 1(3a - 1)$
= $(4a + 1) (3a - 1)$
Now, resubstituting the value of 'a' in the above
= $[4(3x – 2y) + 1] [3(3x – 2y) - 1]$
= $(12x – 8y + 1) (9x – 6y – 1)$

14. $4(2x - 3y)^2 - 8x+12y - 3$
Solution:

We have, $4(2x - 3y)^2 - 8x+12y – 3$
= $4(2x - 3y)^2 – 4(2x + 3y) – 3$
Let's substitute $(2x – 3y) = a$
= $4(a^2) – 4a – 3$
= $4a^2 – 6a + 2a – 3$ [By splitting the middle term]
= $2a(2a - 3) + 1(2a - 3)$

= (2a - 3) (2a + 1)
Now, resubstituting the value of 'a' in the above
= [2(2x – 3y) – 3] [2(2x – 3y) + 1]
= (4x – 6y - 3) (4x – 6y + 1)

15. 3 - 5x + 5y - 12(x - y)^2
Solution:

We have, 3 - 5x + 5y - 12(x - y)^2
= 3 – 5(x – y) - 12(x - y)^2
Let's substitute (x – y) = a
= 3 – 5a – 12a^2
= 3 – 9a + 4a – 12a^2 [By splitting the middle term]
= 3(1 – 3a) + 4a(1 – 3a)
= (1 – 3a) (4a + 3)
Now, resubstituting the value of 'a' in the above
= [1 – 3(x - y)] [4(x - y) + 3]
= (1 – 3x + 3y) (4x – 4y + 3)

16. 9x^2 + 3x - 8y - 64y^2
Solution:

We have, 9x^2 + 3x - 8y - 64y^2
On rearranging,
= 9x^2 - 64y^2 + 3x - 8y
= [(3x)^2 – (8y)^2] + (3x – 8y)
= (3x – 8y) (3x + 8y) + (3x – 8y) [As x^2 – y^2 = (x + y)(x - y)]
Taking (3x – 8y) as common,
= (3x – 8y) (3x + 8y + 1)

17. 2√3x^2 + x - 5√3
Solution:

We have, 2√3x^2 + x - 5√3
By splitting the middle term,
= 2√3x^2 + 6x – 5x - 5√3
= 2√3x(x + √3) – 5(x + √3)
= (2√3x - 5) (x + √3)

18. ¼ (a + b)^2 – 9/16 (2a - b)^2
Solution:

We have, $\frac{1}{4}(a+b)^2 - \frac{9}{16}(2a-b)^2$

$= \frac{1}{4}\left[(a+b)^2 - \frac{9}{4}(2a-b)^2\right]$

$= \frac{1}{4}\left[(a+b)^2 - \left[\frac{3}{2}(2a-b)\right]^2\right]$

$= \frac{1}{4}\left[\left(a+b+\frac{3}{2}(2a-b)\right)\left(a+b-\frac{3}{2}(2a-b)\right)\right]$ [As $x^2 - y^2 = (x + y)(x - y)$]

$= \frac{1}{4}\left[\left(a+b+3a-\frac{3b}{2}\right)\left(a+b-3a+\frac{3b}{2}\right)\right]$

$= \frac{1}{4}\left[\left(4a-\frac{b}{2}\right)\left(\frac{5b}{2}-2a\right)\right]$

$= \frac{1}{4}\left[\left(\frac{8a-b}{2}\right)\left(\frac{5b-4a}{2}\right)\right]$

$= \frac{1}{4}\left[\frac{1}{4}(8a-b)(5b-4a)\right]$

$= \frac{1}{16}(8a-b)(5b-4a)$

19. 2(ab + cd) - a^2 - b^2 + c^2 + d^2
Solution:

We have, 2(ab + cd) - a^2 - b^2 + c^2 + d^2
= 2ab + 2cd – a^2 – b^2 + c^2 + d^2
On rearranging and grouping, we get
= (c^2 + d^2 + 2cd) – (a^2 + b^2 – 2ab)
= $(c + d)^2 – (a – b)^2$
= [c + d – (a - b)] [c + d + (a - b)] [As $x^2 – y^2 = (x + y)(x - y)$]
= (c + d – a + b) (c + d + a - b)

20. Find the value of:
(i) 987^2 - 13^2
(ii) $(67.8)^2 – (32.2)^2$
(iii) [$(6.7)^2 – (3.3)^2$]/ (6.7 – 3.3)
(iv) [$(18.5)^2 – (6.5)^2$]/ (18.5 – 6.5)

Solution:

(i) We have, $987^2 - 13^2$
= (987 + 13) (987 - 13)
= 1000 x 974
= 974000

(ii) We have, $(67.8)^2 – (32.2)^2$
= (67.8 + 32.2) (67.8 – 32.2)
= 100 x 35.6
= 3560

(iii) We have, $\frac{(6.7)^2 - (3.3)^2}{6.7 - 3.3}$

$= \frac{(6.7 + 3.3)(6.7 - 3.3)}{(6.7 - 3.3)}$

= 10

(iv) We have, $\frac{(18.5)^2 - (6.5)^2}{18.5 + 6.5}$

$= \frac{(18.5 + 6.5)(18.5 - 6.5)}{(18.5 + 6.5)}$

= 12