Exercise 8(A)

1. Express each of the following in logarithmic form:
(i) $5^3 = 125$
(ii) $3^{-2} = 1/9$
(iii) $10^{-3} = 0.001$
(iv) $(81)^{3/4} = 27$
Solution:

We know that,
$a^b = c \Rightarrow \log_a c = b$

(i) $5^3 = 125$
$\log_5 125 = 3$

(ii) $3^{-2} = 1/9$
$\log_3 1/9 = -2$

(iii) $10^{-3} = 0.001$
$\log_{10} 0.001 = -3$

(iv) $(81)^{3/4} = 27$
$\log_{81} 27 = 3/4$

2. Express each of the following in exponential form:
(i) $\log_8 0.125 = -1$
(ii) $\log_{10} 0.01 = -2$
(iii) $\log_a A = x$
(iv) $\log_{10} 1 = 0$
Solution:

We know that,
$\log_a c = b \Rightarrow a^b = c$

(i) $\log_8 0.125 = -1$
$8^{-1} = 0.125$

(ii) $\log_{10} 0.01 = -2$
$8^{-1} = 0.125$

(iii) $\log_a A = x$
$a^x = A$

(iv) $\log_{10} 1 = 0$
$10^0 = 1$

3. Solve for x: $\log_{10} x = -2$.

Solution:

We have,
$\log_{10} x = -2$
$10^{-2} = x$ [As $\log_a c = b \Rightarrow a^b = c$]
$x = 10^{-2}$
$x = 1/10^2$
$x = 1/100$
Hence, $x = 0.01$

4. Find the logarithm of:
(i) 100 to the base 10
(ii) 0.1 to the base 10
(iii) 0.001 to the base 10
(iv) 32 to the base 4
(v) 0.125 to the base 2
(vi) 1/16 to the base 4
(vii) 27 to the base 9
(viii) 1/81 to the base 27
Solution;

(i) Let $\log_{10} 100 = x$
So, $10^x = 100$
$10^x = 10^2$
Then,
$x = 2$ [If $a^m = a^n$; then $m = n$]
Hence, $\log_{10} 100 = 2$

(ii) Let $\log_{10} 0.1 = x$
So, $10^x = 0.1$
$10^x = 1/10$
$10^x = 10^{-1}$
Then,
$x = -1$ [If $a^m = a^n$; then $m = n$]
Hence, $\log_{10} 0.1 = -1$

(iii) Let $\log_{10} 0.001 = x$
So, $10^x = 0.001$
$10^x = 1/1000$
$10^x = 1/10^3$
$10^x = 10^{-3}$
Then,
$x = -3$ [If $a^m = a^n$; then $m = n$]
Hence, $\log_{10} 0.001 = -3$

(iv) Let $\log_4 32 = x$

So, $4^x = 32$
$4^x = 2 \times 2 \times 2 \times 2 \times 2$
$(2^2)^x = 2^5$
$2^{2x} = 2^5$
Then,
$2x = 5$ [If $a^m = a^n$; then m = n]
$x = 5/2$
Hence, $\log_4 32 = 5/2$

(v) Let $\log_2 0.125 = x$
So, $2^x = 0.125$
$2^x = 125/1000$
$2^x = 1/8$
$2^x = (½)^3$
$2^x = 2^{-3}$
Then,
$x = -3$ [If $a^m = a^n$; then m = n]
Hence, $\log_2 0.125 = -3$

(vi) Let $\log_4 1/16 = x$
So, $4^x = 1/16$
$4^x = (¼)^{-2}$
$4^x = 4^{-2}$
Then,
$x = -2$ [If $a^m = a^n$; then m = n]
Hence, $\log_4 1/16 = -2$

(vii) Let $\log_9 27 = x$
So, $9^x = 27$
$9^x = 3 \times 3 \times 3$
$(3^2)^x = 3^3$
$3^{2x} = 3^3$
Then,
$2x = 3$ [If $a^m = a^n$; then m = n]
$x = 3/2$
Hence, $\log_9 27 = 3/2$

(viii) Let $\log_{27} 1/81 = x$
So, $27^x = 1/81$
$27^x = 1/9^2$
$(3^3)^x = 1/(3^2)^2$
$3^{3x} = 1/3^4$
$3^{3x} = 3^{-4}$
Then,
$3x = -4$ [If $a^m = a^n$; then m = n]
$x = -4/3$

Hence, $\log_{27} 1/81 = -4/3$

5. State, true or false:
(i) If $\log_{10} x = a$, then $10^x = a$
(ii) If $x^y = z$, then $y = \log_z x$
(iii) $\log_2 8 = 3$ and $\log_8 2 = 1/3$
Solution:

(i) We have,
$\log_{10} x = a$
So, $10^a = x$
Thus, the statement $10^x = a$ is false

(ii) We have,
$x^y = z$
So, $\log_x z = y$
Thus, the statement $y = \log_z x$ is false

(iii) We have,
$\log_2 8 = 3$
So, $2^3 = 8$ … (1)
Now consider the equation,
$\log_8 2 = 1/3$
$8^{1/3} = 2$
$(2^3)^{1/3} = 2$ … (2)
Both equations (1) and (2) are correct
Thus, the given statements, $\log_2 8 = 3$ and $\log_8 2 = 1/3$ are true

6. Find x, if:
(i) $\log_3 x = 0$
(ii) $\log_x 2 = -1$
(iii) $\log_9 243 = x$
(iv) $\log_5 (x - 7) = 1$
(v) $\log_4 32 = x - 4$
(vi) $\log_7 (2x^2 - 1) = 2$
Solution:

(i) We have, $\log_3 x = 0$
So, $3^0 = x$
$1 = x$
Hence, $x = 1$

(ii) we have, $\log_x 2 = -1$
So, $x^{-1} = 2$
$1/x = 2$
Hence, $x = ½$

(iii) We have, $\log_9 243 = x$
$9^x = 243$
$(3^2)^x = 3^5$
$3^{2x} = 3^5$
On comparing the exponents, we get
$2x = 5$
$x = 5/2 = 2\frac{1}{2}$

(iv) We have, $\log_5 (x - 7) = 1$
So, $5^1 = x - 7$
$5 = x - 7$
$x = 5 + 7$
Hence, $x = 12$

(v) We have, $\log_4 32 = x - 4$
So, $4^{(x - 4)} = 32$
$(2^2)^{(x - 4)} = 2^5$
$2^{(2x - 8)} = 2^5$
On comparing the exponents, we get
$2x - 8 = 5$
$2x = 5 + 8$
Hence,
$x = 13/2 = 6\frac{1}{2}$

(vi) We have, $\log_7 (2x^2 - 1) = 2$
So, $(2x^2 - 1) = 7^2$
$2x^2 - 1 = 49$
$2x^2 = 49 + 1$
$2x^2 = 50$
$x^2 = 25$
Taking square root on both side, we get
$x = \pm 5$
Hence, $x = 5$ (Neglecting the negative value)

7. Evaluate:
(i) $\log_{10} 0.01$
(ii) $\log_2 (1 \div 8)$
(iii) $\log_5 1$
(iv) $\log_5 125$
(v) $\log_{16} 8$
(vi) $\log_{0.5} 16$
Solution:

(i) Let $\log_{10} 0.01 = x$
Then, $10^x = 0.01$

$10^x = 1/100 = 1/10^2$
So, $10^x = 10^{-2}$
On comparing the exponents, we get
x = -2
Hence, $\log_{10} 0.01 = -2$

(ii) Let $\log_2 (1 \div 8) = x$
Then, $2^x = 1/8$
$2^x = 1/2^3$
So, $2^x = 2^{-3}$
On comparing the exponents, we get
x = -3
Hence, $\log_{10} (1 \div 8) = -3$

(iii) Let $\log_5 1 = x$
Then, $5^x = 1$
$5^x = 5^0$
On comparing the exponents, we get
x = 0
Hence, $\log_5 1 = 0$

(iv) Let $\log_5 125 = x$
Then, $5^x = 125$
$5^x = (5 \times 5 \times 5) = 5^3$
So, $5^x = 5^3$
On comparing the exponents, we get
x = 3
Hence, $\log_5 125 = 3$

(v) Let $\log_{16} 8 = x$
Then, $16^x = 8$
$(2^4)^x = (2 \times 2 \times 2) = 2^3$
So, $2^{4x} = 2^3$
On comparing the exponents, we get
4x = 3
x = 3/4
Hence, $\log_{16} 8 = 3/4$

(vi) Let $\log_{0.5} 16 = x$
Then, $0.5^x = 16$
$(5/10)^x = (2 \times 2 \times 2 \times 2)$
$(1/2)^x = 2^4$
So, $2^{-x} = 2^4$
On comparing the exponents, we get
-x = 4
⇒ x = -4

Hence, $\log_{0.5} 16 = -4$

8. If $\log_a m = n$, express a^{n-1} in terms in terms of a and m.
Solution:

We have, $\log_a m = n$
So,
$a^n = m$
Dividing by a on both sides, we get
$a^n/a = m/a$
$a^{n-1} = m/a$

9. Given $\log_2 x = m$ and $\log_5 y = n$
(i) Express 2^{m-3} in terms of x
(ii) Express 5^{3n+2} in terms of y
Solution:

Given, $\log_2 x = m$ and $\log_5 y = n$
So,
$2^m = x$ and $5^n = y$
(i) Taking, $2^m = x$
$2^m/2^3 = x/2^3$
$2^{m-3} = x/8$

(ii) Taking, $5^n = y$
Cubing on both sides, we have
$(5^n)^3 = y^3$
$5^{3n} = y^3$
Multiplying by 5^2 on both sides, we have
$5^{3n} \times 5^2 = y^3 \times 5^2$
$5^{3n+2} = 25y^3$

10. If $\log_2 x = a$ and $\log_3 y = a$, write 72^a in terms of x and y.
Solution:

Given, $\log_2 x = a$ and $\log_3 y = a$
So,
$2^a = x$ and $3^a = y$
Now, the prime factorization of 72 is
$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$
Hence,
$(72)^a = (2^3 \times 3^2)^a$
$= 2^{3a} \times 3^{2a}$
$= (2^a)^3 \times (3^a)^2$
$= x^3y^2$ [As $2^a = x$ and $3^a = y$]

11. Solve for x: log (x - 1) + log (x + 1) = $\log_2 1$
Solution:

We have,
$\log (x - 1) + \log (x + 1) = \log_2 1$
$\log (x - 1) + \log (x + 1) = 0$
$\log [(x - 1) (x + 1)] = 0$
Then,
$(x - 1) (x + 1) = 1$ [As log 1 = 0]
$x^2 – 1 = 1$
$x^2 = 1 + 1$
$x^2 = 2$
$x = \pm\sqrt{2}$
The value $-\sqrt{2}$ is not a possible, since log of a negative number is not defined.
Hence, $x = \sqrt{2}$

12. If log (x^2 - 21) = 2, show that x = ± 11.
Solution:

Given, $\log (x^2 - 21) = 2$
So,
$x^2 - 21 = 10^2$
$x^2 - 21 = 100$
$x^2 = 121$
Taking square root on both sides, we get
$x = \pm 11$

Exercise 8(B)

1. Express in terms of log 2 and log 3:
(i) log 36
(ii) log 144
(iii) log 4.5
(iv) log 26/51 - log 91/119
(v) log 75/16 - 2log 5/9 + log 32/243
Solution:

(i) log 36 = log (2 x 2 x 3 x 3)
= log $(2^2 \times 3^2)$
= log 2^2 + log 3^2 [Using $\log_a mn = \log_a m + \log_a n$]
= 2log 2 + 2log 3 [Using $\log_a m^n = n\log_a m$]

(ii) log 144 = log (2 x 2 x 2 x 2 x 3 x 3)
= log $(2^4 \times 3^2)$
= log 2^4 + log 3^2 [Using $\log_a mn = \log_a m + \log_a n$]
= 4log 2 + 2log 3 [Using $\log_a m^n = n\log_a m$]

(iii) log 4.5 = log 45/10
= log (5 x 3 x 3)/ (5 x 2)
= log $3^2/2$
= log 3^2 - log 2 [Using $\log_a m/n = \log_a m - \log_a n$]
= 2log 3 - log 2 [Using $\log_a m^n = n\log_a m$]

(iv) log 26/51 - log 91/119 = log (26/51)/ (91/119) [Using $\log_a m - \log_a n = \log_a m/n$]
= log [(26/51) x (119/91)]
= log (2 x 13 x 7 x 117)/ (3 x 17 x 7 x 13)
= log 2/3
= log 2 – log 3 [Using $\log_a m/n = \log_a m - \log_a n$]

(v) log 75/16 - 2log 5/9 + log 32/243
= log 75/16 - log $(5/9)^2$ + log 32/243 [Using $n\log_a m = \log_a m^n$]
= log 75/16 - log 25/81 + log 32/243
= log [(75/16)/ (25/81)] + log 32/243 [Using $\log_a m - \log_a n = \log_a m/n$]
= log (75 x 81)/ (16 x 25) + log 32/243
= log (3 x 81)/16 + log 32/243
= log 243/16 + log 32/243
= log (243/16) x (32/243) [Using $\log_a m + \log_a n = \log_a mn$]
= log 32/16
= log 2

2. Express each of the following in a form free from logarithm:
(i) 2 log x - log y = 1
(ii) 2 log x + 3 log y = log a
(iii) a log x - b log y = 2 log 3

Solution:

(i) We have, 2 log x - log y = 1
Then,
log x^2 - log y = 1 [Using $n\log_a m = \log_a m^n$]
log x^2/y = 1 [Using $\log_a m - \log_a n = \log_a m/n$]
Now, on removing log we have
$x^2/y = 10^1$
⇒ $x^2 = 10y$

(ii) We have, 2 log x + 3 log y = log a
Then,
log x^2 + log y^3 = log a [Using $n\log_a m = \log_a m^n$]
log x^2y^3 = log a [Using $\log_a m + \log_a n = \log_a mn$]
Now, on removing log we have
$x^2y^3 = a$

(iii) a log x - b log y = 2 log 3
Then,
log x^a - log y^b = log 3^2 [Using $n\log_a m = \log_a m^n$]
log x^a/y^b = log 3^2 [Using $\log_a m - \log_a n = \log_a m/n$]
Now, on removing log we have
$x^a/y^b = 3^2$
⇒ $x^2 = 9y^b$

3. Evaluate each of the following without using tables:
(i) log 5 + log 8 - 2log 2
(ii) $\log_{10} 8 + \log_{10} 25 + 2\log_{10} 3 - \log_{10} 18$
(iii) log 4 + 1/3 log 125 – 1/5 log 32
Solution:

(i) We have, log 5 + log 8 - 2log 2
= log 5 + log 8 - log 2^2 [Using $n\log_a m = \log_a m^n$]
= log 5 + log 8 - log 4
= log (5 x 8) - log 4 [Using $\log_a m + \log_a n = \log_a mn$]
= log 40 - log 4
= log 40/4 [Using $\log_a m - \log_a n = \log_a m/n$]
= log 10
= 1

(ii) We have, $\log_{10} 8 + \log_{10} 25 + 2\log_{10} 3 - \log_{10} 18$
= $\log_{10} 8 + \log_{10} 25 + \log_{10} 3^2 - \log_{10} 18$ [Using $n\log_a m = \log_a m^n$]
= $\log_{10} 8 + \log_{10} 25 + \log_{10} 9 - \log_{10} 18$
= $\log_{10} (8 \times 25 \times 9) - \log_{10} 18$ [Using $\log_a l + \log_a m + \log_a n = \log_a lmn$]
= $\log_{10} 1800 - \log_{10} 18$
= $\log_{10} 1800/18$ [Using $\log_a m - \log_a n = \log_a m/n$]

= $\log_{10} 100$
= $\log_{10} 10^2$
= $2\log_{10} 10$ [Using $\log_a m^n = n\log_a m$]
= 2 x 1
= 2

(iii) We have, log 4 + 1/3log 125 – 1/5log 32
= $\log 4 + \log (125)^{1/3} – \log (32)^{1/5}$ [Using $n\log_a m = \log_a m^n$]
= $\log 4 + \log (5^3)^{1/3} – \log (2^5)^{1/5}$
= log 4 + log 5 – log 2
= log (4 x 5) – log 2 [Using $\log_a m + \log_a n = \log_a mn$]
= log 20 – log 2
= log 20/2 [Using $\log_a m - \log_a n = \log_a m/n$]
= log 10
= 1

4. Prove that:
2log 15/18 – log 25/162 + log 4/9 = log 2
Solution:

Taking L.H.S.,
= 2log 15/18 – log 25/162 + log 4/9
= $\log (15/18)^2$ – log 25/162 + log 4/9 [Using $n\log_a m = \log_a m^n$]
= log 225/324 – log 25/162 + log 4/9
= log [(225/324)/(25/162)] + log 4/9 [Using $\log_a m - \log_a n = \log_a m/n$]
= log (225 x 162)/(324 x 25) + log 4/9
= log (9 x 1)/(2 x 1) + log 4/9
= log 9/2 + log 4/9 [Using $\log_a m + \log_a n = \log_a mn$]
= log (9/2 x 4/9)
= log 2
= R.H.S.

5. Find x, if:
x - log 48 + 3 log 2 = 1/3 log 125 - log 3.
Solution:

We have,
x - log 48 + 3 log 2 = 1/3 log 125 - log 3
Solving for x, we have
x = log 48 – 3 log 2 + 1/3 log 125 - log 3
= $\log 48 – \log 2^3 + \log 125^{1/3} – \log 3$ [Using $n\log_a m = \log_a m^n$]
= $\log 48 – \log 8 + \log (5^3)^{1/3} – \log 3$
= (log 48 – log 8) + (log 5 – log 3)
= log 48/8 + log 5/3 [Using $\log_a m - \log_a n = \log_a m/n$]
= log (48/8 x 5/3) [Using $\log_a m + \log_a n = \log_a mn$]
= log (2 x 5)

= log 10
= 1
Hence, x = 1

6. Express $\log_{10} 2 + 1$ in the form of $\log_{10} x$.
Solution:

Given, $\log_{10} 2 + 1$
= $\log_{10} 2 + \log_{10} 10$ [As, $\log_{10} 10 = 1$]
= $\log_{10} (2 \times 10)$ [Using $\log_a m + \log_a n = \log_a mn$]
= $\log_{10} 20$

7. Solve for x:
(i) $\log_{10} (x - 10) = 1$
(ii) $\log (x^2 - 21) = 2$
(iii) log (x - 2) + log (x + 2) = log 5
(iv) log (x + 5) + log (x - 5) = 4 log 2 + 2 log 3
Solution:

(i) We have, $\log_{10} (x - 10) = 1$
Then,
$x - 10 = 10^1$
x = 10 + 10
Hence, x = 20

(ii) We have, $\log (x^2 - 21) = 2$
Then,
$x^2 - 21 = 10^2$
$x^2 - 21 = 100$
$x^2 = 100 + 21$
$x^2 = 121$
Taking square root on both sides,
Hence, x = ±11

(iii) We have, log (x - 2) + log (x + 2) = log 5
Then,
log (x - 2)(x + 2) = log 5 [Using $\log_a m + \log_a n = \log_a mn$]
$\log (x^2 - 2^2) = \log 5$ [As $(x - a)(x + a) = x^2 - a^2$]
$\log (x^2 - 4) = \log 5$
Removing log on both sides, we get
$x^2 - 4 = 5$
$x^2 = 5 + 4$
$x^2 = 9$
Taking square root on both sides,
x = ±3

(iv) We have, log (x + 5) + log (x - 5) = 4 log 2 + 2 log 3
Then,
log (x + 5) + log (x - 5) = log 2^4 + log 3^2 [Using $n\log_a m = \log_a m^n$]
log (x + 5)(x - 5) = log 16 + log 9 [Using $\log_a m + \log_a n = \log_a mn$]
log (x^2 - 5^2) = log (16 x 9) [As $(x - a)(x + a) = x^2 – a^2$]
log (x^2 – 25) = log 144
Removing log on both sides, we have
$x^2 – 25 = 144$
$x^2 = 144 + 25$
$x^2 = 169$
Taking square root on both sides, we get
x = ±13

8. Solve for x:
(i) log 81/log 27 = x
(ii) log 128/log 32 = x
(iii) log 64/log 8 = log x
(iv) log 225/log 15 = log x
Solution:

(i) We have, log 81/log 27 = x
x = log 81/log 27
= log (3 x 3 x 3 x 3)/ log (3 x 3 x 3)
= log 3^4/log 3^3
= (4log 3)/(3log 3) [Using $\log_a m^n = n\log_a m$]
= 4/3
Hence, x = 4/3

(ii) We have, log 128/log 32 = x
x = log 128/log 32
= log (2 x 2 x 2 x 2 x 2 x 2 x 2)/ log (2 x 2 x 2 x 2 x 2)
= log 2^7/log 2^5
= (7log 2)/(5log 2) [Using $\log_a m^n = n\log_a m$]
= 7/5
Hence, x = 7/5

(iii) log 64/log 8 = log x
log x = log 64/log 8
= log (2 x 2 x 2 x 2 x 2 x 2)/ log (2 x 2 x 2)
= log 2^6/log 2^3
= (6log 2)/(3log 2) [Using $\log_a m^n = n\log_a m$]
= 6/3
= 2
So, log x = 2
Hence, x = 10^2 = 100

(iv) We have, log 225/log 15 = log x
log x = log 225/log 15
= log (15 x 15)/ log 15
= log 15^2/log 15
= (2log 15)/log 15 [Using $\log_a m^n = n\log_a m$]
= 2
So, log x = 2
Hence, x = 10^2 = 100

9. Given log x = m + n and log y = m - n, express the value of log 10x/y^2 in terms of m and n.
Solution:

Given, log x = m + n and log y = m – n
Now consider log 10x/y^2,
log 10x/y^2 = log 10x – log y^2 [Using $\log_a m/n = \log_a m - \log_a n$]
= log 10x – 2log y
= log 10 + log x – 2 log y
= 1 + (m + n) – 2 (m - n)
= 1 + m + n – 2m + 2n
= 1 + 3n - m

10. State, true or false:
(i) log 1 × log 1000 = 0
(ii) log x/log y = log x – log y
(iii) If log 25/log 5 = log x, then x = 2
(iv) log x × log y = log x + log y
Solution:

(i) We have, log 1 × log 1000 = 0
Now,
log 1 = 0 and
log 1000 = log 10^3 = 3log 10 = 3 [Using $\log_a m^n = n\log_a m$]
So,
log 1 × log 1000 = 0 x 3 = 0
Thus, the statement log 1 × log 1000 = 0 is true

(ii) We have, log x/log y = log x – log y
We know that,
log x/y = log x – log y
So,
log x/log y ≠ log x – log y
Thus, the statement log x/log y = log x – log y is false

(iii) We have, log 25/log 5 = log x
log (5 x 5)/log 5 = log x
log 5^2/ log 5 = log x
2log 5/log 5 = log x [Using $\log_a m^n = n\log_a m$]
2 = log x
So, x = 10^2
x = 100
Thus, the statement x = 2 is false

(iv) We know, log x + log y = log xy
So,
log x + log y ≠ log x × log y
Thus, the statement log x + log y = log x × log y is false

11. If $\log_{10} 2$ = a and $\log_{10} 3$ = b; express each of the following in terms of 'a' and 'b':
(i) log 12
(ii) log 2.25
(iii) log $2\frac{1}{4}$
(iv) log 5.4
(v) log 60
(iv) log $3\frac{1}{8}$

Solution:

Given that $\log_{10} 2$ = a and $\log_{10} 3$ = b … (1)
(i) log 12 = log (2 x 2 x 3)
= log (2 x 2) + log 3 [Using $\log_a mn = \log_a m + \log_a n$]
= log 2^2 + log 3
= 2log 2 + log 3 [Using $\log_a m^n = n\log_a m$]
= 2a + b [From 1]

(ii) log 2.25 = log 225/100
= log (25 x 9)/(25 x 4)
= log 9/4
= log $(3/2)^2$
= 2log 3/2 [Using $\log_a m^n = n\log_a m$]
= 2(log 3 – log 2) [Using $\log_a m/n = \log_a m - \log_a n$]
= 2(b - a) [From 1]
= 2b – 2a

(iii) log $2\frac{1}{4}$ = log 9/4
= log $(3/2)^2$

= 2log 3/2 [Using $\log_a m^n = n\log_a m$]
= 2(log 3 – log 2) [Using $\log_a m/n = \log_a m - \log_a n$]
= 2(b - a) [From 1]
= 2b – 2a

(iv) log 5.4 = log 54/10
= log (2 x 3 x 3 x 3)/10
= log (2 x 3^3) – log 10 [Using $\log_a m/n = \log_a m - \log_a n$]
= log 2 + log 3^3 – 1 [Using $\log_a mn = \log_a m + \log_a n$ and log 10 = 1]
= log 2 + 3log 3 – 1 [Using $\log_a m^n = n\log_a m$]
= a + 3b – 1 [From 1]

(v) log 60 = log (10 x 3 x 2)
= log 10 + log 3 + log 2 [Using $\log_a lmn = \log_a l + \log_a m + \log_{10} n$]
= 1 + b + a [From 1]

(vi) log $3\frac{1}{8}$ = log 25/8
= log $5^2/2^3$
= log 5^2 – log 2^3 [Using $\log_a m/n = \log_a m - \log_a n$]
= 2log 5 – 3log 2 [Using $\log_a m^n = n\log_a m$]
= 2log 10/2 – 3log 2
= 2(log 10 – log 2) – 3log 2 [Using $\log_a m/n = \log_a m - \log_a n$]
= 2log 10 – 2log 2 – 3log 2
= 2(1) – 2a – 3a [From 1]
= 2 – 5a

12. If log 2 = 0.3010 and log 3 = 0.4771; find the value of:
(i) log 12
(ii) log 1.2
(iii) log 3.6
(iv) log 15
(v) log 25
(vi) 2/3 log 8
Solution:

Given, log 2 = 0.3010 and log 3 = 0.4771
(i) log 12 = log (4 x 3)
= log 4 + log 3 [Using $\log_a mn = \log_a m + \log_a n$]
= log 2^2 + log 3
= 2log 2 + log 3 [Using $\log_a m^n = n\log_a m$]
= 2 x 0.3010 + 0.4771
= 1.0791

(ii) log 1.2 = log 12/10
= log 12 – log 10 [Using $\log_a m/n = \log_a m - \log_a n$]
= log (4 x 3) – log 10

= log 4 + log 3 – log 10 [Using $\log_a mn = \log_a m + \log_a n$]
= $\log 2^2$ + log 3 – log 10
= 2log 2 + log 3 – log 10 [Using $\log_a m^n = n\log_a m$]
= 2 x 0.3010 + 0.4771 – 1 [As log 10 = 1]
= 0.6020 + 0.4771 – 1
= 1.0791 – 1
= 0.0791

(iii) log 3.6 = log 36/10
= log 36 – log 10 [Using $\log_a m/n = \log_a m - \log_a n$]
= log (2 x 2 x 3 x 3) – 1 [As log 10 = 1]
= log $(2^2 \times 3^2)$ – 1
= $\log 2^2 + \log 3^2$ – 1 [Using $\log_a mn = \log_a m + \log_a n$]
= 2log 2 + 2log 3 – 1 [Using $\log_a m^n = n\log_a m$]
= 2 x 0.3010 + 2 x 0.4771 – 1
= 0.6020 + 0.9542 – 1
= 1.5562 – 1
= 0.5562

(iv) log 15 = log (15/10 x 10)
= log 15/10 + log 10 [Using $\log_a mn = \log_a m + \log_a n$]
= log 3/2 + 1 [As log 10 = 1]
= log 3 – log 2 + 1 [Using $\log_a m/n = \log_a m - \log_a n$]
= 0.4771 - 0.3010 + 1
= 1.1761

(v) log 25 = log (25/4 x 4)
= log 100/4
= log 100 – log 4 [Using $\log_a m/n = \log_a m - \log_a n$]
= $\log 10^2 - \log 2^2$
= 2log 10 – 2log 2 [Using $\log_a m^n = n\log_a m$]
= (2 x 1) – (2 x 0.3010)
= 2 – 0.6020
= 1.398

13. Given 2 $\log_{10} x$ + 1 = $\log_{10} 250$, find:
(i) x
(ii) $\log_{10} 2x$
Solution:

(i) Given equation, $2\log_{10} x + 1 = \log_{10} 250$
$\log_{10} x^2 + \log_{10} 10 = \log_{10} 250$ [Using $n\log_a m = \log_a m^n$ and $\log_{10} 10 = 1$]
$\log_{10} 10x^2 = \log_{10} 250$ [Using $\log_a m + \log_a n = \log_a mn$]
Removing log on both sides, we have
$10x^2 = 250$
$x^2 = 25$

$x = \pm 5$
As x cannot be a negative value, $x = -5$ is not possible
Hence, $x = 5$

(ii) Now, from (i) we have $x = 5$
So,
$\log_{10} 2x = \log_{10} 2(5)$
$= \log_{10} 10$
$= 1$

14. Given 3log x + ½ log y = 2, express y in term of x.
Solution:

We have, $3\log x + \frac{1}{2}\log y = 2$
$\log x^3 + \log y^{1/2} = 2$ [Using $\log_a m + \log_a n = \log_a mn$]
$\log x^3y^{1/2} = 2$
Removing logarithm, we get
$x^3y^{1/2} = 10^2$
$y^{1/2} = 100/x^3$
On squaring on both sides, we get
$y = 10000/x^6$
$y = 10000x^{-6}$

15. If $x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$, find $\log 10\sqrt{y}/x^2z^3$ in terms of a, b and c.
Solution:

We have,
$x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$
So,
$\log x = a \log 100$, $\log y = b \log 10000$ and $\log z = c \log 10$
$\Rightarrow \log x = a \log 10^2$, $\log y = b \log 10^4$ and $\log z = c \log 10$
$\Rightarrow \log x = 2a \log 10$, $\log y = 4b \log 10$ and $\log z = c \log 10$
$\Rightarrow \log x = 2a$, $\log y = 4b$ and $\log z = c$ … (i)
Now,
$\log 10\sqrt{y}/x^2z^3 = \log 10\sqrt{y} - \log x^2z^3$ [Using $\log_a m/n = \log_a m - \log_a n$]
$= (\log 10 + \log \sqrt{y}) - (\log x^2 + \log z^3)$ [Using $\log_a mn = \log_a m + \log_a n$]
$= 1 + \log y^{1/2} - \log x^2 - \log z^3$
$= 1 + \frac{1}{2}\log y - 2\log x - 3\log z$ [Using $\log_a m^n = n\log_a m$]
$= 1 + \frac{1}{2}(4b) - 2(2a) - 3c$ … [Using (i)]
$= 1 + 2b - 4a - 3c$

16. If 3(log 5 – log 3) – (log 5 – 2 log 6) = 2 – log x, find x.
Solution:

We have,
$3(\log 5 - \log 3) - (\log 5 - 2\log 6) = 2 - \log x$

3log 5 – 3log 3 – log 5 + 2log 6 = 2 – log x
3log 5 – 3log 3 – log 5 + 2log (3 x 2) = 2 – log x
2log 5 – 3log 3 + 2(log 3 + log 2) = 2 – log x [Using $\log_a mn = \log_a m + \log_a n$]
2log 5 – log 3 + 2log 3 + 2log 2 = 2 – log x
2log 5 - log 3 + 2log 2 = 2 – log x
2log 5 - log 3 + 2log 2 + log x = 2
log 5^2 - log 3 + log 2^2 + log x = 2 [Using $n\log_a m = \log_a m^n$]
log 25 - log 3 + log 4 + log x = 2
log (25 × 4 × x)/3 = 2 [Using $\log_a m + \log_a n = \log_a mn$ & $\log_a m - \log_a n = \log_a m/n$]
log 100x/3 = 2
On removing logarithm,
100x/3 = 10^2
100x/3 = 100
Dividing by 100 on both sides, we have
x/3 = 1
Hence, x = 3

Exercise 8(C)

1. If $\log_{10} 8 = 0.90$; find the value of:
(i) $\log_{10} 4$
(ii) $\log \sqrt{32}$
(iii) log 0.125
Solution:

Given, $\log_{10} 8 = 0.90$
$\log_{10} (2 \times 2 \times 2) = 0.90$
$\log_{10} 2^3 = 0.90$
$3 \log_{10} 2 = 0.90$
$\log_{10} 2 = 0.90/3$
$\log_{10} 2 = 0.30$ … (1)

(i) $\log 4 = \log_{10} (2 \times 2)$
$= \log_{10} 2^2$
$= 2 \log_{10} 2$
$= 2 \times 0.60$ … [From (1)]
$= 1.20$

(ii) $\log \sqrt{32} = \log_{10} 32^{1/2}$
$= ½ \log_{10} (2 \times 2 \times 2 \times 2 \times 2)$
$= ½ \log_{10} 2^5$
$= ½ \times 5 \log_{10} 2$
$= ½ \times 5 \times 0.30$ … [From (1)]
$= 0.75$

(iii) $\log 0.125 = \log 125/1000$
$= \log_{10} 1/8$
$= \log_{10} 1/2^3$
$= \log_{10} 2^{-3}$
$= -3 \log_{10} 2$
$= -3 \times 0.30$ … [From (1)]
$= -0.90$

2. If log 27 = 1.431, find the value of:
(i) log 9 (ii) log 300
Solution:

Given, log 27 = 1.431
So, $\log 3^3 = 1.431$
$3\log 3 = 1.431$
$\log 3 = 1.431/3$
$= 0.477$ … (1)

(i) $\log 9 = \log 3^2$

= 2log 3
= 2 x 0.477 … [From (1)]
= 0.954

(ii) log 300 = log (3 x 100)
= log 3 + log 100
= log 3 + log 10^2
= log 3 + 2log 10
= log 3 + 2 [As log 10 = 1]
= 0.477 + 2
= 2.477

3. If $\log_{10}$ a = b, find $10^{3b - 2}$ in terms of a.
Solution:

Given, $\log_{10}$ a = b
Now,
Let $10^{3b - 2}$ = x
Applying log on both sides,
log $10^{3b - 2}$ = log x
(3b – 2)log 10 = log x
3b – 2 = log x
3$\log_{10}$ a – 2 = log x
3$\log_{10}$ a – 2log 10 = log x
$\log_{10}$ a^3 – log 10^2 = log x
$\log_{10}$ a^3 – log 100 = log x
$\log_{10}$ a^3/100 = log x
On removing logarithm, we get
a^3/100 = x
Hence, $10^{3b - 2}$ = a^3/100

4. If $\log_5$ x = y, find $5^{2y+ 3}$ in terms of x.
Solution:

Given, $\log_5$ x = y
So, 5^y = x
Squaring on both sides, we get
$(5^y)^2 = x^2$
$5^{2y} = x^2$
$5^{2y} \times 5^3 = x^2 \times 5^3$
Hence,
$5^{2y+3} = 125x^2$

5. Given: $\log_3$ m = x and $\log_3$ n = y.
(i) Express $3^{2x - 3}$ in terms of m.
(ii) Write down $3^{1 - 2y + 3x}$ in terms of m and n.

(iii) If 2 $\log_3 A = 5x - 3y$; find A in terms of m and n.

Solution:

Given, $\log_3 m = x$ and $\log_3 n = y$
So, $3^x = m$ and $3^y = n$ … (1)
(i) Taking the given expression, $3^{2x - 3}$
$3^{2x - 3} = 3^{2x} . 3^{-3}$
$= 3^{2x} . 1/3^3$
$= (3^x)^2/3^3$
$= m^2/3^3$ … [Using (1)]
$= m^2/27$
Hence, $3^{2x - 3} = m^2/27$

(ii) Taking the given expression, $3^{1 - 2y + 3x}$
$3^{1 - 2y + 3x} = 3^1 . 3^{-2y} . 3^{3x}$
$= 3 . (3^y)^{-2} . (3^x)^3$
$= 3 . n^{-2} . m^3$ … [Using (1)]
$= 3m^3/n^2$
Hence, $3^{1 - 2y + 3x} = 3m^3/n^2$

(iii) Taking the given equation,
$2 \log_3 A = 5x - 3y$
$\log_3 A^2 = 5x - 3y$
$\log_3 A^2 = 5\log_3 m - 3\log_3 n$ … [Using (1)]
$\log_3 A^2 = \log_3 m^5 - \log_3 n^3$
$\log_3 A^2 = \log_3 m^5/n^3$
Removing logarithm on both sides, we get
$A^2 = m^5/n^3$
Hence, by taking square root on both sides
$A = \sqrt{(m^5/n^3)} = m^{5/2}/n^{3/2}$

6. Simplify:
(i) $\log (a)^3 - \log a$
(ii) $\log (a)^3 \div \log a$
Solution:

(i) We have, $\log (a)^3 - \log a$
$= 3\log a - \log a$
$= 2\log a$

(ii) We have, $\log (a)^3 \div \log a$
$= 3\log a/ \log a$
$= 3$

7. If $\log (a + b) = \log a + \log b$, find a in terms of b.

Solution:

We have, log (a + b) = log a + log b
Then,
log (a + b) = log ab
So, on removing logarithm, we have
a + b = ab
a – ab = -b
a(1 - b) = -b
a = -b/(1 - b)
Hence,
a = b/(b - 1)

8. Prove that:
(i) $(\log a)^2 - (\log b)^2 = \log (a/b). \log (ab)$
(ii) If a log b + b log a - 1 = 0, then $b^a. a^b = 10$
Solution:

(i) Taking L.H.S. we have,
= $(\log a)^2 - (\log b)^2$
= (log a + log b) (log a - log b) [As $x^2 - y^2 = (x + y)(x - y)$]
= (log ab). (log a/b)
= R.H.S.
- Hence proved

(ii) We have, a log b + b log a - 1 = 0
So,
$\log b^a + \log a^b - 1 = 0$
$\log b^a + \log a^b = 1$
$\log b^a a^b = 1$
On removing logarithm, we get
$b^a a^b = 10$
- Hence proved

9. (i) If log (a + 1) = log (4a - 3) - log 3; find a.
(ii) If 2 log y - log x - 3 = 0, express x in terms of y.
(iii) Prove that: $\log_{10} 125 = 3(1 - \log_{10} 2)$.
Solution:

(i) Given, log (a + 1) = log (4a - 3) - log 3
So,
log (a + 1) = log (4a - 3)/3
On removing logarithm on both sides, we have
a + 1 = (4a - 3)/3
3(a + 1) = 4a – 3

$3a + 3 = 4a – 3$
Hence, $a = 6$

(ii) Given, $2\log y - \log x - 3 = 0$
So,
$\log y^2 – \log x = 3$
$\log y^2/x = 3$
On removing logarithm, we have
$y^2/x = 10^3 = 1000$
Hence, $x = y^2/1000$

(iii) Considering the L.H.S., we have
$\log_{10} 125 = \log_{10} (5 \times 5 \times 5)$
$= \log_{10} 5^3$
$= 3\log_{10} 5$
$= 3\log_{10} 10/2$
$= 3(\log_{10} 10 - \log_{10} 2)$
$= 3(1 - \log_{10} 2)$ [Since, $\log_{10} 10 = 1$]
= R.H.S.
- Hence proved

10. Given $\log x = 2m – n$, $\log y = n – 2m$ and $\log z = 3m – 2n$. Find in terms of m and n, the value of $\log x^2y^3/z^4$.
Solution:

We have, $\log x = 2m – n$, $\log y = n – 2m$ and $\log z = 3m – 2n$
Now, considering
$\log x^2y^3/z^4 = \log x^2y^3 – \log z^4$
$= (\log x^2 + \log y^3) – \log z^4$
$= 2\log x + 3\log y – 4\log z$
$= 2(2m - n) + 3(n – 2m) – 4(3m – 2n)$
$= 4m – 2n + 3n – 6m – 12m + 8n$
$= -14m + 9n$

11. Given $\log_x 25 – \log_x 5 = 2 – \log_x 1/125$; find x.
Solution:

We have, $\log_x 25 – \log_x 5 = 2 – \log_x 1/125$
$\log_x (5 \times 5) – \log_x 5 = 2 – \log_x 1/(5 \times 5 \times 5)$
$\log_x 5^2 – \log_x 5 = 2 – \log_x 1/5^3$
$2\log_x 5 – \log_x 5 = 2 – \log_x 1/5^3$
$\log_x 5 = 2 – 3\log_x 1/5$
$\log_x 5 = 2 + 3\log_x (1/5)^{-1}$
$\log_x 5 = 2 + 3\log_x 5$
$2 = \log_x 5 - 3\log_x 5$
$2 = -2\log_x 5$

$-1 = \log_x 5$
Removing logarithm, we get
$x^{-1} = 5$
Hence, $x = 1/5$

Exercise 8(D)

1. If 3/2 log a + 2/3 log b - 1 = 0, find the value of $a^9.b^4$
Solution:

Given equation,
3/2 log a + 2/3 log b - 1 = 0
log $a^{3/2}$ + log $b^{2/3}$ - 1 = 0
log $a^{3/2} \times b^{2/3}$ - 1 = 0
log $a^{3/2}.b^{2/3}$ = 1
Removing logarithm, we have
$a^{3/2}.b^{2/3} = 10$
On manipulating,
$(a^{3/2}.b^{2/3})^6 = 10^6$
Hence,
$a^9.b^4 = 10^6$

2. If x = 1 + log 2 - log 5, y = 2log3 and z = log a - log 5; find the value of a if x + y = 2z.
Solution:

Given, x = 1 + log 2 - log 5, y = 2log3 and z = log a - log 5
Now, considering the given equation x + y = 2z
(1 + log 2 – log 5) + (2log 3) = 2(log a - log 5)
1 + log 2 – log 5 + 2log 3 = 2log a – 2log 5
1 + log 2 – log 5 + 2log 3 + 2log 5 = 2log a
log 10 + log 2 + log 3^2 + log 5 = log a^2
log 10 + log 2 + log 9 + log 5 = log a^2
log (10 x 2 x 9 x 5) = log a^2
log 900 = log a^2
On removing logarithm on both sides, we have
$900 = a^2$
Taking square root, we get
a = ±30
Since, a cannot be a negative value,
Hence, a = 30

3. If x = log 0.6; y = log 1.25 and z = log 3 - 2log 2, find the values of:
(i) x + y - z (ii) $5^{x + y - z}$
Solution:

Given,
x = log 0.6, y = log 1.25 and z = log 3 - 2log 2
So, z = log 3 - log 2^2
= log 3 – log 4
= log ¾
= log 0.75 … (1)

(i) Considering,
$x + y - z = \log 0.6 + \log 1.25 – \log 0.75$ … [From (1)]
$= \log (0.6 \times 1.25)/0.75$
$= \log 0.75/0.75$
$= \log 1$
$= 0$ … (2)

(ii) Now, considering
$5^{x+y-z} = 5^0$ … [From (2)]
$= 1$

4. If $a^2 = \log x$, $b^3 = \log y$ and $3a^2 - 2b^3 = 6 \log z$, express y in terms of x and z.
Solution:

We have, $a^2 = \log x$ and $b^3 = \log y$
Now, considering the equation
$3a^2 – 2b^3 = 6\log z$
$3\log x – 2\log y = 6\log z$
$\log x^3 – \log y^2 = \log z^6$
$\log x^3/y^2 = \log z^6$
On removing logarithm on both sides, we get
$x^3/y^2 = z^6$
So,
$y^2 = x^3/z^6$
Taking square root on both sides, we get
$y = \sqrt{(x^3/z^6)}$
Hence, $y = x^{3/2}/z^3$

5. If $\log (a - b)/2 = ½ (\log a + \log b)$, show that: $a^2 + b^2 = 6ab$.
Solution:

We have, $\log (a - b)/2 = ½ (\log a + \log b)$
$\log (a - b)/2 = ½ \log a + ½ \log b$
$= \log a^{1/2} + \log b^{1/2}$
$= \log \sqrt{a} + \log \sqrt{b}$
$= \log \sqrt{(ab)}$
Now, removing logarithm on both sides, we get
$(a - b)/2 = \sqrt{(ab)}$
Squaring on both sides, we get
$[(a - b)/2]^2 = [\sqrt{(ab)}]^2$
$(a - b)^2/4 = ab$
$(a – b)^2 = 4ab$
$a^2 + b^2 – 2ab = 4ab$
$a^2 + b^2 = 4ab + 2ab$
$a^2 + b^2 = 6ab$
- Hence proved

6. If $a^2 + b^2 = 23ab$, show that: log (a + b)/5 = ½ (log a + log b).
Solution:

Given, $a^2 + b^2 = 23ab$
Adding 2ab on both sides,
$a^2 + b^2 + 2ab = 23ab + 2ab$
$(a + b)^2 = 25ab$
$(a + b)^2/25 = ab$
$[(a + b)/5]^2 = ab$
Taking logarithm on both sides, we have
$\log [(a + b)/5]^2 = \log ab$
$2\log (a + b)/5 = \log ab$
$2\log (a + b)/5 = \log a + \log b$
Thus,
$\log (a + b)/5 = ½ (\log a + \log b)$

7. If m = log 20 and n = log 25, find the value of x, so that: 2log (x - 4) = 2m - n.
Solution:

Given, m = log 20 and n = log 25
Now, considering the given expression
$2\log (x - 4) = 2m - n$
$2\log (x - 4) = 2\log 20 – \log 25$
$\log (x - 4)^2 = \log 20^2 – \log 25$
$\log (x - 4)^2 = \log 400 – \log 25$
$\log (x - 4)^2 = \log 400/25$
Removing logarithm on both sides,
$(x - 4)^2 = 400/25$
$x^2 – 8x + 16 = 16$
$x^2 – 8x = 0$
$x(x - 8) = 0$
So,
x = 0 or x = 8
If x = 0, then log (x - 4) doesn't exist
Hence, x = 8

8. Solve for x and y; if x > 0 and y > 0; log xy = log x/y + 2log 2 = 2.
Solution:

We have,
$\log xy = \log x/y + 2\log 2 = 2$
Considering the equation,
$\log xy = 2$
$\log xy = 2\log 10$
$\log xy = \log 10^2$

log xy = log 100
On removing logarithm,
xy = 100 … (1)

Now, consider the equation
log x/y + 2log 2 = 2
log x/y + log 2^2 = 2
log x/y + log 4 = 2
log 4x/y = 2
Removing logarithm, we get
4x/y = 10^2
4x/y = 100
x/y = 25
(xy)/y^2 = 25
100/y^2 = 25 … [From (1)]
y^2 = 100/25
y^2 = 4
y = 2 [Since, y > 0]

From log xy = 2
Substituting the value of y, we get
log 2x = 2
On removing logarithm,
2x = 10^2
2x = 100
x = 100/2
x = 50
Thus, the values x and y are 50 and 2 respectively

9. Find x, if:
(i) $\log_x$ 625 = -4
(ii) $\log_x$ (5x - 6) = 2
Solution:

(i) We have, $\log_x$ 625 = -4
On removing logarithm,
x^{-4} = 625
$(1/x)^4 = 5^4$
Taking the fourth root on both sides,
1/x = 5
Hence, x = 1/5

(ii) We have, $\log_x$ (5x - 6) = 2
On removing logarithm,
x^2 = 5x – 6
x^2 – 5x + 6 = 0

$x^2 - 3x - 2x + 6 = 0$
$x(x - 3) - 2(x - 2) = 0$
$(x - 2)(x - 3) = 0$
Hence,
$x = 2$ or 3

10. If p = log 20 and q = log 25, find the value of x, if 2log (x + 1) = 2p – q.
Solution:

Given, p = log 20 and q = log 25
Considering the equation,
$2\log (x + 1) = 2p - q$
$\log (x + 1)^2 = 2p - q$
$\log (x + 1)^2 = 2\log 20 - \log 25$
$\log (x + 1)^2 = \log 20^2 - \log 25$
$\log (x + 1)^2 = \log 400 - \log 25$
$\log (x + 1)^2 = \log 400/25$
Removing logarithm on both sides, we have
$(x + 1)^2 = 400/25 = 16$
$(x + 1)^2 = (4)^2$
Taking square root on both sides, we have
$x + 1 = 4$
$x = 4 - 1$
Hence, $x = 3$

11. If $\log_2 (x + y) = \log_3 (x - y) = \log 25/\log 0.2$, find the value of x and y.
Solution:

Considering the relation, $\log_2 (x + y) = \log 25/\log 0.2$
$\log_2 (x + y) = \log_{0.2} 25$
$= \log_{2/10} 5^2$
$= 2\log_{1/5} 5$
$= 2\log_{5^{-1}} 5$
$= -2\log_5 5$
$= -2 \times 1$
$= -2$
So, we have
$\log_2 (x + y) = -2$
Removing logarithm, we get
$x + y = 2^{-2}$
$x + y = 1/2^2$
$x + y = ¼$ … (i)

Now, considering the relation $\log_3 (x - y) = \log 25/\log 0.2$
$\log_3 (x - y) = = \log_{0.2} 25$
$= \log_{2/10} 5^2$

$= 2\log_{1/5} 5$
$= 2\log_{5^{-1}} 5$
$= -2\log_5 5$
$= -2 \times 1$
$= -2$
So, we have
$\log_3 (x - y) = -2$
Removing logarithm, we get
$x – y = 3^{-2}$
$x – y = 1/3^2$
$x – y = 1/9$ … (ii)

On adding (i) and (ii), we get
$x + y = 1/4$
$x – y = 1/9$

$2x = 1/4 + 1/9$
$2x = (9 + 4)/36$
$2x = 13/36$
$x = 13/(36 \times 2)$
$= 13/72$

Now, substituting the value of x in (i), we get
$13/72 + y = 1/4$
$y = 1/4 - 13/72$
$= (18 - 13)/72$
$= 5/72$

Hence, the values of x and are is 13/72 and 5/72 respectively

12. Given: log x/log y = 3/2 and log xy = 5; find the values of x and y.
Solution:

Given, $\log x/\log y = 3/2$ … (i) and $\log xy = 5$ … (ii)
So,
$\log xy = \log x + \log y = 5$
And, we have $\log y = (2\log x)/3$ … [From (i)]
Now,
$\log x + (2\log x)/3 = 5$
$3\log x + 2\log x = 5 \times 3$
$5\log x = 15$
$\log x = 15/5$
$\log x = 3$
Removing logarithm, we get
$x = 10^3 = 1000$

Substituting value of x in (ii), we get
log xy = 5
Removing logarithm, we get
$xy = 10^5$
$(10^3). y = 10^5$
$y = 10^5/10^3$
$y = 10^2$
y = 100

13. Given $\log_{10} x = 2a$ and $\log_{10} y = b/2$
(i) Write 10^a in terms of x
(ii) Write $10^{2b + 1}$ in terms of y
(iii) If $\log_{10} p = 3a – 2b$, express p in terms of x and y.
Solution:

Given, $\log_{10} x = 2a$ and $\log_{10} y = b/2$
(i) Taking $\log_{10} x = 2a$
Removing logarithm on both sides,
$x = 10^{2a}$
Taking square root on both sides, we get
$x^{1/2} = 10^{2a/2}$
Hence, $10^a = x^{1/2}$

(ii) Taking $\log_{10} y = b/2$
Removing logarithm on both sides,
$y = 10^{b/2}$
On manipulating,
$y^4 = 10^{b/2 \text{ x } 4}$
$y^4 = 10^{2b}$
$10y^4 = 10^{2b}$ x 10
Hence, $10^{2b + 1} = 10y^4$

(iii) We have, $10^a = x^{1/2}$
and $y = 10^{b/2}$
Considering the equation, $\log_{10} p = 3a – 2b$
$\log_{10} p = 3a – 2b$
Removing logarithm, we get
$p = 10^{3a – 2b}$
$p = 10^{3a}/10^{2b}$
$p = (10^a)^3/(10^{b/2})^4$
$p = (x^{1/2})^3/(y)^4$
Hence, $p = x^{3/2}/y^4$

14. Solve:
$\log_5 (x + 1) - 1 = 1 + \log_5 (x - 1)$.
Solution:

Considering the given equation,
$\log_5 (x + 1) - 1 = 1 + \log_5 (x - 1)$
$\log_5 (x + 1) - \log_5 (x - 1) = 1 + 1$
$\log_5 (x + 1)/(x - 1) = 2$
Removing logarithm, we have
$(x + 1)/(x - 1) = 5^2$
$(x + 1)/(x - 1) = 25$
$(x + 1) = 25(x - 1)$
$x + 1 = 25x – 25$
$25x – x = 25 + 1$
$24x = 26$
$x = 26/24$
Hence, $x = 13/12$

15. Solve for x, if:
$\log_x 49 – \log_x 7 + \log_x 1/343 + 2 = 0$
Solution:

We have,
$\log_x 49 – \log_x 7 + \log_x 1/343 + 2 = 0$
$\log_x 49/(7 \times 343) + 2 = 0$
$\log_x 1/49 = -2$
$\log_x 1/7^2 = -2$
$\log_x 7^{-2} = -2$
$-2\log_x 7 = -2$
So,
$\log_x 7 = 1$
Removing logarithm, we get
$x = 7$

16. If $a^2 = \log x$, $b^3 = \log y$ and $a^2/2 – b^3/3 = \log c$, find c in terms of x and y.
Solution:

Given,
$a^2 = \log x$, $b^3 = \log y$
Considering the given equation,
$a^2/2 – b^3/3 = \log c$
$(\log x)/2 – (\log y)/3 = \log c$
$½ \log x – 1/3 \log y = \log c$
$\log x^{1/2} – \log y^{1/3} = \log c$
$\log x^{1/2}/y^{1/3} = \log c$
On removing logarithm, we get
$x^{1/2}/y^{1/3} = c$
Hence, $c = x^{1/2}/y^{1/3}$ is the required relation

17. Given: $x = \log_{10} 12$, $y = \log_4 2 \times \log_{10} 9$ and $z = \log_{10} 0.4$, find
(i) x – y – z
(ii) $13^{x - y - z}$
Solution:

(i) Considering, x – y – z
$= \log_{10} 12 - (\log_4 2 \times \log_{10} 9) - \log_{10} 0.4$
$= \log_{10} 12 - (\log_4 2 \times \log_{10} 9) - \log_{10} 0.4$
$= \log_{10} (4 \times 3) - (\log_{10} 2/ \log_{10} 4 \times \log_{10} 9) - \log_{10} 0.4$
$= \log_{10} 4 + \log_{10} 3 - (\log_{10} 2 \times \log_{10} 3^2)/ \log_{10} 2^2 - \log_{10} 4/10$
$= \log_{10} 4 + \log_{10} 3 - (\log_{10} 2 \times 2\log_{10} 3)/ 2\log_{10} 2 - (\log_{10} 4 - \log_{10} 10)$
$= \log_{10} 4 + \log_{10} 3 - \log_{10} 3 - \log_{10} 4 + \log_{10} 10$
$= \log_{10} 4 + \log_{10} 3 - \log_{10} 3 - \log_{10} 4 + 1$
$= 1$

(ii) Now,
$13^{x - y - z} = 13^1 = 13$

18. Solve for x, $\log_x 15\sqrt{5} = 2 - \log_x 3\sqrt{5}$
Solution:

Considering the given equation,
$\log_x 15\sqrt{5} = 2 - \log_x 3\sqrt{5}$
$\log_x 15\sqrt{5} + \log_x 3\sqrt{5} = 2$
$\log_x (15\sqrt{5} \times 3\sqrt{5}) = 2$
$\log_x (45 \times 5) = 2$
$\log_x 225 = 2$
Removing logarithm, we get
$x^2 = 225$
Taking square root on both sides,
$x = 15$

19. Evaluate:
(i) $\log_b a \times \log_c b \times \log_a c$
(ii) $\log_3 8 \div \log_9 16$
(iii) $\log_5 8/(\log_{25} 16 \times \log_{100} 10)$
Solution:

Using $\log_b a = 1/\log_a b$ and $\log_x a/\log_x b = \log_b a$, we have

(i) $\log_b a \times \log_c b \times \log_a c$
$$= \frac{\log_{10}a}{\log_{10}b} \times \frac{\log_{10}b}{\log_{10}c} \times \frac{\log_{10}c}{\log_{10}a}$$
$= 1$

(ii) $\log_3 8 \div \log_9 16$

$= \log_3 8 / \log_9 16$

$$= \frac{\log_{10}8}{\log_{10}3} \times \frac{\log_{10}9}{\log_{10}16}$$

$$= \frac{3\log_{10}2}{\log_{10}3} \times \frac{2\log_{10}3}{4\log_{10}2}$$

= 3 x ½

= 3/2

(iii) $\log_5 8/(\log_{25} 16 \times \log_{100} 10)$

$$= \frac{\frac{\log_{10}8}{\log_{10}5}}{\frac{\log_{10}16}{\log_{10}25} \times \frac{\log_{10}10}{\log_{10}100}}$$

$$= \frac{\frac{\log_{10}2^3}{\log_{10}5}}{\frac{\log_{10}2^4}{\log_{10}5^2} \times \frac{\log_{10}10}{\log_{10}10^2}}$$

$$= \frac{\log_{10}2^3}{\log_{10}5} \times \frac{\log_{10}5^2}{\log_{10}2^4} \times \frac{\log_{10}10^2}{\log_{10}10}$$

$$= \frac{3\log_{10}2}{\log_{10}5} \times \frac{2\log_{10}5}{4\log_{10}2} \times \frac{2\log_{10}10}{\log_{10}10}$$

= 3 x ½ x 2

= 3

20. Show that:

$\log_a m \div \log_{ab} m = 1 + \log_a b$

Solution:

Considering the L.H.S.,

$\log_a m \div \log_{ab} m = \log_a m/\log_{ab} m$

$= \log_m ab/\log_m a$ [As $\log_b a = 1/\log_a b$]

$= \log_a ab$ [As $\log_x a/\log_x b = \log_b a$]

$= \log_a a + \log_a b$

$= 1 + \log_a b$

21. If $\log_{\sqrt{27}} x = 2\ 2/3$, find x.

Solution:

We have,

$\log_{\sqrt{27}} x = 2\ 2/3$

$\log_{\sqrt{27}} x = 8/3$

Removing logarithm, we get

$x = \sqrt{27^{8/3}}$

$= 27^{1/2 \times 8/3}$

$= 27^{4/3}$

$= 3^{3 \times 4/3}$

$= 3^4$

Hence, x = 81

22. Evaluate:

1/(log$_a$ bc + 1) + 1/(log$_b$ ca + 1) + 1/(log$_c$ ab + 1)

Solution:

We have,

$$\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1}$$

$$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$$

$$= \frac{1}{\log_a abc} + \frac{1}{\log_b aba} + \frac{1}{\log_c abc} \qquad [\because \log_a b + \log_b c = \log_a bc]$$

$$= \frac{1}{\frac{\log abc}{\log a}} + \frac{1}{\frac{\log abc}{\log b}} + \frac{1}{\frac{\log abc}{\log c}}$$

$$= \frac{\log a + \log b + \log c}{\log abc}$$

$$= \frac{\log abc}{\log abc} \qquad [\because \log_a b + \log_b c = \log_a bc]$$

$$= 1$$