

## GA - General Aptitude

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### Q1 - Q5 carry one mark each.

**Q.No. 1** He is known for his unscrupulous ways. He always sheds \_\_\_\_\_ tears to deceive people.

- (A) fox's
- (B) crocodile's
- (C) crocodile
- (D) fox

**Q.No. 2** Jofra Archer, the England fast bowler, is \_\_\_\_\_ than accurate.

- (A) more fast
- (B) faster
- (C) less fast
- (D) more faster

**Q.No. 3** Select the word that fits the analogy:

Build : Building :: Grow : \_\_\_\_\_

- (A) Grown
- (B) Grew
- (C) Growth
- (D) Growed

**Q.No. 4** I do not think you know the case well enough to have opinions. Having said that, I agree with your other point.

What does the phrase "having said that" mean in the given text?

- (A) as opposed to what I have said
- (B) despite what I have said
- (C) in addition to what I have said
- (D) contrary to what I have said

**Q.No. 5** Define  $[x]$  as the greatest integer less than or equal to  $x$ , for each  $x \in (-\infty, \infty)$ . If  $y = [x]$ , then area under  $y$  for  $x \in [1, 4]$  is \_\_\_\_\_.

- (A) 1
- (B) 3
- (C) 4
- (D) 6

### Q6 - Q10 carry two mark each.

**Q.No. 6** Crowd funding deals with mobilisation of funds for a project from a large number of people, who would be willing to invest smaller amounts through web-based platforms in the project.

Based on the above paragraph, which of the following is correct about crowd funding?

- (A) Funds raised through unwilling contributions on web-based platforms.
- (B) Funds raised through large contributions on web-based platforms.
- (C) Funds raised through coerced contributions on web-based platforms.
- (D) Funds raised through voluntary contributions on web-based platforms.

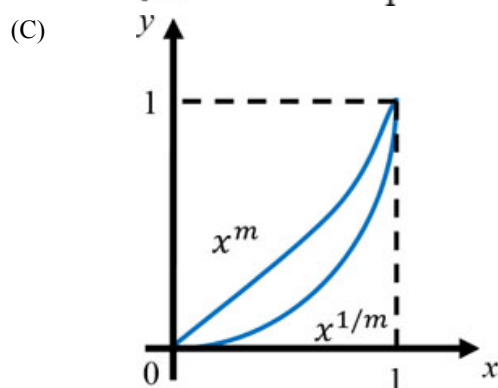
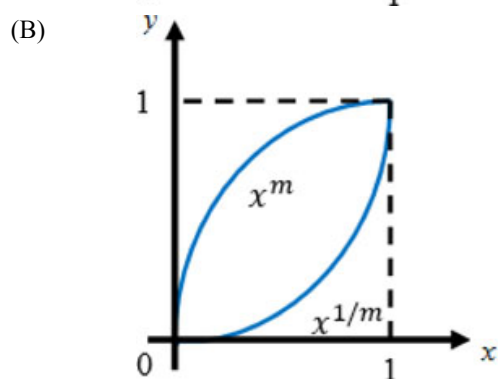
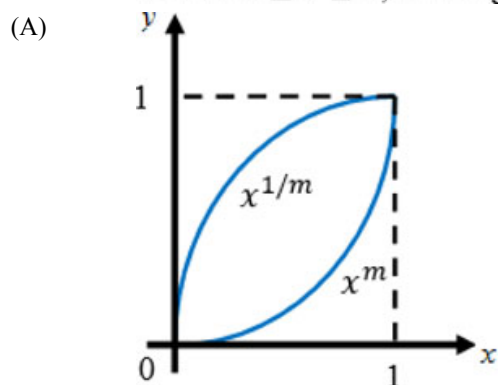
**Q.No. 7** P, Q, R and S are to be uniquely coded using  $\alpha$  and  $\beta$ . If P is coded as  $\alpha\alpha$  and Q as  $\alpha\beta$ , then R and S, respectively, can be coded as \_\_\_\_\_.

- (A)  $\beta\alpha$  and  $\alpha\beta$
- (B)  $\beta\beta$  and  $\alpha\alpha$
- (C)  $\alpha\beta$  and  $\beta\beta$
- (D)  $\beta\alpha$  and  $\beta\beta$

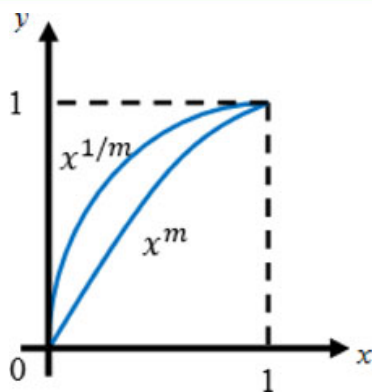
**Q.No. 8** The sum of the first  $n$  terms in the sequence 8, 88, 888, 8888, ... is \_\_\_\_\_.

- (A)  $\frac{81}{80}(10^n - 1) + \frac{9}{8}n$
- (B)  $\frac{81}{80}(10^n - 1) - \frac{9}{8}n$
- (C)  $\frac{80}{81}(10^n - 1) + \frac{8}{9}n$
- (D)  $\frac{80}{81}(10^n - 1) - \frac{8}{9}n$

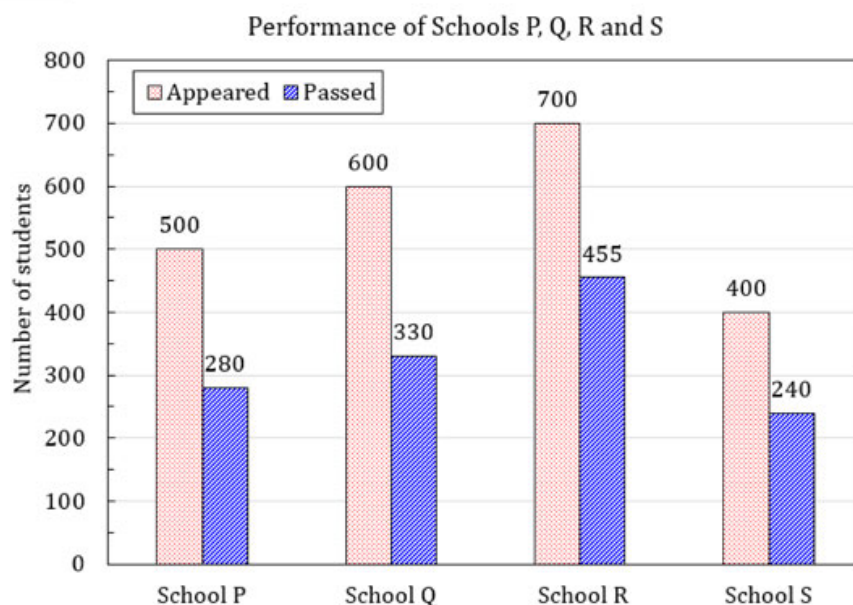
**Q.No. 9** Select the graph that schematically represents BOTH  $y = x^m$  and  $y = x^{1/m}$  properly in the interval  $0 \leq x \leq 1$ , for integer values of  $m$ , where  $m > 1$ .



(D)



- Q.No. 10 The bar graph shows the data of the students who appeared and passed in an examination for four schools P, Q, R and S. The average of success rates (in percentage) of these four schools is \_\_\_\_\_.



- (A) 58.5 %  
 (B) 58.8 %  
 (C) 59.0 %  
 (D) 59.3 %

**PH: Physics**

- Q.No. 1 Which one of the following is a solution of  $\frac{d^2u(x)}{dx^2} = k^2u(x)$ , for  $k$  real?

- (A)  $e^{-kx}$   
 (B)  $\sin kx$   
 (C)  $\cos kx$   
 (D)  $\sinh x$

- Q.No. 2 A real, invertible  $3 \times 3$  matrix  $M$  has eigenvalues  $\lambda_i, (i=1,2,3)$  and the corresponding eigenvectors are  $|e_i\rangle, (i=1,2,3)$  respectively. Which one of the following is correct?

- (A)  $M|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$ , for  $i=1,2,3$   
 (B)  $M^{-1}|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$ , for  $i=1,2,3$

- (C)  $M^{-1}|e_i\rangle = \lambda_i|e_i\rangle$ , for  $i=1,2,3$
- (D) The eigenvalues of  $M$  and  $M^{-1}$  are not related.

**Q.No. 3** A quantum particle is subjected to the potential

$$V(x) = \begin{cases} \infty, & x \leq -\frac{a}{2} \\ 0, & -\frac{a}{2} < x < \frac{a}{2} \\ \infty, & x \geq \frac{a}{2} \end{cases}$$

The ground state wave function of the particle is proportional to

- (A)  $\sin\left(\frac{\pi x}{2a}\right)$
- (B)  $\sin\left(\frac{\pi x}{a}\right)$
- (C)  $\cos\left(\frac{\pi x}{2a}\right)$
- (D)  $\cos\left(\frac{\pi x}{a}\right)$

**Q.No. 4** Let  $\hat{a}$  and  $\hat{a}^\dagger$ , respectively denote the lowering and raising operators of a one-dimensional simple harmonic oscillator. Let  $|n\rangle$  be the energy eigenstate of the simple harmonic oscillator. Given that  $|n\rangle$  is also an eigenstate of  $\hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a}$ , the corresponding eigenvalue is

- (A)  $n(n-1)$
- (B)  $n(n+1)$
- (C)  $(n+1)^2$
- (D)  $n^2$

**Q.No. 5** Which one of the following is a universal logic gate?

- (A) AND
- (B) NOT
- (C) OR
- (D) NAND

**Q.No. 6** Which one of the following is the correct binary equivalent of the hexadecimal F6C?

- (A) 0110 1111 1100
- (B) 1111 0110 1100
- (C) 1100 0110 1111
- (D) 0110 1100 0111

**Q.No. 7** The total angular momentum  $j$  of the ground state of the  $^{17}_8\text{O}$  nucleus is

- (A)  $\frac{1}{2}$
- (B) 1

- (C)  $\frac{3}{2}$   
 (D)  $\frac{5}{2}$

**Q.No. 8** A particle  $X$  is produced in the process  $\pi^+ + p \rightarrow K^+ + X$  via the strong interaction. If the quark content of the  $K^+$  is  $u\bar{s}$ , the quark content of  $X$  is

- (A)  $c\bar{s}$   
 (B)  $uud$   
 (C)  $uus$   
 (D)  $u\bar{d}$

**Q.No. 9** A medium ( $\epsilon_r > 1, \mu_r = 1, \sigma > 0$ ) is semi-transparent to an electromagnetic wave when

- (A) Conduction current  $\gg$  Displacement current  
 (B) Conduction current  $\ll$  Displacement current  
 (C) Conduction current = Displacement current  
 (D) Both Conduction current and Displacement current are zero

**Q.No. 10** A particle is moving in a central force field given by  $\vec{F} = -\frac{k}{r^3} \hat{r}$ , where  $\hat{r}$  is the unit vector pointing away from the center of the field. The potential energy of the particle is given by

- (A)  $\frac{k}{r^2}$   
 (B)  $\frac{k}{2r^2}$   
 (C)  $-\frac{k}{r^2}$   
 (D)  $-\frac{k}{2r^2}$

**Q.No. 11** Choose the correct statement related to the Fermi energy ( $E_F$ ) and the chemical potential ( $\mu$ ) of a metal.

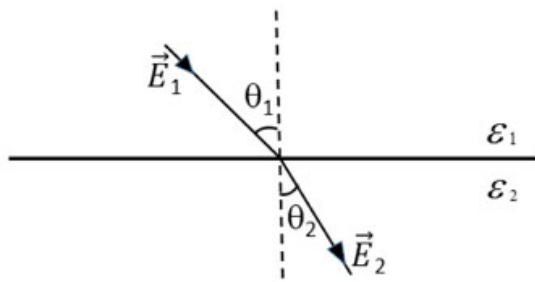
- (A)  $\mu = E_F$  only at 0 K  
 (B)  $\mu = E_F$  at finite temperature  
 (C)  $\mu < E_F$  at 0 K  
 (D)  $\mu > E_F$  at finite temperature

**Q.No. 12** Consider a diatomic molecule formed by identical atoms. If  $E_v$  and  $E_e$  represent the energy of the vibrational nuclear motion and electronic motion respectively, then in terms of the electronic mass  $m$  and nuclear mass  $M$ ,  $\frac{E_v}{E_e}$  is

- proportional to  
 (A)  $\left(\frac{m}{M}\right)^{1/2}$

- (B)  $\frac{m}{M}$
- (C)  $\left(\frac{m}{M}\right)^{3/2}$
- (D)  $\left(\frac{m}{M}\right)^2$

**Q.No. 13** Which one of the following relations determines the manner in which the electric field lines are refracted across the interface between two dielectric media having dielectric constants  $\epsilon_1$  and  $\epsilon_2$  (see figure)?



- (A)  $\epsilon_1 \sin \theta_1 = \epsilon_2 \sin \theta_2$
- (B)  $\epsilon_1 \cos \theta_1 = \epsilon_2 \cos \theta_2$
- (C)  $\epsilon_1 \tan \theta_1 = \epsilon_2 \tan \theta_2$
- (D)  $\epsilon_1 \cot \theta_1 = \epsilon_2 \cot \theta_2$

**Q.No. 14** If  $\vec{E}$  and  $\vec{B}$  are the electric and magnetic fields respectively, then  $\vec{E} \cdot \vec{B}$  is

- (A) odd under parity and even under time reversal
- (B) even under parity and odd under time reversal
- (C) odd under parity and odd under time reversal
- (D) even under parity and even under time reversal

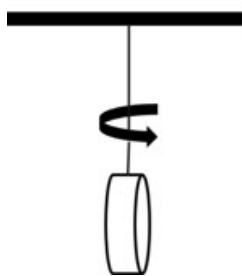
**Q.No. 15**



A small disc is suspended by a fiber such that it is free to rotate about the fiber axis (see figure). For small angular deflections, the Hamiltonian for the disc is given by

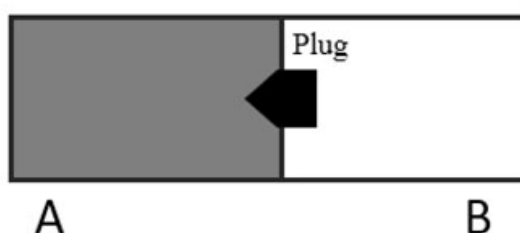
$$H = \frac{p_\theta^2}{2I} + \frac{1}{2}\alpha\theta^2,$$

where  $I$  is the moment of inertia and  $\alpha$  is the restoring torque per unit deflection. The disc is subjected to angular deflections ( $\theta$ ) due to thermal collisions from the surrounding gas at temperature  $T$  and  $p_\theta$  is the momentum conjugate to  $\theta$ . The average and the root-mean-square angular deflection,  $\theta_{avg}$  and  $\theta_{rms}$ , respectively are



- (A)  $\theta_{avg} = 0$  and  $\theta_{rms} = \left(\frac{k_B T}{\alpha}\right)^{3/2}$
- (B)  $\theta_{avg} = 0$  and  $\theta_{rms} = \left(\frac{k_B T}{\alpha}\right)^{1/2}$
- (C)  $\theta_{avg} \neq 0$  and  $\theta_{rms} = \left(\frac{k_B T}{\alpha}\right)^{1/2}$
- (D)  $\theta_{avg} \neq 0$  and  $\theta_{rms} = \left(\frac{k_B T}{\alpha}\right)^{3/2}$

**Q.No. 16** As shown in the figure, an ideal gas is confined to chamber A of an insulated container, with vacuum in chamber B. When the plug in the wall separating the chambers A and B is removed, the gas fills both the chambers. Which one of the following statements is true?



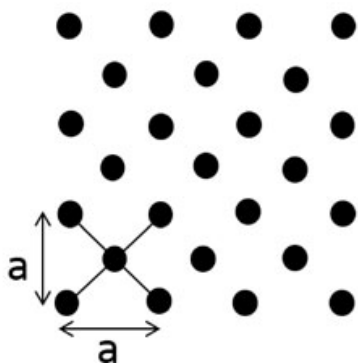
- (A) The temperature of the gas remains unchanged
- (B) Internal energy of the gas decreases
- (C) Temperature of the gas decreases as it expands to fill the space in chamber B
- (D) Internal energy of the gas increases as its atoms have more space to move around

Q.No. 17 Particle  $A$  with angular momentum  $j = \frac{3}{2}$  decays into two particles  $B$  and  $C$  with angular momenta  $j_1$  and  $j_2$ , respectively. If  $\left|\frac{3}{2}, \frac{3}{2}\right\rangle_A = \alpha \left|1, 1\right\rangle_B \otimes \left|\frac{1}{2}, \frac{1}{2}\right\rangle_C$ , the value of  $\alpha$  is \_\_\_\_\_.

Q.No. 18 Far from the Earth, the Earth's magnetic field can be approximated as due to a bar magnet of magnetic pole strength  $4 \times 10^{14}$  Am. Assume this magnetic field is generated by a current carrying loop encircling the magnetic equator. The current required to do so is about  $4 \times 10^n$  A, where  $n$  is an integer. The value of  $n$  is \_\_\_\_\_.

(Earth's circumference:  $4 \times 10^7$  m)

Q.No. 19 The number of distinct ways the primitive unit cell can be constructed for the two dimensional lattice as shown in the figure is \_\_\_\_\_.



Q.No. 20 A hydrogenic atom is subjected to a strong magnetic field. In the absence of spin-orbit coupling, the number of doubly degenerate states created out of the  $d$ -level is \_\_\_\_\_.

Q.No. 21 A particle  $Y$  undergoes strong decay  $Y \rightarrow \pi^- + \pi^-$ . The isospin of  $Y$  is \_\_\_\_\_.

Q.No. 22 For a complex variable  $z$  and the contour  $c: |z|=1$  taken in the counter clockwise direction,  $\frac{1}{2\pi i} \oint_c \left( z - \frac{2}{z} + \frac{3}{z^2} \right) dz =$  \_\_\_\_\_.

Q.No. 23 Let  $p$  be the momentum conjugate to the generalized coordinate  $q$ . If the transformation

$$Q = \sqrt{2} q^m \cos p$$

$$P = \sqrt{2} q^m \sin p$$

is canonical, then  $m =$  \_\_\_\_\_.



- Q.No. 24** A conducting sphere of radius 1 m is placed in air. The maximum number of electrons that can be put on the sphere to avoid electrical breakdown is about  $7 \times 10^n$ , where  $n$  is an integer. The value of  $n$  is \_\_\_\_\_.

Assume:

Breakdown electric field strength in air is  $|\vec{E}| = 3 \times 10^6$  V/m

Permittivity of free space  $\epsilon_0 = 8.85 \times 10^{-12}$  F/m

Electron charge  $e = 1.60 \times 10^{-19}$  C

- Q.No. 25** If a particle is moving along a sinusoidal curve, the number of degrees of freedom of the particle is \_\_\_\_.

- Q.No. 26** The product of eigenvalues of  $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$  is

- (A) -1  
(B) 1  
(C) 0  
(D) 2

- Q.No. 27** Let  $|e_1\rangle \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ ,  $|e_2\rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$  and  $|e_3\rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ . Let  $S = \{|e_1\rangle, |e_2\rangle, |e_3\rangle\}$ . Let  $\mathbb{R}^3$  denote

the three-dimensional real vector space. Which one of the following is correct?

- (A)  $S$  is an orthonormal set  
(B)  $S$  is a linearly dependent set  
(C)  $S$  is a basis for  $\mathbb{R}^3$   
(D)

$$\sum_{i=1}^3 |e_i\rangle \langle e_i| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Q.No. 28**  $\hat{S}_x$  denotes the spin operator defined as  $\hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ . Which one of the

following is correct?

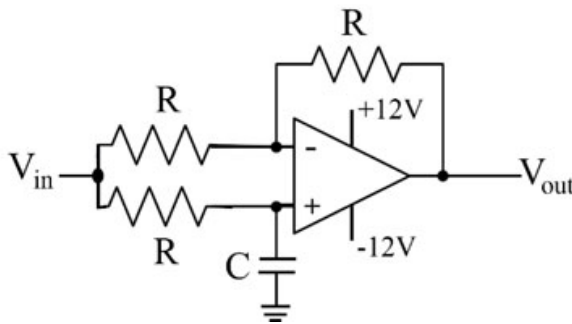
- (A) The eigenstates of spin operator  $\hat{S}_x$  are  $|\uparrow\rangle_x \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $|\downarrow\rangle_x \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$   
(B) The eigenstates of spin operator  $\hat{S}_x$  are  $|\uparrow\rangle_x \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$  and  $|\uparrow\rangle_x \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$   
(C)

In the spin state  $\frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$ , upon the measurement of  $\hat{S}_x$ , the probability for obtaining  $|\uparrow\rangle_x$  is  $\frac{1}{4}$

(D)

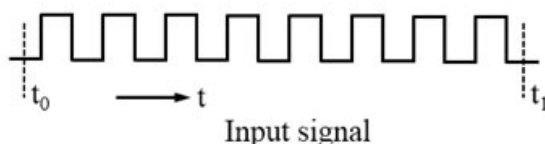
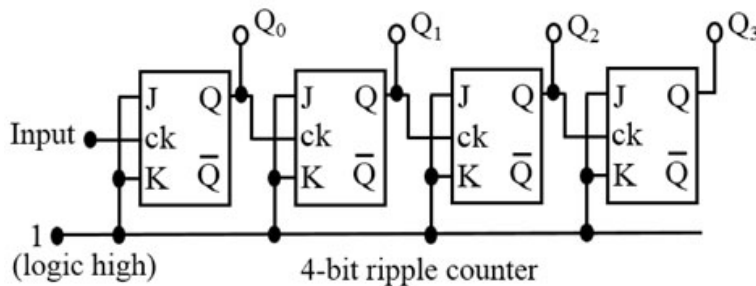
In the spin state  $\frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$ , upon the measurement of  $\hat{S}_x$ , the probability for obtaining  $|\uparrow\rangle_x$  is  $\frac{2+\sqrt{3}}{4}$ .

**Q.No. 29** The input voltage ( $V_{in}$ ) to the circuit shown in the figure is  $2\cos(100t)$  V. The output voltage ( $V_{out}$ ) is  $2\cos(100t - \frac{\pi}{2})$  V. If  $R = 1\text{ k}\Omega$ , the value of  $C$  (in  $\mu\text{F}$ ) is



- (A) 0.1
- (B) 1
- (C) 10
- (D) 100

**Q.No. 30** Consider a 4-bit counter constructed out of four flip-flops. It is formed by connecting the J and K inputs to logic high and feeding the Q output to the clock input of the following flip-flop (see the figure). The input signal to the counter is a series of square pulses and the change of state is triggered by the falling edge. At time  $t = t_0$  the outputs are in logic low state ( $Q_0 = Q_1 = Q_2 = Q_3 = 0$ ). Then at  $t = t_1$ , the logic state of the outputs is



- (A)  $Q_0 = 1, Q_1 = 0, Q_2 = 0$  and  $Q_3 = 0$

- (B)  $Q_0 = 0, Q_1 = 0, Q_2 = 0$  and  $Q_3 = 1$   
 (C)  $Q_0 = 1, Q_1 = 0, Q_2 = 1$  and  $Q_3 = 0$   
 (D)  $Q_0 = 0, Q_1 = 1, Q_2 = 1$  and  $Q_3 = 1$

**Q.No. 31**

Consider the Lagrangian  $L = a\left(\frac{dx}{dt}\right)^2 + b\left(\frac{dy}{dt}\right)^2 + cxy$ , where  $a, b$  and  $c$  are

constants. If  $p_x$  and  $p_y$  are the momenta conjugate to the coordinates  $x$  and  $y$  respectively, then the Hamiltonian is

- (A)  $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - cxy$   
 (B)  $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} - cxy$   
 (C)  $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + cxy$   
 (D)  $\frac{p_x^2}{a} + \frac{p_y^2}{b} + cxy$

**Q.No. 32** Which one of the following matrices does NOT represent a proper rotation in a plane?

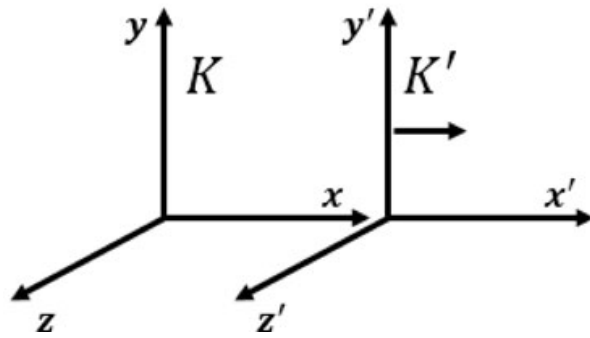
- (A)  $\begin{pmatrix} -\sin \theta & \cos \theta \\ -\cos \theta & -\sin \theta \end{pmatrix}$   
 (B)  $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$   
 (C)  $\begin{pmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$   
 (D)  $\begin{pmatrix} -\sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$

**Q.No. 33**

A uniform magnetic field  $\vec{B} = B_0 \hat{y}$  exists in an inertial frame  $K$ . A perfect conducting sphere moves with a constant velocity  $\vec{v} = v_0 \hat{x}$  with respect to this inertial frame. The rest frame of the sphere is  $K'$  (see figure). The electric and magnetic fields in  $K$  and  $K'$  are related as

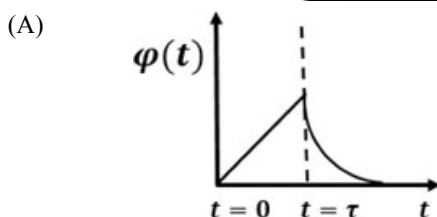
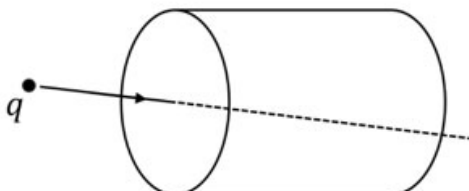
$$\left. \begin{aligned} \vec{E}'_{\parallel} &= \vec{E}_{\parallel} & \vec{E}'_{\perp} &= \gamma (\vec{E}_{\perp} + \vec{v} \times \vec{B}) \\ \vec{B}'_{\parallel} &= \vec{B}_{\parallel} & \vec{B}'_{\perp} &= \gamma \left( \vec{B}_{\perp} - \frac{\vec{v}}{c^2} \times \vec{E} \right) \end{aligned} \right\}, \quad \gamma = \frac{1}{\sqrt{1 - (v/c)^2}}.$$

The induced surface charge density on the sphere (to the lowest order in  $v/c$ ) in the frame  $K'$  is

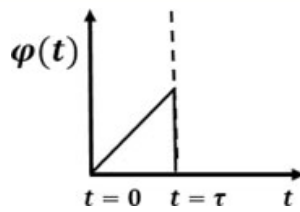


- (A) maximum along  $z'$
- (B) maximum along  $y'$
- (C) maximum along  $x'$
- (D) uniform over the sphere

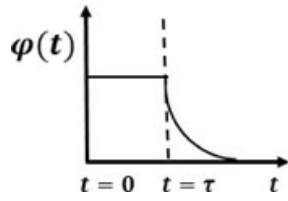
**Q.No. 34** A charge  $q$  moving with uniform speed enters a cylindrical region in free space at  $t = 0$  and exits the region at  $t = \tau$  (see figure). Which one of the following options best describes the time dependence of the total electric flux  $\phi(t)$ , through the entire surface of the cylinder?



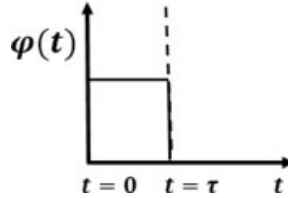
(B)



(C)



(D)



- Q.No. 35** Consider a one-dimensional non-magnetic crystal with one atom per unit cell. Assume that the valence electrons (i) do not interact with each other and (ii) interact weakly with the ions. If  $n$  is the number of valence electrons per unit cell, then at 0 K,
- (A) the crystal is metallic for any value of  $n$
  - (B) the crystal is non-metallic for any value of  $n$
  - (C) the crystal is metallic for even values of  $n$
  - (D) the crystal is metallic for odd values of  $n$

- Q.No. 36** According to the Fermi gas model of the nucleus, the nucleons move in a spherical volume of radius  $R (= R_0 A^{\frac{1}{3}}$ , where  $A$  is the mass number and  $R_0$  is an empirical constant with the dimensions of length). The Fermi energy of the nucleus  $E_F$  is proportional to

- (A)  $R_0^2$
- (B)  $\frac{1}{R_0}$
- (C)  $\frac{1}{R_0^2}$
- (D)  $\frac{1}{R_0^3}$

- Q.No. 37** Consider a two dimensional crystal with 3 atoms in the basis. The number of allowed optical branches ( $n$ ) and acoustic branches ( $m$ ) due to the lattice vibrations are
- (A)  $(n, m) = (2, 4)$
  - (B)  $(n, m) = (3, 3)$
  - (C)  $(n, m) = (4, 2)$
  - (D)  $(n, m) = (1, 5)$

**Q.No. 38**

The internal energy  $U$  of a system is given by  $U(S, V) = \lambda V^{-2/3} S^2$ , where  $\lambda$  is a constant of appropriate dimensions;  $V$  and  $S$  denote the volume and entropy, respectively. Which one of the following gives the correct equation of state of the system?

- (A)  $\frac{PV^{1/3}}{T^2} = \text{constant}$
- (B)  $\frac{PV}{T^{1/3}} = \text{constant}$
- (C)  $\frac{P}{V^{1/3}T} = \text{constant}$
- (D)  $\frac{PV^{2/3}}{T} = \text{constant}$

**Q.No. 39** The potential energy of a particle of mass  $m$  is given by

$$U(x) = a \sin(k^2 x - \pi/2), \quad a > 0, \quad k^2 > 0.$$

The angular frequency of small oscillations of the particle about  $x = 0$  is

- (A)  $k^2 \sqrt{\frac{2a}{m}}$
- (B)  $k^2 \sqrt{\frac{a}{m}}$
- (C)  $k^2 \sqrt{\frac{a}{2m}}$
- (D)  $2k^2 \sqrt{\frac{a}{m}}$

**Q.No. 40** The radial wave function of a particle in a central potential is given by

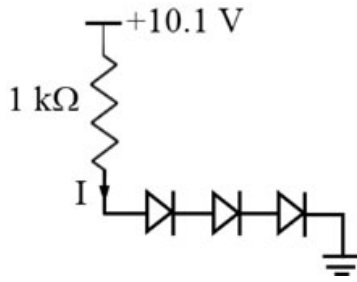
$R(r) = A \frac{r}{a} \exp\left(-\frac{r}{2a}\right)$ , where  $A$  is the normalization constant and  $a$  is positive constant of suitable dimensions. If  $\gamma a$  is the most probable distance of the particle from the force center, the value of  $\gamma$  is \_\_\_\_\_.

**Q.No. 41** A free particle of mass  $M$  is located in a three-dimensional cubic potential well with impenetrable walls. The degeneracy of the fifth excited state of the particle is \_\_\_\_\_.

**Q.No. 42**



Consider the circuit given in the figure. Let the forward voltage drop across each diode be 0.7 V. The current  $I$  (in mA) through the resistor is \_\_\_\_.



- Q.No. 43 Let  $u^\mu$  denote the 4-velocity of a relativistic particle whose square  $u^\mu u_\mu = 1$ . If  $\varepsilon_{\mu\nu\rho\sigma}$  is the Levi-Civita tensor then the value of  $\varepsilon_{\mu\nu\rho\sigma} u^\mu u^\nu u^\rho u^\sigma$  is \_\_\_\_.
- Q.No. 44 Consider a simple cubic monoatomic Bravais lattice which has a basis with vectors  $\vec{r}_1 = 0, \vec{r}_2 = \frac{a}{4}(\hat{x} + \hat{y} + \hat{z})$ ,  $a$  is the lattice parameter. The Bragg reflection is observed due to the change in the wave vector between the incident and the scattered beam as given by  $\vec{K} = n_1 \vec{G}_1 + n_2 \vec{G}_2 + n_3 \vec{G}_3$ , where  $\vec{G}_1$ ,  $\vec{G}_2$ , and  $\vec{G}_3$  are primitive reciprocal lattice vectors. For  $n_1 = 3$ ,  $n_2 = 3$  and  $n_3 = 2$ , the geometrical structure factor is \_\_\_\_.
- Q.No. 45 A plane electromagnetic wave of wavelength  $\lambda$  is incident on a circular loop of conducting wire. The loop radius is  $a$  ( $a \ll \lambda$ ). The angle (in degrees), made by the Poynting vector with the normal to the plane of the loop to generate a maximum induced electrical signal, is \_\_\_\_.
- Q.No. 46 An electron in a hydrogen atom is in the state  $n=3, l=2, m=-2$ . Let  $\hat{L}_y$  denote the  $y$ -component of the orbital angular momentum operator. If  $(\Delta \hat{L}_y)^2 = \alpha \hbar^2$ , the value of  $\alpha$  is \_\_\_\_.
- Q.No. 47 A sinusoidal voltage of the form  $V(t) = V_0 \cos(\omega t)$  is applied across a parallel plate capacitor placed in vacuum. Ignoring the edge effects, the induced *emf* within the region between the capacitor plates can be expressed as a power series in  $\omega$ . The lowest non-vanishing exponent in  $\omega$  is \_\_\_\_.
- Q.No. 48 If  $x = \sum_{k=1}^{\infty} a_k \sin kx$ , for  $-\pi \leq x \leq \pi$ , the value of  $a_2$  is \_\_\_\_.
- Q.No. 49

$$\text{Let } f_n(x) = \begin{cases} 0, & x < -\frac{1}{2n} \\ n, & -\frac{1}{2n} < x < \frac{1}{2n} \\ 0, & \frac{1}{2n} < x. \end{cases}$$

The value of  $\lim_{n \rightarrow \infty} \int_{-\infty}^{\infty} f_n(x) \sin x \, dx$  is \_\_\_\_\_.

Q.No. 50 Consider the Hamiltonian  $\hat{H} = \hat{H}_0 + \hat{H}'$  where

$$\hat{H}_0 = \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix} \text{ and } \hat{H}' \text{ is the time independent perturbation given by}$$

$$\hat{H}' = \begin{pmatrix} 0 & k & 0 \\ k & 0 & k \\ 0 & k & 0 \end{pmatrix}, \text{ where } k > 0. \text{ If, the maximum energy eigenvalue of } \hat{H} \text{ is 3 eV}$$

corresponding to  $E = 2 \text{ eV}$ , the value of  $k$  (rounded off to three decimal places) in eV is \_\_\_\_\_.

Q.No. 51 A hydrogen atom is in an orbital angular momentum state  $|l, m=l\rangle$ . If  $\vec{L}$  lies on a cone which makes a half angle  $30^\circ$  with respect to the z-axis, the value of  $l$  is \_\_\_\_\_.

Q.No. 52 In the center of mass frame, two protons each having energy 7000 GeV, collide to produce protons and anti-protons. The maximum number of anti-protons produced is \_\_\_\_\_.

(Assume the proton mass to be  $1 \text{ GeV}/c^2$ )

Q.No. 53 Consider a gas of hydrogen atoms in the atmosphere of the Sun where the temperature is 5800 K. If a sample from this atmosphere contains  $6.023 \times 10^{23}$  of hydrogen atoms in the ground state, the number of hydrogen atoms in the first excited state is approximately  $8 \times 10^n$ , where  $n$  is an integer. The value of  $n$  is \_\_\_\_\_.

(Boltzmann constant:  $8.617 \times 10^{-5} \text{ eV/K}$ )

Q.No. 54

For a gas of non-interacting particles, the probability that a particle has a speed  $v$  in the interval  $v$  to  $v + dv$  is given by

$$f(v)dv = 4\pi v^2 dv \left( \frac{m}{2\pi k_B T} \right)^{3/2} e^{-mv^2/2k_B T}$$

If  $E$  is the energy of a particle, then the maximum in the corresponding energy distribution in units of  $E/k_B T$  occurs at \_\_\_\_\_ (rounded off to one decimal place).

Q.No. 55

The Planck's energy density distribution is given by  $u(\omega) = \frac{\hbar \omega^3}{\pi^2 c^3 (e^{\hbar \omega / k_B T} - 1)}$ .

At long wavelengths, the energy density of photons in thermal equilibrium with a cavity at temperature  $T$  varies as  $T^\alpha$ , where  $\alpha$  is \_\_\_\_\_.

