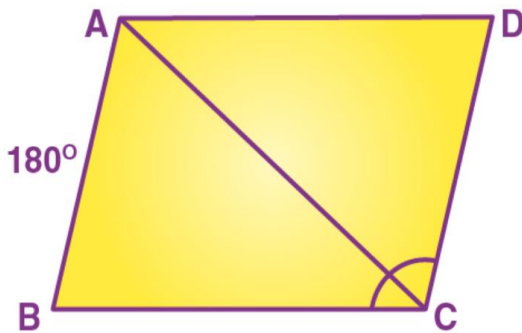


Exercise
Question 1.

In parallelogram ABCD, $\angle A = 3$ times $\angle B$. Find all the angles of the parallelogram. In the same parallelogram if $AB = 5x - 7$ and $CD = 3x + 1$; find the length of CD.

Solution:-



Let $\angle B = x$

$\angle A = 3\angle B = 3x$

$AD \parallel BC$

$\angle A + \angle B = 180^\circ$

$3x + x = 180^\circ$

$\Rightarrow 4x = 180^\circ$

$\Rightarrow x = 45^\circ$

$\angle B = 45^\circ$

$\angle A = 3x = 3 \times 45 = 135^\circ$

And $\angle B = \angle D = 45^\circ$

Opposite angles of parallelogram are equal.

$\angle A = \angle C = 135^\circ$

Opposite sides of parallelogram are equal.

$AB = CD$

$5x - 7 = 3x + 1$

$\Rightarrow 5x - 3x = 1 + 7$

$\Rightarrow 2x = 8$

$\Rightarrow x = 4$

$CD = 3 \times 4 + 1 = 13$

Hence $135^\circ, 45^\circ, 135^\circ$ and $45^\circ:13$

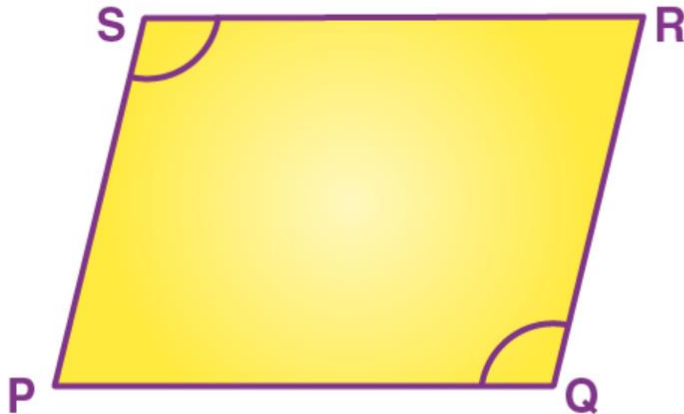
Question 2.

In parallelogram PQRS, $\angle Q = (4x - 5)^\circ$ and $\angle S = (3x + 10)^\circ$. Calculate: $\angle Q$ and $\angle R$.

Solution:-

In parallelogram PQRS,

$\angle Q = (4x - 5)^\circ$ and $\angle S = (3x + 10)^\circ$



Opposite \angle s of parallelogram are equal

$$\angle Q = \angle S$$

$$4x - 5 = 3x + 10$$

$$4x - 3x = 10 + 5$$

$$x = 15$$

$$\angle Q = 4x - 5 = 4 \times 15 - 5 = 55^\circ$$

$$\text{Also } \angle Q + \angle R = 180^\circ$$

$$55^\circ + \angle R = 180^\circ$$

$$\angle R = 180^\circ - 55^\circ = 125^\circ$$

$$\angle Q = 55^\circ; \angle R = 125^\circ$$

Question 3.

In rhombus ABCD:

(i) If $\angle A = 74^\circ$; find $\angle B$ and $\angle C$.

(ii) If $AD = 7.5\text{cm}$; find BC and CD .

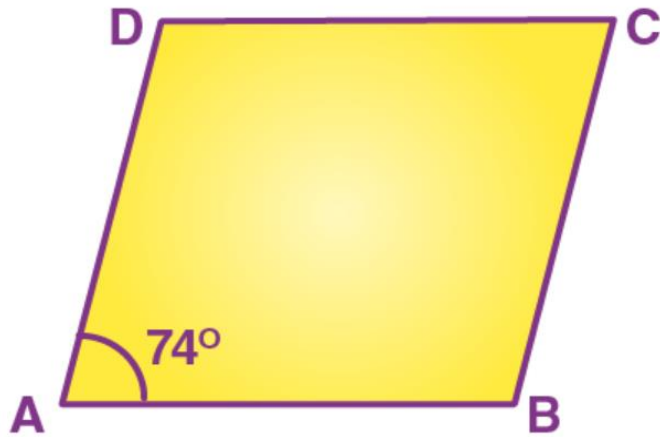
Solution:-

$AD \parallel BC$

$$\angle A + \angle B = 180^\circ$$

$$74^\circ + \angle B = 180^\circ$$

$$\angle B = 180^\circ - 74^\circ = 106^\circ$$



Opposite angles of Rhombus are equal.

$$\angle A = \angle C = 74^\circ$$

Sides of Rhombus are equal.

$$BC = CD = AD = 7.5 \text{ cm}$$

(i) $\angle B = 106^\circ$; $\angle C = 74^\circ$

(ii) Ans: $BC = 7.5 \text{ cm}$ and $CD = 7.5 \text{ cm}$

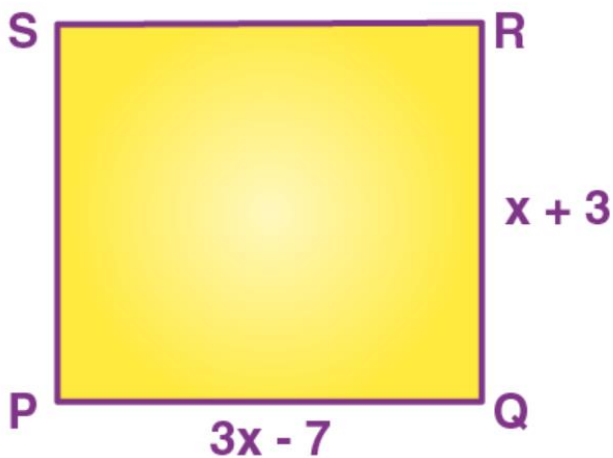
Question 4

In square PQRS:

(i) If $PQ = 3x - 7$ and $QR = x + 3$; find PS

Solution:-

(i) Sides of square are equal.



$$PQ = QR$$

$$3x - 7 = x + 3$$

$$3x - x = 3 + 7$$

$$2x = 10$$

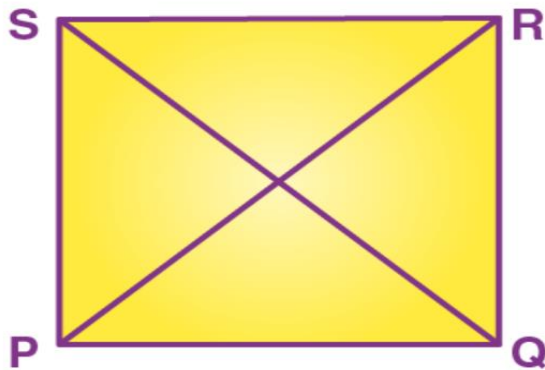
$$x = 5$$

$$PS = PQ = 3x - 7 = 3 \times 5 - 7 = 8$$

(ii) If $PR = 5x$ and $QR = 9x - 8$. Find OS

Solution:-

(ii) $PR = 5x$ and $QR = 9x - 8$.



$$PR = QS$$

$$5x = 9x - 8$$

$$\Rightarrow 5x - 9x = -8$$

$$\Rightarrow -4x = -8$$

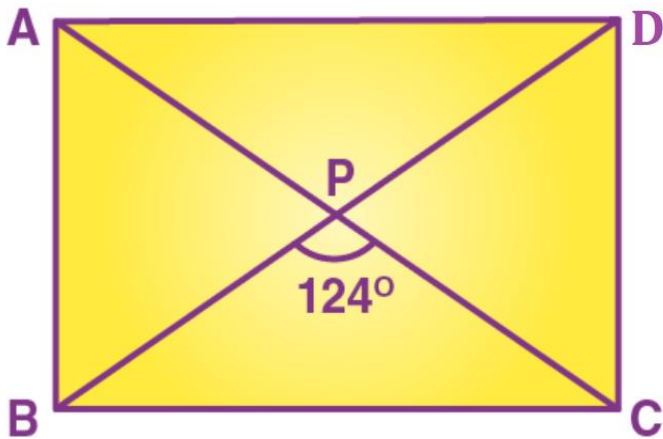
$$\Rightarrow x = 2$$

$$QS = 9x - 8 = 9 \times 2 - 8 = 10$$

Question 5.

ABCD is a rectangle, if $\angle BPC = 124^\circ$

Calculate: (1) $\angle BAP$ (ii) $\angle ADP$



Solution:-

Diagonals of rectangle are equal and bisect each other.

$$\angle PBC = \angle PCB = x \text{ (say)}$$

$$\text{But } \angle BPC + \angle PBC + \angle PCB = 180^\circ$$

$$124^\circ + x + x = 180^\circ$$

$$2x = 180^\circ - 124^\circ$$

$$2x = 56^\circ$$

$$\Rightarrow x = 28^\circ$$

$$\angle PBC = 28^\circ$$

But $\angle PBC = \angle ADP$ [Alternate \angle S]

$$\angle ADP = 28^\circ$$

$$\text{Again } \angle APB = 180^\circ - 124^\circ = 56^\circ$$

$$\angle BAP = \frac{1}{2}(180^\circ - \angle APB)$$

$$\angle BAP = \frac{1}{2}(180^\circ - \angle APB)$$

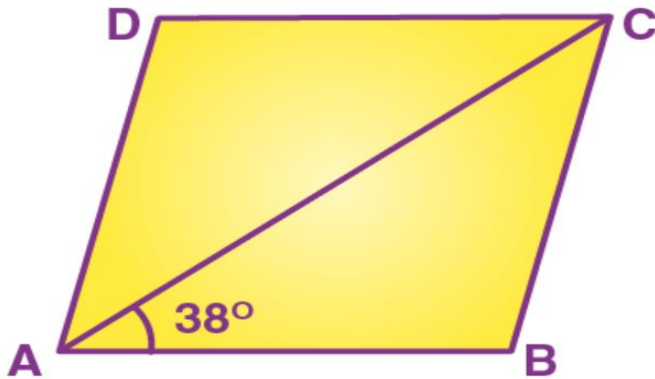
$$= \frac{1}{2} \times (180^\circ - 56^\circ) = \frac{1}{2} \times 124^\circ = 62^\circ$$

$$\text{Hence (i) } \angle BAP = 62^\circ \text{ (ii) } \angle ADP = 28^\circ$$

Question 6.

ABCD is a rhombus. If $\angle BAC = 38^\circ$, find:

- (i) $\angle ACB$
- (ii) $\angle DAC$
- (iii) $\angle ADC$



Solution:-

ABCD is Rhombus (Given)

$AB = BC$ $\angle BAC = \angle ACB$ (\angle S opp. to equal sides)

But $\angle BAC = 38^\circ$ (Given)

$\angle ACB = 38^\circ$

In $\triangle ABC$, $\angle ABC + \angle BAC + \angle ACB = 180^\circ$

$\angle ABC + 38^\circ + 38^\circ = 180^\circ$

$\angle ABC = 180^\circ - 76^\circ = 104^\circ$

$\angle ADC = \angle ABC$ (Opp. \angle s of rhombus)

$\angle ADC = 104^\circ$ $\angle DAC = \angle DCA$ ($AD = CD$)

$\angle DAC = \frac{1}{2} [180^\circ - 104^\circ]$

$\angle DAC = \frac{1}{2} \times 76^\circ = 38^\circ$

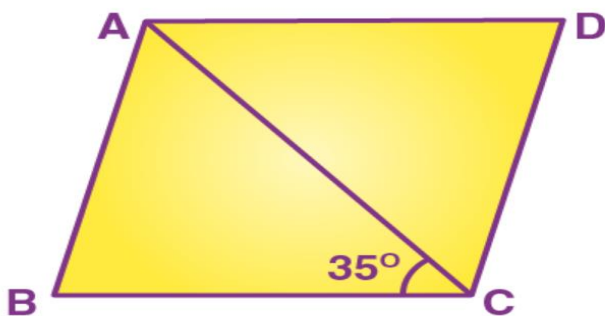
Hence (i) $\angle ACB = 38^\circ$ (ii) $\angle DAC = 38^\circ$ (iii) $\angle ADC = 104^\circ$ Ans

Question 7.

ABCD is a rhombus. If $\angle BCA = 35^\circ$. Find $\angle ADC$.

Solution:-

Given: Rhombus ABCD in which $\angle BCA = 35^\circ$



To find: $\angle ADC$

Proof: AD || BC

$$\angle DAC = \angle BCA \text{ (Alternate } \angle\text{s)}$$

$$\text{But } \angle BCA = 35^\circ \text{ (Given)}$$

$$\angle DAC = 35^\circ$$

$$\text{But } \angle DAC = \angle ACD \text{ (AD = CD) \& } \angle DAC + \angle ACD + \angle ADC = 180^\circ$$

$$\angle B^\circ + 35^\circ + \angle ACD = 180^\circ$$

$$\angle ADC = 180^\circ - 70^\circ = 110^\circ$$

Hence $\angle ADC = 110^\circ$

Question 8.

PQRS is a parallelogram whose diagonals intersect at M.

$$\angle PMS=54^\circ, \angle OSR=25^\circ \text{ and } \angle SOR=30^\circ;$$

(i) $\angle RPS$

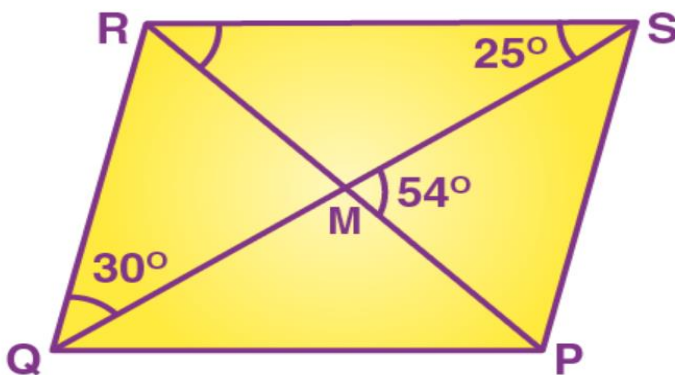
(ii) $\angle PRS$

(iii) $\angle PSR$

Solution:-

Given: Parallelogram PQRS in which diagonals PR & OS intersect at M.

$$\angle PMS=54^\circ; \angle OSR=25^\circ \text{ and } \angle SQR=30^\circ$$



To find: (i) $\angle RPS$ (ii) $\angle PRS$ (iii) $\angle PSR$

Proof: QR || PS

$$\Rightarrow \angle PSQ = \angle SQR \text{ (Alternate } \angle\text{s)}$$

$$\text{But } \angle SQR = 30^\circ$$

$$\angle PSQ = 30^\circ$$

In $\triangle SMP$,

$$\angle PMS + \angle PSM + \angle MPS = 180^\circ \text{ or } 54^\circ + 30^\circ + \angle RPS = 180^\circ$$

$$\angle RPS = 180^\circ - 84^\circ = 96^\circ$$

$$\text{Now, } \angle PRS + \angle RSQ = \angle PMS$$

$$\angle PRS + 25^\circ = 54^\circ$$

$$\angle PRS = 54^\circ - 25^\circ = 29^\circ$$

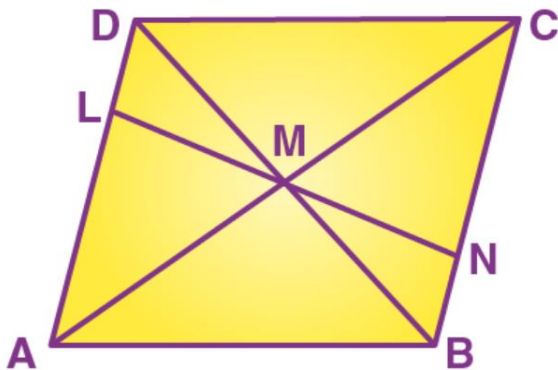
$$\angle PSR = \angle PSQ + \angle RSQ = 30^\circ + 25^\circ = 55^\circ$$

Hence, (i) $\angle RPS = 96^\circ$ (ii) $\angle PRS = 29^\circ$ (iii) $\angle PSR = 55^\circ$

Question 9.

Given: Parallelogram ABCD in which diagonals AC and BD intersect at M. Prove: M is mid-point of LN.

Solution:-



Proof: Diagonals of parallelogram bisect each other

$$MD = MB$$

Also $\angle ADB = \angle DBN$ (Alternate \angle s)

& $\angle DML = \angle BMN$ (vert. opp. \angle s)

$$\triangle DML = \triangle BMN$$

$$LM = MN$$

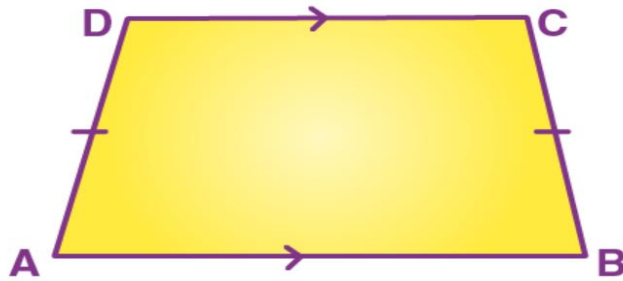
M is mid-point of LN.

Hence proved.

Question 10

In an isosceles trapezium, show that the opposite angles are supplementary.

Solution:



Given: ABCD is isosceles trapezium in which $AD = BC$

To Prove: (i) $\angle A + \angle C = 180$

(ii) $\angle B + \angle D = 180^\circ$

Proof: $AB \parallel CD$

$\Rightarrow \angle A + \angle D = 180$

But $\angle A = \angle B$ [Trapezium is isosceles]

$\angle B + \angle D = 180^\circ$

Similarly $\angle A + \angle C = 180^\circ$

Hence the result.

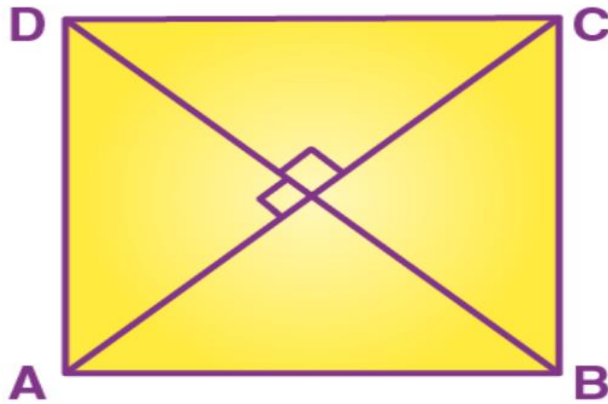
Question 11.

ABCD is a parallelogram. What kind of quadrilateral is it if:

- (i) $AC=BD$ and AC is perpendicular to BD?
- (ii) AC is perpendicular to BD but is not equal to it?
- (iii) $AC=BD$ but AC is not perpendicular to BD?

Solution:-

- (i)



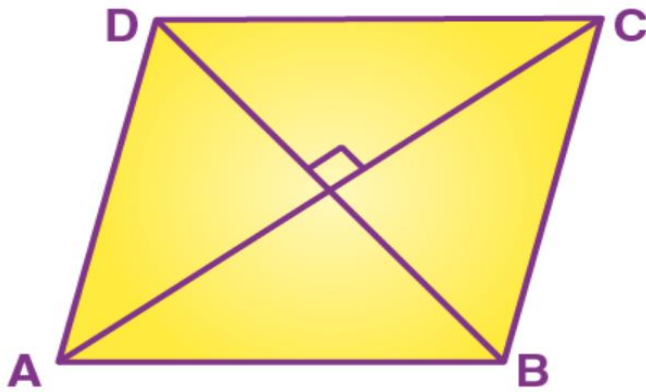
$AC=BD$ (Given)

$\&AC \perp BD$ (Given)

I.e. Diagonals of quadrilateral are equal and they
Are perpendicular to each other.

$\therefore ABCD$ is square

(ii)

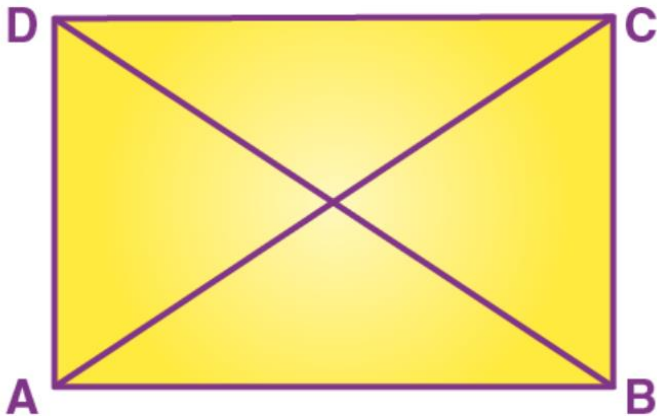


$AC \perp BD$ (Given)

But AC & BD are not equal

$\therefore ABCD$ is a Rhombus.

(iii)



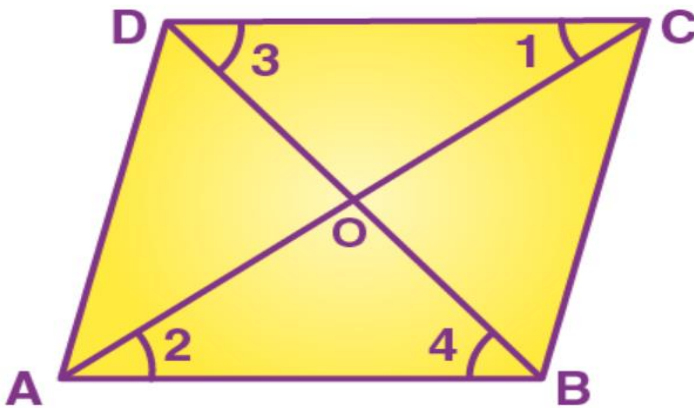
$AC=BD$ but AC & BD are not \perp r to each other.

$\therefore ABCD$ is a Rectangle.

Question 12.

Prove that the diagonals of a parallelogram bisect each other.

Solution:-



Given: Parallelogram $ABCD$ in which diagonals AC and BD bisect each other.

To Prove: $OA=OC$ and $OB=OD$

Proof: $AB \parallel CD$ (Given)

$\angle 1 = \angle 2$ (Alternate \angle s)

$\angle 3 = \angle 4$ (Alternate \angle s)

And $AB=CD$ (opposite sides of parallelogram)

$\triangle COD = \triangle AOB$ (A.S.A rule)

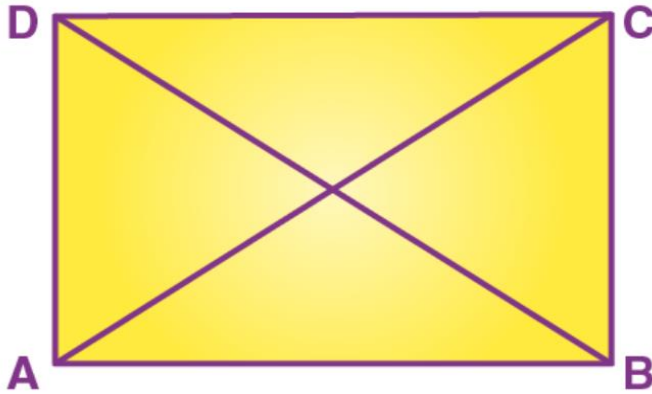
$OA=OC$ and $OB=OD$

Hence the result.

Question 13.

If the diagonals of a parallelogram are of equal lengths, the parallelogram is a rectangle. Prove it.

Solution:-



Given: parallelogram ABCD in which $AC=BD$

To Prove: ABCD is rectangle

Proof: In $\triangle ABC$ and $\triangle ABD$

$AB=AB$ (Common)

$AC=BD$ (Given)

$BC=AD$ (Opposite sides of parallelogram)

$\triangle ABC=\triangle ABD$ (S.S.S. Rule)

$\angle A=\angle B$

But $AD \parallel BC$ (opp. sides of ll gm are

$\angle A+\angle B=180^\circ$

$\angle A=\angle B=90^\circ$

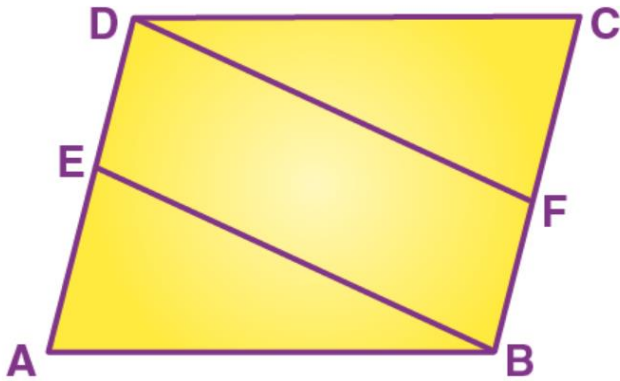
Similarly $\angle D=\angle C=90^\circ$

Hence ABCD is a rectangle.

Question 14.

In parallelogram ABCD, E is the mid-point of AD and F is the mid-point of BC. Prove that BFDE is a parallelogram.

Solution:-



Given: Parallelogram ABCD in which E and F are mid-points of AD and BC

To Prove: BFDE is a parallelogram.

Proof: E is mid-point of AD. (Given)

$$DE = \frac{1}{2}AD$$

Also F is midpoint of BC (Given)

$$BF = \frac{1}{2}BC$$

But $AD=BC$ (opp. sides of llgm)

$$BF=DE$$

Again $AD \parallel BC$

$DE \parallel BF$

Now $DE \parallel BF$ and $DE=BF$

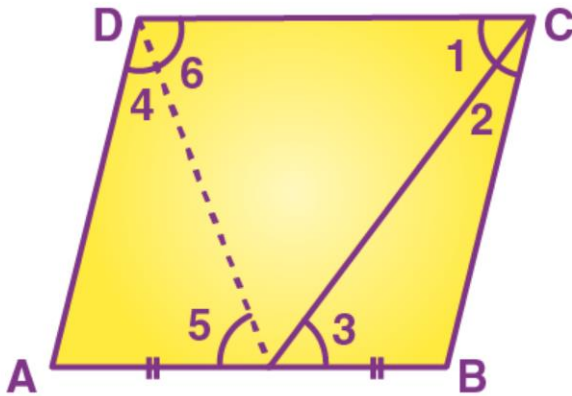
Hence BFDE is a parallelogram.

Question 15.

In parallelogram ABCD, E is the mid-point of side AB and CE bisects angle BCD. Prove that:

- (i) $AE=AD$.
- (ii) DE bisects $\angle ADC$ and
- (iii) Angle DEC is a right angle.

Solution:-



Given: $\parallel\text{gm}$ ABCD in which E is mid-point of AB and CE bisects $\angle BCD$.

To Prove: (i) $AE=AD$

(ii) DE bisects $\angle ADC$

(iii) $\angle DEC=90^\circ$

Const. Join DE

Proof: (i) $AB \parallel CD$ (Given)

And CE bisects it.

$\angle 1 = \angle 3$ (Alternate \angle s)..... (i)

But $\angle 1 = \angle 2$ (Given) (ii)

From (i) & (ii)

$\angle 2 = \angle 3$

$BC = BE$ (Sides opp. to equal angles)

But $BC = AD$ (opp. sides of $\parallel\text{gm}$)

and $BE = AE$ (Given)

$AD = AE$

$\angle 4 = \angle 5$ (\angle s opp. to equal sides)

But $\angle 5 = \angle 6$ (alternate \angle s)

$\Rightarrow \angle 4 = \angle 6$

DE bisects $\angle ADC$.

NOW $AD \parallel BC$

$\Rightarrow \angle D + \angle C = 180^\circ$

$2\angle 6 + 2\angle 1 = 180^\circ$

DE and CE are bisectors.

$$\angle 6 + \angle 1 = \frac{180^\circ}{2}$$

$$\angle 6 + \angle 1 = 90^\circ$$

$$\text{But } \angle DEC + \angle 6 + \angle 1 = 180^\circ$$

$$\angle DEC + 90^\circ = 180^\circ$$

$$\angle DEC = 180^\circ - 90^\circ$$

$$\angle DEC = 90^\circ$$

Hence the result.

