

2.1 Fill in the blanks

- (a) The volume of a cube of side 1 cm is equal tom³
 (b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to...(mm)²
 (c) A vehicle moving with a speed of 18 km h⁻¹ covers....m in 1 s
 (d) The relative density of lead is 11.3. Its density isg cm⁻³ orkg m⁻³.

Solution:

(a) Volume of cube, $V = (1 \text{ cm})^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$.

(b) Surface area = curved area + area on top /base = $2\pi rh + 2\pi r^2 = 2\pi r (h + r)$

$r = 2 \text{ cm} = 20 \text{ mm}$

$h = 10 \text{ cm} = 100 \text{ mm}$

Surface area = $2\pi r (h + r) = 2 \times 3.14 \times 20 (100 + 20) = 15072 \text{ mm}^2$

Hence, answer is 15072 mm²

(c) Speed of vehicle = 18 km/h

1 km = 1000 m

1 hr = 60 x 60 = 3600 s

1 km/hr = 1000 m/3600 s = 5/18 m/s

18 km/h = = (18 x 1000)/3600
 = 5 m/s

Distance travelled by the vehicle in 1 s = 5 m

(d) The Relative density of lead is 11.3 g cm⁻³

=> $11.3 \times 10^3 \text{ kg m}^{-3}$ [1 kilogram = 10³g, 1 meter = 10² cm]

=> $11.3 \times 10^3 \text{ kg m}^{-4}$

2.2 Fill in the blanks by suitable conversion of units

(a) $1 \text{ kg m}^2 \text{ s}^{-2} = \dots \text{g cm}^2 \text{ s}^{-2}$

(b) $1 \text{ m} = \dots \text{ly}$

(c) $3.0 \text{ m s}^{-2} = \dots \text{km h}^{-2}$

(d) $G = 6.67 \times 10^{-11} \text{ N m}^2 (\text{kg})^{-2} = \dots (\text{cm})^3 \text{ s}^{-2} \text{ g}^{-1}$

Solution:

(a) $1 \text{ kg m}^2 \text{ s}^{-2} = \dots \text{g cm}^2 \text{ s}^{-2}$

$1 \text{ kg m}^2 \text{ s}^{-2} = 1\text{kg} \times 1\text{m}^2 \times 1\text{s}^{-2}$

We know that,

$1\text{kg} = 10^3$

$$1\text{m} = 100\text{cm} = 10^2\text{cm}$$

When the values are put together, we get:

$$1\text{kg} \times 1\text{m}^2 \times 1\text{s}^{-2} = 10^3\text{g} \times (10^2\text{cm})^2 \times 1\text{s}^{-2} = 10^3\text{g} \times 10^4 \text{cm}^2 \times 1\text{s}^{-2} = 10^7 \text{gcm}^2\text{s}^{-2}$$

$$\Rightarrow \mathbf{1\text{kg m}^2 \text{s}^{-2} = 10^7 \text{gcm}^2\text{s}^{-2}}$$

(b) $1 \text{ m} = \dots \text{ ly}$

Using the formula,

$$\text{Distance} = \text{speed} \times \text{time}$$

$$\text{Speed of light} = 3 \times 10^8 \text{ m/s}$$

$$\text{Time} = 1 \text{ yr} = 365 \text{ days} = 365 \times 24 \text{ hr} = 365 \times 24 \times 60 \times 60 \text{ sec}$$

Put these values in the formula mentioned above, we get:

$$\text{One light year distance} = (3 \times 10^8 \text{ m/s}) \times (365 \times 24 \times 60 \times 60) = 9.46 \times 10^{15} \text{ m}$$

$$9.46 \times 10^{15} \text{ m} = 1 \text{ ly}$$

$$\text{So that, } 1\text{m} = 1/9.46 \times 10^{15} \text{ ly}$$

$$\Rightarrow 1.06 \times 10^{-16} \text{ ly}$$

$$\Rightarrow \mathbf{1 \text{ meter} = 1.06 \times 10^{-16} \text{ ly}}$$

(c) $3.0 \text{ m s}^{-2} = \dots \text{ km h}^{-2}$

$$1 \text{ km} = 1000\text{m} \text{ so that } 1\text{m} = 1/1000 \text{ km}$$

$$3.0 \text{ m s}^{-2} = 3.0 (1/1000 \text{ km}) (1/3600 \text{ hour})^{-2} = 3.0 \times 10^{-3} \text{ km} \times ((1/3600)^{-2} \text{h}^{-2})$$

$$= 3 \times 10^{-3} \text{ km} \times (3600)^2 \text{ hr}^{-2} = 3.88 \times 10^4 \text{ km h}^{-2}$$

$$\Rightarrow \mathbf{3.0 \text{ m s}^{-2} = 3.88 \times 10^4 \text{ km h}^{-2}}$$

(d) $G = 6.67 \times 10^{-11} \text{ N m}^2 (\text{kg})^{-2} = \dots (\text{cm})^3 \text{s}^{-2} \text{ g}^{-1}$

$$G = 6.67 \times 10^{-11} \text{ N m}^2 (\text{kg})^{-2}$$

We know that,

$$1\text{N} = 1\text{kg m s}^{-2}$$

$$1 \text{ kg} = 10^3 \text{ g}$$

$$1\text{m} = 100\text{cm} = 10^2 \text{ cm}$$

Put the values together, we get:

$$\Rightarrow 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} = 6.67 \times 10^{-11} \times (1\text{kg m s}^{-2}) (1\text{m}^2) (1\text{kg}^{-2})$$

Solve the following and cancelling out the units, we get:

$$\Rightarrow 6.67 \times 10^{-11} \times (1 \text{ kg}^{-1} \times 1 \text{ m}^3 \times 1 \text{ s}^{-2})$$

Put the above values together to convert kg to g and m to cm

$$\Rightarrow 6.67 \times 10^{-11} \times (10^3 \text{ g})^{-1} \times (10^2 \text{ cm})^3 \times (1 \text{ s}^{-2})$$

$$\Rightarrow 6.67 \times 10^{-8} \text{ cm}^3 \text{ s}^{-2} \text{ g}^{-1}$$

$$\Rightarrow G = 6.67 \times 10^{-11} \text{ Nm}^2(\text{kg})^{-2} = 6.67 \times 10^{-8} (\text{cm})^3 \text{ s}^{-2} \text{ g}^{-1}$$

2.3 A calorie is a unit of heat (energy in transit) and it equals about 4.2 J where $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$. Suppose we employ a system of units in which the unit of mass equals $\alpha \text{ kg}$, the unit of length equals $\beta \text{ m}$, the unit of time is $\gamma \text{ s}$. Show that a calorie has a magnitude of $4.2 \alpha^{-1} \beta^{-2} \gamma^2$ in terms of the new units.

Solution:

$$1 \text{ calorie} = 4.2 \text{ J} = 4.2 \text{ kg m}^2 \text{ s}^{-2}$$

The standard formula for the conversion is

$$\frac{\text{Given unit}}{\text{new unit}} = \left(\frac{M_1}{M_2}\right)^x \left(\frac{L_1}{L_2}\right)^y \left(\frac{T_1}{T_2}\right)^z$$

$$\text{Dimensional formula for energy} = [M^1 L^2 T^{-2}]$$

Here, $x = 1$, $y = 2$ and $z = -2$

$$M_1 = 1 \text{ kg}, L_1 = 1 \text{ m}, T_1 = 1 \text{ s}$$

$$\text{and } M_2 = \alpha \text{ kg}, L_2 = \beta \text{ m}, T_2 = \gamma \text{ s}$$

$$\frac{\text{Calorie}}{\text{new unit}} = 4.2 \left(\frac{1}{\alpha}\right)^1 \left(\frac{1}{\beta}\right)^2 \left(\frac{1}{\gamma}\right)^{-2}$$

$$\text{Calorie} = 4.2 \alpha^{-1} \beta^{-2} \gamma^2$$

2.4 Explain this statement clearly:

“To call a dimensional quantity ‘large’ or ‘small’ is meaningless without specifying a standard for comparison”. In view of this, reframe the following statements wherever necessary:

- (a) atoms are very small objects
- (b) a jet plane moves with great speed
- (c) the mass of Jupiter is very large
- (d) the air inside this room contains a large number of molecules
- (e) a proton is much more massive than an electron
- (f) the speed of sound is much smaller than the speed of light.

Solution:

- (a) In comparison with a soccer ball, atoms are very small

- (b) When compared with a bicycle, jet plane travels at high speed.
- (c) When compared with the mass of a cricket ball, the mass of Jupiter is very large.
- (d) As compared with the air inside a lunch box, the air inside the room has a large number of molecules.
- (e) A proton is massive when compared with an electron.
- (f) Like comparing the speed of a bicycle and a jet plane, the speed of light is more than the speed of sound.

2.5 A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

Solution:

Distance between them = Speed of light x Time taken by light to cover the distance

Speed of light = 1 unit

Time taken = $8 \times 60 + 20 = 480 + 20 = 500\text{s}$

The distance between Sun and Earth = $1 \times 500 = 500$ units.

2.6 Which of the following is the most precise device for measuring length:

- (a) a vernier callipers with 20 divisions on the sliding scale
- (b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale
- (c) an optical instrument that can measure length to within a wavelength of light?

Solution:

(a) Least count = $1 - \frac{9}{10} = \frac{1}{10} = 0.01\text{cm}$

(b) Least count = $\frac{\text{pitch}}{\text{number of divisions}}$

= $\frac{1}{10000} = 0.0001\text{ cm}$

(c) least count = wavelength of light = 10^{-5} cm

= 0.00001 cm

We can come to the conclusion that the optical instrument is the most precise device used to measure length.

2.7. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of

the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of the hair?

Solution:

Magnification of the microscope = 100

Average width of the hair in the field of view of the microscope = 3.5 mm

Actual thickness of hair = $3.5 \text{ mm}/100 = 0.035 \text{ mm}$

2. 8. Answer the following:

(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread?

(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?

(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

Solution:

(a) The thread should be wrapped around a pencil a number of times so as to form a coil having its turns touching each other closely. Measure the length of this coil with a metre scale. If L be the length of the coil and n be the number of turns of the coil then the diameter of the thread is given by the relation

Diameter = L/n .

(b) Least count of the screw gauge = Pitch/number of divisions on the circular scale

So, theoretically when the number of divisions on the circular scale is increased the least count of the screw gauge will decrease. Hence, the accuracy of the screw gauge will increase. However, this is only a theoretical idea. Practically, there will be many other difficulties when the number of turns is increased.

(c) The probability of making random errors can be reduced to a larger extent in 100 observations than in the case of 5 observations.

2.9. The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2 . What is the linear magnification of the projector-screen arrangement?

Solution:

Arial Magnification = Area of the image/Area of the object

$$= 1.55/1.75 \times 10^4$$

$$= 8.857 \times 10^3$$

Linear Magnification = $\sqrt{\text{Arial magnification}}$

$$= \sqrt{8.857 \times 10^3}$$

= 94. 1

2.10 State the number of significant figures in the following:

(a) 0.007 m^2

(b) $2.64 \times 10^{24} \text{ kg}$

(c) 0.2370 g cm^{-3}

(d) 6.320 J

(e) 6.032 N m^{-2}

(f) 0.0006032 m^2

Solution:

(a) 0.007 m^2

The given value is 0.007 m^2 .

Only one significant digit. It is 7

(b) $2.64 \times 10^{24} \text{ kg}$

The value is $2.64 \times 10^{24} \text{ kg}$

For the determination of significant values, the power of 10 is irrelevant. The digits 2, 6, and 4 are significant figures. The number of significant digits is 3.

(c) 0.2370 g cm^{-3}

The value is 0.2370 g cm^{-3}

For the given value with decimals, all the numbers 2, 3, 7, and 0 are significant. The 0 before the decimal point is not significant

(d) 6.320 J

All the numbers are significant. The number of significant figures here is 4.

(e) 6.032 N m^{-2}

6, 0, 3, 2 are significant figures. Therefore, the number of significant figures is 4.

(f) 0.0006032 m^2

6, 0, 3, 2 are significant figures. The number of significant figures is 4.

2. 11. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Solution:

Area of the rectangular sheet = length x breadth

$$= 4.234 \times 1.005 = 4.255 \text{ m}^2 = 4.3 \text{ m}^2$$

Volume of the rectangular sheet = length x breadth x thickness = $4.234 \times 1.005 \times 2.01 \times 10^{-2} = 8.55 \times 10^{-2} \text{ m}^3$.

2.12 The mass of a box measured by a grocer's balance is 2.30 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is

(a) the total mass of the box,

(b) the difference in the masses of the pieces to correct significant figures?

Solution:

The mass of the box = 2.30 kg

and the mass of the first gold piece = 20.15 g

The mass of the second gold piece = 20.17 g

The total mass = $2.300 + 0.2015 + 0.2017 = 2.7032 \text{ kg}$

Since 1 is the least number of decimal places, the total mass = 2.7 kg.

The mass difference = $20.17 - 20.15 = 0.02 \text{ g}$

Since 2 is the least number of decimal places, the total mass = 0.02 g.

2.13 A physical quantity P is related to four observables a, b, c and d as follows:

$$P = \frac{a^3 b^2}{\sqrt{cd}}$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P? If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result?

Solution:

$$\frac{a^3 b^2}{\sqrt{cd}} \frac{\Delta P}{P} = \frac{3\Delta a}{a} + \frac{2\Delta b}{b} + \frac{1}{2} \frac{\Delta c}{c} + \frac{\Delta d}{d}$$

$$\left(\frac{\Delta P}{P} \times 100 \right) \% = \left(3 \times \frac{\Delta a}{a} \times 100 + 2 \times \frac{\Delta b}{b} \times 100 + \frac{1}{2} \frac{\Delta c}{c} \times 100 + \frac{\Delta d}{d} \times 100 \right) \%$$

$$= 3 \times 1 + 2 \times 3 + \frac{1}{2} \times 4 + 2$$

$$= 3 + 6 + 2 + 2 = 13 \%$$

$$P = 4.235$$

$$\Delta P = 13 \% \text{ of } P$$

$$= \frac{13P}{100}$$

$$= \frac{13 \times 4.235}{100}$$

$$= 0.55$$

The error lies in the first decimal point, so the value of $p = 4.3$

2.14 A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion:

(a) $y = a \sin \left(\frac{2\pi t}{T} \right)$

(b) $y = a \sin vt$

(c) $y = \frac{a}{T} \sin \frac{t}{a}$

(d) $y = a\sqrt{2} \left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T} \right)$

Solution:

(a) $y = a \sin \frac{2\pi t}{T}$

Dimension of $y = M^0 L^1 T^0$

The dimension of $a = M^0 L^1 T^0$

Dimension of $\sin \frac{2\pi t}{T} = M^0 L^0 T^0$

Since the dimensions on both sides are equal, the formula is dimensionally correct.

(b) It is dimensionally incorrect, as the dimensions on both sides are not equal.

(c) It is dimensionally incorrect, as the dimensions on both sides are not equal.

(d) $y = a\sqrt{2} \left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T} \right)$

Dimension of $y = M^0 L^1 T^0$

The dimension of $a = M^0 L^1 T^0$

Dimension of $\frac{t}{T} = M^0 L^0 T^0$

The formula is dimensionally correct.

2.15 A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light, c . (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c . He writes:

$$m = \frac{m_0}{\sqrt{1-v^2}}$$

Guess where to put the missing c .

Solution:

The relation given is $\frac{m_0}{\sqrt{1-v^2}}$

We can get, $\frac{m_0}{m} = \sqrt{1-v^2} \frac{m_0}{m}$ is dimensionless. Therefore, the right hand side should also be dimensionless.

To satisfy this, $\sqrt{1-v^2}$ should become $\sqrt{1-\frac{v^2}{c^2}}$.

$$\text{Thus, } m = m_0 \sqrt{1-\frac{v^2}{c^2}}$$

2.16 The unit of length convenient on the atomic scale is known as an angstrom and is denoted by \AA : $1 \text{\AA} = 10^{-10} \text{ m}$. The size of a hydrogen atom is about 0.5\AA . What is the total atomic volume in m^3 of a mole of hydrogen atoms?

Solution:

hydrogen atom radius = $0.5 \text{\AA} = 0.5 \times 10^{-10} \text{ m}$

$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.5 \times 10^{-10})^3$$

$$= 0.524 \times 10^{-30} \text{ m}^3$$

1 hydrogen mole contains 6.023×10^{23} hydrogen atoms.

$$\begin{aligned} \text{Volume of 1 mole of hydrogen atom} &= 6.023 \times 10^{23} \times 0.524 \times 10^{-30} \\ &= 3.16 \times 10^{-7} \text{ m}^3. \end{aligned}$$

2.17 One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large?

Solution:

$$\text{Radius} = 0.5 \text{ \AA} = 0.5 \times 10^{-10} \text{ m}$$

$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.5 \times 10^{-10})^3$$

$$= 0.524 \times 10^{-30} \text{ m}^3$$

1 hydrogen mole contains 6.023×10^{23} hydrogen atoms.

$$\begin{aligned} \text{Volume of 1 mole of hydrogen atom} &= 6.023 \times 10^{23} \times 0.524 \times 10^{-30} \\ &= 3.16 \times 10^{-7} \text{ m}^3 \end{aligned}$$

$$V_m = 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3$$

$$\frac{V_m}{V_a} = \frac{22.4 \times 10^{-3}}{3.16 \times 10^{-7}} = 7.1 \times 10^4$$

The molar volume is 7.1×10^4 times more than the atomic volume. Hence, the inter-atomic separation in hydrogen gas is larger than the size of the hydrogen atom.

2.18 Explain this common observation clearly: If you look out of the window of a fast-moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hilltops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

Solution:

An imaginary line which joins the object and the observer's eye is called the line of sight. When we observe the nearby objects, they move fast in the opposite direction as the line of sight changes constantly. Whereas, the distant objects seem to be stationary as the line of sight does not change rapidly.

2.19. The principle of 'parallax' in section 2.3.1 is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit $\approx 3 \times 10^{11}$ m. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1" (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second of arc) from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres?

Solution:

Diameter of Earth's orbit = 3×10^{11} m

Radius of Earth's orbit $r = 1.5 \times 10^{11}$ m

Let the distance parallax angle be $\theta = 1''$ (s) = 4.847×10^{-6} rad.

Let the distance of the star be D.

Parsec is defined as the distance at which the average radius of the Earth's orbit subtends an angle of 1"

Therefore, $D = 1.5 \times 10^{11} / 4.847 \times 10^{-6} = 0.309 \times 10^{17}$

Hence 1 parsec $\approx 3.09 \times 10^{16}$ m.

2. 20. The nearest star to our solar system is 4.29 light-years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun?

Solution:

1 light year is the distance travelled by light in a year

1 light year = $3 \times 10^8 \times 365 \times 24 \times 60 \times 60 = 9.46 \times 10^{15}$ m

Therefore, distance travelled by light in 4.29 light years = $4.29 \times 9.46 \times 10^{15} = 4.058 \times 10^{16}$ m

Parsec is also a unit of distance

1 parsec = 3.08×10^{16} m

Therefore, the distance travelled by light in parsec is given as

4.29 light years = $4.058 \times 10^{16} / 3.08 \times 10^{16} = 1.318$ parsec = 1.32 parsec.

Using the relation,

$$\theta = d / D$$

here,

d is the diameter of Earth's orbit, $d = 3 \times 10^{11}$ m

D is the distance of the star from the earth, $D = 405868.32 \times 10^{11}$ m

$$\therefore \theta = 3 \times 10^{11} / 405868.32 \times 10^{11} = 7.39 \times 10^{-6} \text{ rad}$$

But the angle covered in 1 sec = 4.85×10^{-6} rad

$$\therefore 7.39 \times 10^{-6} \text{ rad} = 7.39 \times 10^{-6} / 4.85 \times 10^{-6} = 1.52''$$

2.21 Precise measurements of physical quantities are a need for science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.

Solution:

Precise measurement is essential for the development of science. The ultra-short laser pulse is used for measurement of time intervals. X-ray spectroscopy is used to find the interatomic separation. To measure the mass of atoms, the mass spectrometer is developed.

2.23 The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: a mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^8 m.

Solution:

$$\text{Mass} = 2 \times 10^{30} \text{ kg}$$

$$\text{Radius} = 7 \times 10^8 \text{ m}$$

$$\text{Volume } V = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (7 \times 10^8)^3$$

$$= \frac{88}{21} \times 512 \times 10^{24} \text{ m}^3 = 2145.52 \times 10^{24} \text{ m}^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{3 \times 10^{30}}{2145.52 \times 10^{24}} = 1.39 \times 10^3 \text{ kg/m}^3.$$

The density is in the range of solids and liquids. Its density is due to the high gravitational attraction on the outer layer by the inner layer of the sun.

2.24. When the planet Jupiter is at a distance of 824.7 million kilometres from the Earth, its angular diameter is measured to be $35.72''$ of arc. Calculate the diameter of Jupiter.

Solution:

Distance of the planet Jupiter from Earth, $D = 824.7$ million kilometres = 824.7×10^6 km

Angular diameter $\theta = 35.72'' = 35.72 \times 4.85 \times 10^{-6} \text{ rad}$
 $= 173.242 \times 10^{-6} \text{ rad}$
Diameter of Jupiter $d = \theta \times D = 173.241 \times 10^{-6} \times 824.7 \times 10^6 \text{ km}$
 $= 142871 = 1.43 \times 10^5 \text{ km}$

2.25. A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v : $\tan \theta = v$ and checks that the relation has a correct limit: as $v \rightarrow 0$, $\theta \rightarrow 0$, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.

Solution:

According to the principle of homogeneity of dimensional equations,
Dimensions of L.H.S = Dimensions of R.H.S

In relation $v = \tan \theta$, $\tan \theta$ is a trigonometric function and it is dimensionless. The dimension of v is $[L^1 T^{-1}]$. Therefore, this relation is incorrect.

To make the relation correct, the L.H.S must be divided by the velocity of rain, u .

Therefore, the relation becomes

$$v/u = \tan \theta$$

This relation is correct dimensionally

2.26. It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s?

Solution:

Total time = 100 years = $100 \times 365 \times 24 \times 60 \times 60 \text{ s}$

Error in 100 years = 0.02 s

Error in 1 second = $0.02/100 \times 365 \times 24 \times 60 \times 60$

$$= 6.34 \times 10^{-12} \text{ s}$$

Accuracy of the standard cesium clock in measuring a time-interval of 1 s is 10^{-12} s

2.27. Estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å. (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the mass density of sodium in its crystalline phase: 970 kg m^{-3} . Are the two densities of the same order of magnitude? If so, why?

Solution:

The diameter of sodium = $2.5 \text{ Å} = 2.5 \times 10^{-10} \text{ m}$

Therefore, the radius is $1.25 \times 10^{-10} \text{ m}$

Volume of sodium atom, $V = (4/3)\pi r^3$

$$= (4/3) \times (22/7) \times (1.25 \times 10^{-10})^3 = 8.177 \times 10^{-30} \text{ m}^3$$

Mass of one mole atom of sodium = 23 g = 23×10^{-3} kg

1 mole of sodium contains 6.023×10^{23} atoms

Therefore, the mass of one sodium atom, $M = 23 \times 10^{-3} / 6.023 \times 10^{23} = 3.818 \times 10^{-26}$ kg

Atomic mass density of sodium, $\rho = M/V$
 $= 3.818 \times 10^{-26} / 8.177 \times 10^{-30}$

$= 0.46692 \times 10^4 = 4669.2 \text{ kg m}^{-3}$

The density of sodium in its solid state is 4669.2 kg m^{-3} but in the crystalline phase, density is 970 kg m^{-3} . Hence, both are in a different order. In solid-phase, atoms are tightly packed but in the crystalline phase, atoms arrange a sequence which contains void. So, density in solid-phase is greater than in the crystalline phase.

2.28. The unit of length convenient on the nuclear scale is a fermi: $1 \text{ f} = 10^{-15} \text{ m}$. Nuclear sizes obey roughly the following empirical relation:

$$r = r_0 A^{1/3}$$

where r is the radius of the nucleus, A its mass number, and r_0 is a constant equal to about, 1.2 f . Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of the sodium nucleus. Compare it with the average mass density of a sodium atom obtained in Exercise. 2.27.

Solution:

Radius of the nucleus

$$r = r_0 A^{1/3}$$

$$r_0 = 1.2 \text{ f} = 1.2 \times 10^{-15} \text{ m}$$

Considering the nucleus is spherical. Volume of nucleus

$$= \frac{4}{3} \pi r^3 = \frac{4}{3} \pi [r_0 A^{1/3}]^3 = \frac{4}{3} \pi r_0^3 A$$

Mass of nucleus = $m A$

m is the average mass of the nucleon

A is the number of nucleons

Nuclear mass density = Mass of nucleus/Volume of nucleus

$$= \frac{m A}{\frac{4}{3} \pi r^3} = \frac{3 m A}{4 \pi r^3} = \frac{3 m A}{4 \pi r_0^3 A}$$

$$= \frac{3 m}{4 \pi r_0^3}$$

Using $m = 1.66 \times 10^{-27} \text{ kg}$ and $r_0 = 1.2 \text{ f} = 1.2 \times 10^{-15} \text{ m}$ in the above equation

$$= \frac{3 \times 1.66 \times 10^{-27}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3} = \frac{4.98 \times 10^{-27}}{21.703 \times 10^{-45}} = 2.29 \times 10^{17} \text{ kg/m}^3$$

So, the nuclear mass density is much larger than atomic mass density for a sodium atom we got in 2.27.

2.29. A LASER is a source of very intense, monochromatic, and the unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

Solution:

Time taken for the laser beam to return to Earth after reflection by the Moon's surface = 2.56 s

The speed of laser light, $c = 3 \times 10^8$ m/s.

Let d be the distance of Moon from the Earth,

The time taken by laser signal to reach the Moon, $t = 2d/c$

Therefore, $d = tc/2 = (2.56 \times 3 \times 10^8)/2 = 3.84 \times 10^8$ m

2. 30. A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects underwater. In a submarine equipped with a SONAR, the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s^{-1}).

Solution:

Speed of sound in water, $v = 1450 \text{ m s}^{-1}$

Time between generation and the reception of the echo after reflection, $2t = 77.0$ s

Time taken for the sound waves to reach the submarine, $t = 77.0/2 = 38.5$ s

Then $v = d/t$

Distance of enemy submarine, $d = tv$

Therefore, $d = vt = (1450 \times 38.5) = 55825 \text{ m} = 55.8 \times 10^3 \text{ m}$ or 55.8 km.

2.31. The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?

Solution:

Time taken by light from the quasar to reach the observer, $t = 3.0$ billion years = 3.0×10^9 years = $3.0 \times 10^9 \times 365 \times 24 \times 60 \times 60$ s

= 94608000×10^9 s

= 9.46×10^{16} m

Speed of light = 3×10^8 m/s

Distance of quasar from Earth = $3.0 \times 10^8 \times 9.46 \times 10^{16}$ m
= 28.38×10^{24} m

2.32. It is a well-known fact that during a total solar eclipse-the disk of the moon almost completely covers the disk of the Sun. From this fact and from the information you can gather from examples 2.3 and 2.4, determine the approximate diameter of the moon.

Solution:

From examples 2.3 and 2.4 we get the following data

Distance of the Moon from Earth = 3.84×10^8 m

Distance of the Sun from Earth = 1.496×10^{11} m

Sun's diameter = 1.39×10^9 m

Sun's angular diameter, $\theta = 1920'' = 1920 \times 4.85 \times 10^{-6}$ rad = 9.31×10^{-3} rad [$1'' = 4.85 \times 10^{-6}$ rad]

During a total solar eclipse, the disc of the moon completely covers the disc of the sun, so the angular diameter of both the sun and the moon must be equal.

Therefore, Angular diameter of the moon, $\theta = 9.31 \times 10^{-3}$ rad

The earth-moon distance, $S = 3.8452 \times 10^8$ m

Therefore, the diameter of the moon, $D = \theta \times S$
 $= 9.31 \times 10^{-3} \times 3.8452 \times 10^8$ m = 35.796×10^5 m

