Exercise 3.4 Page No: 97

Using elementary transformations, find the inverse of each of the matrices, if it exists in Exercises 1 to 17.

1.

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

As we know, A = I A

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$[R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

$$\left[R_2 \to \frac{1}{5} R_2\right.$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

$$[R_1 \to R_1 + R_2]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

Therefore, $A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

2.

$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

As A = AI = $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A \quad [R_2 \to R_2 - R_1]$$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

3. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

As we know, A = AI

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A \quad [R_1 \to R_1 - 3R_2]$$

$$A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

4.

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

As we know, A = AI

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Again,

$$\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} A \quad [R_1 \leftrightarrow R_2]$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 5 & -2 \end{bmatrix} A \quad [R_2 \leftrightarrow R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

Therefore, the inverse of given matrix is:

$$A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

5.

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

As we know, A = AI

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Again,

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} A \quad [R_2 \to R_2 - 3R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

6.

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

As we know, A = AI

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Again,

$$\begin{bmatrix}1 & 3\\2 & 5\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A \quad [R_1 \leftrightarrow R_2]$$

$$\begin{bmatrix}1 & 3\\0 & -1\end{bmatrix}=\begin{bmatrix}0 & 1\\-2 & 1\end{bmatrix}A \quad [R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}=\begin{bmatrix}3 & -5\\1 & -2\end{bmatrix}A \quad [R_1 \to R_1 + 3R_2]$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}3 & -5\\-1 & 2\end{bmatrix}A \quad [R_2 \to (-1)R_2]$$

$$A^{-1}=\begin{bmatrix}3 & -5\\-1 & 2\end{bmatrix}$$

7. $\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}$

Solution:

Let $A=\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$$

$$\begin{bmatrix}6 & 2\\5 & 2\end{bmatrix}=\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}A \quad [R_1 \to 2R_1]$$

$$\begin{bmatrix}1 & 0\\5 & 2\end{bmatrix}=\begin{bmatrix}2 & -1\\0 & 1\end{bmatrix}A \quad [R_1 \to R_1 - R_2]$$

$$\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}=\begin{bmatrix}2 & -1\\-10 & 6\end{bmatrix}A \quad [R_2 \to R_2 - 5R_1]$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}2 & -1\\-5 & 3\end{bmatrix}A \quad \left[R_2 \to \frac{1}{2}R_2\right]$$

$$A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

8.

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A \quad [R_2 \to R_2 - 3R_1]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A \quad [R_1 \to R_1 - R_2]$$

$$A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

9.

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix}1 & 3\\2 & 7\end{bmatrix}=\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}A \quad [R_1 \to R_1 - R_2]$$

$$\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & -1\\-2 & 3\end{bmatrix}A \quad [R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & -10\\-2 & 3\end{bmatrix}A \quad [R_1 \to R_1 - 3R_2]$$

$$A^{-1}=\begin{bmatrix}7 & -10\\-2 & 3\end{bmatrix}$$

10.

$$\begin{bmatrix}3 & -1\\-4 & 2\end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix}3 & -1\\-4 & 2\end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix}3 & -1\\-4 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$$

$$\begin{bmatrix}-1 & 1\\-4 & 2\end{bmatrix}=\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}A \quad [R_1 \to R_1 + R_2]$$

$$\begin{bmatrix}1 & -1\\-4 & 2\end{bmatrix}=\begin{bmatrix}-1 & -1\\0 & 1\end{bmatrix}A \quad [R_1 \to (-1)\,R_1]$$

$$\begin{bmatrix}1 & -1\\0 & -2\end{bmatrix}=\begin{bmatrix}-1 & -1\\-4 & -3\end{bmatrix}A \quad [R_2 \to R_2 + 4R_1]$$

$$\begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}=\begin{bmatrix}-1 & -1\\2 & \frac{3}{2}\end{bmatrix}A \quad \left[R_2 \to \frac{-1}{2}R_2\right]$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & \frac{1}{2}\\2 & \frac{3}{2}\end{bmatrix}A \quad [R_1 \to R_1 + R_2]$$

$$A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

11.

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & -2 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A \quad [R_1 \leftrightarrow R_2]$$

$$\begin{bmatrix} 1 & -2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A \quad [R_2 \to R_2 - 2R_1]$$

$$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-1}{2} & 1 \end{bmatrix} A \quad \left[R_2 \to \frac{-1}{2} R_2\right]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ \frac{-1}{2} & 1 \end{bmatrix} A \quad [R_1 \to R_1 + 2R_2]$$

$$A^{-1} = \begin{bmatrix} -1 & 3 \\ -1/2 & 1 \end{bmatrix}$$

12.

$$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

As we know, A = IA

$$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \quad \left[R_1 \to \frac{1}{6}R_1\right]$$

$$\begin{bmatrix} 1 & \frac{-1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A \quad \left[R_2 \to R_2 + 2R_1\right]$$

All entries in second row of left side are zero, so A^{-1} does not exist.

13.

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

We know that, A = IA

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying: $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$$

Applying: $R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

14.

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

We know that, A = IA

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying: $\left[R_1 \to \frac{1}{2}R_1\right]$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 4R_1$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -2 & 1 \end{bmatrix} A$$

All entries in second row of left side are zero, so inverse of the matrix does not exist.

15.

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

We know that, A = IA

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - R_3$

and $R_1 \to (-1)R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 2R_1$
and $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 5 \\ 0 & -5 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 0 & -2 \end{bmatrix} A$$

Applying: $R_2 \leftrightarrow R_3$

and $R_2 \to \left(\frac{-1}{5}\right) R_2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ \frac{-3}{5} & 0 & \frac{2}{5} \\ 2 & 1 & -2 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - R_2$

and $R_3 \to \frac{1}{5} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ \frac{-3}{5} & 0 & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{bmatrix}$$

Applying: $[R_2 \to R_2 + R_3]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

16.

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

We know that, A = IA

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 + 3R_1$

and $R_3 \to R_3 - 2R_1$

and $R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 4 \\ 0 & 9 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix} A$$

Applying: $R_2 \to (-1)R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -4 \\ 0 & 9 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - 3R_2$

and $R_3 \to R_3 - 9R_2$

and $R_3 \to \frac{1}{25}R_3$

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ 2 & 0 & -1 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

Applying: $R_1 \to R_1 - 10R_3$

and $R_2 \to R_2 + 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

17.

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

We know that, A = IA

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 2R_1$

and $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying: $R_2 \to R_2 - 2R_1$

and $R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & -2 & -5 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 0 \end{bmatrix} A$$

Applying: $R_1 \to R_1 - R_2$

and $R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying: $R_1 \to R_1 + R_3$

and $R_2 \to R_2 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

18.
Matrices A and B will be inverse of each other only if

(A) AB = BA
(B) AB = BA = 0
(C) AB = 0, BA = I
(D) AB = BA = I

Solution:

Option (D) is correct.

Consider A as a square matrix of order m, and if another square matrix of same order exists of order m,
We know that
AB = BA = I where B is the inverse of A.
Here A is the inverse of B.
Hence, the matrices A and B will be inverse of each other only if AB = BA = I.