Q.11.1: Find the:

(a) Maximum frequency, and

(b) The minimum wavelength of X-rays produced by 30 kV electrons.

Solution:

Electron potential, V = 30 kV **= 3×10^4 V**

Hence, electron energy, **E = 3×10^4 eV**

Where, e = Charge on one electron **= 1.6×10^{-19} C**

(a) Maximum frequency by the X-rays = v

The energy of the electrons:

E = hv

Where,

h = Planck's constant **= 6.626×10^{-34} Js**

Therefore, $v = \frac{E}{h}$

$= \frac{1.6\times10^{-19}\times3\times10^{4}}{6.626\times10^{-34}}$ = **7.24×10^{18} Hz**

Hence, 7.24×10^{18} Hz is the maximum frequency of the X-rays.

(b) The minimum wavelength produced:

$\lambda = \frac{c}{v}$

$= \frac{3\times10^{8}}{7.24\times10^{18}}$ = **4.14×10^{-11} m = 0.0414 nm**

Q.11.2: The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the

(a) maximum kinetic energy of the emitted electrons

(b) Stopping potential

(c) maximum speed of the emitted photoelectrons?

Solution:

Work function of caesium, Φ_o = **2.14eV**

Frequency of light, **v = 6.0×10^{14} Hz**

(a) The maximum energy (kinetic) by the photoelectric effect:

K = hv − Φ_o

Where,

h = Planck's constant **= 6.626 x 10^{-34} Js**

Therefore,

$$K = \frac{6.626\times10^{-34}\times6\times10^{14}}{1.6\times10^{-19}} - 2.14$$

= 2.485 – 2.140 **= 0.345 eV**

Hence, 0.345 eV is the maximum kinetic energy of the emitted electrons.

(b) For stopping potential V_o, we can write the equation for kinetic energy as:

K = eV_o

Therefore, $V_o = \frac{K}{e}$

$$= \frac{0.345\times1.6\times10^{-19}}{1.6\times10^{-19}}$$

= 0.345 V

Hence, 0.345 V is the stopping potential of the material.

(c) Maximum speed of photoelectrons emitted **= v**

Following is the kinetic energy relation:

$$K = \frac{1}{2}mv^2$$

Where,

m = mass of electron **= 9.1 x 10^{-31} Kg**

$$v^2 = \frac{2K}{m}$$

$$= \frac{2\times0.345\times1.6\times10^{-19}}{9.1\times10^{-31}}$$

=0.1104 x 10^{12}

Therefore, v = 3.323 x 10^5 m/s **= 332.3 km/s**

Hence**,** 332.3 km/s is the maximum speed of the emitted photoelectrons.

Question 11.3:

The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

Solution:

Photoelectric cut-off voltage, $V_o = 1.5$ V

For emitted photoelectrons, the maximum kinetic energy is:

$K_e = eV_o$

Where,

e = charge on an electron **= 1.6×10^{-19} C**

Therefore, $K_e = 1.6 \times 10^{-19} \times 1.5$ **= 2.4×10^{-19} J**

Therefore, 2.4×10^{-19} J is the maximum kinetic energy emitted by the photoelectrons.

Question 11.4

Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

(a) Find the energy and momentum of each photon in the light beam

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have a uniform cross-section which is less than the target area)

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

Solution:

Monochromatic light having a wavelength, λ = 632.8 nm **= 632.8×10^{-9} m**

Given that the laser emits the power of, P = 9.42 mW **= 9.42×10^{-3} W**

Planck's constant, **$h = 6.626 \times 10^{-34}$ Js**

Speed of light, **$c = 3 \times 10^{8}$ m/s**

Mass of a hydrogen atom, **$m = 1.66 \times 10^{-27}$ kg**

(a) The photons having the energy as:

$E = \frac{hc}{\lambda}$

$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{632.8 \times 10^{-9}}$

$= 3.141 \times 10^{-19}$ J

Therefore, each photon has a momentum of:

$P = \frac{h}{\lambda}$

$= \frac{6.626\times10^{-34}}{632.8\times10^{-9}}$

= 1.047 x 10^{-27} kg m/s

(b) Number of photons/second arriving at the target illuminated by the beam = n

Assuming the uniform cross-sectional area of the beam is less than the target area.

Hence, equation for power is written as:

P = nE

Therefore, n= $\frac{P}{E}$

$= \frac{9.42\times10^{-3}}{3.141\times10^{-19}}$

= 3 x 10^{16} photons/s

(c) Given that, momentum of the hydrogen atom is equal to the momentum of the photon,

P= 1.047 x 10^{-27} kg m/s

Momentum is given as:

P=mv

Where,

v = speed of hydrogen atom

Therefore, v = $\frac{p}{m}$

$= \frac{1.047\times10^{-27}}{1.66\times10^{-27}}$ **= 0.621 m/s**

Question 11.5:

The energy flux of sunlight reaching the surface of the earth is 1.388 × 10^3 W/m^2. How many photons are incident on the Earth per second/square meter? Assume an average wavelength of 550 nm.

Solution:

Sunlight reaching the surface of the earth has an energy flux of

ϕ **= 1.388 × 10^3 W/m^2**

Hence, power of sunlight per square metre, **P = 1.388 × 10^3 W**

Speed of light, **$c = 3 \times 10^8$ m/s**

Planck's constant, **$h = 6.626 \times 10^{-34}$ Js**

λ = 550nm **= 550×10^{-9}m** is the average wavelength of the photons from the sunlight

Number of photons per square metre incident on earth per second = n

Hence, the equation for power be written as:

P = nE

Therefore, n= $\frac{P}{E}$

= $\frac{P\lambda}{hc}$

= $\frac{1.388\times10^3\times550\times10^{-9}}{6.626\times10^{-34}\times3\times10^8}$ **= 3.84×10^{21} photons/m^2/s**

Therefore, 3.84×10^{21} photons are incident on the earth per square meters.

11.6: In an experiment on the photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.

Solution:

Given that the slope of cut-off voltage (V) versus frequency (v) being:

$\frac{V}{v}$ = 4.12×10^{-15} Vs

V and frequency being related by the equation as:

Hv = eV

Where,

e = Charge on an electron = 1.6×10^{-19} C

h = Planck's constant

Therefore, h = e x $\frac{V}{v}$

= $1.6 \times 10^{-19} \times 4.12 \times 10^{-15}$**= 6.592×10^{-34} Js**

Therefore, 6.592×10^{-34} Js is the Planck's constant that is determined from the above equation.

Question 11.7:

A 100W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength

of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?

Solution:

Power of the sodium lamp **P = 100W**

Wavelength of the emitted sodium light, λ = 589nm

= 589 x 10^{-9} m

Planck's constant, **h = 6.626 x 10^{-34} Js**

Speed of light, c = 3 x 10^{8}

(a)

The energy per photon associated with the sodium light is given as:

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.626\times10^{-34}\times3\times10^{8}}{589\times10^{-9}}$$

$$= 3.37 \times 10^{-19}\ J = \frac{3.37\times10^{-19}}{1.6\times10^{-19}} = \mathbf{2.11\ eV}$$

(b)

Number of photons delivered to the sphere = n

The equation for power can be written as:

P = nE

Therefore, $n = \frac{P}{E}$

$$= \frac{100}{3.37\times10^{-19}} = 2.96 \times 10^{20} \text{ photons/s}$$

Therefore, 2.96 x 10^{20} photons are delivered every second to the sphere.

Question 11.8:

The threshold frequency for a certain metal is 3.3 x 10^{14} Hz. If the light of frequency 8.2 x 10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.

Solution:

Threshold frequency of the metal, **v_o= 3.3 × 10^{14} Hz.**

Frequency of light incident on the metal, **v= 8.2 × 10^{14} Hz**

Charge on an electron, **e = 1.6 × 10^{-19} C**

Planck's constant, **h = 6.626 × 10^{-34} Js**

Cut-off voltage for the photoelectric emission from the metal **= V_o**

The equation for the cut –off energy is given as:

$eV_o = h(v - v_o)$

$V_o = \frac{h(v - v_o)}{e}$

$= \frac{6.626 \times 10^{-34} \times (8.2 \times 10^{14} - 3.3 \times 10^{14})}{1.6 \times 10^{-19}}$ **= 2.0291 V**

Question 11.9:

The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Solution:

Work function of the metal, Φ_o **=4.2eV**

Charge on an electron, **e = 1.6 x 10^{-19} C**

Planck's constant, **h = 6.626 × 10^{-34} Js**

Wavelength of the incident radiation, λ = 330nm **= 330 × 10^{-9} m**

Speed of light, **c = 3 × 10^{8} m/s**

The energy of the incident photon is given as:

$E = \frac{hc}{\lambda}$

$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{330 \times 10^{-9}}$

$= 6.0 \times 10^{-19}$ J $= \frac{6.0 \times 10^{-19}}{1.6 \times 10^{-19}}$ **= 3.76 eV**

The energy of the incident radiation is less than the work function of the metal. Hence, there is no photoelectric emission taking place.

Question 11.10:

Light of frequency 7.21 x 10^{14} Hz is incident in a metal surface. Electrons with a maximum speed of 6.0 x 10^{5} m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Solution:

Frequency of the incident photon, **v = 488nm = 488 x 10^{-9}m**

Maximum speed of the electrons, **v = 6.0 x 10^{5} m/s**

Planck's constant, **h= 6.626 x 10^{-34} Js**

Mass of an electron, **m = 9.1 x 10^{-31} Kg**

For threshold frequency v_o, the relation for kinetic energy is written as:

$$\frac{1}{2}mv^2 = h(v - v_o)$$

$$v_o = v - \frac{mv^2}{2h}$$

$$= 7.21 \times 10^{14} - \frac{(9.1\times10^{-31})\times(6\times10^{5})^2}{2\times(6.626\times10^{-34})}$$

$= 7.21 \times 10^{14} - 2.472 \times 10^{14}$ **$= 4.738 \times 10^{14}$ Hz**

Therefore, 4.738 x 10^{14} Hz is the threshold frequency for the photoemission of the electrons.

Question 11.11:

Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

Solution:

Wavelength of light produced by the argon laser,

λ = 488nm **= 488 x 10^{-9} m**

Stopping potential of the photoelectrons, **V_o = 0.38 V**

1eV = 1.6 x 10^{-19} J

Therefore, $V_o = \frac{0.38}{1.6\times10^{-19}}$ eV

Planck's constant, **h = 6.6 x 10^{-34} Js**

Charge on an electron, **e = 1.6 x 10^{-19} C**

Speed of light, **c = 3 x 10^8 m/s**

Using Einstein's photoelectric effect, following is the relation for the work function:

Φ_o of the material of the emitter as:

$eV_o = \frac{hc}{\lambda} - \phi_o$

$\Phi_o = \frac{hc}{\lambda} - eV_o$

$= \frac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times488\times10^{-9}} - \frac{1.6\times10^{-19}\times0.38}{1.6\times10^{-19}}$

= 2.54 – 0.38 **= 2.16 eV**

Therefore, 2.16eV is the work function of the material with which the emitter is made.

Question 11.12: Calculate the

(a) momentum, and

(b) the de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Solution:

Potential difference, **V = 56V**

Planck's constant, **h = 6.6 x 10^{-34} Js**

Mass of an electron, **m = 9.1x 10^{-31} Kg**

Charge on an electron, **e = 1.6 x 10^{-19}C**

(a) At equilibrium, the kinetic energy of each electron is equal to the accelerating potential i.e., we can write the relation of velocity (v) of each electron as:

$\frac{1}{2}mv^2 = eV$

$v^2 = \frac{2eV}{m}$

Therefore, $v = \sqrt{\frac{2\times1.6\times10^{-19}\times56}{9.1\times10^{-31}}}$

$= \sqrt{19.69\times10^{12}}$

= 4.44 x 10^6 m/s

The momentum of each accelerated electron is given as:

p = mv

= 9.1 x 10^{-31} x 4.44 x 10^{6}**= 4.04 x 10^{-24} Kg m/s**

Therefore, 4.04 x 10^{-24} Kg m/s is the momentum of each electron.

(b) de Broglie wavelength of an electron accelerating through a potential V, is given by the relation:

$\lambda = \frac{12.27}{\sqrt{V}}$ A°

= $\frac{12.27}{\sqrt{56}}$ x 10^{-19} m **= 0.1639 nm**

Therefore, 0.1639 nm is the de Broglie wavelength of each electron.

Question 11.13: What is the:

(a) Momentum,

(b) Speed, and

(c) De Broglie wavelength of an electron with a kinetic energy of 120 eV.

Solution:

Kinetic energy of the electron, **E_K = 120 eV**

Planck's constant, **h = 6.6 × 10^{-34} Js**

Mass of an electron, **m = 9.1 × 10^{-31} Kg**

Charge on an electron, **e = 1.6 × 10^{-19} C**

(a) For an electron, we can write the relation for kinetic energy as:

$E_k = \frac{1}{2}mv^2$

Where, v = speed of the electron

Therefore, $v^2 = \sqrt{\frac{2eE_k}{m}}$

$= \sqrt{\frac{2\times1.6\times10^{-19}\times120}{9.1\times10^{-31}}}$

$= \sqrt{42.198\times10^{12}}$

= 6.496 **×** 10^{6} m/s

Momentum of the electron, p = mv = 9.1 **×** 10^{-31}**×** 6.496 **×** 10^{6}

= 5.91 **×** 10^{-24} kg m/s

Therefore, 5.91 × 10^{-24} Kg m/s is the momentum of the electron.

(b) speed of the electron, v = 6.496 × 10^6 m/s

(c) de Broglie wavelength of an electron having a momentum p, is given as:

$$\lambda = \frac{h}{p}$$

$= \frac{6.6\times10^{-34}}{5.91\times10^{-24}}$ = 1.116 x 10^{-10} m **= 0.112 nm**

Therefore, 0.112 nm is the de Broglie wavelength of the electron.

Question 11.14:

The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

(a) an electron, and

(b) a neutron, would have the same de Broglie wavelength.

Solution:

Wavelength of light of a sodium line, λ = 589nm = 589 x 10^{-9} m

Mass of an electron, m_e = 9.1 x 10^{-31} Kg

Mass of a neutron, m_n = 1.66 x 10^{-27} Kg

Planck's constant, h = 6.6 x 10^{-34} Js

(a) For the kinetic energy K, of an electron accelerating with the velocity v, we have the relation:

K = $\frac{1}{2}mv^2$ . . . . **(1)**

We have the relation for de Broglie wavelength as:

$$\lambda = \frac{h}{m_e v}$$

Therefore, $v^2 = \frac{h^2}{\lambda^2 m_e^2}$ . . . . **(2)**

Substituting equation (2) in equation (1), we get the relation:

K = $\frac{1}{2} \frac{m_e h^2}{\lambda^2 m_e^2}$

$= \frac{h^2}{2\lambda^2 m_e}$ . . . . **(3)**

$= \frac{(6.6\times10^{-34})^2}{2\times(589\times10^{-9})^2\times9.1\times10^{-31}}$

= 6.9 x 10^{-25} J = $\frac{6.9\times10^{-25}}{1.6\times10^{-19}}$ **= 4.31 x 10^{-6} eV**

Hence, the kinetic energy of the electron is 6.9 x 10^{-25} J

(b) Using equation (3), we can write the relation for the kinetic energy of the neutron as:

= $\frac{h^2}{2\lambda^2 m_n}$

= $\frac{(6.6\times10^{-34})^2}{2\times(589\times10^{-9})^2\times1.66\times10^{-27}}$

= 3.78x 10^{-28} J

= $\frac{3.78\times10^{-28}}{1.6\times10^{-19}}$

= 2.36 x 10^{-9} eV **= 2.36 neV**

The neutron has the kinetic energy of 3.78 x 10^{-28} J or 2.36 neV.

Question 11.15:

What is the de Broglie wavelength of:

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,

(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and

(c) a dust particle of mass 1.0 × 10^{-9} kg drifting with a speed of 2.2 m/s?

Solution:

(a) Mass of the bullet, **m = 0.040 Kg**

Speed of the bullet, v = 1.0 km/s **= 1000 m/s**

Planck’s constant, **h = 6.6 x 10^{-34} Js**

de Broglie wavelength of the bullet is given by the relation:

$\lambda = \frac{h}{mv}$

= $\frac{6.6\times10^{-34}}{0.040\times1000}$

= 1.65 x 10^{-35} m

(b) Mass of the ball, **m = 0.060 Kg**

Speed of the ball, **v = 1.0 m/s**

de Broglie wavelength of the ball is given by the relation:

$= \lambda = \frac{h}{mv}$

$= \frac{6.6\times10^{-34}}{0.060\times1}$

= 1.1 x 10^{-32} m

(c) Mass of the dust particle, **m = 1 x 10^{-9} Kg**

speed of the dust particle, **v = 2.2 m/s**

de Broglie wavelength of the dust particle is given by the relation:

$= \lambda = \frac{h}{mv}$

$= \frac{6.6\times10^{-34}}{2.2\times1\times10^{-9}}$

= 3.0 x 10^{-25} m

Question 11.16:

An electron and a photon each have a wavelength of 1.00 nm. Find:

(a) Their momenta,

(b) The energy of the photon, and

(c) The kinetic energy of the electron.

Solution:

Wavelength of an electron λ_e and a photon λ_p ,

λ_e = λ_p = λ = 1 nm **= 1 x 10^{-9} m**

Planck's constant, h = 6.63 x 10^{-34} Js

(a) The momentum of an elementary particle is given by de Broglie relation:

$\lambda = \frac{h}{p}$

$p = \frac{h}{\lambda}$

It is clear that momentum depends only on the wavelength of the particle. Since the wavelengths of an electron and a photon are equal, both have an equal momentum.

Therefore, p = $\frac{6.63 \times 10^{-34}}{1 \times 10^{-9}}$

= 6.63 x 10^{-25} Kg m/s

(b) The energy of a photon is given by the relation:

E = $\frac{hc}{\lambda}$

Where,

Speed of light, c = 3 x 10^{8} m/s

Therefore, E = $\frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1 \times 10^{-9} \times 1.6 \times 10^{-19}}$

= 1243.1 eV = 1.243 keV

Therefore, the energy of the photon is 1.243 keV.

(c) The kinetic energy (K) of an electron having momentum p, is given by the relation:

K = $\frac{1}{2}\frac{p^2}{m}$

Where, m = Mass of the electron = 9.1 x 10^{-31} Kg

p = 6.63 x 10^{-25} Kg m/s

Therefore, K = $\frac{1}{2} \times \frac{(6.63 \times 10^{-25})^2}{9.1 \times 10^{-31}}$

= 2.415 x 10^{-19} J

= $\frac{2.415 \times 10^{-19}}{1.6 \times 10^{-19}}$ **= 1.51 eV**

1.51eV is the kinetic energy of the electron.

Question 11.17:

(a) For what kinetic energy of a neutron will the associated de Broglie wavelength be 1.40 x 10^{-10} m?

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of (3/2) kT at 300 K.

Solution:

(a) de Broglie wavelength of the neutron, λ = 1.40 x 10^{-10} m

Mass of a neutron, m_n = 1.66 x 10^{-27} Kg

Planck's constant, h = 6.63 x 10^{-34} Js

Kinetic energy (K) and velocity (v) are related as:

K = $\frac{1}{2}m_n v^2$ .... (1)

de Broglie wavelength (λ) and velocity (v) are related as:

$\lambda = \frac{h}{m_n v}$ .... (2)

Using equation (2) and equation (1), we get:

K = $\frac{1}{2}\frac{m_n h^2}{\lambda^2 m_n^2}$

= $\frac{h^2}{2\lambda^2 m_n}$

= $\frac{(6.63\times10^{-34})^2}{2\times(1.40\times10^{-10})^2\times1.66\times10^{-27}}$

= 6.75 x 10^{-21} J

= $\frac{6.75\times10^{-21}}{1.6\times10^{-19}}$

= 4.219 x10^{-2} eV

Hence, the kinetic energy of the neutron is 6.75 x 10^{-21} J or 4.219 x 10^{-2} eV.

(b) Temperature of the neutron, **T = 300K**

Boltzmann constant, k = 1.38 x 10^{-23} Kg m^2 s^{-2} K^{-1}

Average kinetic energy of the neutron:

K' = $\frac{3}{2}$ kT

= $\frac{3}{2}$ x 1.38 x 10^{-23} x 300

= 6.21 x 10^{-21} J

The relation for the de Broglie wavelength is given as:

$\lambda' = \frac{h}{\sqrt{2K' m_n}}$

Where,

m_n = 1.66 x 10^{-27} Kg

h = 6.63 x 10^{-34} Js

K' = 6.21 x 10^{-21} J

Therefore, $\lambda' = \frac{6.63\times10^{-34}}{\sqrt{2\times6.21\times10^{-21}\times1.66\times10^{-27}}}$

= 1.46 **×** 10^{-10} m **= 0.146 nm**

Therefore, 0.146nm is the de Broglie wavelength of the neutron.

Question 11.18:

Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Solution:

The momentum of a photon having energy (hv) is given as:

p = $\frac{hv}{c}$ = $\frac{h}{\lambda}$

$\lambda = \frac{h}{p}$ .... **(i)**

Where,

λ = wavelength of the electromagnetic radiation

c = speed of light

h = Planck's constant

De Broglie wavelength of the photon is given as:

$\lambda = \frac{h}{mv}$

But, **p = mv**

Therefore, $\lambda = \frac{h}{p}$ ....**(ii)**

Where, m = mass of the photon

v = velocity of the photon

From equation (i) and (ii) it can be concluded that the wavelength of the electromagnetic radiation and the de Broglie wavelength of the photon are equal.

Question 11.19:

What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean-square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

Solution:

Temperature of the nitrogen molecule, **T = 300 K**

Atomic mass of nitrogen **= 14.0076 u**

Hence, mass of the nitrogen molecule, m = 2 × 14.0076 **= 28.0152 u**

But, **1 u = 1.66×10^{-27} kg**

Therefore, **m = $28.0152 \times 1.66 \times 10^{-27}$ kg**

Planck's constant, h = 6.63×10^{-34} Js

Boltzmann constant, k = 1.38×10^{-23} J/K

We have the expression that relates mean kinetic energy $\left(\frac{3}{2}kT\right)$ of the nitrogen molecule with the root mean square speed (V_{rms}) as:

$$\frac{1}{2}mv_{rms}^2 = \left(\frac{3}{2}kT\right)$$

$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

For nitrogen molecule, the de Broglie wavelength is given as:

$$\lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{3\times28.0152\times1.66\times10^{-27}\times1.38\times10^{-23}\times300}}$$

= 0.028×10^{-9} m **= 0.028 nm**

Therefore, the de Broglie wavelength of the nitrogen molecule is 0.028 nm.

Question 11.20:

(a) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons. The specific charge of the electron, i.e., its e/m is given to be 1.76×10^{11} C kg^{-1}.
(b) Use the same formula you employ in (a) to obtain electron speed for a collector potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified?

Solution:

(a) Potential difference of the evacuated tube = 500 V

Specific charge of the electron, e/m = 1.76×10^{11} C kg^{-1}

Kinetic energy = (1/2) mv^2 = eV

Speed of the emitted electron, v = $(2Ve/m)^{1/2}$

= $(2 \times 500 \times 1.76 \times 10^{11})^{1/2}$

= 1.32×10^7 m/s

(b) Collector potential, V = 10 MV = 10×10^6 V.

Speed of electron = v = $(2Ve/m)^{1/2}$

= $(2 \times 10^7 \times 1.76 \times 10^{11})^{1/2}$

= 1.88×10^9 m/s

This answer is not correct. Since the value is greater than the speed of light (c). The expression (1/2) mv^2 for energy should be used in the non -relativistic limit. i.e., v << c.

In the relativistic limits, the total energy is given as

E = mc^2

Here,

m is the relativistic mass

$m = m_0 (1- v^2/c^2)^{1/2}$

m_0 = mass of the particle at rest

Kinetic energy is given as

$K = mc^2 – m_0c^2$

Question 11.21:

(a) A monoenergetic electron beam with an electron speed of 5.20×10^6 m s^{-1} is subject to a magnetic field of 1.30×10^{-4} T normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals 1.76×10^{11}C kg^{-1}.
(b) Is the formula you employ in (a) valid for calculating the radius of the path of a 20 MeV electron beam? If not, in what way is it modified?

[Note: Exercises 11.20(b) and 11.21(b) take you to relativistic mechanics which is beyond the scope of this book. They have been inserted here simply to emphasise the point that the formulas you use in part (a) of the exercises are not valid at very high speeds or energies. See answers at the end to know what 'very high speed or energy' means.]

Solution:

Speed of the electron, v= 5.20×10^6 m s^{-1}

Magnetic field experienced by the electron, B = 1.30×10^{-4} T

Specific charge, e/m = 1.76×10^{11} Ckg^{-1}

Here,

e = charge on the electron = 1.6×10^{-19} C

m = mass of the electron = 9.1×10^{-31} kg^{-1}

Force exerted on the electron is given as

$$F = e\left|\vec{v} \times \vec{B}\right|$$

= evBsinθ

θ is the angle between the magnetic field and the velocity of the beam. The magnetic field is normal to the direction of the beam.

θ = 90^0

F = evB

The normal magnetic field provides the centripetal force.

Therefore, evB = mv^2/r

r = mv/eB = v/(e/m)B

= $(5.20 \times 10^6)/(1.76 \times 10^{11}) \times (1.30 \times 10^{-4})$ = 0.227 m = 22.7 cm

Therefore, the radius of the circular path is 22.7 cm

(b) Energy of the electron beam, E = 20 Mev = $20 \times 10^6 \times 1.6 \times 10^{-19}$ J

The energy of the electron beam is, E = (1/2) mv^2

$$\Rightarrow v = \sqrt{\frac{2E}{m}}$$

$$v = \sqrt{\frac{2\times 20\times 10^6\times 1.6\times 10^{-19}}{9.1\times 10^{-31}}} = 2.652 \times 10^9 m/s$$

The result is greater than the speed of light. Therefore, it is wrong. The expression (1/2) mv^2 for energy should be used in the non -relativistic limit. i.e., v << c.

In the relativistic limits, the total mass is given as

m = $m_0 (1- v^2/c^2)^{1/2}$

m_0 = mass of the particle at rest

Therefore, the radius of the circular path is

r = mv/eB

$$r = \frac{m_0 v}{eB\sqrt{\frac{c^2-v^2}{c^2}}}$$

Question 11.22:

An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure (~10^{-2} mm of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path can be viewed because the gas ions in the path focus the beam by attracting electrons, and emitting light by electron capture; this method is known as the 'fine beam tube' method.) Determine e/m from the data.

Solution:

Potential of the collector, V= 100 V

Magnetic field experienced by the electron, B =2.83×10^{-4} T

Radius of the circular orbit, r = 12 cm = 12.0×10^{-2} m

Kinetic energy, $(1/2)mv^2 = eV$

$v^2 = 2eV/m$ ———(1)

The magnetic field that curves the path of the electron provides the centripetal force

$evB = mv^2/r$

$eB = mv/r$

$v = eBr/m$ ———(2)

Substituting (2) in (1)

$$\frac{2eV}{m} = \frac{e^2B^2r^2}{m^2}$$

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

$$\frac{e}{m} = \frac{2\times100}{(2.83\times10^{-4})^2(12\times10^{-2})^2} = 1.73\times\ 10^{11}Ckg^{-1}$$

Therefore, the specific charge ration e/m is 1.73×10^{11} Ckg^{-1}

Question 11. 23:

(a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 Å. What is the maximum energy of a photon in the radiation?
(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Solution:

(a) Wavelength produced by the X-ray tube, λ= 0.45 Å = 0.45×10^{-10} m

Speed of light, c = 3×10^8 m/s

Planck's constant, h = 6.626×10^{-34} Js

The maximum energy of a photon is given as

$E_{max} = hc/\lambda_{min}$

= (6.626 x 10^{-34})(3 x 10^{8} m/s)/(0.45 x 10^{-10} m x 1.6 x 10^{-19})

= 19.878 x 10^{-26}/0.72 x 10^{-29}

= 27.60 x 10^{3} eV = 27.6 keV

(b) To incident electron should have an energy of 27.6 keV to get a X-ray of 27.6 keV. Therefore, the accelerating voltage of the order of 30 keV is required for producing X-rays.

Question 11.24:

In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as the annihilation of an electron-positron pair of total energy 10.2 BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1BeV = 10^{9} eV)

Solution:

Total energy of the electron-positron pair, E= 10.2 BeV = 10.2 x 10^{9} eV = 10.2 x 10^{9} x 1.6 x 10^{-19} J

Hence the energy of each γ-ray , E' = E/2 = (10.2 x 10^{9} x 1.6 x 10^{-19})/2 = 8.16 x 10^{-10} J

Energy and wavelength relation is given as,

E' = hc/λ

Therefore, λ = hc/E'

Here, h = 6.626 x 10^{-34} Js

c = 3 x 10^{8} m/s

$$\lambda = \frac{6.626\times10^{-34}\times3\times10^{8}}{8.16\times10^{-10}} = 2.436\times\ 10^{-16}m$$

The wavelength associated with each γ-ray is 2.436 x 10^{-16} m

Question 11.25:

Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about photons! The second number tells you why our eye can never 'count photons', even in barely detectable light.
(a) The number of photons emitted per second by a Medium wave transmitter of 10 kW power, emitting radio waves of wavelength 500 m.
(b) The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive (~10^{-10} W m^{-2}). Take the area of the pupil to be about 0.4 cm^{2}, and the average frequency of white light to be about 6 × 10^{14} Hz

Solution:

(a) Power of the medium wave transmitter, P = 10 kW = 10^{4} W

Energy emitted by the transmitter per second, E = 10^{4}

Wavelength of the radio waves, λ = 500 m

The energy of the wave is given as, E' = hc/λ

$E' = (6.6 \times 10^{-34} \times 3 \times 10^{8})/500$

$= 3.96 \times 10^{-28}$ J

Let n be the number of photons emitted by the transmitter

nE' = E

n = E/E'

$= 10^{4}/(3.96 \times 10^{-28})$

$= 0.2525 \times 10^{32}$

The energy E' of the radio photon is very less, but the number of photons emitted is large. The total energy of the radio ways can be considered as continuous and the existence of the minimum quantum energy can be ignored.

(b) Intensity of the light perceived by the human eye, $I = 10^{-10}$ W m^{-2}

Area of the pupil, $A = 0.4\ cm^{2} = 0.4 \times 10^{-4}\ m^{2}$

Frequency of the white light, $v = 6 \times 10^{14}$ Hz

h = Planck's constant = 6.6×10^{-34} Js

Energy of the emitted photon, E = hv

$= 6.6 \times 10^{-34} \times 6 \times 10^{14}$

$= 3.96 \times 10^{-19}$ J

Let n be the total number of photons falling per unit area per unit time. The total energy per unit for n photons is

$E = n \times 3.96 \times 10^{-19}\ J/s/m^{2}$

Total energy per unit for n photons is equal to the intensity of the light.

E = I

$I = n \times 3.96 \times 10^{-19}\ J/s/m^{2}$

$n = I/3.96 \times 10^{-19}$

$= 10^{-10}/3.96 \times 10^{-19}$

$= 2.52 \times 10^{8}\ m^{2}/s$

The total number of photons entering the pupil is given as,

$nA = 2.52 \times 10^{8} \times 0.4 \times 10^{-4}$

$= 1.008 \times 10^{4}\ s^{-1}$

The number is large. So the human eye can never count the number of individual photons.

Question 11.26:

Ultraviolet light of wavelength 2271 Å from a 100 W mercury source irradiates a photo-cell made of molybdenum metal. If the stopping potential is –1.3 V, estimate the work function of the metal. How would the photo-cell respond to high intensity (~10^5 W m^{-2}) red light of wavelength 6328 Å produced by a He-Ne laser?

Solution:

Wavelength of ultraviolet light, λ = 2271 Å = 2271×10^{-10} m

Stopping potential of the metal, V_0 = 1.3 V

Planck's constant, h = 6.6×10^{-34} J

Charge on an electron, e = 1.6×10^{-19} C

From photoelectric effect we have the photon-electron relation as

Work function of the metal, $\Phi_0 = hv - eV_0$

$= (hc/\lambda) - eV_0$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{2271\times10^{-10}} - 1.6\times10^{-19}\times1.3$$

$= 8.72 \times 10^{-19} - 2.08 \times 10^{-19}$

$= 6.64 \times 10^{-19}$ J

Threshold frequency of the metal

Therefore, $\Phi_0 = h\upsilon_0$

$\upsilon_0 = \Phi_0/h = 6.64 \times 10^{-19} / 6.6 \times 10^{-34}$

$= 1.006 \times 10^{15}$ Hz

Wavelength of the red light, $\lambda_r = 6328 \times 10^{-10}$ m

Frequency of the red light, $\upsilon_r = c/\lambda_r = 3 \times 10^8/6328 \times 10^{-10}$

$= 4.74 \times 10^{14}$ Hz

$\upsilon_0 > \upsilon_r$

Photocell will not respond to the red light produced by the He-Ne laser.

Question 11.27:

Monochromatic radiation of wavelength 640.2 nm (1nm = 10^{-9} m) from a neon lamp irradiates photosensitive material made of caesium on tungsten. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source and its 427.2 nm line irradiates the same photo-cell. Predict the new stopping voltage.

Solution:

Wavelength of the monochromatic radiation, λ = 640.2 nm = 640.2×10^{-9} m

Stopping potential of the neon lamp, $V_0 = 0.54$ V

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

From photoelectric effect we have the photon-electron relation as $eV_0 = hv - \Phi_0$

Work function of the metal, $\Phi_0 = hv - eV_0$

$= (hc/\lambda) - eV_0$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{640.2\times10^{-9}} - 1.6\times 10^{-19}\times 0.54$$

$= 3.093 \times 10^{-19} - 0.864 \times 10^{-19}$

$= 2.229 \times 10^{-19}$ J

Wavelength of the radiation emitted by the iron source, $\lambda' = 427.2$ nm $= 427.2 \times 10^{-9}$ m

Let V_0' be the new stopping potential

Therefore, $eV_0' = (hc/\lambda') - \Phi_0$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{427.2\times10^{-9}} - 2.229\times 10^{-19}$$

$= 4.63 \times 10^{-19} - 2.229 \times 10^{-19}$

$= 2.401 \times 10^{-19}$ J

$V_0' = 2.401 \times 10^{-19}$ J/ 1.6×10^{-19} J

$= 1.5$ eV

The new stopping potential = 1.50 eV

Question 11.28:

A mercury lamp is a convenient source for studying the frequency dependence of photoelectric emission since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used:

$\lambda_1 = 3650$ Å,

$\lambda_2 = 4047$ Å,

$\lambda_3 = 4358$ Å,

$\lambda_4 = 5461$ Å,

$\lambda_5 = 6907$ Å,

The stopping voltages, respectively, were measured to be: $V_{01} = 1.28$ V, $V_{02} = 0.95$ V, $V_{03} = 0.74$ V, $V_{04} = 0.16$ V, $V_{05} = 0$ V. Determine the value of Planck's constant h, the threshold frequency and work function for the material.

[Note: You will notice that to get h from the data, you will need to know e (which you can take to

be 1.6 × 10^{-19} C). Experiments of this kind on Na, Li, K, etc. were performed by Millikan, who, using his own value of e (from the oil-drop experiment) confirmed Einstein's photoelectric equation and at the same time gave an independent estimate of the value of h.]

Solution:

From photoelectric effect we have the photon-electron relation as $eV_0 = hv - \Phi_0$

Work function of the metal, $\Phi_0 = hv - eV_0$

$\Phi_0 = (hc/\lambda) - eV_0$

$$V_0 = \frac{h}{e}\nu - \frac{\phi_0}{e} \text{ ---(1)}$$

Here,

V_0 = Stopping potential
h = Planck's constant
e = Charge on an electron
v = Frequency of radiation
Φ_0 = Work function of a material

Stopping proportional is directly proportional to the frequency.
Frequency, v = Speed of light (c)/Wavelength (λ)

Using this equation we can find the frequency of various lines

$v_1 = c/\lambda_1 = 3 \times 10^8/3650 \times 10^{-10}$

$= 8.219 \times 10^{14}$ Hz

$v_2 = c/\lambda_2 = 3 \times 10^8/4047 \times 10^{-10}$

$= 7.412 \times 10^{14}$ Hz

$v_3 = c/\lambda_3 = 3 \times 10^8/4358 \times 10^{-10}$

$= 6.88 \times 10^{14}$ Hz

$v_4 = c/\lambda_4 = 3 \times 10^8/5461 \times 10^{-10}$

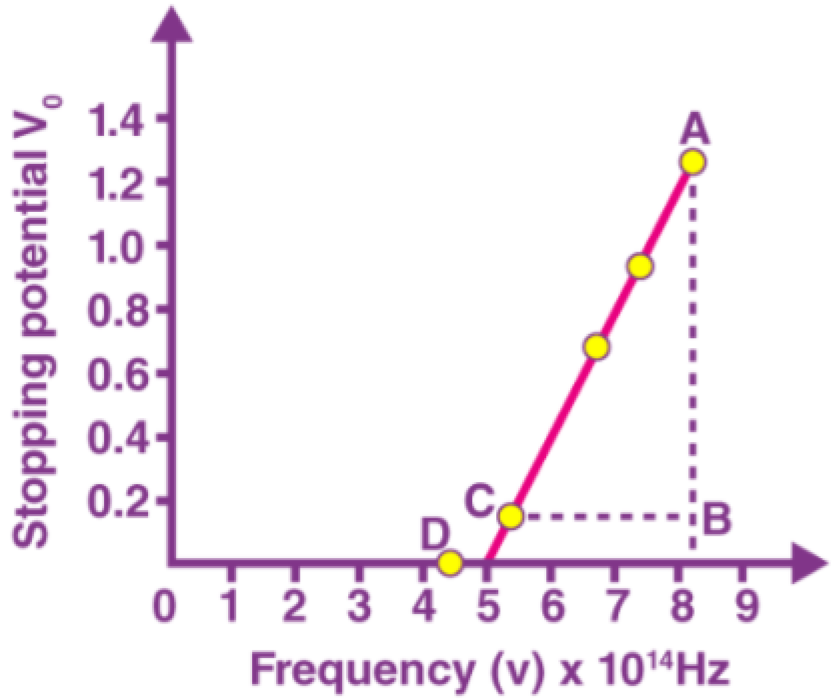
$= 5.493 \times 10^{14}$ Hz

$v_5 = c/\lambda_5 = 3 \times 10^8/6907 \times 10^{-10}$

$= 4.343 \times 10^{14}$ Hz

Frequency	8.219	7.412	6.884	5.493	4.343
Stopping potential	1.28	0.95	0.74	0.16	o

The above values can be plotted in a graph



The graph is a straight line. It intersects the y-axis at 5 x 10^{14} Hz. This is the threshold frequency. The point D is the frequency less than the threshold frequency.

Slope of the straight line = AB/CB = (1.28 – 0.16)/(8.214 – 5.493) x 10^{14}

From equation (1), the slope is written as

h/e = (1.28 – 0.16)/(8.214 – 5.493) x 10^{14}

h = (1.12 x 1.6 x 10^{-19})/(2.726 x 10^{14})

= 6.573 x 10^{-34} Js

The work function of the metal is,

$\Phi_0 = h\nu_0$

= (6.573 x 10^{-34} x 5 x 10^{14})

= 3.286 x 10^{-19} J

= 3.286 x 10^{-19} /1.6 x 10^{-19}

Φ_0= 2.054 eV

Question 11.29:

The work function for the following metals is given: Na: 2.75 eV; K: 2.30 eV; Mo: 4.17 eV; Ni: 5.15 eV. Which of these metals will not give photoelectric emission for radiation of wavelength 3300 Å from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?

Solution:

Wavelength λ = 3300 Å

Speed of light = 3 x 10^8 m/s

Planck's constant = 6.63 x 10^{-34} Js

Energy of the photon of the incident light

E = hc/λ = (6.63 x 10^{-34} x 3 x 10^8)/3300x 10^{-10}

⇒ 6.018 x 10^{-19} J

⇒ (6.018 x 10^{-19} J)/1.6 x 10^{-19}

= 3.7 eV

The energy of the incident radiation is greater than the work function of Na and K. It is lesser for Mo and Ni. Therefore, Mo and Ni will not show photoelectric effect.

If the laser is brought nearer and placed 50 cm away, then the intensity of the radiation will increase. The energy of the radiation will not be affected. Therefore, the result will be the same. However, the photoelectrons from Na and K will increase in proportion to intensity.

Question 11.30:

Light of intensity 10^{-5} W m^{-2} falls on a sodium photo-cell of surface area 2 cm^2. Assuming that the top 5 layers of sodium absorb the incident energy, estimate the time required for photoelectric emission in the wave-picture of radiation. The work function for the metal is given to be about 2 eV. What is the implication of your answer?

Solution:

Intensity of the light = 10^{-5} W m^{-2}

Surface area of the sodium photocell, A = 2 cm^2

Incident power of the light, P = I x A

= 10^{-5} x 2 x 10^{-4}

= 2 x 10^{-9} W

Work function of the metal, Φ_0 = 2 eV

= 2 x 1.6 x 10^{-19}

= 3.2 x 10^{-19} J

The number of layers of sodium that absorbs the incident energy, n = 5

Atomic area of the sodium atom, A_e is 10^{-20} m^2

Hence, the number of conduction electrons in n layers is given as:

n' = n x (A/A_e)

= 5 x [(2 x 10^{-4})/10^{-20}] = 10^{17}

The incident power is absorbed by all the electrons continuously. The amount of energy absorbed per electron per second is

E = P/n'

= $(2 \times 10^{-9})/10^{17}$

= 2×10^{-26} J/s

The time for photoelectric emission

t = Φ_0/E

= $(3.2 \times 10^{-19})/(2 \times 10^{-26}) = 1.6 \times 10^{7}$ s ≈ 0.507 years

The time required for the photoelectric emission is almost half a year. This is not practical. Therefore, the wave function is in disagreement with the given experiment.

Question 11. 31:

Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy? (For quantitative comparison, take the wavelength of the probe equal to 1 Å, which is of the order of interatomic spacing in the lattice) (m_e =9.11 × 10^{-31} kg).

Solution:

For Electrons, Kinetic energy, K.E= (1/2) $m_e v^2$

= $(m_e v)^2/2m$

K.E = $p^2/2m_e$

$$\Rightarrow p = \sqrt{2m_e K.E}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K.E}}$$

$$\lambda^2 = \frac{h^2}{2m_e K.E}$$

$$K.E = \frac{h^2}{2m_e \lambda^2}$$

$$K.E = \frac{(6.64\times10^{-34})^2}{2\times9.1\times10^{-31}\times(10^{-10})^2}$$

K.E = 2.4×10^{-17} J

$$K.E = \frac{2.4\times10^{-17}}{1.6\times10^{-19}}$$

= 149.375 eV

For photon of X-rays, Energy, E = hc/λe

= $(6.63 \times 10^{-34} \times 3 \times 10^{8})/ (10^{-10} \times 1.6 \times 10^{-19})$

= 12.375×10^{3} eV

= 12.375 keV

The energy of the photons of X-rays is more than the energy of the electron.

Question 11.32:

(a) Obtain the de Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in question 11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain. ($m_n = 1.675 \times 10^{-27}$ kg).

(b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature (27 °C). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Solution:

(a) Kinetic energy of the neutron = 150 eV

= $150 \times 1.6 \times 10^{-19}$

= 2.4×10^{-17} J

Mass of the neutron, $m_n = 1.675 \times 10^{-27}$ kg

The kinetic energy of the neutron is given by the relation

K.E = (1/2) $m_e v^2$

K.E = $p^2/2m_e$

$$\Rightarrow p = \sqrt{2m_e K.E}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K.E}}$$

Wavelength and mass are inversely proportional. Wavelength decreases with increase in the mass and vice versa.

$$\lambda = \frac{(6.63 \times 10^{-34})}{\sqrt{2.24 \times 10^{-17} \times 1.675 \times 10^{-27}}}$$

= 2.327×10^{-12} m

In question 11.31 it is given inter-atomic spacing of the crystal is about 1 Å, i.e., 10^{-10} m. The interatomic spacing is about 100 times greater. Therefore, a neutron of kinetic energy is 150 eV is not good for a diffraction experiment.

(b) Room temperature = 27^0 C = 27 + 273 = 300 K

Average kinetic energy of the neutron, E = (3/2) kT

here, k = Boltzmann constant = 1.38×10^{-23} J/mol/K

The wavelength of the neutron is

$$\lambda = \frac{h}{\sqrt{2m_n E}}$$

$$\lambda = \frac{h}{\sqrt{2m_n \frac{3}{2} kT}}$$

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{3\times1.675\times10^{-27}\times1.38\times10^{-23}\times300}}$$

= 1.447×10^{-10} m

This wavelength is comparable to the inter-atomic spacing of the crystal. Hence a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Question 11.33:

An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Solution:

Electrons are accelerated by a voltage = 50 kV

Charge on an electron, e = 1.6×10^{-19} C

Mass of the electron, $m_e = 9.11 \times 10^{-31}$ kg

Wavelength of the yellow light = 5.9×10^{-7} m

The kinetic energy of the electron, E = eV

= $(1.6 \times 10^{-19}) \times (50 \times 10^{3})$

= 8×10^{-15} J

De Broglie wavelength of electron is given as

$$\lambda = \frac{h}{\sqrt{2m_e E}}$$

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times8\times10^{-15}}}$$

= 5.467×10^{-12} m

The wavelength is 10^5 times lesser than the wavelength of the yellow light. The resolving power of the microscope and the wavelength of the light used is inversely proportional. Therefore, the resolving power of the electron microscope is 10^5 times greater than the optical microscope.

Question 11.34:

The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length-scale of 10^{-15} m or less. This structure was first probed in the early 1970s using high energy electron beams produced by a linear accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV).

Solution:

Wavelength of the proton or neutron, $\lambda \approx 10^{-15}$ m

Rest mass-energy of an electron:

$m_0c^2 = 0.511$ MeV

$= 0.511 \times 10^6 \times 1.6 \times 10^{-19}$

$= 0.8176 \times 10^{-13}$ J

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Speed of light, $c = 3 \times 10^8$ m/s

The momentum of the proton or a neutron is given as

$p = h/\lambda$

$= 6.6 \times 10^{-34} / 10^{-15}$

$= 6.6 \times 10^{-19}$ kg m/s

The relativistic relation for energy (E) is given as

$E^2 = p^2c^2 + m^2{}_0C^4$

$= (6.6 \times 10^{-19} \times 3 \times 10^8)^2 + (0.8176 \times 10^{-13})^2$

$= 392.04 \times 10^{-22} + 0.6685 \times 10^{-26}$

$\approx 392.04 \times 10^{-22}$

$\Rightarrow E = 19.8 \times 10^{-11}$

$= 19.8 \times 10^{-11}/1.6 \times 10^{-19}$

$= 12.375 \times 10^8$ eV

Order of energy of these electron beams is 12.375×10^8 eV

Question 11. 35:

Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature (27 °C) and 1 atm pressure, and compare it with the mean separation between two atoms under these conditions.

Solution:

Room temperature, $T = 27^0$ C = 27 + 273 = 300 K

Atmospheric pressure, P = 1 atm = 1.01 x 10^5 Pa

Atomic weight of He atom = 4

Avogadro's number, N_A = 6.023 x 10^{23}

Boltzmann's constant, k = 1.38 x 10^{-23} J/mol/K

Average energy of a gas at temperature T is given as

De Broglie wavelength is given as

$$\lambda = \frac{h}{\sqrt{2mE}}$$

E = (3/2) kT

m = mass of the He atom

= Atomic weight/N_A

= 4/(6.023 x 10^{23})

= 6.64 x 10^{-24} g

m = 6.64 x 10^{-27} kg

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{3\times6.64\times10^{-27}\times1.38\times10^{-23}\times300}}$$

= 0.7268 x 10^{-10} m

We have the ideal gas formula

PV = RT

PV = kNT

V/N = kT/P

Here,

V is the volume of the gas

N is the number of moles of the gas

Mean separation between the two atoms of the gas is given as

$$r = \left[\frac{V}{N}\right]^{1/3} = \left[\frac{kT}{P}\right]^{1/3}$$

$$r = \left[\frac{1.38\times10^{-23}\times300}{1.01\times10^{5}}\right]^{1/3}$$

= 3.35 x 10^{-9} m

The mean separation between the atom is greater than the de Broglie wavelength.

Question 11. 36:

Compute the typical de Broglie wavelength of an electron in metal at 27 °C and compare it with the mean separation between two electrons in a metal which is given to be about 2 × 10^{-10} m. [Note: Exercises 11.35 and 11.36 reveal that while the wave-packets associated with gaseous molecules under ordinary conditions are non-overlapping, the electron wave-packets in a metal strongly overlap with one another. This suggests that whereas molecules in an ordinary gas can be distinguished apart, electrons in a metal cannot be distinguished apart from one another. This indistinguishability has many fundamental implications which you will explore in more advanced Physics courses.]

Solution:

Temperature, T = 27°C = 27 + 273 = 300 K

Mean separation between two electrons, r = 2 × 10^{-10} m

De Broglie wavelength of an electron is

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

h = Planck's constant = 6.6 × 10^{-34} Js

m = Mass of an electron = 9.11 × 10^{-31} kg

k = Boltzmann constant = 1.38 × 10^{-23} J/mol/K.

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{3\times9.11\times10^{-31}\times1.38\times10^{-23}\times300}}$$

≈ 6.2 x 10^{-9} m

Hence, the de Broglie wavelength is much greater than the given inter-electron separation.

Question 11.37:

Answer the following questions:
(a) Quarks inside protons and neutrons are thought to carry fractional charges [(+2/3)e ; (–1/3)e]. Why do they not show up in Millikan's oil-drop experiment?
(b) What is so special about the combination e/m? Why do we not simply talk of e and m separately?
(c) Why should gases be insulators at ordinary pressures and start conducting at very low pressures?
(d) Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?

(e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations:
E = h v, p = h/λ
But while the value of λ is physically significant, the value of v (and therefore, the value of the phase speed v λ) has no physical significance. Why?

Solution:

(a) Quarks inside protons and neutrons are thought to carry fractional charges [(+2/3)e ; (–1/3)e]. This is because the nuclear forces grow stronger if they are pulled apart. Therefore, it seems that fractional charges may exist in nature. The observable charges are an integral multiple of electrical charges (e).

(b) The relation between electric field and the magnetic field,

$eV = (1/2)\, mv^2$ and $eBv = mv^2/r$

Here,

e = electric charge

v = velocity

V = potential

r = Radius

B = magnetic field

From these equations, it can be understood that the dynamics of an electron can be determined only by the ratio e/m and not by e and m separately.

(c) At the atmospheric pressure, the ions in the gas does not have a chance of reaching their respective electrons due to collision and recombination with other molecules in the gas. At low pressures, ions have a chance to reach their respective electrons and results in the flow of current.

(d) The minimum energy required for an electron in the conduction band to get out of the metal is called work function. These electrons occupy different energy levels, because of which for the same incident radiation, electrons come out with different energies.

(e) The absolute value of the energy of a particle is arbitrary within the addictive constant. Therefore, the wavelength(λ) is significant, but the frequency (v) of the electron does not have direct physical significance. Therefore, product vλ has no physical significance.