

**Q1: Choose the correct alternative from the clues given at the end of each statement:**

- (a) The size of the atom in Thomson's model is ..... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)
- (b) ) In the ground state of ..... electrons are in stable equilibrium, while in ..... electrons always experience a net force. (Thomson's model/ Rutherford's model.)
- (c) A classical atom based on ..... is doomed to collapse. (Thomson's model/ Rutherford's model.)
- (d) An atom has a nearly continuous mass distribution in a .....but has a highly non-uniform mass distribution in .....(Thomson's model/ Rutherford's model.)
- (e) The positively charged part of the atom possesses most of the mass in ..... (Rutherford's model/both the models.)

**Solution:**

- (a) The size of the atom in Thomson's model is no different from the atomic size in Rutherford's model.
- (b) In the ground state of Thomson's model electrons are in stable equilibrium, while in Rutherford's model electrons always experience a net force.
- (c) A classical atom based on Rutherford's model is doomed to collapse.
- (d) An atom has a nearly continuous mass distribution in a Thomson's model but has a highly non-uniform mass distribution in Rutherford's model.
- (e) The positively charged part of the atom possesses most of the mass in both the models.

**Q2: Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?**

**Solution:**

We know that the mass of the incident alpha particle ( $6.64 \times 10^{-27}$ kg) is more than the mass of hydrogen ( $1.67 \times 10^{-27}$ Kg). Hence, the target nucleus is lighter, from which we can conclude that the alpha particle would not rebound. Implying to the fact that solid hydrogen isn't a suitable replacement to gold foil for the alpha particle scattering experiment.

**Q3: What is the shortest wavelength present in the Paschen series of spectral lines?**

**Solution:**

We know the Rydberg's formula is written as:

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19} \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

Where,

$h$  = Planck's constant =  $6.6 \times 10^{-34}$

$c = \text{Speed of light} = 3 \times 10^8 \text{ m/s}$

$n_1$  and  $n_2$  are integers

So the shortest wavelength present in the Paschen series of the spectral lines is given for  $n_1 = 3$  and  $n_2 = \infty$

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19} \left[ \frac{1}{3^2} - \frac{1}{\infty^2} \right]$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 9}{21.76 \times 10^{-19}}$$

$$= 8.189 \times 10^7 \text{ m}$$

$$= 818.9 \text{ nm}$$

**Q4: A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?**

**Solution:**

It is given that

Separation of two energy levels in an atom,

$$E = 2.3 \text{ eV} = 2.3 \times 1.6 \times 10^{-19}$$

$$= 3.68 \times 10^{-19} \text{ J}$$

Consider  $\nu$  as the frequency of radiation emitted when the atom transits from the upper level to the lower level.

So the relation for energy can be written as:

$$E = h\nu$$

Where,

$$h = \text{Planck's constant} = 6.62 \times 10^{-34} \text{ Js}$$

$$\nu = E/h$$

$$= \frac{3.68 \times 10^{-19}}{6.62 \times 10^{-34}}$$

$$= 5.55 \times 10^{14} \text{ Hz}$$

$$= 5.6 \times 10^{14} \text{ Hz}$$

Therefore, the frequency of radiation is  $5.6 \times 10^{14} \text{ Hz}$ .

**Q5: The ground state energy of hydrogen atom is  $-13.6 \text{ eV}$ . What are the kinetic and potential energies of the electron in this state?**

**Solution:**

Ground state energy of hydrogen atom,  $E = -13.6 \text{ eV}$

The total energy of hydrogen atom is  $-13.6 \text{ eV}$ . The kinetic energy is equal to the negative of the total energy.

Kinetic energy =  $-E = -(-13.6) = 13.6 \text{ eV}$

Potential energy = negative of two times of kinetic energy.

Potential energy =  $-2 \times (13.6) = -27.2 \text{ eV}$

**Q6: A hydrogen atom initially in the ground level absorbs a photon, which excites it to the  $n = 4$  level. Determine the wavelength and frequency of photon.**

**Solution:**

For ground level,  $n_1 = 1$

Let,  $E_1$  be the energy of this level. It is known that  $E_1$  is related with  $n_1$  as:

$$E_1 = \frac{-13.6}{(n_1)^2} \text{ eV} = \frac{-13.6}{1^2} = -13.6 \text{ eV}$$

The atom is excited to a higher level,  $n_2 = 4$

Let,  $E_2$  be the energy of this level:

$$E_2 = \frac{-13.6}{(n_2)^2} \text{ eV} = \frac{-13.6}{4^2} = \frac{-13.6}{16} \text{ eV}$$

Following is the amount of energy absorbed by the photon:

$$E = E_2 - E_1$$

$$= \frac{-13.6}{16} - \left( \frac{-13.6}{1} \right)$$

$$= \frac{-13.6 \times 15}{16} \text{ eV}$$

$$= \frac{-13.6 \times 15}{16} \times 1.6 \times 10^{-19}$$

$$= 2.04 \times 10^{-18} \text{ J}$$

For a photon of wavelength  $\lambda$ , the expression of energy is written as:

$$E = \frac{hc}{\lambda}$$

Where,

$h$  = Planck's constant =  $6.6 \times 10^{-34} \text{ Js}$

$c$  = Speed of light =  $3 \times 10^8 \text{ m/s}$

$$\lambda = \frac{hc}{E}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.04 \times 10^{-18}}$$

$$\lambda = 97 \text{ nm}$$

$$c = \lambda v$$

$$v = c/\lambda$$

$$= \frac{3 \times 10^8}{9.7 \times 10^{-8}}$$

$$v = 3.1 \times 10^{15} \text{ Hz}$$

Therefore, 97nm and  $3.1 \times 10^{15}$ Hz is the wavelength and frequency of the photon.

**Q.7: (a) Using the Bohr's model, calculate the speed of the electron in a hydrogen atom in the  $n = 1, 2,$  and  $3$  levels. (b) Calculate the orbital period in each of these levels.**

**Solution:**

**(a)** Let  $v_1$  be the orbital speed of the electron in a hydrogen atom in the ground state level,  $n_1 = 1$ . For charge ( $e$ ) of an electron,  $v_1$  is given by the relation,

$$v_1 = \frac{e^2}{n_1 4 \pi \epsilon_0 \left(\frac{h}{2\pi}\right)} = \frac{e^2}{2 \epsilon_0 h}$$

Where,  $e = 1.6 \times 10^{-19} \text{ C}$

$\epsilon_0 =$  Permittivity of free space  $= 8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$

$h =$  Planck's constant  $= 6.62 \times 10^{-34} \text{ J s}$

$$v_1 = \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}} = 0.0218 \times 10^8 = 2.18 \times 10^6 \text{ m/s}$$

For level  $n_2 = 2$ , we can write the relation for the corresponding orbital speed as:

$$v_2 = \frac{e^2}{n_2 2 \epsilon_0 h} = \frac{(1.6 \times 10^{-19})^2}{2 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}} = 1.09 \times 10^6 \text{ m/s}$$

For level  $n_3 = 3$ , we can write the relation for the corresponding orbital speed as:

$$v_3 = \frac{e^2}{n_3 2 \epsilon_0 h} = \frac{(1.6 \times 10^{-19})^2}{2 \times 3 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}} = 7.27 \times 10^5 \text{ m/s}$$

Therefore, in a hydrogen atom, the speed of the electron at different levels that is  $n=1$ ,  $n=2$ , and  $n=3$  is  $2.18 \times 10^6 \text{ m/s}$  and  $1.09 \times 10^6 \text{ m/s}$  and  $7.27 \times 10^5 \text{ m/s}$ .

(b) Let,  $T_1$  be the orbital period of the electron when it is in level  $n_1 = 1$ .

Orbital period is related to orbital speed as:

$$T_1 = \frac{2\pi r_1}{v_1} \quad [\text{Where, } r_1 = \text{Radius of the orbit}] = \frac{n_1^2 h^2 \epsilon_0}{\pi m e^2}$$

$h$  = Planck's constant =  $6.62 \times 10^{-34}$  J s

$e$  = Charge on an electron =  $1.6 \times 10^{-19}$  C

$\epsilon_0$  = Permittivity of free space =  $8.85 \times 10^{-12}$  N<sup>-1</sup> C<sup>2</sup> m<sup>-2</sup>

$m$  = Mass of an electron =  $9.1 \times 10^{-31}$  kg

$$T_1 = \frac{2\pi r_1}{v_1} = \frac{2\pi \times (1)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{2.18 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} = 15.27 \times 10^{-17} = 1.527 \times 10^{-16} \text{ s}$$

For level  $n_2 = 2$ , we can write the period as:

$$T_2 = \frac{2\pi r_2}{v_2} \quad [\text{Where, } r_2 = \text{Radius of the electron in } n_2 = 2] = \frac{(n_2)^2 h^2 \epsilon_0}{\pi m e^2}$$

$$= \frac{2\pi \times (2)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{1.09 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} = 1.22 \times 10^{-15} \text{ s}$$

For level  $n_3 = 3$ , we can write the period as:

$$T_3 = \frac{2\pi r_3}{v_3} \quad [\text{Where, } r_3 = \text{Radius of the electron in } n_3 = 3] = \frac{(n_3)^2 h^2 \epsilon_0}{\pi m e^2}$$

$$= \frac{2\pi \times (3)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{7.27 \times 10^5 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} = 4.12 \times 10^{-15} \text{ s}$$

Therefore,  $1.52 \times 10^{-16}$ s,  $1.22 \times 10^{-15}$ s and  $4.12 \times 10^{-15}$ s are the orbital periods in each levels.

**Q8: The radius of the innermost electron orbit of a hydrogen atom is  $5.3 \times 10^{-11}$  m. What are the radii of the  $n = 2$  and  $n = 3$  orbits?**

**Solution:**

The radius of the innermost orbit of a hydrogen atom,  $r_1 = 5.3 \times 10^{-11}$  m.

Let  $r_2$  be the radius of the orbit at  $n = 2$ . It is related to the radius of the inner most orbit as:

$$r_2 = (n)^2 r_1 = 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10} \text{ m}$$

For  $n = 3$ , we can write the corresponding electron radius as:

$$r_3 = (n)^2 r_1 = 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10} \text{ m}$$

Therefore,  $2.12 \times 10^{-10}$  m and  $4.77 \times 10^{-10}$  m are the radii of an electron for  $n = 2$  and  $n = 3$  orbits respectively.

**Q9: A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?**

**Solution:**

It is given that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5 eV. Also, the energy of the gaseous hydrogen in its ground state at room temperature is -13.6 eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes  $-13.6 + 12.5$  eV i.e., -1.1 eV.

Orbital energy is related to orbit level (n) as:

$$E = \frac{-13.6}{n^2} \text{ eV}$$

For  $n = 3$ ,  $E = -13.6 / 9 = -1.5$  eV

This energy is approximately equal to the energy of gaseous hydrogen.

It can be concluded that the electron has jumped from  $n = 1$  to  $n = 3$  level.

During its de-excitation, the electrons can jump from  $n = 3$  to  $n = 1$  directly, which forms a line of the Lyman series of the hydrogen spectrum.

We have the relation for wave number for Lyman series as:

$$\frac{1}{\lambda} = R_y \left( \frac{1}{1^2} - \frac{1}{n^2} \right)$$

Where,  $R_y =$  Rydberg constant  $= 1.097 \times 10^7 \text{ m}^{-1}$

$\lambda =$  Wavelength of radiation emitted by the transition of the electron

For  $n = 3$ , we can obtain  $\lambda$  as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left( \frac{1}{1^2} - \frac{1}{3^2} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left( 1 - \frac{1}{9} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left( \frac{8}{9} \right)$$

$$\lambda = \frac{9}{8 \times 1.097 \times 10^7} = 102.55 \text{ nm}$$

If the electron jumps from  $n = 2$  to  $n = 1$ , then the wavelength of the radiation is given as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left( \frac{1}{1^2} - \frac{1}{2^2} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left( 1 - \frac{1}{4} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{3}{4}\right)$$

$$\lambda = \frac{4}{3 \times 1.097 \times 10^7} = 121.54 \text{ nm}$$

If the transition takes place from  $n = 3$  to  $n = 2$ , then the wavelength of the radiation is given as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{5}{36}\right)$$

$$\lambda = \frac{36}{5 \times 1.097 \times 10^7} = 656.33 \text{ nm}$$

This radiation corresponds to the Balmer series of the hydrogen spectrum.

Therefore, there are two wavelengths that are emitted in Lyman series and they are approximately 102.5 nm and 121.5 nm and one wavelength in the Balmer series which is 656.33 nm.

**Q10: In accordance with the Bohr's model, find the quantum number that characterizes the earth's revolution around the sun in an orbit of radius  $3 \times 10^{11}$  m with orbital speed  $3 \times 10^4$  m/s. (Mass of earth =  $6.0 \times 10^{24}$  kg.)**

**Solution:**

Radius of the orbit of the Earth around the Sun,  $r = 1.5 \times 10^{11}$  m

Orbital speed of the Earth,  $v = 3 \times 10^4$  m/s

Mass of the Earth,  $m = 6.0 \times 10^{24}$  kg

According to Bohr's model, angular momentum is quantized and given as:

$$mvr = \frac{n\hbar}{2\pi}$$

Where,

$h$  = Planck's constant =  $6.62 \times 10^{-34}$  J s

$n$  = Quantum number

$$n = \frac{mvr2\pi}{\hbar} = \frac{2\pi \times 6 \times 10^{24} \times 3 \times 10^4 \times 1.5 \times 10^{11}}{6.62 \times 10^{-34}} = 25.61 \times 10^{73} = 2.6 \times 10^{74}$$

Therefore, the earth revolution can be characterized by the quantum number  $2.6 \times 10^{74}$ .

**Additional Questions:**

**Q11: Choose a suitable solution to the given statements which justify the difference between Thomson's model and Rutherford's model**

**(a) In the case of scattering of alpha particles by a gold foil, average angle of deflection of alpha particles stated by Rutherford's model is (less than, almost the same as, much greater than) stated by Thomson's model.**

**(b) Is the likelihood of reverse scattering (i.e., dispersing of  $\alpha$ -particles at points more prominent than  $90^\circ$ ) anticipated by Thomson's model ( considerably less, about the same, or much more prominent ) than that anticipated by Rutherford's model?**

**(c) For a small thickness T, keeping other factors constant, it has been found that amount of alpha particles scattered at direct angles is proportional to T. This linear dependence implies?**

**(d) To calculate average angle of scattering of alpha particles by thin gold foil, which model states its wrong to skip multiple scattering?**

**Solution:**

**(a)** almost the same

The normal point of diversion of alpha particles by a thin gold film anticipated by Thomson's model is about the same as from anticipated by Rutherford's model. This is on the grounds that the average angle was taken in both models.

**(b)** much less

The likelihood of scattering of alpha particles at points more than  $90^\circ$  anticipated by Thomson's model is considerably less than that anticipated by Rutherford's model.

**(c)** Dispersing is predominantly because of single collisions. The odds of a single collision increment linearly with the amount of target molecules. Since the number of target particles increment with an expansion in thickness, the impact likelihood depends straightly on the thickness of the objective.

**(d)** Thomson's model

It isn't right to disregard multiple scattering in Thomson's model for figuring out the average angle of scattering of alpha particles by a thin gold film. This is on the grounds that a solitary collision causes almost no deflection in this model. Subsequently, the watched normal scattering edge can be clarified just by considering multiple scattering.

**Q12: The gravitational attraction amongst proton and electron in a hydrogen atom is weaker than the coulomb attraction by a component of around  $10^{-40}$ . Another option method for taking a gander at this case is to assess the span of the first Bohr circle of a hydrogen particle if the electron and proton were bound by gravitational attraction. You will discover the appropriate response fascinating.**

**Solution:**

Radius of the first Bohr orbit is given by the relation:

$$r_1 = \frac{4\pi\epsilon_0 \left(\frac{\hbar}{2\pi}\right)^2}{m_e e^2} \quad \text{---(1)}$$

Where,

$\epsilon_0$  = Permittivity of free space

$h$  = Planck's constant =  $6.63 \times 10^{-34}$  J s

$m_e$  = Mass of an electron =  $9.1 \times 10^{-31}$  kg

$e$  = Charge of an electron =  $1.9 \times 10^{-19}$  C

$m_p$  = Mass of a proton =  $1.67 \times 10^{-27}$  kg

$r$  = Distance between the electron and the proton

Coulomb attraction between an electron and a proton is:

$$F_C = \frac{e^2}{4\pi\epsilon_0 r^2} \text{ ---(2)}$$

Gravitational force of attraction between an electron and a proton is:

$$F_G = \frac{Gm_e m_p}{r^2} \text{ ---(3)}$$

Where,

$G$  = Gravitational constant =  $6.67 \times 10^{-11}$  N m<sup>2</sup>/kg<sup>2</sup>

Considering electrostatic force and the gravitational force between an electron and a proton to be equal, we get:

$$\therefore F_G = F_C$$

$$\frac{Gm_e m_p}{r^2} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$Gm_e m_p = \frac{e^2}{4\pi\epsilon_0} \text{ ---(4)}$$

Putting the value of equation (4) in equation (1), we get:

$$r_1 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_e m_p}$$

$$r_1 = \frac{\left(\frac{6.63 \times 10^{-34}}{2 \times 3.14}\right)^2}{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times (9.1 \times 10^{-31})^2} = 1.21 \times 10^{29} \text{ m}$$

It is known that the universe is 156 billion light years wide or  $1.5 \times 10^{27}$  m wide. Hence, we can conclude that the radius of the first Bohr orbit is much greater than the estimated size of the whole universe.

**Q13: Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level  $n$  to level  $(n-1)$ . For large  $n$ , show that this frequency equals the classical frequency of revolution of the electron in the orbit.**

**Solution:**

It is given that the hydrogen atom de-excites from the level  $n$  to level  $(n-1)$ .

The equation for energy ( $E_1$ ) of the radiation at the level  $n$  is

$$E_1 = hv_1 = \frac{hme^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \times \left(\frac{1}{n^2}\right) \text{ -----(1)}$$

Here,

$v_1$  is the frequency at level  $n$

$h$  = Planck's constant

$m$  = mass of the hydrogen atom

$e$  = Electron charge

$\epsilon_0$  = Permittivity of free space

The energy ( $E_2$ ) of the radiation at level  $(n-1)$  is

$$E_2 = hv_2 = \frac{hme^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \times \left(\frac{1}{(n-1)^2}\right) \text{ -----(2)}$$

Here,

$v_2$  is the frequency at level  $(n - 1)$

Energy ( $E$ ) is released as a result of de-excitation

$$E = E_2 - E_1$$

$$hv = E_2 - E_1 \text{ -----(3)}$$

Here,

$v$  = Frequency of the radiation emitted

Substituting (1) and (2) in (3) we get

$$\begin{aligned} v &= \frac{me^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \left[ \frac{1}{(n-1)^2} - \frac{1}{n^2} \right] \\ &= \frac{me^4(2n-1)}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^2(n-1)^2} \end{aligned}$$

For large  $n$ , we can write  $(2n - 1) \approx 2n$  and  $(n-1) \approx n$

$$\text{Therefore, } v = \frac{me^4}{32\pi^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \text{ -----(4)}$$

Classical relation of the frequency of revolution of an electron is given as

$$v_c = v/2\pi r \text{ -----(5)}$$

In the  $n^{\text{th}}$  orbit, the velocity of the electron is

$$v = \frac{e^2}{4\pi\epsilon_0 \left(\frac{h}{2\pi}\right) n} \quad \text{---(6)}$$

The radius of the  $n^{\text{th}}$  orbit  $r$  is given as

$$r = \frac{4\pi\epsilon_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2} n^2 \quad \text{-----(7)}$$

Putting the equation (6) and equation (7) in equation (5) we get

$$v = \frac{m_e e^4}{32\pi^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad \text{-----(8)}$$

Therefore, the frequency of radiation emitted by the hydrogen atom is equal to the classical orbital frequency.

**Q 14:** Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom ( $\sim 10^{-10}\text{m}$ ).

(a) Construct a quantity with the dimensions of length from the fundamental constants  $e$ ,  $m_e$ , and  $c$ . Determine its numerical value.

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves  $c$ . But energies of atoms are mostly in a non-relativistic domain where  $c$  is not expected to play any role. This is what may have suggested Bohr discard  $c$  and look for 'something else' to get the right atomic size. Now, the Planck's constant  $h$  had already made its appearance elsewhere. Bohr's great insight lay in recognising that  $h$ ,  $m_e$ , and  $e$  will yield the right atomic size. Construct a quantity with the dimension of length from  $h$ ,  $m_e$ , and  $e$  and confirm that its numerical value has indeed the correct order of magnitude.

**Solution:**

(a)

Charge of an electron,  $e = 1.6 \times 10^{-19} \text{ C}$

Mass of the electron,  $m_e = 9.1 \times 10^{-31} \text{ kg}$

Speed of light,  $c = 3 \times 10^8 \text{ m/s}$

The equation from the given quantities is given as,

$$\frac{e^2}{4\pi\epsilon_0 m_e c^2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

$\epsilon_0$  is the permittivity of free space

The numerical value of the quantity is

$$= 9 \times 10^9 \times [(1.6 \times 10^{-19})^2 / 9.1 \times 10^{-31} \times (3 \times 10^8)^2]$$

$$= 2.81 \times 10^{-15} \text{ m}$$

The numerical value of the equation taken is much lesser than the size of the atom.

(b)

Charge of an electron,  $e = 1.6 \times 10^{-19} \text{ C}$

Mass of the electron,  $m_e = 9.1 \times 10^{-31} \text{ kg}$

Planck's constant,  $h = 6.63 \times 10^{-34} \text{ Js}$

Considering a quantity involving all these values as

$$\frac{4\pi\epsilon_0 \left[ \frac{h}{2\pi} \right]^2}{m_e e^2}$$

Where,  $\epsilon_0 =$  Permittivity of free space

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{C}^{-2}$$

The numerical value of the above equation is

$$4\pi\epsilon_0 \times \frac{\left( \frac{h}{2\pi} \right)^2}{m_e e^2}$$

$$= \frac{1}{9 \times 10^9} \times \frac{\left( \frac{6.63 \times 10^{-34}}{2 \times 3.14} \right)^2}{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$= 0.53 \times 10^{-10} \text{ m}$$

The numerical value of the quantity is of the order of the atomic size.

**Q 15: The total energy of an electron in the first excited state of the hydrogen atom is about  $-3.4 \text{ eV}$ .**

**(a) What is the kinetic energy of the electron in this state?**

**(b) What is the potential energy of the electron in this state?**

**(c) Which of the answers above would change if the choice of the zero of potential energy is changed?**

**Solution:**

(a) Total energy of the electron,  $E = -3.4 \text{ eV}$

The kinetic energy of the electron is equal to the negative of the total energy.

$$\text{K.E} = -E$$

$$= -(-3.4) = +3.4 \text{ eV}$$

$$\text{Kinetic energy} = +3.4 \text{ eV}$$

(b) Potential energy (U) of the electron is equal to the negative of twice of the kinetic energy,

$$\text{P.E} = -2 (\text{K.E})$$

$$= -2 \times 3.4 = -6.8 \text{ eV}$$

$$\text{Potential energy} = -6.8 \text{ eV}$$

(c) The potential energy of the system depends on the reference point. If the reference point is changed from zero then the potential energy will change. The total energy is given by the sum of the potential energy and kinetic energy. Therefore, the total energy will also change.

**Q16: If Bohr's quantization postulate (angular momentum =  $nh/2\pi$ ) is a basic law of nature, it should be equally valid for the case of planetary motion as well. Why then do we never speak of quantization of orbits of planets around the sun?**

**Solution:**

The quantum level for a planetary motion is considered to be continuous. This is because the angular momentum associated with planetary motion is largely relative to the value of Planck's constant ( $h$ ).  $10^{70}h$  is the order of the angular momentum of the Earth in its orbit. As the values of  $n$  increase, the angular momenta decreases. So, the planetary motion is considered to be continuous.

**Q17: Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon ( $\mu^-$ ) of mass about  $207m_e$  orbits around a proton].**

**Solution:**

Mass of a negatively charged muon,  $m_\mu = 207 m_e$

According to Bohr's model:

$$\text{Bohr radius, } r_e \propto \left(\frac{1}{m_e}\right)$$

And, energy of a ground state electronic hydrogen atom,  $E_e \propto m_e$

Also, the energy of a ground state muonic hydrogen atom,  $E_\mu \propto m_\mu$

We have the value of the first Bohr orbit,  $r_e = 0.53 \text{ \AA} = 0.53 \times 10^{-10} \text{ m}$

Let,  $r_\mu$  be the radius of muonic hydrogen atom.

Following is the relation at equilibrium:

$$m_\mu r_\mu = m_e r_e$$

$$207 m_e \times r_\mu = m_e r_e$$

$$r_\mu = (0.53 \times 10^{-10}) / 207 = 2.56 \times 10^{-13} \text{ m}$$

Hence, for a muonic hydrogen atom  $2.56 \times 10^{-13}$  m is the value of first Bohr radius.

We have,

$$E_e = -13.6 \text{ eV}$$

Considering the ratio of the energies we get:

$$\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu} = \frac{m_e}{207 m_e}$$

$$E_\mu = 207 E_e = 207 \times (-13.6) = -2.81 \text{ k eV}$$

-2.81 k eV is the ground state energy of a muonic hydrogen atom.

