Q 13.1 (a) Lithium has two stable isotopes $^{6}_{3}Li$ and $^{7}_{3}Li$ have respective abundances of 7.5% and 92.5%.

These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, $^{10}_{5}B$ and $^{11}_{5}B$. Their respective masses are 10.01294 u and 11.00931 u,

and the atomic mass of boron is 10.811 u. Find the abundances of $^{10}_{5}B$ and $^{11}_{5}B$.

Solution:

(a) Mass of $^{6}_{3}Li$ lithium isotope, m_1 = 6.01512 u

Mass of $^{7}_{3}Li$ lithium isotope , m_2 = 7.01600 u

Abundance of $^{6}_{3}Li$, n_1= 7.5%

Abundance of $^{7}_{3}Li$, n_2= 92.5%

The atomic mass of lithium atom is:

$$m = \frac{m_1 n_1 + m_2 n_2}{n_1 + n_2}$$

$$m = \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{7.5 + 92.5}$$

= 6.940934 u

(b) Mass of boron isotope $^{10}_{5}B$ m_1 = 10.01294 u

Mass of boron isotope $^{11}_{5}B$ m_2 = 11.00931 u

Abundance of $^{10}_{5}B$, n_1 = x%

Abundance of $^{11}_{5}B$, n_2= (100 − x)%

Atomic mass of boron, m = 10.811 u

The atomic mass of boron atom is:

$$m = \frac{m_1 n_1 + m_2 n_2}{n_1 + n_2}$$

$$10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + 100 - x}$$

1081.11 = 10.01294x + 1100.931 – 11.00931 x

x = 19.821/0.99637 = 19.89 %

And 100 − x = 80.11%

Hence, the abundance of ${}_{5}^{10}B$ is 19.89% and that of ${}_{5}^{11}B$ is 80.11%.

Q 13.2 :The three stable isotopes of neon: ${}_{10}^{20}Ne$, ${}_{10}^{21}Ne$ and ${}_{10}^{22}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Solution:

Atomic mass of ${}_{10}^{20}Ne$, m_1 = 19.99 u

Abundance of ${}_{10}^{20}Ne$, n_1 = 90.51%

Atomic mass of ${}_{10}^{21}Ne$, m_2 = 20.99 u

Abundance of ${}_{10}^{21}Ne$, n_2 = 0.27%

Atomic mass of ${}_{10}^{22}Ne$, m_3 = 21.99 u

Abundance of ${}_{10}^{22}Ne$, n_3 = 9.22%

Below is the average atomic mass of neon:

$$m = \frac{m_1 n_1 + m_2 n_2 + m_3 n_3}{n_1 + n_2 + n_3}$$

$$= \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{90.51 + 0.27 + 9.22}$$

= 20.1771 u

Q 13.3: Obtain the binding energy in MeV of a nitrogen nucleus ${}_{7}^{14}N$, given m(${}_{7}^{14}N$)=14.00307 u

Solution:

Atomic mass of nitrogen ${}_{7}^{14}N$, m = 14.00307 u

A nucleus of ${}_{7}^{14}N$ nitrogen contains 7 neutrons and 7 protons.

$\Delta m = 7m_H + 7m_n - m$ is the mass defect the nucleus

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

= 7.054775 + 7.06055 − 14.00307

= 0.11236 u

But 1 u = 931.5 MeV/c^2

$\Delta m = 0.11236 \times 931.5$ MeV/c^2

$E_b = \Delta mc^2$ is the binding energy of the nucleus

Where, c =Speed of light

$$E_b = 0.11236 \times 931.5\left(\frac{MeV}{c^2}\right) \times c^2$$

= 104.66334 Mev

Therefore, 104.66334 MeV is the binding energy of the nitrogen nucleus.

Q 13.4: Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data:

m ($^{56}_{23}Fe$) = 55.934939 u

m($^{209}_{83}Bi$) = 208.980388 u

Solution:

Atomic mass of $^{56}_{26}Fe$, m_1 = 55.934939 u

$^{56}_{26}Fe$ nucleus has 26 protons and (56 − 26) = 30 neutrons

Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$

= 26.20345 + 30.25995 − 55.934939

= 0.528461 u

But 1 u = 931.5 MeV/c^2

Δm = 0.528461 × 931.5 MeV/c^2

$E_{b1} = \Delta mc^2$ is the binding energy of the nucleus.

Where, c = Speed of light

$$E_{b_1} = 0.528461 \times 931.5 \left(\frac{MeV}{c^2}\right) \times c^2$$

= 492.26 MeV

Average binding energy per nucleon = $\frac{492.26}{56} = 8.79 MeV$

Atomic mass of $^{209}_{83}Bi$, m_2 = 208.980388 u

$^{209}_{83}Bi$ nucleus has 83 protons and (209 − 83) 126 neutrons.

The mass defect of the nucleus is given as:

$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\Delta m'$ = 83 × 1.007825 + 126 × 1.008665 − 208.980388

= 83.649475 + 127.091790 − 208.980388

= 1.760877 u

But 1 u = 931.5 MeV/c^2

$\Delta m'$ = 1.760877 × 931.5 MeV/c^2

$E_{b2} = \Delta m'c^2$ is the binding energy of the nucleus.

$$= 1.760877 \times 931.5 \left(\frac{MeV}{c^2}\right) \times c^2$$

= 1640.26 MeV

Average binding energy per nucleon = 1640.26/209 = 7.848 MeV

Q 13.5: A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ with mass = 62.92960 u.

Solution:

Mass of a copper coin, m' = 3 g

Atomic mass of $^{63}_{29}Cu$ atom, m = 62.92960 u

The total number of $^{63}_{29}Cu$ atoms in the coin, $N = \frac{N_A \times m'}{Mass\ number}$

Where,

N_A = Avogadro's number = 6.023×10^{23} atoms /g

Mass number = 63 g

$N = \frac{6.023 \times 10^{23} \times 3}{63}$ = 2.868×10^{22} atoms

$^{63}_{29}Cu$ nucleus has 29 protons and (63 – 29) 34 neutrons

Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$

= 0.591935 u

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22}$

$= 1.69766958 \times 10^{22}$ u

But 1 u = 931.5 MeV/c^2

$\Delta m = 1.69766958 \times 10^{22} \times 931.5$ MeV/c^2

$E_b = \Delta mc^2$ is the binding energy of the nuclei of the coin

$= 1.69766958 \times 10^{22} \times 931.5$ MeV/c^2 x c^2

$= 1.581 \times 10^{25}$ MeV

But 1 MeV = 1.6×10^{-13} J

$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$

$= 2.5296 \times 10^{12}$ J

Therefore, the energy required to separate all the neutrons and protons from the given coin is 2.5296 × 10^{12} J

Q 13.6: Write nuclear reaction equations for

(i) α-decay of $^{226}_{88}Ra$ (ii) α-decay of $^{242}_{94}Pu$

(iii) β^--decay of $^{32}_{15}P$ (iv) β^- -decay of $^{210}_{83}Bi$

(v) β^+-decay of $^{11}_{6}C$ (vi) β^+ -decay of $^{97}_{43}Tc$

(vii) Electron capture of $^{120}_{54}Xe$

Solution:

In helium, α is a nucleus $^{4}_{2}He$ and β is an electron (e^- for β^- and e^+ for β^+). 2 protons and 4 neutrons is lost in

every α decay. Whereas 1 proton and a neutrino is emitted from the nucleus in every β^+ decay. In every β^- decay, there is a gain of 1 proton and an anti-neutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

(i) $^{226}_{88}Ra \rightarrow \ ^{222}_{86}Rn + \ ^{4}_{2}He$

(ii) $^{242}_{94}Pu \rightarrow \ ^{238}_{92}U + \ ^{4}_{2}He$

(iii) $^{32}_{15}P \rightarrow \ ^{32}_{16}S + e^- + \bar{v}$

(iv) $^{210}_{83}B \rightarrow \ ^{210}_{84}PO + e^- + \bar{v}$

(v) $^{11}_{6}C \rightarrow \ ^{11}_{5}B + e^+ + v$

(vi) $^{97}_{43}Tc \rightarrow \ ^{97}_{42}MO + e^+ + v$

(vii) $^{120}_{54}Xe + e^+ \rightarrow \ ^{120}_{53}I + v$

Q 13.7: A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

Solution:

The half-life of the radioactive isotope = T years

N_0 is the actual amount of radioactive isotope.

(a) After decay, the amount of the radioactive isotope = N

It is given that only 3.125% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

But, $\frac{N}{N_0} = e^{-\lambda t}$

Where, λ = Decay constant

t = Time

$$e^{-\lambda t} = \frac{1}{32}$$

$$-\lambda t = ln1 – ln32$$

$$-\lambda t = 0 – 3.4657$$

Since, $-\lambda = \frac{0.693}{T}$

t = $\frac{3.466}{\frac{0.693}{T}} \approx 5T years$

Hence, the isotope will take about 5T years to reduce to 3.125% of its original value.

(b) After decay, the amount of the radioactive isotope = N

It is given that only 1% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$e^{-\lambda t} = \frac{1}{100}$$

$$-\lambda t = ln1 – ln100$$

$e^{-\lambda t} = 0 - 4.6052$

t = 4.6052/ λ

Since, λ = 0.693/T

$t = \frac{4.6052}{\frac{0.693}{T}}$ = 6.645 T years

Hence, the isotope will take about 6.645T years to reduce to 1% of its original value.

Q 13.8: The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive $_{6}^{14}C$ present with the stable carbon isotope $_{6}^{12}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of $_{6}^{14}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of $_{6}^{14}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Solution:

Decay rate of living carbon-containing matter, R = 15 decay/min

Let N be the number of radioactive atoms present in a normal carbon-containing matter.

Half life of $_{6}^{14}C, T_{\frac{1}{2}}$ = 5730 years

The decay rate of the specimen obtained from the Mohenjo-Daro site:

R' = 9 decays/min

Let N' be the number of radioactive atoms present in the specimen during the Mohenjo-Daro period.

Therefore, the relation between the decay constant, λ and time, t is:

$\frac{N}{N'} = \frac{R}{R'} = e^{-\lambda t}$

$e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$

$-\lambda t = log_e \frac{3}{5} = -0.5108$

$t = \frac{0.5108}{\lambda}$

But $\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{5730}$

$t = \frac{0.5108}{\frac{0.693}{5730}} = 4223.5 years$

So, the approximate age of the Indus-Valley civilization is 4223.5 years.

Q 13.9: Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.

Solution:

The strength of the radioactive source is given as:

$\frac{dN}{dt} = 8.0 mCi$

= 8 x 10^{-3} x 3.7 x 10^{10}

= 29.6 x 10^{7} decay/s

Where,

N = Required number of atoms

Half-life of $^{60}_{27}Co$ T 1/2 = 5.3 years

= 5.3 × 365 × 24 × 60 × 60

= 1.67 × 10^{8} s

The rate of decay for decay constant λ is:

$\frac{dN}{dt} = \lambda N$

Where, $\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{1.67 \times 10^{8}} s^{-1}$

$N = \frac{1}{\lambda} \frac{dN}{dt}$

$= \frac{29.6 \times 10^7}{\frac{0.693}{1.67 \times 10^8}}$

= 7.133 x 10^{16} atoms

For $^{60}_{27}Co$:

Mass of 6.023 × 10^{23} (Avogadro's number) atoms = 60 g

Mass of atoms 7.133 x 10^{16} atoms = $= \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}}$

= 7.106 x 10^{-6} g

Hence, the amount of $^{60}_{27}Co$ necessary for the purpose is 7.106 × 10^{-6} g.

Q 13.10 : The half-life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

Solution:

Half life of $^{90}_{38}Sr$, $t_{\frac{1}{2}}$ = 28 years

= 28 × 365 × 24 × 60 × 60

= 8.83 × 10^8 s

Mass of the isotope, m = 15 mg

90 g of $^{90}_{38}Sr$ atom contains 6.023 × 10^{23} (Avogadro's number) atoms.

Therefore, 15 mg of $^{90}_{38}Sr$ contains atoms:

= $\frac{6.023 \times 10^{23} \times 15 \times 10^{-3}}{90}$

i.e 1.0038 x 10^{20} number of atoms

Rate of disintegration, $\frac{dN}{dt} = \lambda N$

Where,

λ = decay constant = $\frac{0.693}{8.83 \times 10^8} s^{-1}$

$\frac{dN}{dt} = \frac{0.693 \times 1.0038 \times 10^{20}}{8.83 \times 10^8}$

= 7.878 x 10^{10} atoms/s

Hence, the disintegration rate of 15 mg of $^{90}_{38}Sr$ is 7.878 × 10^{10} atoms/s.

Q 13.11: Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the $^{107}_{47}Ag$ silver isotope .

Solution:

Nuclear radius of the gold isotope $^{197}_{79}Au$ = R_{Au}

Nuclear radius of the silver isotope $^{107}_{47}Ag$ = R_{Ag}

Mass number of gold, A_{Au} = 197

Mass number of silver, A_{Ag} = 107

Following is the relationship of the radii of the two nuclei and their mass number:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{R_{Au}}{R_{Ag}}\right)^{\frac{1}{3}} = \left(\frac{197}{107}\right)^{\frac{1}{3}}$$

= 1.2256

Hence, 1.23 is the ratio of the nuclear radii of the gold and silver isotopes.

Q 13.12: Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of

(a) $^{226}_{88}Ra$

(b) $^{220}_{86}Rn$

Given

$m(^{226}_{88}Ra)$ **= 226.02540 u,** $m(^{222}_{86}Rn)$ **= 222.01750 u,**

$m(^{220}_{86}Rn)$ **= 220.01137 u,** $m(^{216}_{84}Po)$ **= 216.00189 u.**

Solution:

(a) Alpha particle decay of $^{226}_{88}Ra$ emits a helium nucleus. As a result, its mass number reduces to 222 (226 − 4) and its atomic number reduces to 86 (88 − 2). This is shown in the following nuclear reaction.

$$^{226}_{88}Ra \rightarrow ^{222}_{86}Ra + ^{4}_{2}He$$

Q-value of emitted α-particle = (Sum of initial mass − Sum of final mass) c^2

Where, c = Speed of light

It is given that:

$m(^{226}_{88}Ra)$ = 226.02540 u

$m(^{222}_{86}Rn)$ = 222.01750 u

$m(^{4}_{2}He)$ = 4.002603 u

Q-value = [226.02540 − (222.01750 + 4.002603)] u c^2

= 0.005297 u c^2

But 1 u = 931.5 MeV/c^2

Q = 0.005297 × 931.5 ≈ 4.94 MeV

Kinetic energy of the α-particle = $\left(\frac{Mass\ number\ after\ decay}{Mass\ number\ before\ decay}\right) \times Q = \frac{222}{226} \times 4.94$ = 4.85 MeV

(b) Alpha particle decay of $^{220}_{86}Rn$

$$^{220}_{86}Ra \rightarrow ^{216}_{84}Po + ^{4}_{e}He$$

It is given that:

Mass of $^{220}_{86}Rn$ = 220.01137 u

Mass of $^{216}_{84}Po$ = 216.00189 u

Q-value =[220.01137 – (216.00189 + 4.00260)] x 931.5

≈ 641 MeV

Kinetic energy of the α-particle = $= \left(\frac{220-4}{220}\right) \times 6.41$

= 6.29 MeV

Q 13.13: The radionuclide ^{11}C decays according to

$$^{11}_{6}C \rightarrow ^{11}_{5}B + e^{+} + v : \quad T_{1/2} = 20.3 \text{ min}$$

The maximum energy of the emitted positron is 0.960 MeV.
Given the mass values:
m ($^{11}_{6}$ C) = 11.011434 u and

m ($^{11}_{6}$B) = 11.009305 u,
calculate Q and compare it with the maximum energy of the positron emitted.

Solution:

$$^{11}_{6}C \rightarrow ^{11}_{5}B + e^{+} + v : \quad T_{1/2} = 20.3 \text{ min}$$

Mass defect in the reaction is $\Delta m = m(_6C^{11}) - m(_5B^{11}) - m_e$

This is given in terms of atomic masses. To express in terms of nuclear mass, we should subtract $6m_e$ from carbon, $5m_e$ from boron.

$\Delta m = m(_6C^{11}) - 6m_e - (m(_5B^{11}) - 5m_e) - m_e$

$= m(_6C^{11}) - 6m_e - m(_5B^{11}) + 5m_e - m_e$

$= m(_6C^{11}) - m(_5B^{11}) - 2m_e$

$\Delta m = [11.011434 - 11.009305 - 2 \times 0.000548]$ u

= 0.002129 – 0.001096 = 0.001033

Q = Δm x 931 MeV

= 0.001033 x 931

Q= 0.9617 MeV

The Q-factor of the reaction is equal to the maximum energy of the emitted positron.

Q 13.14: The nucleus $^{23}_{10}Ne$ decays β^- by emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m(^{23}_{10}Ne)$ **= 22.994466 u**

$m(^{23}_{11}Na)$ **= 22.989770 u.**

Solution:

In β^- emission, the number of protons increases by 1, and one electron and an antineutrino is emitted from the parent nucleus.

β^- emission from the nucleus $^{23}_{10}Ne$.

$$^{23}_{10}Ne \rightarrow ^{23}_{11}Na + e^- + \bar{v} + Q$$

It is given that:

Atomic mass of $m(^{23}_{10}Ne)$ = 22.994466 u

Atomic mass of $m(^{23}_{11}Na)$ = 22.989770 u

Mass of an electron, m_e = 0.000548 u

Q-value of the given reaction is given as:

$$Q = [m(^{23}_{10}Ne) - [m(^{23}_{11}Na) + m_e]]c^2$$

There are 10 electrons and 11 electrons in $^{23}_{10}Ne$ and $^{23}_{11}Na$ respectively. Hence, the mass of the electron is cancelled in the Q-value equation.

$Q = [\ 22.994466 - 22.989770\ ]\ c^2$

$= (0.004696\ c^2)$ u

But 1 u = 931.5 MeV/c^2

Q = 0.004696 x 931.5 = 4.374 MeV

The daughter nucleus is too heavy as compared to e^- and $\bar{v}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the Q-value, i.e., 4.374 MeV.

Q 13.15: The Q value of a nuclear reaction A + b → C + d is defined by $Q = [\ m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) ${}_1^1H +_1^3 \ H \rightarrow_1^2 \ H +_1^2 \ H$

(ii) ${}_6^{12}C +_6^{12} \ C \rightarrow_{10}^{20} \ Ne +_2^4 \ He$

Atomic masses are given to be

$m({}_1^2H) = \ 2.014102 \ u$

$m({}_1^3H) = \ 3.016049 \ u$

$m({}_6^{12}C) = \ 12.000000 \ u$

$m({}_{10}^{20}Ne) = \ 19.992439 \ u$

Solution:

(i) The given nuclear reaction is:

${}_1^1H +_1^3 \ H \rightarrow_1^2 \ H +_1^2 \ H$

It is given that:

Atomic mass $m({}_1^1H) = \ 1.007825 \ u$

Atomic mass $m({}_1^3H) = \ 3.016049 \ u$

Atomic mass $m({}_1^2H) = \ 2.014102 \ u$

According to the question, the Q-value of the reaction can be written as:

Q = [$m({}_1^1H)$ + $m({}_1^3H)$ – 2m $m({}_1^2H)$] c^2

= [1.007825 + 3.016049 – 2 x 2.014102] c^2

Q = (– 0.00433 c^2) u

But 1 u = 931.5 MeV/c^2

Q = -0.00433 x 931.5 = – 4.0334 MeV

The negative Q-value of the reaction shows that the reaction is endothermic.

(ii) The given nuclear reaction is:

${}_6^{12}C +_6^{12} \ C \rightarrow_{10}^{20} \ Ne +_2^4 \ He$

It is given that:

Atomic mass of $m(^{12}_{6}C) = \quad 12.000000\ u$

Atomic mass of $m(^{20}_{10}Ne) = \quad 19.992439\ u$

Atomic mass of $m(^{4}_{2}He) = \quad 4.002603\ u$

The Q-value of this reaction is given as :

Q = [$2m(^{12}_{6}C)$ – $m(^{20}_{10}Ne)$ – $m(^{4}_{2}He)$] c^2

= [2 x 12.000000 – 19.992439 – 4.002603] c^2

= [0.004958 c^2] u

= 0.004958 x 931.5 = 4.618377 MeV

Since we obtained positive Q-value, it can be concluded that the reaction is exothermic.

Q 13.16: Suppose, we think of fission of a $^{56}_{26}Fe$ nucleus into two equal fragments, $^{28}_{13}Al$. Is the fission energetically possible? Argue by working out Q of the process. Given

$m(^{56}_{26}Fe) = \quad 55.93494\ u$

$m(^{28}_{13}Al) = \quad 27.98191\ u$

Solution:

The fission of $^{56}_{26}Fe$ can be given as :

$^{56}_{26}Fe$ -> 2 $^{28}_{13}Al$

It is given that:

Atomic mass of $m(^{56}_{26}Fe) = \quad 55.93494\ u$

Atomic mass of $m(^{28}_{13}Al) = \quad 27.98191\ u$

The Q-value of this nuclear reaction is given as:

Q = [$m(^{56}_{26}Fe) – 2m(^{28}_{13}Al)$] c^2

= [55.93494 – 2 x 27.98191] c^2

= ($-0.02888\ c^2$) u

But 1 u = 931.5 MeV/c^2

Q = – 0.02888 x 931.5 = -26.902 MeV

Since the Q-value is negative for the fission, it is energetically not possible.

Q 13.17: The fission properties $^{239}_{94}Pu$ of are very similar to those of $^{235}_{92}U$ .The average energy released per fission is 180 MeV. How much energy is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission ?

Solution:

Average energy released per fission of $^{239}_{94}Pu$, E_{av} = 180MeV

Amount of pure $^{239}_{94}Pu$, m = 1 kg = 1000 g

N_A= Avogadro number = 6.023×10^{23}

Mass number of $^{239}_{94}Pu$ = 239 g

1 mole of $^{239}_{94}Pu$ contains N_A atoms.

Therefore, mg of $^{239}_{94}Pu$ contains $\left(\frac{N_A}{Mass\ Number} \times m\right)$ atoms

$\frac{6.023 \times 10^{23}}{239} \times\ 1000 =\ 2.52 \times\ 10^{24}$ atoms

Total energy released during the fission of 1 kg of $^{239}_{94}Pu$ is calculated as:

E = E_{av} x 2.52 x 10^{24}

= 180 x 2.52 x 10^{24}

= 4.536 x 10^{26} MeV

Hence, 4.536 x 10^{26} MeV is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission.

Q 13.18: A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much $_{92}^{235}$ U did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $_{92}^{235}$U and that this nuclide is consumed only by the fission process.

Solution:

Reactor consumes half its fuel is 5 years. Therefore, the half-life of the fuel of the fission reactor, $t_{1/2}$ = 5 x 365 x 24 x 60 x 60 s

200 MeV is released during the fission of 1 g of $_{92}^{235}U$.

1 mole, i.e., 235 g of $_{92}^{235}U$ contains 6.023 x 10^{23} atoms.

Number of atoms in 1 g of $_{92}^{235}U$ is (6.023 x 10^{23}/235) atoms.

The total energy generated per gram of $_{92}^{235}U$ is (6.023 x 10^{23}/235) x 200 MeV/g

$$= \frac{200 \times 6.023 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^{6}}{235}$$

= 8.20 x 10^{10} J/g

The reactor operates only 80% of the time.

Therefore, the amount of $_{92}^{235}U$ consumed in 5 years by the 1000 MW fission reactor is

$$= \frac{5 \times 365 \times 24 \times 60 \times 60 \times 1000 \times 10^{6}}{8.20 \times 10^{10}} \times \frac{80}{100}$$

= 12614400/8.20

= 1538341 g

= 1538 Kg

Initial amount of $_{92}^{235}U$ is 2 x 1538 = 3076 Kg

Q 13. 19: How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$_1^2H + \ _1^2H \rightarrow \ _2^3He + \ n + \ 3.27 MeV$$

Solution:

The fusion reaction given is

$$_1^2H + \ _1^2H \rightarrow \ _2^3He + \ n + \ 3.27 MeV$$

Amount of deuterium, m = 2 kg

1 mole, i.e., 2 g of deuterium contains 6.023 x 10^{23} atoms.

Therefore, 2 Kg of deuterium contains (6.023 x 10^{23}/2) x 2000 = 6.023 x 10^{26} atoms

From the reaction given, it can be understood that 2 atoms of deuterium fuse, 3.27 MeV energy are released. The total energy per nucleus released during the fusion reaction is

E = (3.27/2) x 6.023 x 10^{26} MeV

= (3.27/2) x 6.023 x 10^{26} x 1.6 x 10^{-19} x 10^{6}

= 1.576 x 10^{14} J

Power of the electric lamp, P = 100 W = 100 J/s

The energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is 1.576 x 10^{14} /100 = 1.576 x 10^{12}s

= (1.576 x 10^{12})/ (365 x 24 x 60 x 60)

= (1.576 x 10^{12})/3.1536 x10^{7}

= 4.99 x 10^{4} years

Q 13.20: Calculate the height of the potential barrier for a head-on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Solution:

When two deuterons collide head-on, the distance between their centers, d is given as:

Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 f m = 2 × 10^{-15} m

d = 2 × 10^{-15} + 2 × 10^{-15} = 4 × 10^{-15} m

Charge on a deuteron nucleus = Charge on an electron = e = 1.6 × 10^{-19} C

Potential energy of the two-deuteron system:

$$v = \frac{e^2}{4\pi\epsilon_0 d}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 Nm^2C^{-2}$$

Therefore,

$$V = \frac{9\times10^9\times(1.6\times10^{-19})^2}{4\times10^{-15}} J$$

$$V = \frac{9\times10^9\times(1.6\times10^{-19})^2}{4\times10^{-15}\times(1.6\times10^{-19})} eV$$

= 360 k eV

Hence, the height of the potential barrier of the two-deuteron system is 360 k eV.

Q 13.21: From the relation $R = R_0A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Solution:

Nuclear radius is given as

$R = R_0A^{1/3}$

Here,

R_0 is Constant

A is the mass number of the nucleus

Nuclear matter density, ρ = Mass of the nucleus/Volume of the nucleus

Mass of the nucleus = mA

Density of the nucleus = $(4/3)\pi R^3$

$= (4/3)\pi(R_0A^{1/3})^3$

$= (4/3)\pi R_0^3A$

$\rho = mA/[(4/3)\pi R_0^3A]$

$= 3mA/(4\pi R_0^3A)$

$\rho = 3m/(4\pi R_0^3)$

Nuclear matter density is nearly a constant and is independent of A.

Q 13.22: For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K-shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + {}_Z^A X \rightarrow {}_{Z-1}^{A} Y + v$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

Solution:

The chemical equations of the two competing process are written as follows:

$${}_A^Z\mathrm{X} \rightarrow {}_A^{Z-1}\mathrm{Y} + e + \nu + Q_1$$ [Positron Emission]

$$e^{-1} + {}_A^Z\mathrm{X} \rightarrow {}_A^{Z-1}\mathrm{Y} + v + Q_2$$ [Electron Capture]

The Q-value of the positron emission reaction is determined as follows :

$$Q_1 = [m_N(^A_ZX) - m_N(^A_{Z-1}Y) - m_e]c^2$$

$$Q_1 = [m(^A_ZX) - Zm_e - m(^A_{Z-1}Y) + (Z-1)m_e - m_e]c^2$$

$$Q_1 = [m(^A_ZX) - m(^A_{Z-1}Y) - 2m_e]c^2$$

The Q-value of the electron capture reaction is determined as follows:

$$Q_2 = [m_N(^A_ZX) + m_e - m_N(^A_{Z-1}Y)]c^2$$

$$Q_2 = [m(^A_ZX) - m(^A_{Z-1}Y)]c^2$$

In the equation, m_N is the mass of nucleus and m denotes the mass of atom.

From equation (1) and (2), we understand that,

$Q_1 = Q_2 - 2m_ec^2$

From the above relation, we can infer that if Q1 > 0, then Q2 > 0; Also, if Q2> 0, it does not necessarily mean that Q1 > 0.

In other words, this means that if $\beta +$ emission is energetically allowed, then the electron capture process is

necessarily allowed, but not vice-versa. This is because the Q-value must be positive for an energetically allowed nuclear reaction.

Q 13.23: In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are $^{24}_{12}Mg$ (23.98504u), $^{25}_{12}Mg$(24.98584u) and $^{26}_{12}Mg$ (25.98259u). The natural abundance of $^{24}_{12}Mg$ is 78.99% by mass. Calculate the abundances of the other two isotopes.

Solution:

Let the abundance of $^{25}_{12}Mg$ be x%

The abundance of $^{26}_{12}Mg$ = (100 – 78.99 -x)%

= (21.01 – x)%

The average atomic mass of magnetic

$$24.312 = \frac{23.98504 \times 78.99 + 24.98584x + 25.98259(21.01 - x))}{100}$$

24.312 x 100= 1894.57 + 24.98584x + 545. 894 – 25.98259x

2431.2 = 2440.46 – 0.99675x

0.99675x = 2440.46 – 2431.2

0.99675x = 9.26

x = 9.26/0.99675

x = 9.290%

Abundance of $^{25}_{12}Mg$ is 9.290%

The abundance of $^{26}_{12}Mg$ = (21.01 – x)%

= (21.01 -9.290)% = 11.71%

Q 13. 24: The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei $^{41}_{20}Ca$ and $^{27}_{13}Al$ from the following data:
$m(^{40}_{20}Ca)$ = 39.962591 u
$m(^{41}_{20}Ca)$ = 40.962278 u
$m(^{26}_{13}Al)$ = 25.986895 u
$m(^{27}_{13}Al)$ = 26.981541 u

Solution:

When the nucleons are separated from $_{20}Ca^{41} \rightarrow {}_{20}Ca^{40} + {}_0n^1$

Mass defect, Δm = m $(_{20}Ca^{40})$ + m_n – m$(_{20}Ca^{41})$

= 39.962591 + 1.008665 – 40.962278

= 0.008978 amu

Neutron separation energy = 0.008978 x 931 MeV = 8.362 MeV

Similarly, $_{13}Al^{27} \rightarrow {}_{13}Al^{26} + {}_0n^1$

Mass defect, Δm = m $(_{13}Al^{26})$ + m_n – m$(_{13}Al^{27})$

= 25.986895 + 1.008665 – 26.981541

= 26.99556 – 26.981541 = 0.014019 amu

Neutron separation energy = 0.014019 x 931 MeV

= 13.051 MeV

Q 13. 25: A source contains two phosphorous radionuclides $^{32}_{15}P$ ($T_{1/2}$ = 14.3d) and $^{33}_{15}P$($T_{1/2}$ = 25.3d). Initially, 10% of the decays come from $^{33}_{15}P$. How long one must wait until 90% do so?

Solution:

Rate of disintegration is

– (dN/dt) ∝ N

Initially 10% of the decay is due to $^{33}_{15}P$ and 90% of the decay comes from $^{32}_{15}P$.

We have to find the time at which 90% of the decay is due to ${}^{33}_{15}P$ and 10% of the decay comes from ${}^{32}_{15}P$.

Initially, if the amount of ${}^{33}_{15}P$ is N, then the amount of ${}^{32}_{15}P$ is 9N.

Finally, if the amount of ${}^{33}_{15}P$ is 9N', then the amount of ${}^{32}_{15}P$ is N'.

For ${}^{32}_{15}P$,

$$\frac{N'}{9N} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$N' = 9N\left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$N' = 9N\,(2)^{-t/14.3} \text{ --(1)}$$

For ${}^{33}_{15}P$

$$\frac{9N'}{N} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$9N' = N\left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$9N' = N\,(2)^{-t/25.3} \text{ --(2)}$$

On dividing, (1) by (2)

$$\frac{1}{9} = 9 \times 2^{\frac{t}{25.3} - \frac{t}{14.3}}$$

$$\frac{1}{81} = 2^{-\frac{11t}{25.3 \times 14.3}}$$

$$log1 - log81 = -\frac{11t}{25.3 \times 14.3} log2$$

$$-\frac{11t}{25.3 \times 14.3} = \frac{0 - 1.908}{0.301}$$

$$t = \frac{25.3 \times 14.3 \times 1.908}{11 \times 0.301} \approx 208.5\ days$$

Hence, it will take about 208.5 days for 90% decay of ${}_{15}P^{33}$.

Q 13. 26: Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$$^{223}_{88}\text{Ra} \rightarrow \ ^{209}_{82}\text{Pb} + \ ^{14}_{6}\text{C}$$

$$^{223}_{88}\text{Ra} \rightarrow \ ^{219}_{86}\text{Rn} + \ ^{4}_{2}\text{He}$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Solution:

$$^{223}_{88}\text{Ra} \rightarrow \ ^{209}_{82}\text{Pb} + \ ^{14}_{6}\text{C}$$

$\Delta m = m\,({}_{88}Ra^{223}) - m\,({}_{82}Pb^{209}) - m({}_{6}C^{14})$

= 223.01850 – 208.9817 – 14.00324

= 0.03419 u

The amount of energy released is given as

Q = Δm x 931 MeV

= 0.03419 x 931 MeV = 31.83 MeV

$$^{223}_{88}\text{Ra} \rightarrow \ ^{219}_{86}\text{Rn} + \ ^{4}_{2}\text{He}$$

$\Delta m = m\,({}_{88}Ra^{223}) - m\,({}_{86}Rn^{219}) - m({}_{2}He^{4})$

= 223.01850 – 219.00948 – 4.00260

= 0.00642 u

Q = 0.00642 x 931 MeV = 5.98 MeV

As both the Q-factor values are positive the reaction is energetically allowed.

Q 13. 27: Consider the fission of ${}^{238}_{92}U$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are ${}^{140}_{58}Ce$ and ${}^{99}_{44}Ru$. Calculate Q for this fission process. The relevant atomic and particle masses are

$m({}^{238}_{92}U)$ =238.05079 u
$m({}^{140}_{58}Ce)$ =139.90543 u
$m({}^{99}_{44}Ru)$ = 98.90594 u

Solution:

In the fission of ${}^{238}_{92}U$, 10 β^- particles decay from the parent nucleus. The nuclear reaction can be written as:

$$^{238}_{92}\mathrm{U} + \ ^{1}_{0}\mathrm{n} \longrightarrow \ ^{140}_{58}\mathrm{Ce} + \ ^{99}_{44}\mathrm{Ru} + \ 10^{0}_{-1}e$$

Given:

$m_1 = (^{238}{}_{92}U) = 238.05079$ u
$m_2 = (^{140}{}_{58}Ce) = 139.90543$ u
$m_3 = (^{99}{}_{44}Ru) = 98.90594$ u

$m_4 = {}^1{}_0n = 1.008665$ u

The Q value is given as

$$Q = [\,m'(^{238}{}_{92}U) + m\,(^1{}_0n) - m'(^{140}{}_{58}Ce) - m'(^{99}{}_{44}Ru) - 10m_e\,]\,c^2$$

Here,

m' is the atomic masses of the nuclei

$m'(^{238}{}_{92}U) = m_1 - 92\ m_e$

$m'(^{140}{}_{58}Ce) = m_2 - 58m_e$

$m'(^{99}{}_{44}Ru) = m_3 - 44m_e$

Therefore, $Q = [m_1 - 92\ m_e + m_4 - m_2 + 58m_e - m_3 + 44m_e - 10m_e]\ c^2$

$= [m_1 + m_4 - m_2 - m_3]c^2$

$= [238.0507 + 1.008665 - 139.90543 - 98.90594]c^2$

$= 0.247995\ c^2$ u

$[1\ u = 931.5\ MeV/c^2]$
$Q = 0.247995 \times 931.5 = 231.007$ MeV

The Q- value of the fission process is 231.007 MeV

Q 13. 28: Consider the D–T reaction (deuterium–tritium fusion)

$$^{2}_{1}\mathrm{H} + \ ^{3}_{1}\mathrm{H} \rightarrow \ ^{4}_{2}\mathrm{He} + \ \mathrm{n}$$

(a) Calculate the energy released in MeV in this reaction from the data:
$m(^2{}_1H) = 2.014102$ u
$m(^3{}_1H) = 3.016049$ u
(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles = 2(3kT/2); k = Boltzman's constant, T = absolute temperature.)

Solution:

$${}_{1}^{2}\mathrm{H} + {}_{1}^{3}\mathrm{H} \rightarrow {}_{2}^{4}\mathrm{He} + \mathrm{n}$$

The Q value is given as

Q = Δm x 931 MeV

= (m ($_1H^2$) + m($_1H^3$) − m($_2He^4$) − m_n) x 931

=(2.014102 + 3.016049- 4.002603-1.00867) x 931

Q = 0.0188 x 931 = 17.58 MeV

(b) Repulsive potential energy of two nuclei when they touch each other is

$$= \frac{e^2}{4\pi\epsilon_0(2r)}$$

Where ε_0 = Permittivity of free space

$\frac{1}{4\Pi\epsilon_0}$ =9 x 10^9 Nm^2C^{-2}

$$= \frac{9\times10^9\times(1.6\times10^{-19})^2}{2\times2\times10^{-15}} joule$$

= (23.04 x 10^{-29})/(4 x 10^{-15})

= 5. 76 x 10^{-14} J

Kinetic energy needed to overcome the coulomb repulsion between the two nuclei is

K.E = 5. 76 x 10^{-14} J

K.E= 2 x (3/2) KT

K is the Boltzmann constant = 1.38 x 10^{-23}

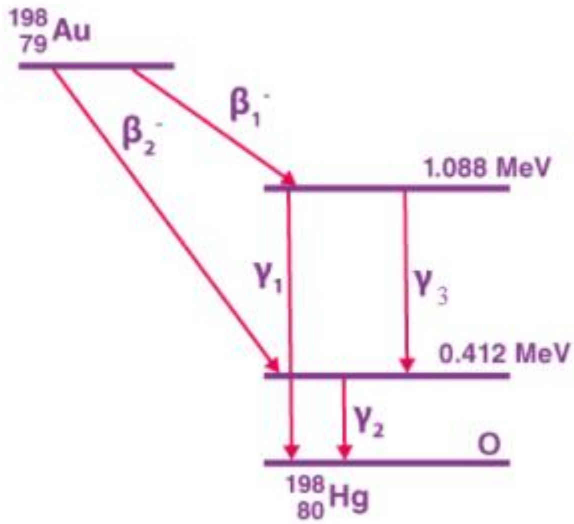
T = K.E/3K = 5. 76 x 10^{-14}/(3 x 1.38 x 10^{-23})

= 5. 76 x 10^{-14}/(4.14 x 10^{-23})

= 1.3913 x 10^9 K

Q 13. 29: Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 13.6. You are given that

m(^{198}Au) = 197.968233 u
m(^{198}Hg) =197.966760 u



Solution:

From the γ decay diagram it can be seen that γ_1 decays from the 1.088 MeV energy level to the 0 MeV energy level.
Energy corresponding to γ_1 decay is given as
$E_1 = 1.088 - 0 = 1.088 MeV$
$h\nu_1 = 1.088 \times 1.6 \times 10^{-19} \times 10^6 J$
Where,
h= Planck's constant = 6.6×10^{-34} Js

ν_1 = frequency of radiation radiated by γ_1 decay.

$$v_1 = \frac{E_1}{h} = \frac{1.088 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 2.637 \times 10^{20} Hz$$

From the diagram it can observed that γ_2 decays from the 0.412 MeV energy level to the 0 MeV energy level.
Energy corresponding to γ_2 decay is given as
$E_2 = 0.412 - 0 = 0.412 MeV$
$h\nu_2 = 0.412 \times 1.6 \times 10^{-19} \times 10^6 J$
Where ν_2 is the frequency of the radiation radiated by γ_2 decay.

Therefore, $\nu_2 = E_2/h$

$$v_2 = \frac{E_2}{h} = \frac{0.412 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}}$$

$= 9.988 \times 10^{19}$ Hz
From the gamma decay diagram, it can be γ_3 decays from the 1.088 MeV energy level to the 0.412 MeV energy level.
Energy corresponding to γ_3 is given as
$E_3 = 1.088 - 0.412 = 0.676$ MeV

$hv_3 = 0.676\times10^{-19}\times10^6$ J
Where v_3 = frequency of the radiation radiated by γ_3 decay

Therefore, $v_3 = E_3/h$

$$v_3 = \frac{E_3}{h} = \frac{0.676\times1.6\times10^{-19}\times10^6}{6.6\times10^{-34}}$$

$=1.639\times10^{20}$ Hz
The mass of Au is 197.968233 u

Mass of Hg = 197.966760 u

[$1u=931.5MeV/c^2$]
The energy of the highest level is given as :
$E=[m(^{198}_{78}Au - m(^{190}_{80}Hg)]$

=197.968233−197.96676=0.001473u

=0.001473×931.5=1.3720995MeV

β_1 decays from the 1.3720995 MeV level to the 1.088 MeV level

Therefore, the maximum kinetic energy of the β_1 particle =1.3720995−1.088=0.2840995MeV

β_2 decays from the 1.3720995 MeV level to the 0.412 MeV level
Therefore, the maximum kinetic energy of the β_2 particle =1.3720995−0.412=0.9600995

Q 13.30. Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within the sun and b) the fission of 1.0 kg of ^{235}U in a fission reactor.

Solution:

Amount of hydrogen, m = 1 kg = 1000 g

(a) 1 mole, i.e., 1 g of hydrogen contains 6.023×10^{23} atoms. Therefore, 1000 g of hydrogen contains $6.023 \times 10^{23} \times 1000$ atoms.

In Sun, four hydrogen nuclei fuse to form a helium nucleus and will release energy of 26 MeV

Energy released by the fusion of 1 kg of hydrogen,

$E_1 = (6.023 \times 10^{23} \times 1000 \times 26)/4$

$E_1 = 39.16 \times 10^{26}$ MeV

(b) Amount of ^{235}U = 1 kg = 1000 g

1 mole, i.e., 235 g of uranium contains 6.023×10^{23} atoms. Therefore, 1000 g of uranium contains $(6.023 \times 10^{23} \times 1000)/235$ atoms.

The energy released during the fission of one atom of ^{235}U = 200 MeV

Energy released by the fission of 1 Kg of ^{235}U,

E_2= (6.023 x 10^{23} x 1000 x 200)/235

E_2 = 5.1 x 10^{26} MeV

(E_1/E_2) = (39.16 x 10^{26}/5.1 x 10^{26})

= 7.67

The energy released in the fusion of hydrogen is 7.65 times more than the energy released during the fission of Uranium.

Q 13.31. Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten per cent of which was to be obtained from nuclear power plants. Suppose we are given that, on average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200MeV.

Solution:

Electric power to be generated = 2 x 10^5 MW = 2 x 10^5 x 10^6 J/s = 2 x 10^{11} J/s

10 % of the amount is obtained from the nuclear power plant

P_1 = (10/100) x 2 x 10^{11} x 60 x 60 x 24 x 365 J/year

Heat energy released during per fission of a ^{235}U nucleus, E = 200 MeV

Efficiency of the reactor = 25%

The amount of energy converted to electrical energy in fission is (25/100) x 200 = 50 MeV

= 50 x 1.6 x 10^{-19} x 10^6 = 80 x 10^{-13} J

Required number of atoms for fission per year

= [(10/100) x 2 x 10^{11} x 60 x 60 x 24 x 365]/80 x 10^{-13}

= 788400 x 10^{23} atoms

1 mole, i.e., 235 g of ^{235}U contains 6.023 x 10^{23} atoms

Mass of 6.023 x 10^{23} atoms of ^{235}U = 235 x 10^{-3} kg

Mass of 788400 x 10^{23} atoms of ^{235}U

=[(235 x 10^{-3})/(6.023 x 10^{23})] x 78840 x 10^{24}

= 3. 076 x 10^4 Kg

The mass of uranium needed per year is 3.076 x 10^4 Kg