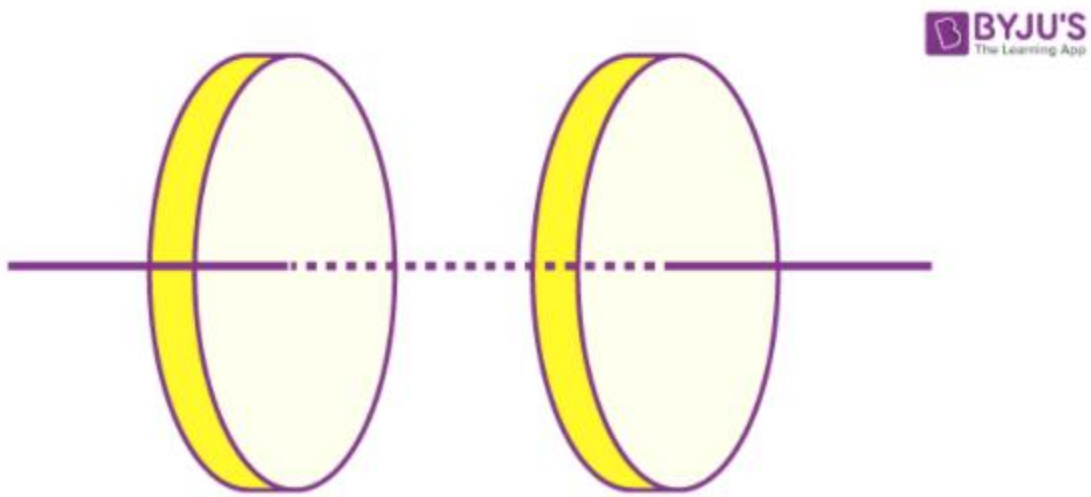
Q 8.1) The Figure shows a capacitor made of two circular plates each of radius 12 cm and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15A.

(a) Calculate the capacitance and the rate of change of the potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.



Solution:

Given Values:

The radius of each circular plate (r) is 12 cm or 0.12 m

The distance between the plates (d) is 5 cm or 0.05 m

The charging current (I) is 0.15 A

The permittivity of free space is $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

(a) The capacitance between the two plates can be calculated as follows:

$$C = \frac{\varepsilon_0 A}{d}$$

where,

A = Area of each plate = πr^2 $C = \frac{\varepsilon_0 \pi r^2}{d}$

= $\frac{8.85 \times 10^{-12} \times \pi (0.12)^2}{0.05}$

= $8.0032 \times 10^{-12}\ F$

= 80.032 pF

The charge on each plate is given by,

q = CV

where,

V is the potential difference across the plates

Differentiation on both sides with respect to time (t) gives:

$$\frac{dq}{dt} = C\frac{dV}{dt}$$

But, $\frac{dq}{dt}$ = Current (I)

$$\therefore \frac{dV}{dt} = \frac{I}{C}$$

=> $\frac{0.15}{80.032\times10^{-12}} = 1.87 \times 10^{9}\ V/s$

Therefore, the change in the potential difference between the plates is $1.87 \times 10^{9}\ V/s$.

(b) The displacement current across the plates is the same as the conduction current. Hence, the displacement current, id is 0.15 A.

(c) Yes

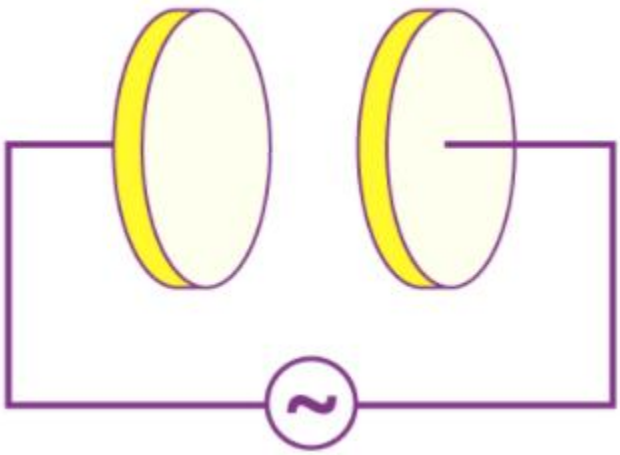
Kirchhoff's first rule is valid at each plate of the capacitor provided that we take the sum of conduction and displacement for current.

Q 8.2) A parallel plate capacitor made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 pF. The capacitor is connected to a 230 V ac supply with an (angular) frequency of 300 rad s^{-1}.

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.



Solution:

Radius of each circular plate, R = 6.0 cm = 0.06 m

Capacitance of a parallel plate capacitor, C = 100 pF = $100 \times 10^{-12}\ F$

Supply voltage, V = 230 V

Angular frequency, $\omega = 300\ rad\ s^{-1}$

(a) Rms value of conduction current, I = $\frac{V}{X_c}$

Where,

X_c = Capacitive reactance = $\frac{1}{\omega C}$

$\therefore\ I = V \times \omega C$

= $230 \times 300 \times 100 \times 10^{-12}$

= $6.9 \times 10^{-6} A$

= $6.9\ \mu A$

Hence, the rms value of conduction current is $6.9\ \mu A$.

(b) Yes, conduction current is equivalent to displacement current.

(c) Magnetic field is given as:

$$B = \frac{\mu_0 r}{2\pi R^2} I_0$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ N\ A^{-2}$

I_0 = Maximum value of current = $\sqrt{2}\ I$

r = Distance between the plates from the axis = 3.0 cm = 0.03 m

$$\therefore\ B = \frac{4\pi \times 10^{-7} \times 0.03 \times \sqrt{2} \times 6.9 \times 10^{-6}}{2\pi \times (0.06)^2}$$

= $1.63 \times 10^{-11}\ T$

Hence, the magnetic field at that point is $1.63 \times 10^{-11}\ T$.

Q 8.3) What physical quantity is the same for X-rays of wavelength 10^{-10}m, the red light of wavelength 6800 Å and radiowaves of wavelength 500m?

Solution:

The speed of light (3 x 10^8 m/s) in a vacuum is the same for all wavelengths. It is independent of the wavelength in the vacuum.

Q 8.4) A plane electromagnetic wave travels in vacuum along the z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

Solution:

The electromagnetic wave travels in a vacuum along the z-direction. The electric field (E) and the magnetic field (H) are in the x-y plane. They are mutually perpendicular.

Frequency of the wave, v = 30 MHz = 30 x 10^6 s^{-1}

Speed of light in vacuum, C = 3 x 10^8 m/s

Wavelength of a wave is given as:

$$\lambda = \frac{c}{v}$$

= $\frac{3 \times 10^8}{30 \times 10^6}$ = 10 m

Q 8.5) A radio can tune in to any station in the 7.5 MHz to 12 MHz bands. What is the corresponding wavelength band?

Solution:

A radio can tune to minimum frequency, $v_1 = 7.5\ MHz = 7.5 \times 10^6\ Hz$

Maximum frequency, $v_2 = 12\ MHz = 12 \times 10^6\ Hz$

Speed of light, c = $3 \times 10^8\ m/s$

Corresponding wavelength for v_1 can be calculated as:

$$\lambda_1 = \frac{c}{v_1}$$

$$= \frac{3 \times 10^3}{7.5 \times 10^6} = 40\ m$$

Corresponding wavelength for v_2 can be calculated as:

$$\lambda_2 = \frac{c}{v_2}$$

$$= \frac{3 \times 10^3}{12 \times 10^6} = 25\ m$$

Thus, the wavelength band of the radio is 40 m to 25 m.

Q 8.6) A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

Solution:

The frequency of an electromagnetic wave produced by the oscillator is the same as that of a charged particle oscillating about its mean position i.e., 10^9 Hz.

Q 8.7) The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is B_0=510 nT. What is the amplitude of the electric field part of the wave?

Solution:

Amplitude of magnetic field of an electromagnetic wave in a vacuum,

$$B_0 = 510\ nT = 510 \times 10^{-9}\ T$$

Speed of light in vacuum, c = $3 \times 10^8\ m/s$

Amplitude of electric field of an electromagnetic wave is given by the relation,

$$E = cB_0 = 3 \times 10^8 \times 510 \times 10^{-9} = 153\ N/C$$

Therefore, the electric field part of the wave is 153 N/C.

Q 8.8) Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120\ N/C$ and that its frequency is v = 50 MHz.(a) Determine $B_0,\ \omega,\ k\ and\ \lambda$ (b) Find expressions for E and B.

Solution:

Electric field amplitude, $E_0 = 120\ N/C$

Frequency of source, v = 50 MHz = 50×10^6 Hz

Speed of light, c = 3×10^8 m/s

(a) Magnitude of magnetic field strength is given as:

$$B_0 = \frac{E_0}{c}$$

$$= \frac{120}{3 \times 10^8}$$

$$= 40 \times 10^{-8} = 400 \times 10^{-9} T = 400\ nT$$

Angular frequency of source is given by:

$$\omega = 2\pi v = 2\pi \times 50 \times 10^6 = 3.14 \times 10^8\ rads^{-1}$$

$= 3.14 \times 10^8$ rad/s

Propagation constant is given as:

$$k = \frac{\omega}{c}$$

$$= \frac{3.14 \times 10^8}{3 \times 10^8} = 1.05\ rad/m$$

Wavelength of wave is given by:

$\lambda = \frac{c}{v}$

$= \frac{3\times10^8}{50\times10^6} = 6.0$ m

(b) Suppose the wave is propagating in the positive x-direction. Then, the electric field vector will be in the positive y-direction and the magnetic field vector will be in the positive z-direction. This is because all three vectors are mutually perpendicular.

Equation of electric field vector is given as:

$$\overline{E} = E_0 \, sin(kx - \omega t) \, \widehat{j}$$

$$= 120 \, sin[1.05x - 3.14 \times 10^8 t] \, \widehat{j}$$

And, magnetic field vector is given as:

$$\overline{B} = B_0 \, sin(kx - \omega t) \, \widehat{k}$$

$$\overline{B} = (400 \times 10^{-9}) sin[1.05x - 3.14 \times 10^8 t] \, \widehat{k}$$

Q 8.9) The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula E = hv (for the energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

Solution:

The energy of a photon is given as:

E = hv = $\frac{hc}{\lambda}$

Where,

h = Planck's constant = $6.6 \times 10^{-34} \, Js$

c = Speed of light = $3 \times 10^8 \, m/s$

If the wavelength λ is in metre and the energy is in Joule, then by dividing E by 1.6×10^{-19} will convert the energy into eV.

$$E = \frac{hc}{\lambda \times 1.6 \times 10^{-19}}\, eV$$

a) For Gamma rays, the wavelength ranges from 10^{-10} to 10^{-14} m, therefore the photon energy can be calculated as follows:

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{10^{-10} \times 1.6 \times 10^{-19}} = 12.4 \times 10^{3} \approx 10^{4}\, eV$$

Therefore,

$$\lambda = 10^{-10}\, m, energy = 10^{4}\, eV$$

$$\lambda = 10^{-14}\, m, energy = 10^{8}\, eV$$

The energy for Gamma rays ranges from 10^4 to 10^8 eV.

b) The wavelength for X-rays ranges between 10^{-8} m to 10^{-13} m

For $\lambda = 10^{-8}$,

$$Energy = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{10^{-8} \times 1.6 \times 10^{-19}} = 124 \approx 10^{2}\, eV$$

For $\lambda = 10^{-13}$ m, energy = 10^7 eV

c) For ultraviolet radiation, the wavelength ranges from 4×10^{-7} m to 6×10^{-7} m.

For 4×10^{-7} m,

$$Energy = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{4 \times 10^{-7} \times 1.6 \times 10^{-19}} = 3.1 \approx 10^{10}\, eV$$

For 6×10^{-7} m, the energy is equal to 10^3 eV.

The energy of the ultraviolet radiation varies between 10^{10} to 10^3 eV.

d) For visible light, the wavelength ranges from 4×10^{-7} m to 7×10^{-7} m.

For 4×10^{-7}, the energy is the same as above, that is 10^{10} eV

For 7×10^{-7} m, the energy is 100 eV

e) For infrared radiation, the wavelength ranges between 7×10^{-7} m to 7×10^{-14} m.

The energy for 7×10^{-7} m is 100 eV

The energy for 7×10^{-14} m is 10^{-3} eV

f) For microwaves, the wavelength ranges from 1 mm to 0.3 m.

For 1 mm, the energy is 10^{-3} eV.
For 0.3 m, the energy is 10^{-6} eV.

g) For radio waves, the wavelength ranges from 1 m to few km.

For 1 m, the energy is 10^{-6} eV.

The photon energies for the different parts of the spectrum of a source indicate the spacing of the relevant energy levels of the source

Q 8.10) In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 V m^{-1}.
(a) What is the wavelength of the wave?
(b) What is the amplitude of the oscillating magnetic field?
(c) Show that the average energy density of the E field equals the average energy density of the B field. [c = 3×10^8 m s^{-1}]

Solution:

Frequency of the electromagnetic wave, v = $2 \times 10^{10}\ Hz$

Electric field amplitude, E_0 = 48 V m^{-1}

Speed of light, c = 3×10^8 m/s

(a) Wavelength of a wave is given as:

$$\lambda = \frac{c}{v}$$

$$= \frac{3\times10^8}{2\times10^{10}} = 0.015\ m$$

(b) Magnetic field strength is given as:

$$B_0 = \frac{E_0}{c}$$

$$= \frac{48}{3\times10^8} = 1.6 \times 10^{-7}\ T$$

(c) Energy density of the electric field is given as:

$$U_E = \frac{1}{2}\epsilon_0 E^2$$

And, energy density of the magnetic field is given as:

$$U_B = \frac{1}{2\mu_0}B^2$$

Where,

ϵ_0 = Permittivity of free space

μ_0 = Permeability of free space

E = cB …(1)

Where,

$$c = \frac{1}{\sqrt{\epsilon_0 \ \mu_0}} \quad ...(2)$$

Putting equation (2) in equation (1), we get

$$E = \frac{1}{\sqrt{\epsilon_0 \ \mu_0}} \ B$$

Squaring on both sides, we get

$$E^2 = \frac{1}{\epsilon_0 \ \mu_0} \ B^2$$

$$\epsilon_0 \ E^2 = \frac{B^2}{\mu_0}$$

$$\frac{1}{2} \ \epsilon_0 \ E^2 = \frac{1}{2} \ \frac{B^2}{\mu_0}$$

$$\Rightarrow U_E = U_B$$

Q 8.11) Suppose that the electric field part of an electromagnetic wave in vacuum is E = {(3.1 N/C) cos [(1.8 rad/m) y + (5.4 × 10^6 rad/s)t]} ˆi .
(a) What is the direction of propagation?
(b) What is the wavelength λ?
(c) What is the frequency v?
(d) What is the amplitude of the magnetic field part of the wave?
(e) Write an expression for the magnetic field part of the wave.

Solution:

(a) The direction of motion is along the negative y-direction. i.e., along -j.

(b) The given equation is compared with the equation,

$E = E_0 \cos (ky + \omega t)$

⇒ k = 1.8 rad/s

$\omega = 5.4 \times 10^6$ rad/s

$\lambda = 2\pi/k = (2 \times 3.14)/1.8 = 3.492$ m

(c) Frequency, $v = \omega/2\pi = 5.4 \times 10^6/(2 \times 3.14) = 0.859 \times 10^6$ Hz

(d) Amplitude of the magnetic field, $B_0 = E_0/c$

$= 3.1/(3 \times 10^8) = 1.03 \times 10^{-8}$ T= 10.3×10^{-9} T= 10.3 nT

(e) $B_z = B_0 \cos (ky + \omega t)\hat{k}$ ={(10.3 nT) cos[(1.8 rad/m)y + (5.4 × 10^6 rad/s)t]} kˆ

Q 8. 12) About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation
(a) at a distance of 1m from the bulb?
(b) at a distance of 10 m?
Assume that the radiation is emitted isotropically and neglect reflection.

Solution:

(a) Average intensity of the visible radiation, $I = P'/4\pi d^2$

Here, the power of the visible radiation, $P' = (5/100) \times 100 = 5$ W

At d = 1 m

$I = P'/4\pi d^2 = 5/(4 \times 3.14 \times 1^2) = 5/12.56 = 0.39$ W/m^2

(b) At d = 10 m

$I = P'/4\pi d^2 = 5/(4 \times 3.14 \times 10^2) = 5/1256 = 0.39 \times 10^{-2}$ W/m^2

Q 8.13) Use the formula $\lambda_m T = 0.29$ cm K to obtain the characteristic temperature ranges for different parts of the electromagnetic spectrum. What do the numbers that you obtain tell you?

Solution:

We have the equation, $\lambda_m T = 0.29$ cm K

$\Rightarrow T = (0.29/\lambda_m)$ cm K

Here, T is the temperature

λ_m is the maximum wavelength of the wave

For $\lambda_m = 10^{-4}$ cm

$T = (0.29/10^{-4})$ cm K = 2900 K

For the visible light, $\lambda_m = 5 \times 10^{-5}$ cm

$T = (0.29/ 5 \times 10^{-5})$ cm K ≈ 6000 K

Note: a lower temperature will also produce wavelength but not with maximum intensity.

Q 8. 14) Given below are some famous numbers associated with electromagnetic radiations in different contexts in physics. State the part of the electromagnetic spectrum to which each belongs.
(a) 21 cm (wavelength emitted by atomic hydrogen in interstellar space).
(b) 1057 MHz (frequency of radiation arising from two close energy levels in hydrogen; known as Lamb shift).
(c) 2.7 K [temperature associated with the isotropic radiation filling all space-thought to be a relic of the 'big-bang' origin of the universe].
(d) 5890 Å – 5896 Å [double lines of sodium]
(e) 14.4 keV [energy of a particular transition in ^{57}Fe nucleus associated with a famous high-resolution spectroscopic method (Mössbauer spectroscopy)].

Solution:

(a) Radio waves (short-wavelength end)
(b) Radio waves (short-wavelength end)
(c) Microwave
(d) Visible light (Yellow)
(e) X-rays (or soft γ-rays) region

Q 8.15) Answer the following questions:
(a) Long-distance radio broadcasts use short-wave bands. Why?
(b) It is necessary to use satellites for long-distance TV transmission. Why?
(c) Optical and radio telescopes are built on the ground but X-ray astronomy is possible only from satellites orbiting the earth. Why?
(d) The small ozone layer on top of the stratosphere is crucial for human survival. Why?
(e) If the earth did not have an atmosphere, would its average surface temperature be higher or lower than what it is now?
(f) Some scientists have predicted that a global nuclear war on the earth would be followed by a severe 'nuclear winter' with a devastating effect on life on earth. What might be the basis of this prediction?

Solution:

(a) Ionosphere reflects waves in the shortwave bands.
(b) Television signals have high frequency and high energy. Therefore, it is not properly reflected by the ionosphere. Satellites are used to reflect the TV signals.
(c) Atmosphere absorbs X-rays, while visible and radio waves can penetrate it.
(d) Ozone layer absorbs the ultraviolet radiations from the sunlight and prevents it from reaching the surface of the earth and causing damage to life.
(e) If the atmosphere is not present, there would be no greenhouse effect. As a result, the temperature of the earth would decrease.
(f) The smoke clouds produced by global nuclear war would perhaps cover substantial parts of the sky preventing solar light from reaching many parts of the globe. This would cause a 'winter'.