Question 1:

A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Solution:

Height of the candle, h =2.5 cm

Image size=h'

Object distance, u= -27 cm

The radius of the concave mirror, R= -36 cm

Focal length of the concave mirror, $f = \frac{R}{2} = -18cm$

Image distance=v

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$= \frac{1}{-18} - \frac{1}{-27} = \frac{-3+2}{54} = -\frac{1}{54}$

$\therefore$ v=-54cm

Therefore, to obtain a sharp image, the distance between the screen and the mirror should be 54cm.

Image magnification is:

$m = \frac{h'}{h} = -\frac{v}{u}$

$\therefore\ h' = \frac{-v}{u} \times h$

$= -\left(\frac{-54}{-27}\right) \times 2.5$ =-5cm

5cm is the height of the image of the candle. As there is a negative sign, the image is inverted and virtual.

If the candle is moved closer to the mirror, then the screen needs to be moved away from the mirror so as to obtain the image.

Question 2:

A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

Solution:

Size of the needle, h_1 =4.5 cm

Object distance, u=-12 cm

Focal length of the convex mirror, f= 15 cm

Image distance= v

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$\frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60}$

$\therefore \frac{60}{9}$ =6.7cm

The distance between the needle image and the mirror is 6.7cm and the image is obtained on the other side of the mirror.

Using magnification formula:

$m = \frac{h_2}{h_1} = -\frac{v}{u}$

$h_2 = \frac{-v}{u} \times h_1$

$= \frac{-6.7}{-12} \times 4.5$ =+2.5cm

$m = \frac{h_2}{h_1} = \frac{2.5}{4.5}$ =0.56

2.5cm is the height of the image. As it has a positive sign, the image is erect, virtual and diminished.

As the needle is moved farther from the mirror, the image also moves away from the mirror resulting in a reduction in the size of the image.

Question 3:

A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Solution:

Actual depth of the needle in water, h_1 =12.5cm

Apparent depth of the needle in water, h_2 =9.4 cm

Refractive Index of water = μ

The value of μ can be obtained as follows:

$$\mu = \frac{h_1}{h_2}$$

$$= \frac{12.5}{9.4} = 1.33$$

Hence, 1.33 is the refractive index of water.

Now water is replaced by a liquid of refractive index 1.63

The actual depth of the needle remains the same, but its apparent depth changes.

Let x be the new apparent depth of the needle.

$$\mu' = \frac{h_1}{x}$$

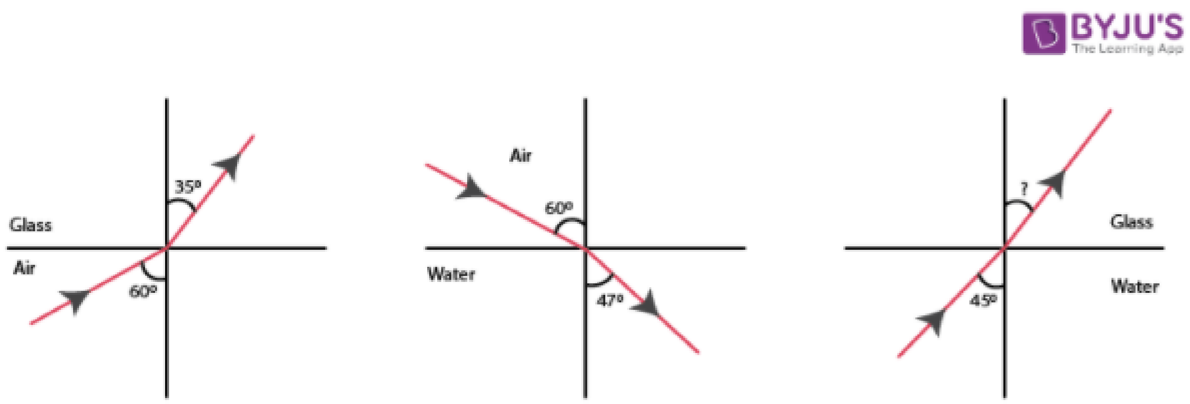
Therefore y= $\frac{h_1}{\mu'}$

$$= \frac{12.5}{1.63} = 7.67\text{cm}$$

The new apparent depth of the needle is 7.67cm. It is observed that the value is less than h_2, therefore the needle needs to be moved up to the focus again.

Distance to be moved to focus=9.4-7.67=1.73cm

Question 4:



The figures above show the refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface.

Solution:

For the glass-air interface:

Angle of incidence, i =60°

Angle of refraction, r=35°

From Snell's law we know that the refractive index of the glass with respect to air is given as:

$$\mu_g^a = \frac{sin\ i}{sin\ r}$$

$$= \frac{sin\ 60^\circ}{sin\ 35^\circ} = \frac{0.8660}{0.5736} = 1.51 \text{ ----(i)}$$

For the air-water interface:

Angle of incidence, i= 60°

Angle of refraction, r= 47°

From Snell's law we know that the refractive index of the water with respect to air is given as:

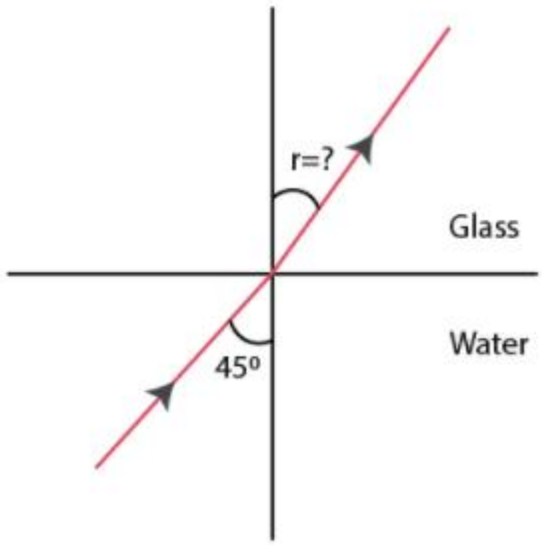
$$\mu_w^a = \frac{sin\ i}{sin\ r}$$

$$= \frac{sin\ 60^\circ}{sin\ 47^\circ} = \frac{0.8660}{0.7314} = 1.184 \text{ -------(ii)}$$

With the help of equations (i) and (ii) the relative refractive index of glass with respect to water can be found:

$$\mu_g^w = \frac{\mu_g^a}{\mu_w^a}$$

$$= \frac{1.51}{1.184} = 1.275$$



Angle of incidence, i= 45°

Angle of refraction= r

r can be calculated from Snell's law as

$$\frac{sin\ i}{sin\ r} = \mu_g^w$$

$$\frac{sin 45^{\circ}}{sin\ r} = 1.275$$

$$sin\ r = \frac{\frac{1}{\sqrt{2}}}{1.275} = 0.5546$$

$$Therefore\ r = sin^{-1}(0.5546) = 38.68^{\circ}$$

Therefore, 38.68° is the angle of refraction at the water-glass interface.

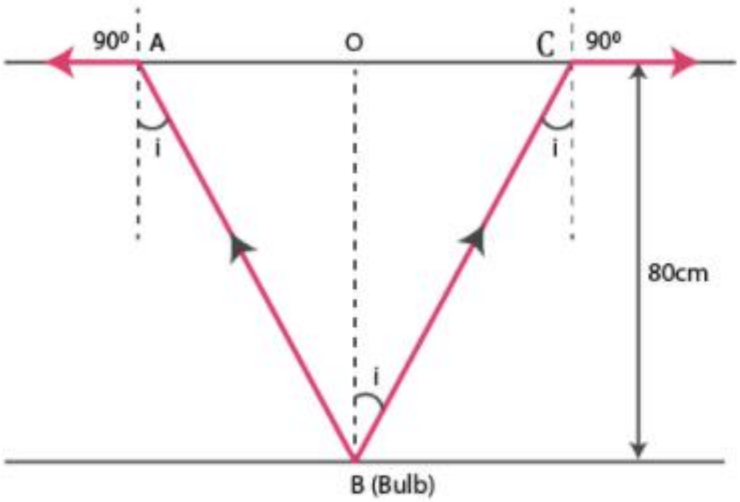
Question 5:

A small bulb is placed at the bottom of a tank containing water to a depth of 80cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

Solution:

Actual depth of the bulb in water, d_1=80 cm =0.8 m

Refractive Index of water, μ = 1.33



I = Angle of incidence

r =Angle of refraction – 90°

As the bulb is used as a point source, the emergent light is considered to be a circle.

R= $\frac{AC}{2}$ =AO=OB

Using Snell's law, we can write the relation for the refractive index of water as:

$$\mu = \frac{sin\ r}{sin\ i}$$

$$1.33 = \frac{sin\ 90^{\circ}}{sin\ i}$$

$$Therefore\ i = sin^{-1}(\frac{1}{1.33}) = 48.75^{\circ}$$

The relation:

tan i= $\frac{OC}{OB} = \frac{R}{d_1}$

Therefore R= $tan\ 48.75^{\circ} \times 0.8 = 0.91m$

Area of the surface of water= $nR^2 = n(0.91)^2 = 2.61m^2$

$2.61m^2$ is found to be the area of water through which the light from the water can emerge out.

Question 6:

A prism is made of glass of unknown refractive index. A parallel beam of light is incident on the face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

Solution:

Angle of minimum deviation, $\delta_m = 40^\circ$

Angle of the prism, A = 60°

Refractive Index of water, $\mu = 1.33$

Refractive Index of the material= μ'

The angle of deviation is related to refractive index (μ')

$$\mu' = \frac{sin\frac{A+\delta_m}{2}}{sin\frac{A}{2}}$$

$$= \frac{sin\frac{60^\circ+40^\circ}{2}}{sin\frac{60^\circ}{2}} = \frac{sin50^\circ}{sin30^\circ} = 1.532$$

1.532 is the refractive index of the material of the prism

δ_m is the new angle of minimum deviation since the prism is immersed in water.

The relation shows the refractive index of glass with respect to the water:

$$\delta_g^w = \frac{\mu'}{\mu} = \frac{sin\frac{(A+\delta_m')}{2}}{sin\frac{A}{2}}$$

$$sin\frac{(A+\delta_m')}{2} = \frac{\mu'}{\mu} sin\frac{A}{2}$$

$$sin\frac{(A+\delta_m')}{2} = \frac{1.532}{1.33} \times sin\frac{60^\circ}{2} = 0.5759$$

$$\frac{(A+\delta_m')}{2} = sin^{-1}0.5759 = 35.16^\circ$$

$$60^\circ + \delta_m' = 70.32^\circ$$

$$Therefore\ \delta_m' = 70.32^\circ - 60^\circ = 10.32^\circ$$

10.32° is the new minimum angle of deviation.

Question 7:

Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20cm?

Solution:

Refractive Index of glass, $\mu = 1.55$

Focal length of the double-convex lens, f=20 cm

Radius of curvature of one face of the lens=R_1

Radius of curvature of the other face of the lens=R_2

Radius of curvature of the double-convex lens=R

Therefore, $R_1 = R\ and\ R_2 = -R$

The value of R is calculated as:

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} + \frac{1}{R}\right]$$

$$\frac{1}{20} = 0.55 \times \frac{2}{R}$$

$$Therefore\ R = 0.55 \times 2 \times 20 = 22cm$$

22cm is the radius of curvature of the double-convex lens.

Question 8:

A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20cm, and (b) a concave lens of focal length 16cm?

Solution:

The object is virtual and the image formed is real.

Object distance, u= +12 cm

(i) The focal length of the convex lens, f =20 cm

Image distance=v

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{12} = \frac{1}{20}$

$\frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60} = \frac{8}{60}$

$Therefore\ v = \frac{60}{8} = 7.5cm$

The image will be formed 7.5cm away from the lens, to the right.

(ii) Focal length of the concave lens, f =-16 cm

Image distance =v

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = -\frac{1}{16} + \frac{1}{12}$

$= \frac{-3+4}{48} = \frac{1}{48}$

∴ v=48cm.

Therefore, the image obtained is 48cm away from the lens towards the right.

Question 9:

An object of size 3.0cm is placed 14cm in front of a concave lens of focal length 21cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Solution:

Size of the object, $h_1 = 3cm$

Object distance, u = -14 cm

Focal length of the concave lens, f = -21 cm

Image distance = v

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = -\frac{1}{21} - \frac{1}{14}$

$$= \frac{-2-3}{42} = \frac{-5}{42}$$

$$Therefore\ v = -\frac{42}{5} = -8.4cm$$

Therefore, the image obtained is 8.4cm away and is obtained on the other side of the lens. Since there is negative sign, it is understood that the image is virtual and erect.

$$m = \frac{Image\ height(h_2)}{Object\ height(h_1)} = \frac{v}{u}$$

$$Therefore\ h_2 = \frac{-8.4}{-14} \times 3 = 0.6 \times 3 = 1.8cm$$

The height of the image is 1.8cm.

If the object is moved further away from the lens, then the virtual image will move towards the mirror. As the object distance increases, the image size decreases.

Question 10:

What is the focal length of a convex lens of focal length 30cm in contact with a concave lens of focal length 20cm? Is the system a converging or a diverging lens? Ignore the thickness of the lenses.

Solution:

Focal length of the convex lens, f_1 =30cm

Focal length of the concave lens, f_2 = -20cm

Focal length of the system of lenses= f

The equivalent focal length of a system of two lenses in contact is given as:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}$$

Therefore f=-60cm

Therefore, 60cm is the focal length of the combination of lenses. Also, the negative sign is an indication that the system of lenses used act as a diverging lens.

Question 11:

A compound microscope consists of an objective lens of focal length 2.0cm and an eyepiece of focal length 6.25cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?

Solution:

Focal length of the objective lens, f_1 =2.0 cm

Focal length of the eyepiece, f_2 =6.25 cm

Distance between the objective lens and the eyepiece, d =15 cm

(a) Least distance of distinct vision, d' = 25 cm

Image distance for the eyepiece, v_2 = -25cm

Object distance for the eyepiece= u_2

According to the lens formula, we have the relation:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{u_2} = \frac{1}{v_2} - \frac{1}{f_2}$$

$$\frac{1}{-25} - \frac{1}{6.25} = \frac{-1-4}{25} = \frac{-5}{25}$$

$$u_2 = -5cm$$

Image distance for the objective lens, $v_1 = d + u_2 = 15–5 = 10cm$

Object distance for the objective lens = u_1

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1}$$

$$\frac{1}{10} - \frac{1}{2} = \frac{1-5}{10} = \frac{-4}{10}$$

$$u_1 = -2.5cm$$

Magnifying power

$$m = \frac{v_0}{|u_0|}(1 + \frac{d}{f_e})$$

$$= \frac{10}{2.5}(1 + \frac{25}{6.25})$$

$$= 20$$

Hence, 20 is the magnifying power of the microscope.

(b) The final Image is formed at infinity.

Therefore image distance for the eyepiece, $v_2 = \infty$

Object distance for the eyepiece= u_2

According to the lens formula, we have the relation:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{\infty} - \frac{1}{u_2} = \frac{1}{6.25}$$

$$u_2 = -6.25cm$$

Image distance for the objective lens, $v_1 = d + u_2 = 15 - 6.25 = 8.75cm$

Object distance for the objective lens=u_1

Following relation is obtained from the lens formula:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{u_1} = \frac{1}{v_1} - \frac{1}{f_1}$$

$$\frac{1}{8.75} - \frac{1}{2.0} = \frac{2-8.75}{17.5}$$

$$u_1 = -\frac{17.5}{6.75} = -2.59cm$$

Magnitude of the object distance, $|u_1|$ =2.59 cm

Following is the relation explaining the magnifying power of a compound microscope:

m= $\frac{v_1}{|u_1|}\left(\frac{d'}{|u_2|}\right)$

= $\frac{8.75}{2.59} \times \frac{25}{6.25}$ =13.51

Hence, 13.51 is the magnifying power of the microscope.

Question 12:

A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope,

Solution:

Focal length of the objective lens, f_o = 8 mm = 0.8 cm

Focal length of the eyepiece, f_e = 2.5 cm

Object distance for the objective lens, u_o = -9.0 mm = -0.9 cm

Least distance of distant vision, d=25 cm

Image distance for the eyepiece, v_e = -d=-25 cm

Object distance for the eyepiece = u_e .

Using the lens formula, we can obtain the value of u_e as:

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$\frac{1}{-25} - \frac{1}{2.5} = \frac{-1-10}{25}$$

$$u_e = -\frac{25}{11} = -2.27cm$$

With the help of lens formula, the value of the image distance for the objective (v) lens is obtained:

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o}$$

$$= \frac{1}{0.8} - \frac{1}{0.9}$$

$$= \frac{1}{7.2}$$

v_o = 7.2 cm

The separation between two lenses are determined as follows:

$= v_0 + |u_e|$

$= 7.2 + 2.27$

$= 9.47$ cm

The magnifying power of the microscope is calculated as follows:

Magnifying power, $M = M_o \times M_e$

$$M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

$$M = \frac{7.2}{0.9}\left(1 + \frac{25}{25}\right)$$

$= 8 \times 11$

$= 88$

Question 13:

A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Solution:

Focal length of the objective lens, f_o = 144 cm

Focal length of the eyepiece, f_e = 6.0 cm

The magnifying power of the telescope is given as:

$m = \frac{f_o}{f_e}$

$= \frac{144}{6} = 24$

The separation between the objective lens and the eyepiece is calculated as: $f_o + f_e = 144 + 6 = 150$cm

Therefore, 24 is the magnifying power of the telescope and the separation between the objective lens and the eyepiece is 150cm.

Question 14:

(i) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?

(ii)If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is $3.48 \times 10^6 m$,and the radius of lunar orbit is $3.8 \times 10^8 m$.

Solution:

Focal length of the objective lens, f_o =15m = $15 \times 10^2 cm$

Focal length of the eyepiece, f_e = 1.0 cm

(i) $\alpha = \frac{f_o}{f_e}$ is the angular magnification of a telescope.

= $\frac{15 \times 10^2}{1}$ =1500

Hence, the angular magnification of the given refracting telescope is 1500.

(ii) Diameter of the moon, d = $3.48 \times 10^6 m$

Radius of the lunar orbit, r_o = $3.8 \times 10^8 m$

Consider d' to be the diameter of the image of the moon formed by the objective lens.
The angle subtended by the diameter of the moon is equal to the angle subtended by the image.

$\frac{d}{r_o} = \frac{d'}{f_o}$

$\frac{3.48 \times 10^6}{3.8 \times 10^8} = \frac{d'}{15}$

Therefore d'= $\frac{3.48}{3.8} \times 10^{-2} \times 15$

$13.74 \times 10^{-2} m = 13.74 cm$

Therefore 13.74cm is the diameter of the image of the moon formed by the objective lens.

Question 15:
Use the mirror equation to deduce that:
(i) an object placed between f and 2f of a concave mirror produces a real image beyond 2f.
(ii) a convex mirror always produces a virtual image independent of the location of the object.
(iii) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.
(iv) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.
[Note: This exercise helps you deduce algebraically properties of images that one obtains from explicit ray diagrams.]

Solution:

(i) The focal length is negative for a concave mirror.
Therefore f < 0
The object distance (u) is negative when the object is placed on the left side of the mirror.
Therefore u < 0
Lens formula for image distance v is written as:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \text{------(i)}$$

For a concave mirror, f is negative, i.e., f < 0

For a real object, i.e., which is on the left side of the mirror.

For u between f and 2f implies that 1/u lies between 1/f and 1/2f

$$\frac{1}{2f} > \frac{1}{u} > \frac{1}{f}(as\, u,\; f\, are\, negative)$$

$$\frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < 0$$

$$\frac{1}{2f} < \frac{1}{v} < 0$$

1/v is negative

which implies v is negative and greater than 2f. Hence, image lies beyond 2f and it is real.

(ii) The focal length is positive for a convex mirror.
Therefore f > 0
The object distance (u) will be negative if the object is placed on the left side of the mirror.
Therefore u < 0
Using mirror formula, we can calculate the image distance v:

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Using eqn (ii), we can conclude that:

$\frac{1}{v} < 0$

v>0

Thus, the image is obtained on the back side of the mirror, Therefore, it can be concluded that a convex mirror always produces a virtual image irrespective of the object distance.

(iii) The focal length is positive for a convex mirror.
Therefore f > 0
The object distance (u) is negative when the object is placed on the left side of the mirror
Therefore u < 0
For image distance v, we have the mirror formula:

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

But we have u<0

Therefore $\frac{1}{v} > \frac{1}{f}$

v<f

Therefore, the image is formed between the focus and the pole and is diminished.
(iv) The focal length of the concave mirror is negative.

Therefore f < 0

The object distance, u is negative for an object that is placed on the left side of the mirror.
Therefore u < 0
It is placed between the focus (f) and the pole.

Therefore f>u>0

$\frac{1}{f} < \frac{1}{u} < 0$

$\frac{1}{f} - \frac{1}{u} < 0$

For image distance v, we have the mirror formula:

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Therefore $\frac{1}{v} < 0$

v>0
The image obtained is virtual since it is formed on the right side of the mirror.
For u < 0 and v > 0, we can write:

$\frac{1}{u} > \frac{1}{v}$

v>u

Magnification, m= $\frac{v}{u}$ > 1

Hence, the formed image is enlarged.

Question 16:
A small pin fixed on a tabletop is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

Solution:

The actual depth of the pin, d = 15 cm
Apparent depth of the pin =d'

Refractive index of glass, μ =1.5

The ratio of actual depth to the apparent depth and the refractive index of the glass are equal. i.e.

$\mu = \frac{d}{d'}$

Therefore d'= $\frac{d}{\mu}$

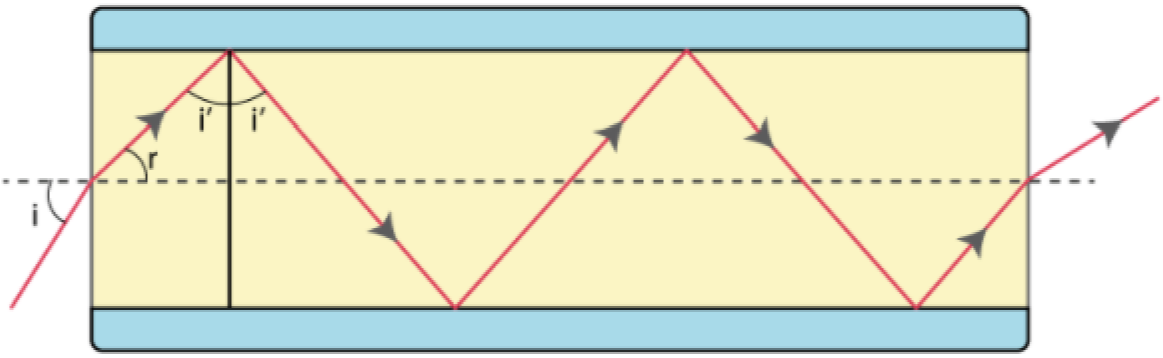
= $\frac{15}{1.5}$ =10cm

The distance at which the pin appears to be raised =d'-d=15-10=5cm
When the angle of incidence is small, the distance is independent of the location of the slab.

Question 17:

(i) Figure below shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.

(ii) What is the answer if there is no outer covering of the pipe?



Solution:

(i) Refractive index of the glass – fibre, μ_2 = 1.68

Refractive index of the outer covering, μ_1 = 1.44

Angle of incidence = i
Angle of refraction = r
Angle of incidence at the interface = i'

The refractive index (μ) of the inner core – outer core interface is given as:

$$\mu = \frac{\mu_2}{\mu_1} = \frac{1}{sini'}$$

$$sini' = \frac{\mu_1}{\mu_2}$$

$$= \frac{1.44}{1.68} = 0.8571$$

$$Therefore\ i' = 59^\circ$$

For the critical angle, total internal reflection (TIR) takes place only when i>i', i.e., i> 59°

Maximum angle of reflection, $r_{max} = 90^\circ - i' = 90^\circ - 59^\circ = 31^\circ$

Let, i_{max} be the maximum angle of incidence.

The refractive index at the air − glass interface, $\mu_1 = 1.68$

We know that

$\mu_1 = \frac{sin i_{max}}{sin r_{max}}$

Rearranging the equation, we get:

sin i_{max} = 1.68 × sin r_{max}

Substituting the values in the above equation, we get:

= 1.68 sin 31°

= 1.68×0.5150 =0.8652

Therefore $i_{max} = sin^{-1} 0.8652 = 60^\circ$

Thus, all the rays incident at angles lying in the range 0 < i < 60° will suffer total internal reflection.

(ii) If the outer covering of the pipe is not present, then:

Refractive index of the outer pipe, μ_1 =Refractive index of air=1

For the angle of incidence i = 90° ,

We can write Snell's law at the air − pipe interface as:

$\frac{sin i}{sin r} = \mu_2 = 1.68$

$sin r = \frac{sin 90^\circ}{1.68} = \frac{1}{1.68}$

$r = sin^{-1}(0.5932)$

$= 36.5^\circ$

Therefore $i' = 90^\circ - 36.5^\circ = 53.5^\circ$

Since i'>r, all incident rays will suffer total internal reflection.

Question 18:
Answer the following questions:
(i) You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.
(ii) A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?
(iii) A diver underwater, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?
(iv) Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?
(v) The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

Solution:

(i) Yes.

Real images can be obtained from plane and convex mirrors too. When the light rays converge at a point behind the plane or convex mirror, the object is said to be virtual. The real image of this object is obtained on the screen which is placed in the front of the mirror. This is where the real image is obtained.

(ii) No

To obtain a virtual image, the light rays must diverge. In the eyes, the convex lens help in converging these divergent rays at the retina. This is an example where the virtual image acts as an object for the lens to produce a real image.

(iii) The diver is in the water while the fisherman is on the land. Air is less dense when compared to the water as a medium. It is mentioned that the diver is viewing the fisherman. This explains that the light rays are travelling from the denser medium to the rarer medium. This means that the refracted rays move away from the normal making fisherman appear taller.

(iv) Yes; Decrease

The reason behind the change in depth of the tank when viewed obliquely is because the light rays bend when they travel from one medium to another. This also means that the apparent depth is less than the near-normal viewing.

(v) Yes

2.42 is the refractive index of the diamond while 1.5 is the refractive index of the ordinary glass. Also, the critical angle of the diamond is less than the glass. The sparkling effect of a diamond is possible because of the large cuts in the angle of incidence. Larger cuts ensures that the light entering the diamond is totally reflected from all the faces.

Question 19:
The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

Solution:

Distance between the object and the image, d = 3 m

Maximum focal length of the convex lens = f_{max}

For real images, the maximum focal length is given as:

$$f_{max} = \frac{d}{4}$$

$$= \frac{3}{4} = 0.75m$$

Hence, for the required purpose, the maximum possible focal length of the convex lens is 0.75 m.

Question 20:
A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

Solution:

Distance between the image (screen) and the object, D = 90 cm
Distance between two locations of the convex lens, d = 20 cm
Focal length of the lens = f
Focal length is related to d and D as:

$$f = \frac{D^2 - d^2}{4D}$$

$$= \frac{90^2 - (20^2)}{4 \times 90} = \frac{770}{36} = 21.39cm$$

Therefore, the focal length of the convex lens is 21.39 cm.

Question 21:
(i) Determine the 'effective focal length' of the combination of the two lenses in Exercise 9.10, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?

(ii) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.

Solution:

Focal length of the convex lens, f_1 = 30 cm

Focal length of the concave lens, f_2 = −20 cm

Distance between the two lenses, d = 8.0 cm
(i) When the parallel beam of light is incident on the convex lens first:
Using lens formula, we can write:

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

Where,

u_1 = Object distance

v_1 = Image distance

$$\frac{1}{v_1} = \frac{1}{30} - \frac{1}{\infty} = \frac{1}{30}$$

Therefore $v_1 = 30cm$

For a concave lens, the image acts as a virtual object
Using lens formula for the concave lens, we can write:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

Where,

u_2 =Object distance

= (30 − d) = 30 − 8 = 22 cm

v_2 = Image distance

$$\frac{1}{v_2} = \frac{1}{22} - \frac{1}{20} = \frac{10-11}{220} = \frac{-1}{220}$$

Therefore $v_2 = -220cm$

The parallel incident beam appears to diverge from a point that is $(220 - \frac{d}{2} = 220 - 4)216cm$ from the centre of the combination of the two lenses.
When the parallel beam of light is incident, from the left, on the concave lens first:
Using the lens formula, we can write:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2}$

Where,

u_2 =Object distance = −∞

v_2 =Image distance

$\frac{1}{v_2} = \frac{1}{-20} + \frac{1}{-\infty} = -\frac{1}{20}$

Therefore $v_2 = -20cm$

For a convex lens, the image will act as a real object.
Applying lens formula to the convex lens, we have:

$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$

Where,

u_1 = Object distance

= −(20 + d) = −(20 + 8) = −28 cm

v_1 = Image distance

$\frac{1}{v_1} = \frac{1}{30} + \frac{1}{-28} = \frac{14-15}{420} = \frac{-1}{420}$

Therefore $v_2 = -420cm$

The parallel incident beam appears to diverge from (420-4) 416cm. The diversion happens from the left of the centre of the combination of the two lenses. The answer is dependent on the side of the combination where the incident beam of light is parallel.

(ii) Height of the image, h_1 = 1.5 cm

Object distance from the side of the convex lens, u_1 = -40 cm

$|u_1| = 40cm$

Using the lens formula:

$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$

Where,

v_1 = Image distance

$\frac{1}{v_1} = \frac{1}{30} + \frac{1}{-40} = \frac{4-3}{120} = \frac{1}{120}$

Therefore $v_1 = 120cm$

Magnification, m= $\frac{v_1}{|u_1|} = \frac{120}{40} = 3$

Therefore, 3 is the magnification of the convex lens.
The image of the convex lens is an object for the concave lens.
According to the lens formula:

$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$

Where,

u_2 = Object distance

= +(120 − 8) = 112 cm.

v_2 =Image distance

$\frac{1}{v_2} = \frac{1}{-20} + \frac{1}{112} = \frac{-112+20}{2240} = \frac{-92}{2240}$

Therefore $v_2 = \frac{-2240}{92}$

Magnification, m'= $\left|\frac{v_2}{u_2}\right| = \frac{2240}{92} \times \frac{1}{112} = \frac{20}{92}$

Hence, the magnification due to the concave lens is $\frac{20}{92}$.

The magnification produced by the combination of the two lenses is calculated as:

$m \times m'$ = 3 × 20/92 = 60/92 = 0.652

The magnification of the combination is given as:

$\frac{h_2}{h_1} = 0.652$

$h_2 = h_1 \times 0.652$

Where,

h_1 = Object size = 1.5 cm

h_2 = Size of the image

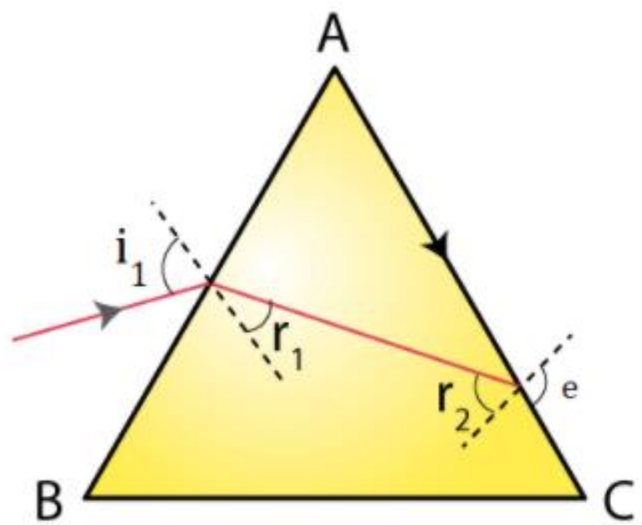
Therefore $h_2 = 0.652 \times 1.5 = 0.98cm$

Hence, the height of the image is 0.98 cm.

Question 22:
At what angle should a ray of light be incident on the face of a prism of refracting angle 60^0 so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Solution:

The incident, refracted and emergent rays associated with a glass prism ABC are shown in the given figure.



Angle of prism, Therefore A = 60°

Refractive index of the prism, μ = 1.524

i_1 = Incident angle

r_1 = Refracted angle

r_2 = Angle of incidence at the face AC

e= Emergent angle = 90°

According to Snell's law, for face AC, we can have:

$\frac{sin e}{sin r_2} = \mu$

$sin r_2 = \frac{1}{\mu} \times sin\ 90^\circ$

= $\frac{1}{1.524} = 0.6562$

Therefore $r_2 = sin^{-1} 0.6562 = 41^\circ$

Therefore $r_1 = A - r_2 = 60 - 41 = 19^\circ$

It is clear from the figure that angle A= $r_1 + r_2$

According to Snell's law, we have the relation:

$\mu = \frac{sin i_1}{sin r_1}$

$sin i_1 = \mu sin r_1$

= $1.524 \times sin 19^\circ$ =0.496

Therefore $i_1 = 29.75^\circ$

Therefore, the angle of incidence is 29.75°

Question 23:
You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will
(i) deviate a pencil of white light without much dispersion,
(ii) disperse (and displace) a pencil of white light without much deviation.

Solution:

(i) The two prisms must be place next to each other. The bases of these two prisms must be such that they are on the opposite sides of the incident white light. The dispersion of white light first takes place when it is incident on the first prism. The dispersed ray then enters the second prism as an incident ray. The dispersion of light from both the prisms emerges as a white light.

(ii) Consider the two prisms from (i). The deviations from this combination of prisms become equal by adjusting the angle between them. When the angle is maintained between these two prisms, the pencil of white light will disperse without much deviation.

Question 24:

For a normal eye, the far point is at infinity and the near point of distinct vision is about 25cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.

Solution:

Least distance of distinct vision, d = 25 cm
Far point of a normal eye, d' = ∞
Converging power of the cornea,

Least converging power of the eye-lens, $P_c = 40D$

Least converging power of the eye-lens, $P_e = 20D$

Eyes use the least converging power to see the objects that are at infinity.

Power of the eye-lens, P = $P_c + P_e = 40 + 20 = 60D$

Power of the eye-lens is given as:

P= $\frac{1}{Focal\ length\ of\ the\ eye\ lens\ (f)}$

$f = \frac{1}{P}$

$\frac{1}{60D}$

$\frac{100}{60} = \frac{5}{3} cm$

To focus an object at the near point, object distance (u) = −d = −25 cm
Focal length of the eye-lens = Distance between the cornea and the retina = Image distance

Hence, image distance, $v = \frac{5}{3}$

Using the lens formula, we can write,

$\frac{1}{f'} = \frac{1}{v} + \frac{1}{u}$

Where,
f’=focal length

$$\frac{1}{f'} = \frac{3}{5} + \frac{1}{25} = \frac{15+1}{25} = \frac{16}{25} cm^{-1}$$

Power $P' = \frac{1}{f'} \times 100$

$$P' = \frac{16}{25} \times 100 = 64D$$

Therefore, Power of the eye-lens = 64 − 40 = 24 D
Hence, 20D to 24D is the range of accommodation of the eye-lens.

Question 25:

Does the human eye partially lose its ability of accommodation when it undergoes short-sightedness (myopia) or long-sightedness (hypermetropia)? If not, what might cause these defects of vision?

Solution:

When a person is suffering from myopia or hypermetropia, the eye-lenses are used. Myopia is a condition when the eyeballs start to elongate from the front to the back. Hypermetropia is a condition when the eyeballs start to shorten. While presbyopia is a condition where the eyeballs loses its ability of accommodation.

Question 26:

Spectacles of power −1.0 dioptre is being used by a person suffering from myopia for distant vision. He also needs to use separate reading glass of power + 2.0 dioptres when he turns old. Explain what may have happened.

Solution:

The myopic person uses a spectacle of power, P = -1.0 D

Focal length of the given spectacles, f = $\frac{1}{P}$ = $\frac{1}{-1 \times 10^{-2}}$

= -100 cm

Therefore, 100cm is the far point of the person. The normal near point of the person is 25cm. When the objects are placed at infinity, virtual images are produced at 100cm. This is possible when the spectacles are used. When ability of accommodation of the eye-lens is used, he will be able to see the objects that are placed between 100cm and 25cm.

During old age, the person uses reading glasses of power, P’ = +2 D

As the age increases, the ability of accommodation decreases. This is known as presbyopia. This is the reason why he is finding it difficult to see the objects placed at 25cm.

Question 27:

A person looking at a cloth with a pattern consisting of vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

Solution:

In the given situation, the person is having difficulty in seeing the horizontal lines while the vertical lines are distinctly visible. This occurs when the refracting system of the eye is not working the same for different planes. This is known as astigmatism. For the vertical plane, the curvature of the eye is sufficient. But the same curvature is not sufficient for the horizontal plane. Hence, the formation of sharp images from the vertical line is possible on the retina and the horizontal lines appears blurred. With the help of cylindrical lenses, this defect can be corrected.

Question 28:

A child with normal near point (25 cm) reads a book with small size print using a magnifying glass: a thin convex lens of focal length 5 cm.

(a) What would be the shortest and the longest distance at which the lens should be placed from the page so that the book can be read easily when viewing through the magnifying glass?

(b) What is the max and the mini angular magnification (magnifying power) possible using the above given simple microscope?

Solution:

(a) Focal length of the magnifying glass, f = 5 cm

Distance vision has the least distance, d = 25 cm

Closest object distance = u

Image distance, v = -d = -25 cm

According to the lens formula, we have:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{-25} - \frac{1}{5}$$

$$= \frac{-6}{25}$$

Therefore, $u = \frac{-25}{6}$

= -4.167 cm

Hence, the closest distance at which the person can read the book is 4.167 cm.

For the object at the longest distance (u’), the image distance (v’) = ∞

According to the lens formula, we have:

$\frac{1}{f} = \frac{1}{v'} - \frac{1}{u'}$

$\frac{1}{u'} = \frac{1}{\infty} - \frac{1}{5}$

$= - \frac{1}{5}$

Therefore, u’ = -5 cm

Hence, the farthest distance at which the person can read the book is 5 cm.

(b) Maximum angular magnification is given by the relation:

$\alpha_{max} = \frac{d}{|u|}$

$= \frac{25}{\frac{25}{6}}$

= 6

Minimum angular magnification is given by the relation:

$= \alpha_{max} = \frac{d}{|u'|}$

$= \frac{25}{5}$

= 5

Question 29:

A large card divided into squares each of size 1 mm^2 is being viewed from a distance of 9 cm through a magnifying glass (converging lens has a focal length of 9 cm) held close to the eye. Determine:

(a) the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Solution:

(a)Area of each square, A = 1 mm^2

Object distance, u = −9 cm

Focal length of a converging lens, f = 10 cm

The lens formula for the image distance v, can be written as:

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{10} = \frac{1}{v} + \frac{1}{9}$

$\frac{1}{v} = -\frac{1}{90}$

Therefore, v = -90 cm

Magnification, m = $\frac{v}{u}$

$= \frac{-90}{-9}$

= 10

Therefore, area of each square in the virtual image = $(10)^2$ A

= 10^2 x 1 = 100 mm^2

= 1 cm^2

(b) The lens has a magnifying power of = $\frac{d}{|u|}$

$= \frac{25}{9}$

= 2.8

(c) The magnification in (a) is not the same as the magnifying power in (b).

The magnification magnitude is $\left(\left|\frac{v}{u}\right|\right)$ and the magnifying power is $\left(\frac{d}{|u|}\right)$.

The two quantities will be equal when the image is formed at the near point (25 cm).

Question 30:

(a)Determine the distance in which the lens should be held from the figure in Exercise 9.29 in order to view the squares distinctly with the maximum possible magnifying power.

(b)Determine the magnification in the following situation.

(c) Find if the magnifying power is equal to magnification.

Explain.

Solution:

(a)When the image is formed at the near point, we get the maximum possible magnification.

So, Image distance, v = −d = −25 cm

Focal length, f = 10 cm

Object distance = u

According to the lens formula, we have:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{-25} - \frac{1}{10} = -\frac{7}{50}$$

Therefore, u = $\frac{-50}{7}$

= – 7.14 cm

Hence, the lens should be kept 7.14 cm away in order to view the squares distinctly.

(b) Magnification = $\left|\frac{v}{u}\right|$

$$= \frac{25}{\frac{50}{7}} = 3.5$$

(c) Magnifying power = $\frac{d}{u}$

$$= \frac{25}{\frac{50}{7}}$$

= 3.5

Therefore, the magnifying power is equal to the magnitude of magnification since the image is formed at the near point (25 cm).

Question 31:

The virtual image of each square in the figure is to have an area of 6.25 mm^2. Find out, what should be the distance between the object in Exercise 9.30 and the magnifying glass? If the eyes are too close to the magnifier, would you be able to see the squares distinctly?

Solution:

Area of the virtual image of each square, A = 6.25 mm^2

Area of each square, A_0 = 1 mm^2

Hence, the linear magnification of the object can be calculated as:

m = $\sqrt{\frac{A}{A_o}}$

= $\sqrt{\frac{6.25}{1}}$

= 2.5

But m = $\frac{Image distance(v)}{Object distance(u)}$

Therefore, v = mu

= 2.5u ………… (1)

The magnifying glass has a focal length of, f = 10 cm

According to the lens formula, we have the relation:

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{10} = \frac{1}{2.5u} - \frac{1}{u}$

$= \frac{1}{u}\left(\frac{1}{2.5} - \frac{1}{1}\right)$

$= \frac{1}{u}\left(\frac{1-2.5}{2.5}\right)$

Therefore, u = $-\frac{1.5\times10}{2.5}$ = -6 cm

And v = 2.5u

= 2.5 x 6

= -15 cm

The virtual image cannot be seen by the eyes distinctly, because the image is formed at a distance of 15 cm, which is less than the near point (i.e. 25 cm) of a normal eye.

Question 32:

Answer the following questions:

(a) An object subtends an angle at the eye which is equal to the angle subtended at the eye by the virtual image that is produced by a magnifying glass. Does the magnifying glass provide angular magnification? Explain.

(b)A person's eyes are very close to the lens when he is viewing through a magnifying glass. Does angular magnification change if the eye is moved back?

(c) The focal length of the lens is inversely proportional to the magnifying power of a simple microscope. Why don't we achieve greater and greater magnifying power by using a convex lens of smaller and smaller focal length?

(d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

(e) Our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing, when viewing from a compound microscope. Explain why? How much should be that short distance between the eye and eyepiece?

Solution:

(a) The angular size of the image and of the object are equal. When the objects are placed at the least distance of distinct vision, they can be viewed with the help of magnifying glass. As the object gets closer, the angular size increases. A magnifying glass is used because it provides an angular magnification. Without magnification, it is difficult to place an object closer to the eye.

(b) Yes, the angular magnification changes. As the distance between the eye and the magnifying glass increases, the angular magnification decreases. This is because the angle subtended by the eye is less than that of the angle subtended by the lens. Angular magnification is independent of the image distance.

(c) Manufacturing lenses with small focal length is difficult. When the focal length is small, there are chances of production of spherical and chromatic aberrations. Therefore, there is no such reduction in the focal length of a convex lens.

(d) A compound microscope produces an angular magnification of $\left[\left(\frac{25}{f_e}\right)+1\right]$

where,

f_e = focal length of the eyepiece

It can be inferred that if f_e is small, then angular magnification of the eyepiece will be large.

The angular magnification of the objective lens of a compound microscope is given as

$$\frac{1}{(|u_o|f_o)}$$

Where, u_o = object distance for the objective lens

f_o = focal length of the objective

When $u_o > f_o$, the magnification is large. While using a microscope, the object is kept closer to the objective lens. So, the object distance is very less. As u_0 decreases, f_0 also decreases. Therefore, in the given situation, both f_e and f_0 are small.

(e) We are unable to collect much-refracted light when we place our eyes too close to the eyepiece of a compound microscope. As a result, there is substantial decrements in the field of view. Hence, the clarity of the image gets blurred. The eye-ring attached to the eyepiece gives the best position for viewing through a compound microscope. The precise location of the eye depends on the separation between the objective lens and the eyepiece.

Question 33:

An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?

Solution:

Focal length of the objective lens, f_0 = 1.25 cm

The focal length of the eyepiece, f_e = 5 cm

Least distance of distinct vision, d = 25 cm

Angular magnification of the compound microscope = 30X

Total magnifying power of the compound microscope, m = 30

The angular magnification of the eyepiece is given by the relation:

$$m_e = \left(1 + \frac{d}{f_e}\right)$$

$$= \left(1 + \frac{25}{5}\right)$$

= 6

The angular magnification of the objective lens (m_o) is related to m_e as:

$m_o m_e$ = m

$$m_o = \frac{m}{m_e}$$

$$= \frac{30}{6}$$

= 5

We also have the relation:

$$m_o = \frac{Image distance for the objective lens (v_o)}{Object distance for the objective lens (-u_o)}$$

$$5 = \frac{v_o}{-u_o}$$

Therefore, $v_o = -5u_o$ ………. (1)

Applying the lens formula for the objective lens:

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\frac{1}{1.25} = \frac{1}{-5u_o} - \frac{1}{u_o}$$

$= \frac{-6}{5u_o}$

Therefore, $u_o = \frac{-6}{5} \times 1.25$

= -1.5 cm

And $v_o = -5u_o$

= -5 x (-1.5)

= 7.5 cm

The object should be placed 1.5 cm away from the objective lens to obtain the desired magnification.

Applying the lens formula for the eyepiece:

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

Where,

v_e = Image distance for the eyepiece = -d = -25 cm

u_e = Object distance for the eyepiece

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$= \frac{-1}{25} - \frac{1}{5}$

$= \frac{-6}{25}$

Therefore, u_e = -4.17 cm

Separation between the objective lens and the eyepiece = $|u_e| + |v_o|$

= 4.17 + 7.5

= 11.67 cm

Therefore, 11.67cm is the separation between the objective lens and the eyepiece.

Question 34:

A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25 cm)?

Solution:

Focal length of the objective lens, f_0 = 140 cm

Focal length of the eyepiece, f_e = 5 cm

Least distance of distinct vision, d = 25 cm

(a)When the telescope is in normal adjustment, its magnifying power is given as:

$$m = \frac{f_o}{f_e}$$

$$= \frac{140}{5} = 28$$

(b) When the final image is formed at d, the magnifying power of the telescope is given as:

$$\frac{f_o}{f_e}\left[1 + \frac{f_e}{d}\right]$$

$$= \frac{140}{5}\left[1 + \frac{5}{25}\right]$$

= 28[1 + 0.2]

= 28 x 1.2 = 33.6

Question 35:

(a)For a telescope, what is the separation between the objective lens and the eyepiece?

(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?

(c)What is the height of the final image of the tower if it is formed at 25 cm?

Solution:

Focal length of the objective lens, f_0 = 140 cm

Focal length of the eyepiece, f_e = 5 cm

(a) In normal adjustment, the separation between the objective lens and the eyepiece = $f_o + f_e$

= 140 + 5

= 145 cm

(b) Height of the tower, h_1 =100 m

Distance of the tower (object) from the telescope, u = 3 Km = 3000 m

The angle subtended by the tower at the telescope is given as:

$$\Theta = \frac{h_1}{u}$$

$= \frac{100}{3000}$

$= \frac{1}{30}$ rad

The angle subtended by the image produced by the objective lens is given as:

$\Theta = \frac{h_2}{f_o} = \frac{h_2}{140}$ rad

Where, h_2 = height of the image of the tower formed by the objective lens

$\frac{1}{30} = \frac{h_2}{140}$

Therefore, $h_2 = \frac{140}{30}$

= 4.7 cm

Therefore, 4.7cm tall is the image of the tower obtained from the objective lens

(c) Image is formed at a distance, d = 25cm

The magnification of the eyepiece is given by the relation:

$m = 1 + \frac{d}{f_e}$

$= 1 + \frac{25}{5}$

= 1 + 5 = 6

Height of the final image = mh_2 = 6 x 4.7 = 28.2 cm

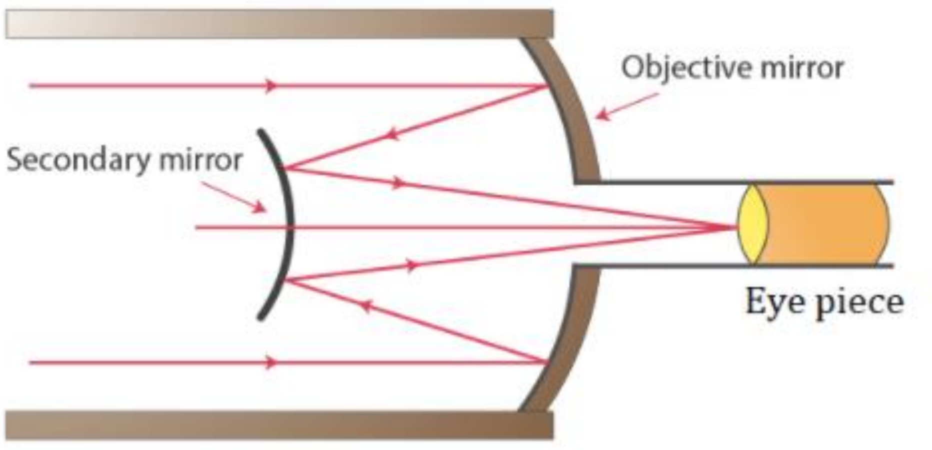
Hence, 28.2cm is the height of the final image of the tower.

Question 36:

A Cassegrain telescope uses two mirrors as shown in Fig. 9.33. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and the small mirror is 140 mm, where will the final image of an object at infinity be?

Solution:

Below is the figure of a Cassegrain telescope. It consists of a concave and a convex mirror.



Distance between the objective mirror and the secondary mirror, d = 20 mm

Radius of curvature of the objective mirror, R_1 = 220 mm

Hence, focal length of the objective mirror, $f_1 = \frac{R_1}{2}$ = 110 mm

Radius of curvature of the secondary mirror, R_2 = 140 mm

Hence, focal length of the secondary mirror, $f_2 = \frac{R_2}{2}$

$= \frac{140}{2}$

= 70 mm

The image of an object placed at infinity, formed by the objective mirror, will act as a virtual object for the secondary mirror.

hence, the virtual object distance for the secondary mirror, $u = f_1 - d$

= 110 – 20

= 90 mm

Applying the mirror formula for the secondary mirror, we can calculate image distance (v) as:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_2}$$

$$\frac{1}{v} = \frac{1}{f_2} - \frac{1}{u}$$

$$= \frac{1}{70} - \frac{1}{90} = \frac{2}{630}$$

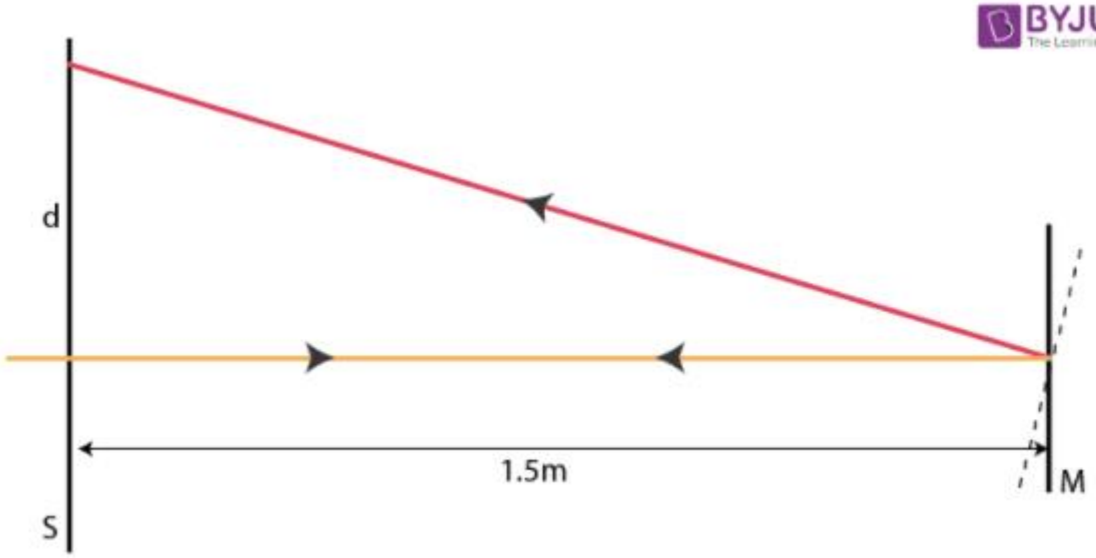
Therefore, $v = \frac{630}{2} = 315$ mm

Hence, 315 mm is the distance between the final image and the secondary mirror.

Question 37:

Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Fig. 9.36. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?

Solution:



Solution:

Angle of deflection, θ = 3.5°

The distance of the screen from the mirror, D = 1.5 m

The deflection experienced by the reflected rays are twice the angle of deflection:

2θ = 7.0°

Following is the displacement (d) of the reflected spot of light:

$$\tan 2\Theta = \frac{d}{1.5}$$

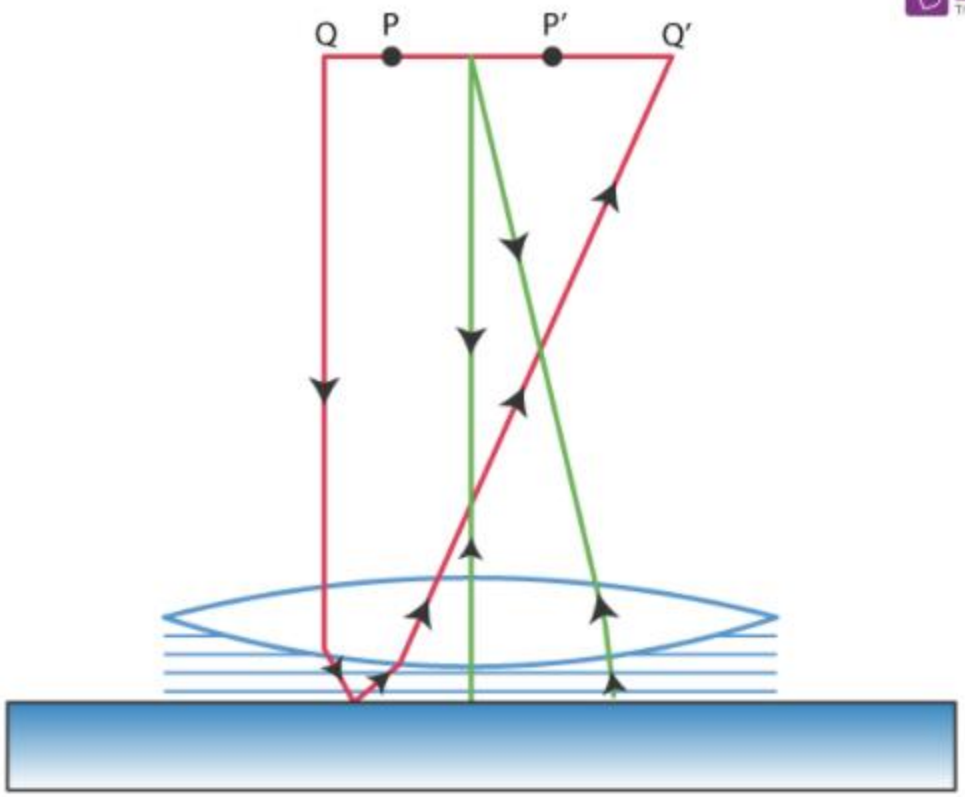
Therefore, d = 1.5 x tan7° = 0.184 m = 18.4 cm

Hence, 18.4 cm is the displacement of the refracted spot of light.

Question 38:

Figure 9.37 shows a biconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?

Solution:



Solution:

Focal length of the convex lens, f_1 = 30 cm

The liquid acts as a mirror. Focal length of the liquid = f_2

Focal length of the system (convex lens + liquid), f = 45 cm

The equivalent focal length of the pair of optical systems that are in contact are given as:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$$

$= \frac{1}{45} - \frac{1}{30}$

$= -\frac{1}{90}$

Therefore, f_2 = -90 cm

Let μ_1 be the refractive index and the radius of curvature of one surface be R. Hence, -R will be the radius of curvature of the other surface.

R can be obtained using the relation : $\frac{1}{f_1} = (\mu_1 - 1)(\frac{1}{R} + \frac{1}{-R})$

$\frac{1}{30}$ = (1.5 – 1) $\left(\frac{2}{R}\right)$

Therefore, R = $\frac{30}{1}$

R = 30 cm

Let μ_2 be the refractive index of the liquid.

Radius of curvature of the liquid on the side of the plane mirror = ∞ Radius of curvature of the liquid on the side of the lens, R = −30 cm

The value of μ_2 can be calculated using the relation:

$$\frac{1}{f_2} = (\mu_2 - 1)\left[\frac{1}{-R} - \frac{1}{infinity}\right]$$

μ_2 – 1 = $\frac{1}{3}$

Therefore, μ_2 = 1.33

Therefore, 1.33 is the refractive index of the liquid.