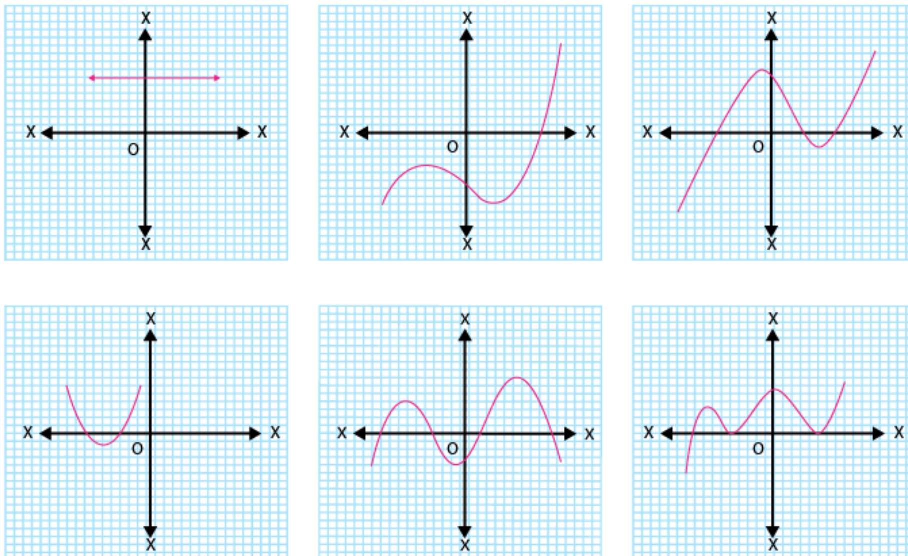
Exercise 2.1 Page: 28

1. The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.



Solutions:

Graphical method to find zeroes:-

Total number of zeroes in any polynomial equation = total number of times the curve intersects x-axis.

(i) In the given graph, the number of zeroes of p(x) is 0 because the graph is parallel to x-axis does not cut it at any point.

(ii) In the given graph, the number of zeroes of p(x) is 1 because the graph intersects the x-axis at only one point.

(iii) In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at any three points.

(iv) In the given graph, the number of zeroes of p(x) is 2 because the graph intersects the x-axis at two points.

(v) In the given graph, the number of zeroes of p(x) is 4 because the graph intersects the x-axis at four points.

(vi) In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at three points.

Exercise 2.2 Page: 33

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solutions:

(i)x^2–2x –8
⇒x^2– 4x+2x–8 = x(x–4)+2(x–4) = (x-4)(x+2)

Therefore, zeroes of polynomial equation x^2–2x–8 are (4, -2)

Sum of zeroes = 4–2 = 2 = -(-2)/1 = -(Coefficient of x)/(Coefficient of x^2)

Product of zeroes = 4×(-2) = -8 =-(8)/1 = (Constant term)/(Coefficient of x^2)

(ii)4s^2–4s+1

⇒4s^2–2s–2s+1 = 2s(2s–1)–1(2s-1) = (2s–1)(2s–1)

Therefore, zeroes of polynomial equation 4s^2–4s+1 are (1/2, 1/2)

Sum of zeroes = (½)+(1/2) = 1 = -4/4 = -(Coefficient of s)/(Coefficient of s^2)

Product of zeros = (1/2)×(1/2) = 1/4 = (Constant term)/(Coefficient of s^2)

(iii) 6x^2–3–7x

⇒6x^2–7x–3 = 6x^2 – 9x + 2x – 3 = 3x(2x - 3) +1(2x - 3) = (3x+1)(2x-3)

Therefore, zeroes of polynomial equation 6x^2–3–7x are (-1/3, 3/2)

Sum of zeroes = -(1/3)+(3/2) = (7/6) = -(Coefficient of x)/(Coefficient of x^2)

Product of zeroes = -(1/3)×(3/2) = -(3/6) = (Constant term) /(Coefficient of x^2)

(iv)4u^2+8u

⇒ 4u(u+2)

Therefore, zeroes of polynomial equation 4u^2 + 8u are (0, -2).

Sum of zeroes = 0+(-2) = -2 = -(8/4) = = -(Coefficient of u)/(Coefficient of u^2)

Product of zeroes = $0\times-2 = 0 = 0/4$ = (Constant term)/(Coefficient of u^2)

(v) t^2–15

$\Rightarrow t^2 = 15$ or $t = \pm\sqrt{15}$
Therefore, zeroes of polynomial equation $t^2 - 15$ are ($\sqrt{15}$, -$\sqrt{15}$)

Sum of zeroes =$\sqrt{15}+(-\sqrt{15}) = 0= -(0/1)=$ -(Coefficient of t) / (Coefficient of t^2)

Product of zeroes = $\sqrt{15}\times(-\sqrt{15}) = -15 = -15/1$ = (Constant term) / (Coefficient of t^2)

(vi) $3x^2$–x–4

$\Rightarrow 3x^2-4x+3x-4 = x(3x-4)+1(3x-4) = (3x - 4)(x + 1)$

Therefore, zeroes of polynomial equation $3x^2 - x - 4$ are (4/3, -1)

Sum of zeroes = $(4/3)+(-1) = (1/3)= -(-1/3)$ = -(Coefficient of x) / (Coefficient of x^2)

Product of zeroes=$(4/3)\times(-1) = (-4/3)$ = (Constant term) /(Coefficient of x^2)

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.
(i) 1/4 , -1

Solution:

From the formulas of sum and product of zeroes, we know,
Sum of zeroes = $\alpha+\beta$
Product of zeroes = $\alpha\ \beta$
Sum of zeroes = $\alpha+\beta = 1/4$
Product of zeroes = $\alpha\ \beta = -1$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-
$x^2-(\alpha+\beta)x +\alpha\beta = 0$

$x^2-(1/4)x +(-1) = 0$

$4x^2-x-4 = 0$

Thus,$4x^2-x-4$ is the quadratic polynomial.

(ii) $\sqrt{2}$, 1/3

Solution:

Sum of zeroes = $\alpha + \beta = \sqrt{2}$
Product of zeroes = $\alpha\beta = 1/3$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2-(\alpha+\beta)x+\alpha\beta = 0$

$x^2-(\sqrt{2})x+(1/3) = 0$

$3x^2-3\sqrt{2}x+1 = 0$

Thus, $3x^2-3\sqrt{2}x+1$ is the quadratic polynomial.

(iii) 0, $\sqrt{5}$

Solution:
Given,
Sum of zeroes = $\alpha+\beta = 0$
Product of zeroes = $\alpha\beta = \sqrt{5}$
∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2-(\alpha+\beta)x+\alpha\beta = 0$

$x^2-(0)x+\sqrt{5} = 0$

Thus, $x^2+\sqrt{5}$ is the quadratic polynomial.

(iv) 1, 1

Solution:

Given,
Sum of zeroes = $\alpha+\beta = 1$
Product of zeroes = $\alpha\beta = 1$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-
$x^2–(\alpha+\beta)x +\alpha\beta = 0$

$x^2–x+1 = 0$

Thus , $x^2–x+1$is the quadratic polynomial.

(v) -1/4, 1/4

Solution:

Given,
Sum of zeroes = α+β = -1/4
Product of zeroes = α β = 1/4

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2–(\alpha+\beta)x +\alpha\beta = 0$

$x^2–(-1/4)x +(1/4) = 0$

$4x^2+x+1 = 0$

Thus,$4x^2+x+1$ is the quadratic polynomial.

(vi) 4, 1

Solution:

Given,
Sum of zeroes = α+β = 4
Product of zeroes = αβ = 1

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2–(\alpha+\beta)x+\alpha\beta = 0$

$x^2–4x+1 = 0$

Thus,$x^2–4x+1$ is the quadratic polynomial.

Exercise 2.3 Page: 36

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) $p(x) = x^3-3x^2+5x-3$, $g(x) = x^2-2$

Solution:

Given,

Dividend = $p(x) = x^3-3x^2+5x-3$

Divisor = $g(x) = x^2-2$

$$
\begin{array}{r|rrrr}
 & x & -3 & & \\
\hline
x^2-2 & x^3 & -3x^2 & +5x & -3 \\
 & - & & & \\
 & x^3 & +0x^2 & -2x & \\
\hline
 & & -3x^2 & +7x & -3 \\
 & & - & & \\
 & & -3x^2 & +0x & +6 \\
\hline
 & & & 7x & -9
\end{array}
$$

Therefore, upon division we get,

Quotient = $x-3$

Remainder = $7x-9$

(ii) $p(x) = x^4-3x^2+4x+5$, $g(x) = x^2+1-x$

Solution:

Given,

Dividend = $p(x) = x^4 - 3x^2 + 4x +5$

Divisor = $g(x) = x^2 +1-x$

$$
\begin{array}{r|rrrrr}
 & x^2 & +x & -3 & & \\
\hline
x^2-x+1 & x^4 & +0x^3 & -3x^2 & +4x & +5 \\
 & - & & & & \\
 & x^4 & -x^3 & +x^2 & & \\
\hline
 & & x^3 & -4x^2 & +4x & +5 \\
 & & - & & & \\
 & & x^3 & -x^2 & +x & \\
\hline
 & & & -3x^2 & +3x & +5 \\
 & & & - & & \\
 & & & -3x^2 & +3x & -3 \\
\hline
 & & & & & 8
\end{array}
$$

Therefore, upon division we get,
Quotient = x^2+x-3
Remainder = 8

(iii) $p(x) = x^4-5x+6$, $g(x) = 2-x^2$
Solution:
Given,

Dividend = $p(x) = x^4 - 5x + 6 = x^4+0x^2-5x+6$
Divisor = $g(x) = 2-x^2 = -x^2+2$

$$
\begin{array}{r|rrrrr}
 & -x^2 & -2 & & & \\
\hline
-x^2+2 & x^4 & +0x^3 & +0x^2 & -5x & +6 \\
 & - & & & & \\
 & x^4 & +0x^3 & -2x^2 & & \\
\hline
 & & & 2x^2 & -5x & +6 \\
 & & & - & & \\
 & & & 2x^2 & +0x & -4 \\
\hline
 & & & & -5x & +10
\end{array}
$$

Therefore, upon division we get,
Quotient = $-x^2-2$

Remainder = -5x + 10

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) t^2-3, 2t^4 +3t^3-2t^2-9t-12
Solutions:
Given,

First polynomial = t^2-3
Second polynomial = 2t^4 +3t^3-2t^2 -9t-12

$$
\begin{array}{l|rrrrr}
 & 2t^2 & +3t & +4 & & \\
\cline{2-6}
t^2-3 & 2t^4 & +3t^3 & -2t^2 & -9t & -12 \\
 & - & & & & \\
 & 2t^4 & +0t^3 & -6t^2 & & \\
\cline{2-6}
 & & 3t^3 & +4t^2 & -9t & -12 \\
 & & - & & & \\
 & & 3t^3 & +0t^2 & -9t & \\
\cline{2-6}
 & & & 4t^2 & +0t & -12 \\
 & & & - & & \\
 & & & 4t^2 & +0t & -12 \\
\cline{2-6}
 & & & & & 0
\end{array}
$$

As we can see, the remainder is left as 0. Therefore, we say that, t^2-3 is a factor of 2t^2+3t+4.

(ii)x^2+3x+1 , 3x^4+5x^3-7x^2+2x+2

Solutions:
Given,

First polynomial = x^2+3x+1
Second polynomial = 3x^4+5x^3-7x^2+2x+2

$$
\begin{array}{l|llllll}
 & 3x^2 & -4x & +2 & & & \\
\hline
x^2+3x+1 & 3x^4 & +5x^3 & -7x^2 & +2x & +2 & \\
 & - & & & & & \\
 & 3x^4 & +9x^3 & +3x^2 & & & \\
\hline
 & & -4x^3 & -10x^2 & +2x & +2 & \\
 & & - & & & & \\
 & & -4x^3 & -12x^2 & -4x & & \\
\hline
 & & & 2x^2 & +6x & +2 & \\
 & & & - & & & \\
 & & & 2x^2 & +6x & +2 & \\
\hline
 & & & & & 0 &
\end{array}
$$

As we can see, the remainder is left as 0. Therefore, we say that, $x^2 + 3x + 1$ is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) x^3-3x+1, x^5-4x^3+x^2+3x+1

Solutions:
Given,

First polynomial = x^3-3x+1

Second polynomial = $x^5-4x^3+x^2+3x+1$

$$
\begin{array}{l|llllll}
 & x^2 & -1 & & & & \\
\hline
x^3-3x+1 & x^5 & +0x^4 & -4x^3 & +x^2 & +3x & +1 \\
 & - & & & & & \\
 & x^5 & +0x^4 & -3x^3 & +x^2 & & \\
\hline
 & & & -x^3 & +0x^2 & +3x & +1 \\
 & & & - & & & \\
 & & & -x^3 & +0x^2 & +3x & -1 \\
\hline
 & & & & & & 2
\end{array}
$$

As we can see, the remainder is not equal to 0. Therefore, we say that, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

3. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{(5/3)}$ and - $\sqrt{(5/3)}$.

Solutions:

Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

$\sqrt{(5/3)}$ and - $\sqrt{(5/3)}$ are zeroes of polynomial f(x).

$\therefore \left(x-\sqrt{(5/3)}\right)\left(x+\sqrt{(5/3)}\right) = x^2-(5/3) = 0$

$(3x^2-5)=0$, is a factor of given polynomial f(x).

Now, when we will divide f(x) by $(3x^2-5)$ the quotient obtained will also be a factor of f(x) and the remainder will be 0.

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
\hline
3x^2-5 & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4 \qquad -5x^2 \\
 & (-) \qquad\quad (+) \\
\hline
 & +6x^3+3x^2-10x-5 \\
 & -6x^3 \qquad -10x \\
 & (+) \qquad\quad (-) \\
\hline
 & 3x^2 \qquad -5 \\
 & 3x^2 \qquad -5 \\
 & (-) \qquad\quad (+) \\
\hline
 & 0
\end{array}
$$

Therefore, $3x^4+6x^3-2x^2-10x-5 = (3x^2-5)(x^2+2x+1)$

Now, on further factorizing (x^2+2x+1) we get,

$x^2+2x+1 = x^2+x+x+1 = 0$

$x(x+1)+1(x+1) = 0$

$(x+1)(x+1) = 0$

So, its zeroes are given by: $x= -1$ and $x = -1$.

Therefore, all four zeroes of given polynomial equation are:

√(5/3),- √(5/3) , −1 and −1**.**

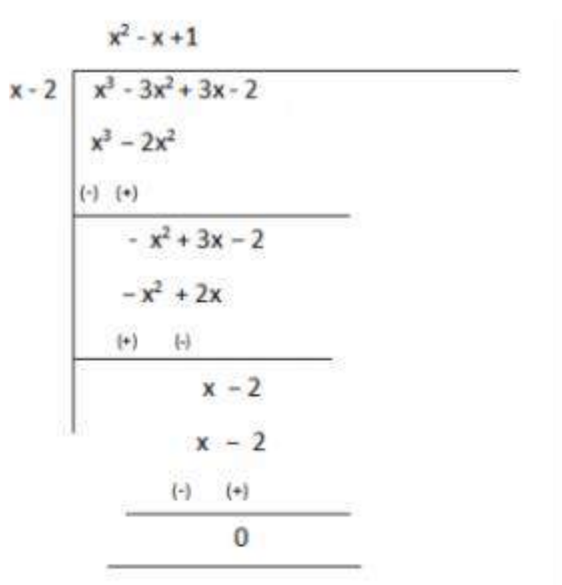
Hence, is the answer.

4. On dividing x^3-$3x^2$+x+2 by a polynomial g(x), the quotient and remainder were x–2 and –2x+4, respectively. Find g(x).

Solutions:
Given,
Dividend, p(x) = x^3-3x^2+x+2
Quotient = x-2
Remainder = –2x+4
We have to find the value of Divisor, g(x) =?

As we know,
Dividend = Divisor × Quotient + Remainder

∴ $x^3-3x^2+x+2 = g(x)\times(x-2) + (-2x+4)$
$x^3-3x^2+x+2-(-2x+4) = g(x)\times(x-2)$
Therefore, g(x) × (x-2) = x^3-3x^2+x+2
Now, for finding g(x) we will divide x^3-3x^2+x+2 with (x-2)



Therefore, **g(x) = (x^2–x+1)**

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and
(i) deg p(x) = deg q(x)
(ii) deg q(x) = deg r(x)
(iii) deg r(x) = 0

Solutions:
According to the division algorithm, dividend p(x) and divisor g(x) are two polynomials, where g(x)≠0. Then we

can find the value of quotient q(x) and remainder r(x), with the help of below given formula;

Dividend = Divisor × Quotient + Remainder
∴ p(x) = g(x)×q(x)+r(x)
Where r(x) = 0 or degree of r(x)< degree of g(x).
Now let us proof the three given cases as per division algorithm by taking examples for each.

(i) deg p(x) = deg q(x)
Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.
Let us take an example, p(x) = $3x^2 + 3x + 3$ is a polynomial to be divided by g(x) = 3.
So, $(3x^2 + 3x + 3)/3 = x^2 + x + 1$ = q(x)
Thus, you can see, the degree of quotient q(x) = 2, which also equal to the degree of dividend p(x).
Hence, division algorithm is satisfied here.

(ii) deg q(x) = deg r(x)
Let us take an example, p(x) = $x^2 + 3$ is a polynomial to be divided by g(x) = x - 1.
So, $x^2 + 3 = (x - 1)\times(x) + (x + 3)$
Hence, quotient q(x) = x
Also, remainder r(x) = x + 3
Thus, you can see, the degree of quotient q(x) = 1, which is also equal to the degree of remainder r(x).
Hence, division algorithm is satisfied here.

(iii) deg r(x) = 0
The degree of remainder is 0 only when the remainder left after division algorithm is constant.
Let us take an example, p(x) = $x^2 + 1$ is a polynomial to be divided by g(x) = x.
So, $x^2 + 1 = (x)\times(x) + 1$
Hence, quotient q(x) = x
And, remainder r(x) = 1
Clearly, the degree of remainder here is 0.
Hence, division algorithm is satisfied here.

Exercise 2.4 Page: 36

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2$; -1/2, 1, -2

Solution:
Given, $p(x) = 2x^3+x^2-5x+2$

And zeroes for p(x) are = 1/2, 1, -2

$\therefore p(1/2) = 2(1/2)^3+(1/2)^2-5(1/2)+2 = (1/4)+(1/4)-(5/2)+2 = 0$

$p(1) = 2(1)^3+(1)^2-5(1)+2 = 0$

$p(-2) = 2(-2)^3+(-2)^2-5(-2)+2 = 0$

Hence, proved 1/2, 1, -2 are the zeroes of $2x^3+x^2-5x+2$.

Now, comparing the given polynomial with general expression, we get;

$\therefore ax^3+bx^2+cx+d = 2x^3+x^2-5x+2$

a=2, b=1, c= -5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

$\alpha+\beta+\gamma = -b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = c/a$

$\alpha\beta\gamma = -d/a$.

Therefore, putting the values of zeroes of the polynomial,

$\alpha+\beta+\gamma = ½+1+(-2) = -1/2 = -b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = (1/2\times1)+(1\times-2)+(-2\times1/2) = -5/2 = c/a$

$\alpha\beta\gamma = ½\times1\times(-2) = -2/2 = -d/a$

Hence, the relationship between the zeroes and the coefficients are satisfied.

(ii) x^3-4x^2+5x-2 ;2, 1, 1

Solution:
Given, p(x) = x^3-4x^2+5x-2

And zeroes for p(x) are 2,1,1.

∴ $p(2)= 2^3-4(2)^2+5(2)-2 = 0$

$p(1) = 1^3-(4\times1^2)+(5\times1)-2 = 0$
Hence proved, 2, 1, 1 are the zeroes of x^3-4x^2+5x-2

Now, comparing the given polynomial with general expression, we get;

∴ $ax^3+bx^2+cx+d = x^3-4x^2+5x-2$
a = 1, b = -4, c = 5 and d = -2

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

$\alpha + \beta + \gamma = -b/a$
$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$
$\alpha \beta \gamma = - d/a$.

Therefore, putting the values of zeroes of the polynomial,

$\alpha +\beta+\gamma = 2+1+1 = 4 = -(-4)/1 = -b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = 2\times1+1\times1+1\times2 = 5 = 5/1= c/a$

$\alpha\beta\gamma = 2\times1\times1 = 2 = -(-2)/1 = -d/a$

Hence, the relationship between the zeroes and the coefficients are satisfied.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Solution:
Let us consider the cubic polynomial is ax^3+bx^2+cx+d and the values of the zeroes of the polynomials be α, β, γ.

As per the given question,

$\alpha+\beta+\gamma = -b/a = 2/1$

$\alpha\beta +\beta\gamma+\gamma\alpha = c/a = -7/1$

$\alpha \beta\gamma = -d/a = -14/1$

Thus, from above three expressions we get the values of coefficient of polynomial.
$a = 1, b = -2, c = -7, d = 14$

Hence, the cubic polynomial is $x^3-2x^2-7x+14$

3. If the zeroes of the polynomial x^3-3x^2+x+1 are a – b, a, a + b, find a and b.

Solution:
We are given with the polynomial here,
$p(x) = x^3-3x^2+x+1$

And zeroes are given as a – b, a, a + b

Now, comparing the given polynomial with general expression, we get;

$\therefore px^3+qx^2+rx+s = x^3-3x^2+x+1$

$p = 1$, $q = -3$, $r = 1$ and $s = 1$

Sum of zeroes = a – b + a + a + b

$-q/p = 3a$

Putting the values q and p.

$-(-3)/1 = 3a$

$a=1$

Thus, the zeroes are 1-b, 1, 1+b.

Now, product of zeroes = 1(1-b)(1+b)

$-s/p = 1-b^2$

$-1/1 = 1-b^2$

$b^2 = 1+1 = 2$

$b = \sqrt{2}$

Hence, $1-\sqrt{2}$, 1, $1+\sqrt{2}$ are the zeroes of x^3-3x^2+x+1.

4. If two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2 \pm\sqrt{3}$, find other zeroes.

Solution:
Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

Let $f(x) = x^4-6x^3-26x^2+138x-35$

Since $2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of given polynomial f(x).

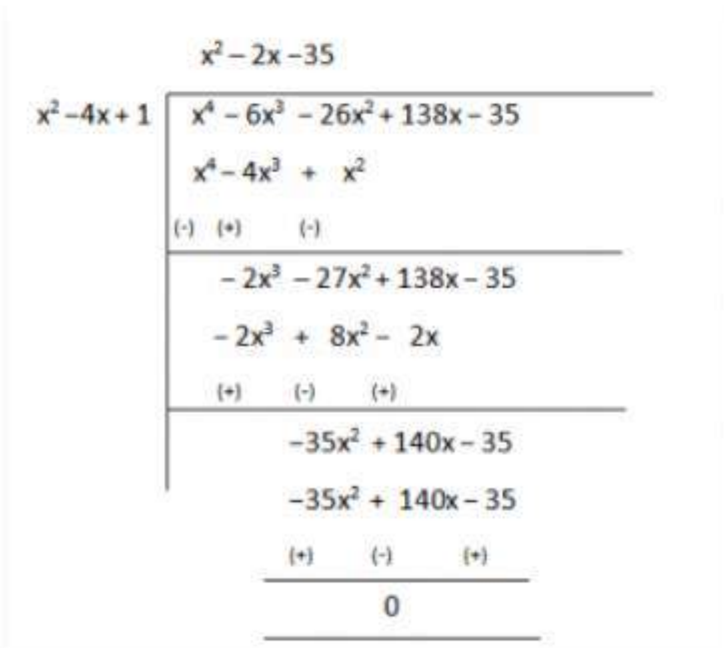
$\therefore [x-(2+\sqrt{3})]\,[x-(2-\sqrt{3})] = 0$

$(x-2-\sqrt{3})(x-2+\sqrt{3}) = 0$

On multiplying the above equation we get,

x^2-4x+1, this is a factor of a given polynomial f(x).

Now, if we will divide f(x) by g(x), the quotient will also be a factor of f(x) and the remainder will be 0.



So, $x^4-6x^3-26x^2+138x-35 = (x^2-4x+1)(x^2-2x-35)$

Now, on further factorizing $(x^2-2x-35)$ we get,

$x^2-(7-5)x-35 = x^2-7x+5x+35 = 0$

$x(x-7)+5(x-7) = 0$

$(x+5)(x-7) = 0$

So, its zeroes are given by:

$x = -5$ and $x = 7$.

Therefore, all four zeroes of given polynomial equation are: $2+\sqrt{3}$, $2-\sqrt{3}$, -5 and 7.