EXERCISE 14A PAGE: 168

1. The sum of the interior angles of a polygon is four times the sum of its exterior angles. Find the number of sides in the polygon.

Solution:
The sum of the interior angle=4 times the sum of the exterior angles.
Therefore, the sum of the interior angles = $4\times360° = 1440°$.
Now we have
$(2n - 4) \times 90° = 1440°$
$2n - 4 = 16$
$2n = 16 + 4$
$2n = 20$
$n = 20/2$
$n = 10$
Thus, the number of sides in the polygon is 10.

2. The angles of a pentagon are in the ratio 4 : 8 : 6 : 4 : 5. Find each angle of the pentagon.

Solution:
Let the angles of the pentagon are 4x, 8x, 6x, 4x and 5x.
Thus, we can write
$4x + 8x + 6x + 4x + 5x = 540°$
$27x = 540°$
$x = 20°$
Hence the angles of the pentagon are:
$4\times20° = 80°$, $8\times20° = 160°$, $6\times20°= 120°$, $4\times20°= 80°$, $5\times20°= 100°$

3. One angle of a six-sided polygon is 140° and the other angles are equal. Find the measure of each equal angle.

Solution:
Let the measure of each equal angles are x.
Then we can write
$140° + 5x = (2 \times 6 - 4) \times 90°$
$140° + 5x = 720°$
$5x = 580°$
$x = 116°$
Therefore, the measure of each equal angles are 116°

4. In a polygon there are 5 right angles and the remaining angles are equal to 195° each. Find the number of sides in the polygon.

Solution:
Let the number of sides of the polygon is *n* and there are k angles with measure 195^o.
Therefore, we can write:
$5 \times 90^o + k \times 195^o = (2n - 4) 90^o$
$180^o n - 195^o k = 450^o - 360^o$
$180^o n - 195^o k = 90^o$
$12n - 13k = 90^o$
In this linear equation n and k must be integer.
Therefore, to satisfy this equation the minimum value of k must be 6 to get n as integer.
Hence the number of sides are: 5 + 6 = 11.

5. Three angles of a seven-sided polygon are 132^o each and the remaining four angles are equal. Find the value of each equal angle.

Solution:
Let the measure of each equal angles are x.
Then we can write
$3 \times 132^o + 4x = (2 \times 7 - 4) 90^o$
$4x = 900^o - 396^o$
$4x = 504^o$
$x = 126^o$
Thus, the measure of each equal angles is 126^o.

6. Two angles of an eight-sided polygon are 142^o and 176^o. If the remaining angles are equal to each other; find the magnitude of each of the equal angles.

Solution:
Let the measure of each equal sides of the polygon is x.
Then we can write:
$142^o + 176^o + 6x = (2 \times 8 - 4) 90^o$
$6x = 1080^o - 318^o$
$6x = 762^o$
$x = 127^o$
Thus, the measure of each equal angles is 127^o

7. In a pentagon ABCDE, AB is parallel to DC and ∠A : ∠E : ∠D = 3 : 4 : 5. Find angle E.
Solution:

Let the measure of the angles are 3x, 4x and 5x.
Thus
$\angle A + \angle B + \angle C + \angle D + \angle E = 540^o$
$3x + (\angle B + \angle C) + 4x + 5x = 540^o$
$12x + 180^o = 540^o$

$12x = 360^o$
$x = 30^o$
Thus, the measure of angle E will be $4 \times 30^o = 120^o$

8. AB, BC and CD are the three consecutive sides of a regular polygon. If ∠BAC = 15°; find,
(i) Each interior angle of the polygon.
(ii) Each exterior angle of the polygon.
(iii) Number of sides of the polygon.

Solution:
(i) Let each angle of measure x degree.
Therefore, measure of each angle will be:
$x – 180^o – 2 \times 15^o = 150^o$

(ii) Let each angle of measure x degree.
Therefore, measure of each exterior angle will be:
$x = 180^o – 150^o$
$= 30^o$

(iii) Let the number of each sides is n.
Now we can write
$n . 150^o = (2n – 4) \times 90^o$
$180^o n – 150^o n = 360^o$
$30^0 n = 360^o$
$n = 12$
Thus, the number of sides are 12.

9. The ratio between an exterior angle and an interior angle of a regular polygon is 2 : 3. Find the number of sides in the polygon.

Solution:
Let measure of each interior and exterior angles are 3k and 2k.
Let number of sides of the polygon is n.
Now we can write:
$n. 3k = (2n – 4) \times 90^o$
$3nk = (2n – 4) 90^o$ ……… (1)
Again
$n. 2k = 360^o$
$nk = 180^o$
from (1)
$3. 180^o = (2n – 4)90^o$
$3 = n – 2$
$n = 5$

Thus, the number of sides of the polygon is 5.

10. The difference between an exterior angle of (n - 1) sided regular polygon and an exterior angle of (n + 2) sided regular polygon is 6° find the value of n.

Solution:

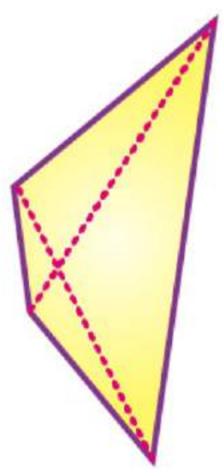
For (n-1) sided regular polygon:
Let measure of each angle is x.
Therefore
$(n - 1)\ x = (2\ (n - 1) - 4)\ 90^o$
$x = (n-3/\ n - 1)\ 180^o$
For (n+1) sided regular polygon:
Let measure of each angle is y.
Therefore
$(n + 2)\ y = (2\ (n + 2) - 4)\ 90^o$
$y = (n/\ n + 2)\ 180^o$
now we have
$y - x = 6^o$
$(n/\ n + 2)\ 180^o - (n-3/\ n - 1)\ 180^o = 6^o$
$(n\ /n + 2) - (n - 3/\ n - 1) = 1/30$
$30\ n(n - 1) - 30\ (n - 3)\ (n + 2) = (n + 2)\ (n - 1)$
$n^2 + n - 182 = 0$
$(n - 13)\ (n + 14) = 0$
$n = 13, -14$
Thus, the value of n is 13.

EXERCISE 14B PAGE: 175

1. State, 'true' or 'false'
(i) The diagonals of a rectangle bisect each other.
(ii) The diagonals of a quadrilateral bisect each other.
(iii) The diagonals of a parallelogram bisect each other at right angle.
(iv) Each diagonal of a rhombus bisects it.
(v) The quadrilateral, whose four sides are equal, is a square.
(vi) Every rhombus is a parallelogram.
(vii) Every parallelogram is a rhombus.
(viii) Diagonals of a rhombus are equal.
(ix) If two adjacent sides of a parallelogram are equal, it is a rhombus.
(x) If the diagonals of a quadrilateral bisect each other at right angle, the quadrilateral is a square.

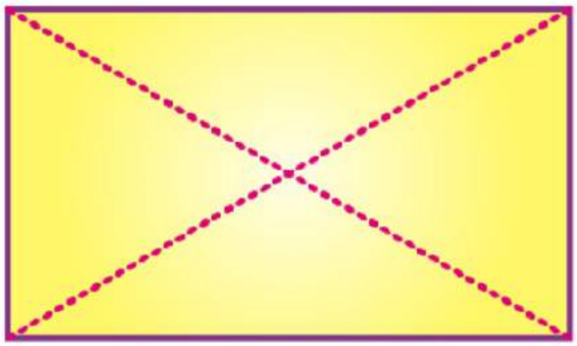
Solution:
(i)True.
This is true, because we know that a rectangle is a parallelogram. So, all the properties of a parallelogram are true for a rectangle. Since the diagonals of a parallelogram bisect each other, the same holds true for a rectangle.

(ii)False
This is not true for any random quadrilateral. Observe the quadrilateral shown below.



Clearly the diagonals of the given quadrilateral do not bisect each other. However, if the quadrilateral was a special quadrilateral like a parallelogram, this would hold true.

(iii)False
Consider a rectangle as shown below.

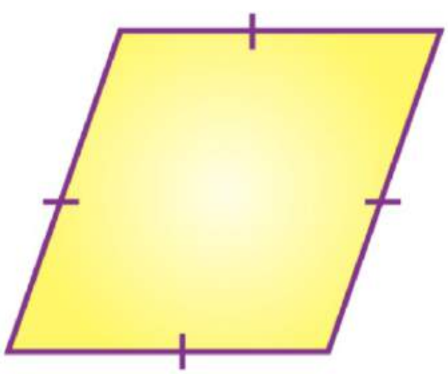


It is a parallelogram. However, the diagonals of a rectangle do not intersect at right angles, even though they bisect each other.

(iv)True
Since a rhombus is a parallelogram, and we know that the diagonals of a parallelogram bisect each other, hence the diagonals of a rhombus too, bisect other.

(v)False
This need not be true, since if the angles of the quadrilateral are not right angles, the quadrilateral would be a rhombus rather than a square.

(vi)True



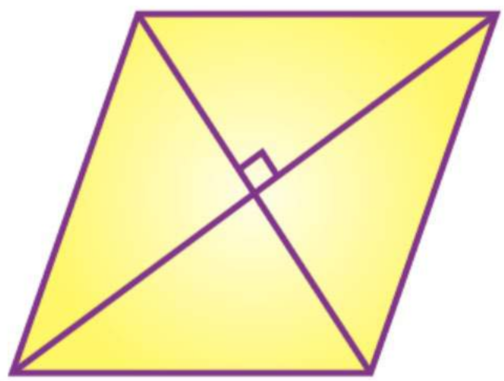
A parallelogram is a quadrilateral with opposite sides parallel and equal.
Since opposite sides of a rhombus are parallel, and all the sides of the rhombus are equal, a rhombus is a parallelogram.

(vii)False
This is false, since a parallelogram in general does not have all its sides equal. Only opposite sides of a parallelogram are equal. However, a rhombus has all its sides equal. So, every parallelogram cannot be a rhombus, except those parallelograms that have all equal sides.

(viii)False
This is a property of a rhombus. The diagonals of a rhombus need not be equal.

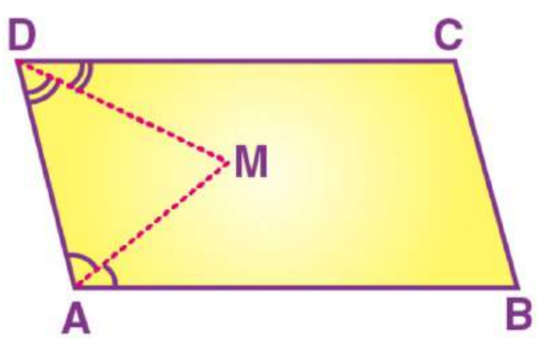
(ix)True
A parallelogram is a quadrilateral with opposite sides parallel and equal.
A rhombus is a quadrilateral with opposite sides parallel, and all sides equal.
If in a parallelogram the adjacent sides are equal, it means all the sides of the parallelogram are equal, thus forming a rhombus.

(x)False



Observe the above figure. The diagonals of the quadrilateral shown above bisect each other at right angles, however the quadrilateral need not be a square, since the angles of the quadrilateral are clearly not right angles.

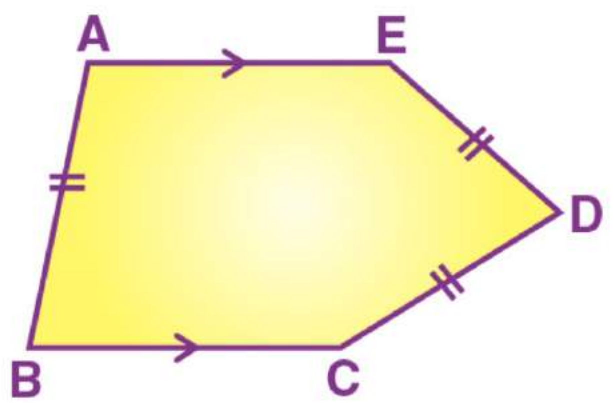
2. In the figure, given below, AM bisects angle A and DM bisects angle D of parallelogram ABCD. Prove that: ∠AMD = 90°.



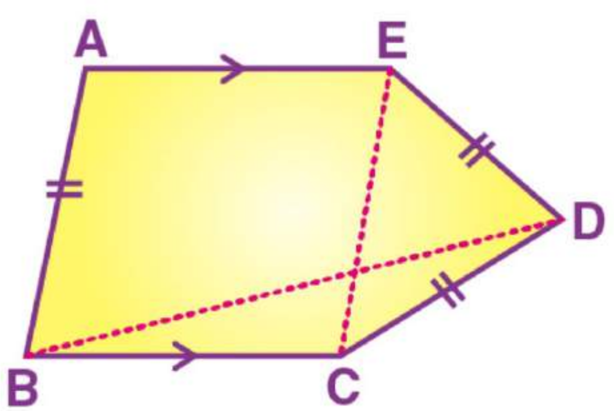
Solution:
From the given figure we can conclude that
∠A + ∠D = 180° [since consecutive angles are supplementary]

$\angle A/2 + \angle D/2 = 90$
Again, from triangle ADM
$\angle A/2 + \angle D/2 + \angle M = 180^o$
$90^o + \angle M = 180^o$
$\angle M = 180^o = 90^o$
Hence $\angle AMD = 90^o$

3. In the following figure, AE and BC are equal and parallel and the three sides AB, CD and DE are equal to one another. If angle A is 102^o. Find angles AEC and BCD.



Solution:
According to the question,

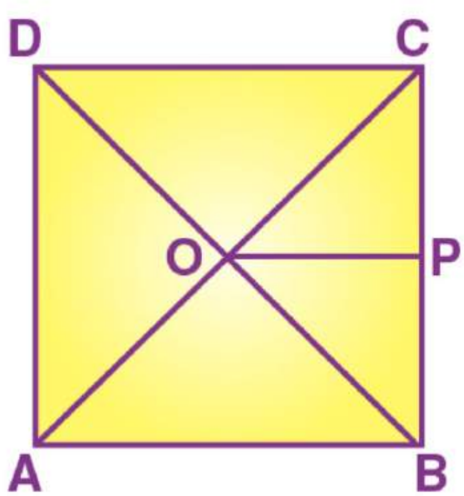


Given that AE = BC
We have to find
$\angle AEC$ and $\angle BCD$
Let us join EC and BD
In the quadrilateral AECB
AE = BC and AB = EC
Also, AE parallel to BC
So quadrilateral is a parallelogram.
In parallelogram consecutive angles are supplementary

$\angle A + \angle B = 180^o$
$102^o + \angle B = 180^o$
$\angle B = 78^o$
In parallelogram opposite angles are equal
$\angle A = \angle BEC$ and $\angle B = \angle AEC$
$\angle BEC = 102^o$ and $\angle AEC = 78^o$
Now consider triangle ECD
EC = ED = CD [since AB = EC]
Therefore, triangle ECD is an equilateral triangle.
$\angle ECD = 60^o$
$\angle BCD = \angle BEC + \angle ECD$
$\angle BCD = 102^o + 60^o$
$\angle BCD = 162^o$
Therefore, $\angle AEC = 78^o$ and $\angle BCD = 162^o$

4. In a square ABCD, diagonals meet at O. P is a point on BC such that OB = BP.
Show that:
(i) $\angle POC = 22\frac{1}{2}^o$
(ii) $\angle BDC = 2\angle POC$
(iii) $\angle BOP = 3\angle CPO$

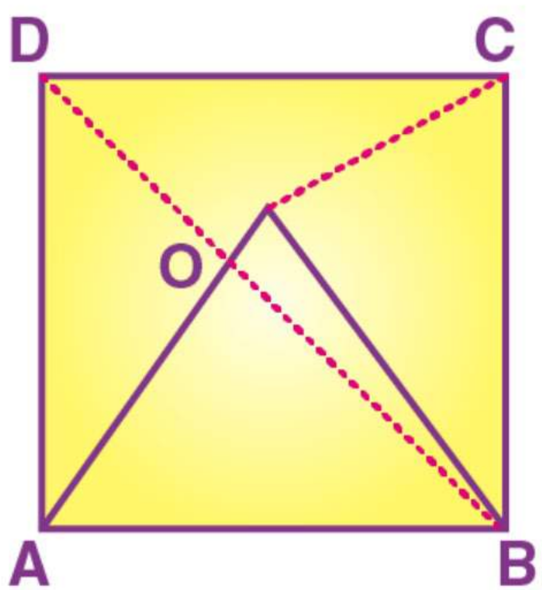
Solution:
Given ABCD is a square and diagonal meet at o. P is a point on BC such that OB = BP



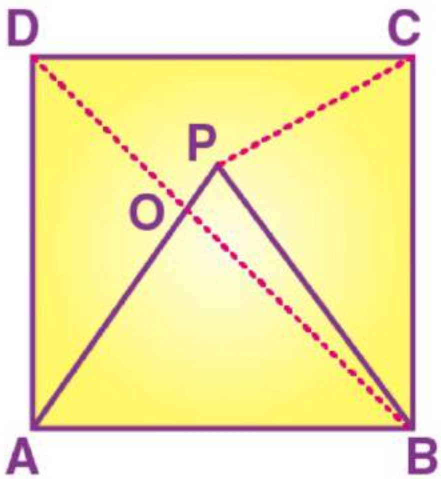
In the triangle BOC and triangle DOC
BD = BD [common side]
BO = CO
OD = OC [since diagonals cuts at O]
$\triangle BOC \cong \triangle DOC$ [By SSS postulate]
Therefore,
$\angle BOC = 90^o$

Now ∠POC = 22.5
∠BOP = 67.5
Again, in triangle BDC
∠BDC = 45° [since ∠B = 45° and ∠C = 90°]
Therefore
∠BDC = 2∠POC
∠BOP = 67.5°
∠BOP = 2 ∠POC
Hence the proof.

5. The given figure shows a square ABCD and an equilateral triangle ABP. Calculate:
(i) ∠AOB
(ii) ∠BPC
(iii) ∠PCD
(iv) Reflex ∠APC



Solution:

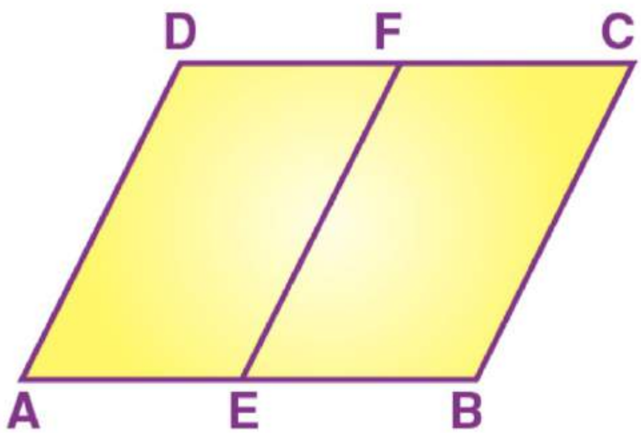


In the given figure, triangle APB is an equilateral triangle
Therefore, all its angles are 60^o
Again, in the triangle ADB
$\angle ABD = 45^o$
$\angle AOB = 180^o - 60^o - 45^o$
$= 75^o$
Again, in triangle BPC
$\angle BPC = 75^o$ [since BP = CB]
Now,
$\angle C = \angle BCP + \angle PCD$
$\angle PCD = 90^o - 75^o$
$\angle PCD = 15^o$
Therefore
$\angle APC = 60^o + 75^o$
$\angle APC = 135^o$
Reflex $\angle APD = 360^o - 135^o = 225^o$
Therefore
(i) $\angle AOB = 75^o$
(ii) $\angle BPC = 75^o$
(iii) $\angle PCD = 15^o$
(iv) Reflex $\angle APC = 225^o$

EXERCISE 14C PAGE: 181

1. E is the mid-point of side AB and F is the midpoint of side DC of parallelogram ABCD. Prove that AEFD is a parallelogram.

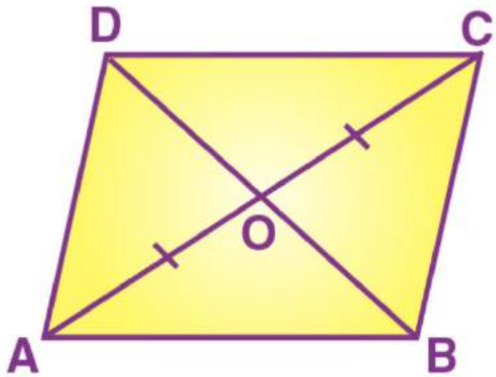
Solution:
Let us draw a parallelogram ABCD Where F is the midpoint
Of side DC of parallelogram ABCD
To prove: AEFD is a parallelogram



Proof:
In quadrilateral ABCD
AB parallel to DC
BC parallel to AD
AB = DC
Multiply both side by ½
½ AB = ½ DC
AE = DF
Also, AD parallel to EF
Therefore, AEFC is a parallelogram.

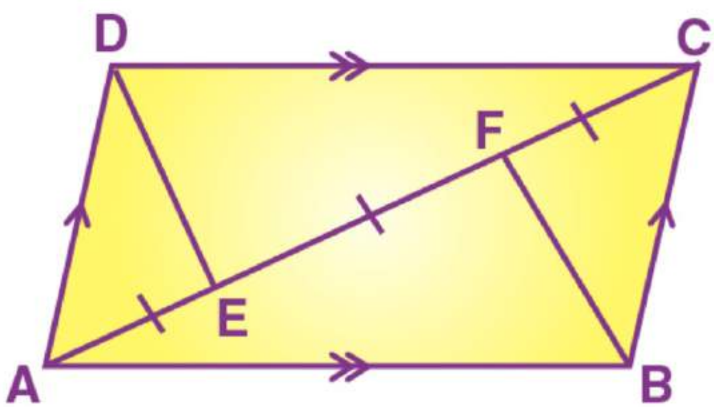
2. The diagonal BD of a parallelogram ABCD bisects angles B and D. Prove that ABCD is a rhombus.

Solution:
Given ABCD is a parallelogram where the diagonal BD bisects parallelogram ABCD at angle B and D
To prove: ABCD is a rhombus
Proof: Let us draw a parallelogram ABCD where the diagonal BD bisects the parallelogram at angle B and D.
Construction: Let us join AC as a diagonal of the parallelogram ABCD

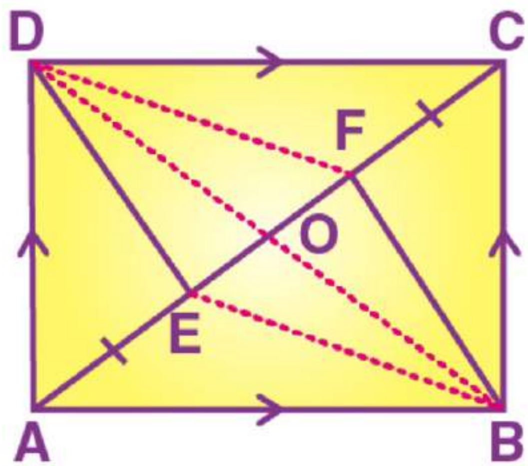


Since ABCD is a parallelogram
Therefore, AB = DC
AD = BC
Diagonal BD bisects angle B and D
So ∠COD = ∠DOA
Again, AC also bisects at A and C
Therefore ∠AOB = ∠BOC
Thus, ABCD is a rhombus.
Hence proved

3. The alongside figure shows a parallelogram ABCD in which AE = EF = FC. Prove that:
(i) DE is parallel to FB
(ii) DE = FB
(iii) DEBF is a parallelogram.

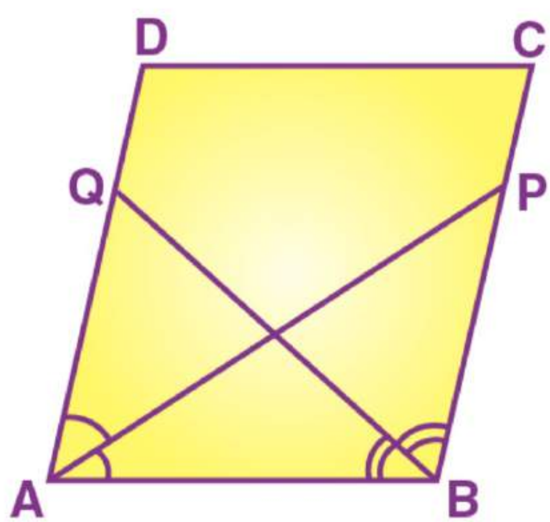


Solution:
Construction:
Join DF and EB
Join diagonal BD



Since the diagonals of a parallelogram bisect each other
Therefore, OA = OC and OB = OD
Also, AE = EF = FC
Now, OA = OC and AE = FC
OA - OC = OC – FC
OE = OF
Thus, in quadrilateral DEFB, we have
OB = OD and OE = OF
Diagonals of a quadrilateral DEFB bisect each other.
DEFB is a parallelogram
DE is parallel to FB
DE = FB (opposite sides are equal)

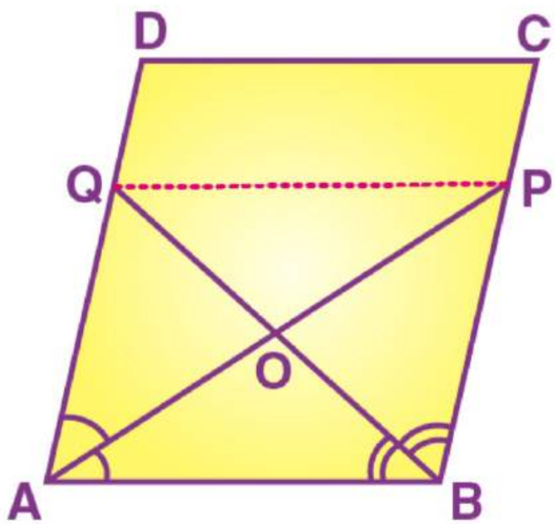
4. In the alongside diagram, ABCD is a parallelogram in which AP bisects angle AP bisects angle A and BQ bisects angle B. Prove that:



(i) AQ = BP
(ii) PQ = CD.

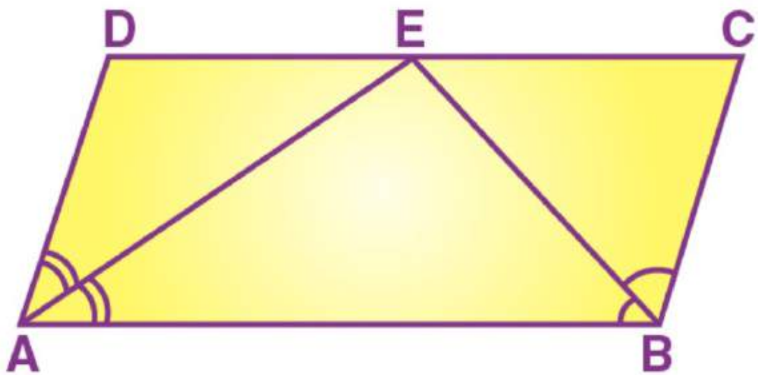
(iii) ABPQ is a parallelogram.

Solution:



Consider the triangle AOQ and triangle BOP
∠AOQ = ∠BOP [opposite angles]
∠OAQ = ∠BPO [alternate angles]
△ AOQ ≅ △ BOP
Hence AQ = BP
Consider the triangle QOP and triangle AOB
∠AOB = ∠QOP [opposite angles]
∠OAB = ∠APQ [alternate angles]
△ QOP ≅ △ AOB
Hence PQ = AB = CD
Consider the quadrilateral QPCD
DQ = CP and DQ parallel to CP
Also, QP = DC and AB parallel to QP parallel to DC
Hence quadrilateral QPCD is a parallelogram.

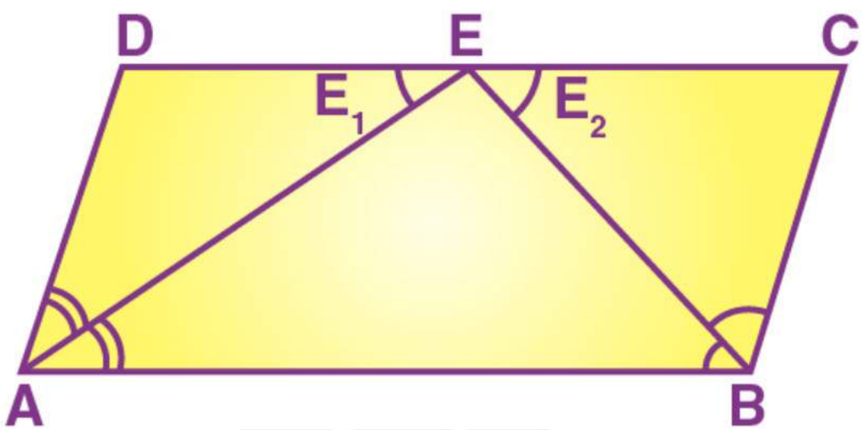
5. In the given figure, ABCD is a parallelogram. Prove that: AB = 2 BC.



Solution:

Given: ABCD is a parallelogram

To prove: AB = 2BC



Proof: ABCD is a parallelogram

$\angle A + \angle D = \angle B + \angle C = 180^o$

From the triangle AEB we have

$\angle A/2 + \angle B/2 + \angle E = 180^o$

$\angle A - \angle A/2 + \angle D + \angle E_1 = 180^o$ [taking E_1 as a new angle]

$\angle A + \angle D + \angle E_1 = 180^o + \angle A/2$

$\angle E_1 = \angle A/2$ [since $\angle A + \angle D = 180^o$]

Again, similarly

$\angle E_2 = \angle B/2$

Now,

AB = DE + EC

= AD + BC

= 2 BC [since AD = BC]