EXERCISE 25

1. Evaluate:

(i) $\cos 22^\circ/\sin 68^\circ$

(ii) $\tan 47^\circ/\cot 43^\circ$

(iii) $\sec 75^\circ/\text{cosec } 15^\circ$

(iv) $\frac{\cos 55^\circ}{\sin 35^\circ} + \frac{\cot 35^\circ}{\tan 55^\circ}$

(v) $\sin^2 40^\circ - \cos^2 50^\circ$

(vi) $\sec^2 18^\circ - \text{cosec}^2 72^\circ$

(vii) $\sin 15^\circ \cos 15^\circ - \cos 75^\circ \sin 75^\circ$

(viii) $\sin 42^\circ \cos 48^\circ - \cos 42^\circ \sin 48^\circ$

Solution:

(i) According to the question we have,
$\cos 22^\circ/\sin 68^\circ = \cos (90^\circ - 68^\circ)/ \sin 68^\circ$
by using complementary angle identity,
$= \sin 68^\circ / \sin 68^\circ$
$= 1$

(ii) According to the question we have,
$\tan 47^\circ/\cot 43^\circ = \tan (90^\circ - 43^\circ)/ \cot 43^\circ$
by using complementary angle identity,
$= \cot 43^\circ / \cot 43^\circ$
$= 1$

(iii) According to the question we have,
$\sec 75^\circ/\text{cosec } 15^\circ = \sec (90^\circ - 15^\circ)/ \text{cosec } 15^\circ$
by using complementary angle identity,
$= \text{cosec } 15^\circ / \text{cosec } 15^\circ$
$= 1$

(iv) According to the question we have,

$$\frac{\cos 55^\circ}{\sin 35^\circ} + \frac{\cot 35^\circ}{\tan 55^\circ}$$

$$= \frac{\cos(90^\circ - 35^\circ)}{\sin 35^\circ} + \frac{\cot(90^\circ - 55^\circ)}{\tan 55^\circ}$$

$$= \frac{\sin 35^\circ}{\sin 35^\circ} + \frac{\tan 55^\circ}{\tan 55^\circ}$$

$= 1 + 1$
$= 2$

(v) $\sin^2 40^\circ - \cos^2 50^\circ$
$= \sin^2 (90^\circ - 50^\circ) - \cos^2 50^\circ$

$= \cos^2 50^o - \cos^2 50^o$
$= 0$

(vi) $\sec^2 18^o - \text{cosec}^2 72^o$
$= [\sec (90^o - 72^o)]^2 - \text{cosec}^2 72^o$
$= \text{cosec}^2 72^o - \text{cosec}^2 72^o$
$= 0$

(vii) $\sin 15^o \cos 15^o - \cos 75^o \sin 75^o$
$= \sin (90^o - 75^o) \cos 15^o - \cos 75^o \sin (90^o - 15^o)$
$= \cos 75^o \cos 15^o - \cos 75^o \cos 15^o$
$= 0$

(viii) $\sin 42^o \sin 48^o - \cos 42^o \cos 48^o$
$= \sin (90^o - 48^o) \sin 48^o - \cos (90^o - 48^o) \cos 48^o$
$= \cos 48^o \sin 48^o - \sin 48^o \cos 48^o$
$= 0$

2. Evaluate:
(i) $\sin (90^o - A) \sin A - \cos (90^o - A) \cos A$
(ii) $\sin^2 35^o - \cos^2 55^o$
(iii) $\frac{\cot 54^o}{\tan 36^o} + \frac{\tan 20^o}{\cot 70^o} - 2$
(iv) $\frac{2 \tan 53^o}{\cot 37^o} - \frac{\cot 80^o}{\tan 10^o}$
(v) $\cos^2 25^o - \sin^2 65^o - \tan^2 45^o$
(vi) $\left(\frac{\sin 77^o}{\cos 13^o}\right)^2 + \left(\frac{\cos 77^o}{\sin 13^o}\right) - 2 \cos^2 45^o$

Solution:
(i) Given $\sin (90^o - A) \sin A - \cos (90^o - A) \cos A$
$= \cos A \sin A - \sin A \cos A$
$= 0$

(ii) Given $\sin^2 35^o - \cos^2 55^o$
$= \sin^2 35^o - [\cos (90^o - 35^o)]^2$
$= \sin^2 35^o - \sin^2 35^o$
$= 0$

(iii) According to the question we have,

$\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$

$= \frac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} - 2$

$= \frac{\tan 36^\circ}{\tan 36^\circ} + \frac{\cot 70^\circ}{\cot 70^\circ} - 2$

On simplifying we get
= 1 + 1 – 2
= 2 – 2
= 0

(iv) According to the question we have

$\frac{2\tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$

$= \frac{2\tan(90^\circ - 37^\circ)}{\cot 37^\circ} - \frac{\cot(90^\circ - 10^\circ)}{\tan 10^\circ}$

$= \frac{2\cot 37^\circ}{\cot 37^\circ} - \frac{\tan 10^\circ}{\tan 10^\circ}$

On simplifying we get
= 2 – 1
= 1

(v) According to the question we have
$\text{Cos}^2 25^\circ - \sin^2 65^\circ - \tan^2 45^\circ$
$= [\cos (90^\circ - 65^\circ)]^2 - \sin^2 65^\circ - (\tan^2 45^\circ)^2$
$= \sin^2 65^\circ - \sin^2 65^\circ - 1^2$
= 0 – 1
= -1

(vi) According to the question we have,

$\left(\frac{\sin 77^\circ}{\cos 13^\circ}\right)^2 + \left(\frac{\cos 77^\circ}{\sin 13^\circ}\right)^2 - 2\cos^2 45^\circ$

$= \left(\frac{\sin(90^\circ - 13^\circ)}{\cos 13^\circ}\right)^2 + \left(\frac{\cos(90^\circ - 13^\circ)}{\sin 13^\circ}\right)^2 - 2(\cos 45^\circ)^2$

$= \left(\frac{\cos 13^\circ}{\cos 13^\circ}\right)^2 + \left(\frac{\sin 13^\circ}{\sin 13^\circ}\right)^2 - 2\left(\frac{1}{\sqrt{2}}\right)^2$

On simplifying we get
$= 1^2 + 1^2 - 2$ (½)
= 1 + 1 – 1
= 1

3. Show that:

(i) tan 10° tan 15° tan 75° tan 80° = 1
(ii) sin 42° sec 48° + cos 42° cosec 48° = 2

Solution:
(i) L.H.S.
= tan 10° tan 15° tan 75° tan 80°
= tan (90° - 80°) tan (90° - 75°) tan 75° tan 80°
= cot 80° cot 75 ° tan 75° tan 80°
= (cot 80° tan 80°) (cot 75° tan 75°)
= (1)(1)
= 1
= R.H.S.

(ii) consider LHS

$$= \sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ$$

$$= \sin(90^\circ - 48^\circ) \times \frac{1}{\cos 48^\circ} + \cos(90^\circ - 48^\circ) \times \frac{1}{\sin 48^\circ}$$

$$= \cos 48^\circ \times \frac{1}{\cos 48^\circ} + \sin 48^\circ \times \frac{1}{\sin 48^\circ}$$

On simplifying we get,
= 1 + 1
= 2
= RHS

4. Express each of the following in terms of angles between 0°and 45°:
(i) sin 59°+ tan 63°
(ii) cosec 68°+ cot 72°
(iii) cos 74°+ sec 67°

Solution:
(i) Given sin 59°+ tan 63°
It can be written as
sin 59°+ tan 63° = sin (90 – 31)° + tan(90 – 27)°
= cos 31° + cos 27°

(ii) Given cosec 68°+ cot 72°
It can be written as
cosec 68°+ cot 72° = = cosec (90 – 22)° + cot (90 – 18)°
= sec 22° + tan 18°

(iii) Given cos 74°+ sec 67°
It can be written as
cos 74°+ sec 67° = cos (90 – 16)° + sec(90 – 23)°

= sin 16° + cosec 23°

5. Question 5
For triangle ABC, show that:

(i) $\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$

(ii) $\tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$

Solution:
(i) We know that for △ ABC
∠ A + ∠ B + ∠ C = 180°
∠ B + ∠ A = 180° = ∠ C
(∠ B + ∠ A) /2 = 90° - ∠ C/2
Sin [(A+B)/2] = sin (90° – C/2)
= cos (C/2)

(ii) We know that for △ ABC
∠ A + ∠ B + ∠ C = 180°
∠ B + ∠ C = 180° = ∠ A
(∠ B + ∠ C) /2 = 90° - ∠ A/2
tan [(B + C)/2] = tan (90° – A/2)
= cot (A/2)

6. Evaluate:

(i) $3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(ii) 3 cos 80° cosec 10° + 2 sin 59° sec 31°

(iii) $\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$

(iv) tan (55° – A) – cot (35° + A)

(v) cosec (65° + A) – sec (25° – A)

(vi) $2\frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2}\cos 45^\circ$

(vii) $\frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\frac{\sin^2 75^\circ}{\cos^2 15^\circ}$

(viii) $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$

(ix) 14 sin 30° + 6 cos 60° – 5 tan 45°

Solution:

(i) Given

$$3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$

$$= 3\frac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ}$$

$$= 3\frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ} = 3 - 1 = 2$$

(ii) Given 3 cos 80° cosec 10° + 2 sin 59° sec 31°

= 3 cos (90° – 19°) cosec 10° + 2 sin (90° – 31°) sec 31°

= 3 sin 10° cosec 10° + 2 cos 31° sec 31°

= 3 (1) + 2 (1)

= 3 + 2

= 5

(iii) Given

$$\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$$

$$= \frac{\sin\left(90^\circ - 10^\circ\right)}{\cos 10^\circ} + \sin(90^\circ - 31^\circ)\sec 31^\circ$$

$$= \frac{\cos 10^\circ}{\cos 10^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ}$$

On simplifying we get

= 1 + 1

= 2

(iv) Given tan (55° – A) – cot (35° + A)

It can be written as

tan (55° – A) – cot (35° + A) = tan [90 - (35° + A) – cot (35° + A)

= cot (35° + A) – cot (35° + A)

= 0

(v) Given cosec (65° + A) – sec (25° – A)

It can be written as

cosec (65° + A) – sec (25° – A) = cosec [90° - (25° - A) – sec (25° – A)

= sec (35° + A) – sec (25° – A)

= 0

(vi) $2\dfrac{\tan 57^\circ}{\cot 33^\circ} - \dfrac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2}\cos 45^\circ$

$= 2\dfrac{\tan(90^\circ - 33^\circ)}{\cot 33^\circ} - \dfrac{\cot(90^\circ - 20^\circ)}{\tan 20^\circ} - \sqrt{2}\left(\dfrac{1}{\sqrt{2}}\right)$

$= 2\dfrac{\cot 33^\circ}{\cot 33^\circ} - \dfrac{\tan 20^\circ}{\tan 20^\circ} - 1$

On simplifying we get,
= 2 – 1 – 1
= 0

(vii) Given

$\dfrac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\dfrac{\sin^2 75^\circ}{\cos^2 15^\circ}$

$= \dfrac{[\cot(90^\circ - 49^\circ)]^2}{\tan^2 49^\circ} - 2\dfrac{[\sin(90^\circ - 15^\circ)]^2}{\cos^2 15^\circ}$

$= \dfrac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2\dfrac{\cos^2 15^\circ}{\cos^2 15^\circ}$

On simplifying we get
= 1 – 2
= -1

(viii) Given

$\dfrac{\cos 70^\circ}{\sin 20^\circ} + \dfrac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$

$= \dfrac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \dfrac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8\left(\dfrac{1}{2}\right)^2$

$= \dfrac{\sin 20^\circ}{\sin 20^\circ} + \dfrac{\sin 31^\circ}{\sin 31^\circ} - 2$

On simplifying we get
= 1 + 1 – 2
= 2 – 2
= 0

(ix) Given 14 sin 30° + 6 cos 60° – 5 tan 45°
It can be written as
14 sin 30° + 6 cos 60° – 5 tan 45° = 14 (½) + 6 (½) – 5 (1)
= 7 + 3 – 5
= 10 – 5
= 5

7. A triangle ABC is right angled at B. Find the value of

$$\frac{\sec A.\sin C - \tan A.\tan C}{\sin B}.$$

Solution:
Since triangle ABC is a right-angled triangle, and right angle at B
Therefore, A + C = 90°

$$\therefore \frac{\sec A.\sin C - \tan A.\tan C}{\sin B}$$
$$= \frac{\sec A(90^\circ - C)\sin C - \tan(90^\circ - C)\tan C}{\sin 90^\circ}$$
$$= \frac{\operatorname{cosec} C \sin C - \cot C \tan C}{1}$$
$$= \frac{1}{\sin C} \times \sin C - \frac{1}{\tan C} \times \tan C$$

On simplifying we get
= 1 – 1
= 0

8. In each case, given below, find the value of angle A, where 0° ≤ A ≤ 90°.
(i) sin (90° - 3A). cosec 42° = 1
(ii) cos (90° - 3A).sec 77° = 1

Solution:
(i) Given sin (90° - 3A). cosec 42° = 1
On rearranging we get
sin (90° - 3A) = 1/cosec 42°
cos 3A = 1/cosec (90° – 48°)
cos 3A = 1/sec 48°
cos 3A = cos 48°
3A = 48°
Therefore, A = 16°

(ii) Given cos (90° - 3A).sec 77° = 1
cos (90° - 3A) = 1/ sec 77°
sin 3A = 1/ sec (90° – 12°)
sin 3A = 1/cosec 12°
sin 3A = sin 12°
3A = 12°
A = 3°