Exercise 6(A)

Solve the pairs of linear (simultaneous) equations by the method of elimination by substitution:

1. 8x + 5y = 9
3x + 2y = 4
Solution:

Given equations,
8x + 5y = 9... (1)
3x + 2y = 4... (2)
From (1), we have
y = (9 – 8x)/5
Now,
Putting this value of y in (2)
3x + 2[(9 – 8x)/5] = 4
On simplifying, we get
15x + 18 - 16x = 20
x = -2
Substituting the value of x in (1), we get
y = (9 – 8x)/5
= (9 – 8 × -2)/5
= (9 + 16)/5
= 25/5
= 5
Hence, the values of x and y are -2 and 5 respectively.

2. 2x - 3y = 7
5x + y = 9
Solution:

Given equations,
2x - 3y = 7... (1)
5x + y = 9... (2)
From (2), we have
y = 9 - 5x
Putting this value of y in (1), we get
2x - 3 (9 - 5x) = 7
2x - 27 + 15x = 7
17x = 34
x = 2
Now, from (2) we get
y = 9 - 5(2)
y = -1
Hence, the values of x and y are 2 and -1 respectively.

3. $2x + 3y = 8$
$2x = 2 + 3y$
Solution:

Given equation,
$2x + 3y = 8$... (1)
$2x = 2 + 3y$... (2)
From (2), we have
$2x = 2 + 3y$
Putting this value of 2x in (1), we get
$2 + 3y + 3y = 8$
$6y = 6$
$y = 1$
Now from (2), we get
$2x = 2 + 3 (1)$
$x = 5/2$
$x = 2.5$
Hence, the values of x and y are 2.5 and 1 respectively.

4. $0.2x + 0.1y = 25$
$2(x - 2) - 1.6y = 116$
Solution:

Given equation,
$0.2 + 0.1y = 25$ …(i)
$2(x-2)-1.6y = 116$ …(ii)
From (i), we have
$0.2x = 25 – 0.1y$
$x = [(25 - 0.1y)/0.2]$ …(iii)
On substituting the value of x from equation (iii) in equation (ii), we get
$2(x - 2) – 1.6y = 116$
$2\{[(25 - 0.1y)/0.2] - 2\} – 1.6y = 116$
$10(25 - 0.1y) - 4 - 1.6y = 116$
$250 – y – 4 – 1.6y = 116$
$-2.6y = - 130$
$y = 50$
Now, on substituting the value of y in (iii), we get
$x = (25 – 0.1y)/0.2$
$x = [25 – 0.1(50)]/0.2$
$x = (25 – 5)/0.2$
$x = 100$
Hence, the values of x and y are 100 and 50 respectively.

5. $6x = 7y + 7$
$7y - x = 8$
Solution:

Given equations,
$6x = 7y + 7$... (1)
$7y - x = 8$... (2)
From (2), we have
$x = 7y - 8$
On putting this value of x in (1), we get
$6(7y - 8) = 7y + 7$
$42y - 48 = 7y + 7$
$35y = 55$
$y = 11/7$
Now, on substituting the value of y in (2) we get
$x = 7(11/7) - 8$
$x = 3$
Hence, the values of x and y are 3 and 11/7 respectively.

6. $y = 4x - 7$
$16x - 5y = 25$
Solution:

Given equations,
$y = 4x - 7$... (1)
$16x - 5y = 25$ … (2)
From (1), we have
$y = 4x - 7$
On putting this value of y in (2)
$16x - 5 (4x - 7) = 25$
$16x - 20x + 35 = 25$
$-4x = -10$
$x = 5/2$
Now, on substituting the value of x in (1), we get
$y = 4(5/2) – 7$
$y = 10 – 7$
$y = 3$
Hence, the values of x and y are 5/2 and 3 respectively.

7. $2x + 7y = 39$
$3x + 5y = 31$
Solution:

Given equations,
$2x + 7y = 39$... (1)
$3x + 5y = 31$... (2)
From (1), we have
$x = (39 - 7y)/2$
On putting the value of x in (2), get

3[(39 - 7y)/2] + 5y = 31
117 - 21y + 10y = 62
-11y = -55
y = 5
Now, on substituting the value of y in (1), we get
x = [39 - 7(5)]/2
x = 4/2
x = 2
Hence, the values of x and y are 2 and 5 respectively.

8. 1.5x + 0.1y = 6.2
3x - 0.4y = 11.2
Solution:

Given equations,
1.5 + 0.1y = 6.2 … (i)
3x-0.4y = 11.2 … (ii)
From (i), we have
1.5x + 0.1y = 6.2
1.5x = 6.2 – 0.1y
x = (6.2 - 0.1y)/1.5 … (iii)
On substituting the value of x in (ii), we get
3x – 0.4y = 11.2
3{[(6.2-0.1y)/1.5]-2} – 0.4y = 11.2
2(6.2-0.1y) – 0.4y = 11.2
12.4 – 0.2y – 0.4y = 11.2
- 0.6y = -1.2
y = 2
Now, on substituting the value of y in (iii), we get
x = (6.2 – 0.1y)/1.5
x = [6.2 – 0.1(2)]/1.5
x = (6.2 – 0.2)/1.5
x = 4

Hence, the values of x and y are 4 and 2 respectively.

9. 2(x - 3) + 3(y - 5) = 0
5(x - 1) + 4(y - 4) = 0
Solution:

Given equations,
2(x - 3) + 3(y - 5) = 0 … (1)
5(x - 1) + 4(y - 4) = 0 … (2)
From (1), we have
2x – 6 + 3y – 15 = 0
2x – 21 + 3y = 0

2x = 21 – 3y
x = (21 – 3y)/2 … (3)
And from (2), we have
5(x - 1) + 4(y - 4) = 0
5x – 5 + 4y – 16 = 0
5x + 4y – 21 = 0 … (4)
On substituting x from (3) in (4), we get
5[(21 – 3y)/2] + 4y – 21 =0
(105 – 15y)/2 + 4y – 21 = 0
105 – 15y +8y – 42 = 0
-7y + 63 = 0
7y = 63
y = 9
Now, substituting y = 9 in (3), we get
x = [21 – 3(9)]/2
= (21 – 27)/2
= (-6/2)
= -3
Hence, the values of x and y are -3 and 9 respectively.

10. (2x + 1)/7 + (5y - 3)/3 = 12
(3x + 2)/2 - (4y + 3)/9 = 13
Solution:

Given equations,
2x+1)/7 + (5y - 3)/3 = 12 … (1)
(3x + 2)/2 - (4y + 3)/9 = 13 … (2)
From (1), we have
(2x + 1)/7 + (5y - 3)/3 = 12
[3(2x + 1) + 7(5y - 3)]/21 = 12
6x + 3 + 35y – 21 = 252
6x + 35y – 18 = 252
6x + 35y = 270
6x = 270 – 35y
x = (270 – 35y)/6 … (3)
And from (2), we have
(3x + 2)/2 - (4y + 3)/9 = 13
[9(3x + 2) - 2(4y + 3)]/18 = 13
27x + 18 – 8y - 6 = 234
27x – 8y + 12 = 234
27x – 8y = 222 … (4)

On substituting x from (3) in (4), we get
27[(270 – 35y)/6] – 8y = 222
7290 – 945y – 48y = 1332
-993y = -5958

y = 6
Now, on substituting value of y in (3), we get
x = (270 -35 × 6)/6
= (270 – 210)/6
= 60/6
= 10
Hence, the values of x and y are 10 and 6 respectively.

11. 3x + 2y = 11
2x - 3y + 10 = 0
Solution:

Given equations,
3x + 2y = 11
2x - 3y + 10 = 0
From (i), we have
3x + 2y = 11
3x = 11 – 2y
x = (11- 2y)/3 …(iii)
On substituting x from (iii) in (ii), we get
2[(11 - 2y)/3] – 3y + 10 = 0
[(22 – 4y)/3] – 3y = -10
(22 – 4y – 9y)/3 = -10
22 – 13y = -30
13y = 52
y = 4
On substituting the value of y in (iii), we get
x = [11 – 2(4)]/3
= (11- 8)/3
= 3/3
= 1
Hence, the values of x and y are 1 and 4 respectively.

12. 2x - 3y + 6 = 0
2x + 3y - 18 = 0
Solution:

Given equations,
2x - 3y + 6 = 0 … (i)
2x + 3y - 18 = 0 … (ii)
From (i), we have
2x – 3y + 6 = 0
2x = 3y – 6
x = (3y – 6)/2 …(iii)
On substituting x from (iii) in (ii), we get
2[(3x - 6)/2] + 3y = 18

3y – 6 + 3y = 18
6y = 24
y = 4
On substituting the value of y in (iii), we get
x = (3 × 4 – 6)/2
= (12 – 6)/2
= 6/2
= 3
Hence, the values of x and y are 3 and 4 respectively.

13. (3x/2) – (5y/3) + 2 = 0
x/3 + y/2 = 2⅙
Solution:

Given equations,
(3x/2) – (5y/3) + 2 = 0 … (i)
x/3 + y/2 = 2⅙ … (ii)
From (ii), we have
(x/3) + (y/2) = (13/6)
(2x + 3y)/6 = (13/6)
2x + 3y = 13
2x = 13 – 3y
x = (13 – 3y)/2 … (iii)
On substituting x from (iii) in (i), we get
(3/2)[(13 – 3y)/2] – (5y/3) = -2
[(39 – 9y)/4] – (5y/3) = -2
(117 – 27y – 20y)/12 = -2
(117 – 47y) / 12 = -2
117 – 47y = -24
47y = 141
y = 3
On substituting the value of y in (iii), we have
x = [(13 – 3 × 3)/2]
= (13 – 9)/2
= 4/2
= 2
Hence, the values of x and y are 2 and 3 respectively.

14. x/6 + y/15 = 4
x/3 – y/12 = 4¾
Solution:

Given equations,
x/6 + y/15 = 4 … (i)
x/3 – y/12 = 4¾ … (ii)
From (i), we have

$x/6 + y/15 = 4$
$(5x+2y)/30 = 4$
$5x + 2y = 120$
$5x + 120 – 2y$
$x = (120 – 2y)/5$ …(iii)
On substituting x from (iii) in (ii), we get
$x/3 – y/12 = 4\frac{3}{4}$
$1/3(x - y/4) = 19/4$
$1/3\{[(120 – 2y)/5] – (y/4)\} = 19/4$
$(480 – 8y – 5y)/20 = 57/4$
$(480 – 13y)/20 = 57/4$
$480 – 13y = 285$
$13y = 195$
$y = 15$
Now, on substituting the value of y in (iii), we get
$x = (120 – 2 \times 15)/5$
$= (120 – 30)/5$
$= 90/5$
$= 18$
Hence, the values of x and y are 18 and 15 respectively.

Exercise 6(B)

For solving each pair of equations, in this exercise use the method of elimination by equating coefficients:

1. 13 + 2y = 9x
3y = 7x
Solution:

Given equations,
13 + 2y = 9x … (1)
3y = 7x … (2)
Performing (1)×3 - (2)×2, we get
39 + 6y = 27x
6y = 14x
__(-)____(-)____
39 = 13x
So,
x = 39/13
x = 3
Now, substituting the value of x in (2), we get
3y = 7(3)
y = 7
Hence, the values of x and y are 3 and 7 respectively.

2. 3x - y = 23
(x/3) + (y/4) = 4
Solution:

Given equations,
3x - y = 23 ... (1)
(x/3) + (y/4) = 4
⇒ 4x + 3y = 48 ... (2)
Performing (1)×3 + (2), we get
9x – 3y = 69
4x + 3y = 48
____
13x = 117
So,
x = 117/13
x = 9
Now, substituting the value of x in (1), we get
3(9) - y = 23
y = 27 - 23
y = 4
Hence, the values of x and y are 9 and 4 respectively.

3. (5y/2) – (x/3) = 8

(y/2) + (5x/3) = 12
Solution:

Given equations,
(5y/2) – (x/3) = 8
⇒ -(x/3) + (5y/2) = 8 … (i)
(y/2 + (5x/3) = 12
⇒ (5x/3) + (y/2) = 12 … (ii)

Performing (i)×5 + (ii), we get
-(5x/3) + (25y/2) = 40
(5x/3) + (y/2) = 12
(+)_____(+)___(+)___
(26y/2) = 52
So,
13y = 52
y = 4
Now, substituting y = 4 in (i), we get
-(x/3) + 5(4)/2 = 8
-(x/3) = 8 – 10
x = 6
Hence, the values of x and y are 6 and 4 respectively.

4.1/5(x - 2) = 1/4(1 - y)
26x + 3y + 4 = 0
Solution:

Given equations,
1/5(x - 2) = 1/4(1 - y)
⇒ 4x + 5y = 13 … (1)
26x + 3y = - 4 … (2)
Performing (1)×3 - (2)×5, we get
12x + 15y = 39
130x + 15y = -20
(-)___(-)____(+)___
-115x = 59
So,
x = - (59/118)
x = -½
Now, substituting x in (1), we get
4(-1/2) + 5y = 13
5y = 13 + 2
y = 3
Hence, the values of x and y are –½ and 3 respectively.

5. y = 2x - 6

y = 0
Solution:

Given equations,
y = 2x - 6
⇒ 2x – y = 6 … (1)
y = 0 … (2)
Adding (1) and (2), we get
x – y = 6
y = 0
____
2x = 6
So,
x = 6/2
x = 3
Hence, the values of x and y are 3 and 0 respectively.

6. (x - y)/6 = 2(4 – x)
2x + y = 3(x - 4)
Solution:

Given equations,
(x - y)/6 = 2(4 - x)
⇒ 13x – y = 48 … (i) [On simplifying]
2x + y = 3(x - 4)
⇒ x – y = 12 … (ii) [On simplifying]
Performing (ii)×13 – (i),
13x – 13y = 156
13x – y = 48
(-) (+) (-)
____
-12y = 108
So,
y = -108/12
y = - 9
Now, substituting y = -9 in (i), we get
13x – (-9) = 48
13x = 39
x = 3
Hence, the values of x and y are 3 and -9 respectively.

7. 3 - (x - 5) = y + 2
2 (x + y) = 4 - 3y
Solution:

Given equations,

3 - (x - 5) = y + 2
⇒ x + y = 6... (1)
2(x + y) = 4 - 3y
⇒ 2x + 5y = 4 ... (2)

Performing (1)×2 – (2), we get
2x + 2y = 12
2x + 5y = 12
(-)   (-)   (-)

-3y = 8
So,
y = -8/3
Now, substituting value of y in (1), we get
x – (8/3) = 6
x = 26/3
Hence, the values of x and y are 26/3 and -8/3 respectively.

8. 2x - 3y - 3 = 0
(2x/3) + 4y + 1/2 = 0
Solution:

Given equations,
2x -3y - 3 = 0
⇒ 2x - 3y = 3 ... (1)
(2x/3) + 4y + 1/2 = 0
⇒ 4x + 24y = -3 ... (2)

Performing (1)×8 + (2), we get
16x – 24y = 24
4x + 24y = -3

20x = 21
So, x = 21/20
Now, substituting the value of x in (1), we get
2(21/20) – 3y = 3
-3y = 3 - (21/20)
y = -3/10
Hence, the values of x and y are 21/20 and -3/10 respectively.

9. 13x+ 11y = 70
11x + 13y = 74
Solution:

Given equations,

$13x + 11y = 70...$ (1)
$11x + 13y = 74...$ (2)
On adding (1) and (2), we get
$24x + 24y = 144$
$x + y = 6...$ (3)
And,
On subtracting (2) from (1), we get
$2x - 2y = -4$
$x - y = -2$ … (4)
Now, adding (3) and (4) we get
$x + y = 6$
$x - y = -2$

$2x = 4 \Rightarrow x = 2$
On substituting the value of x in (3), we get
$2 + y = 6$
$\Rightarrow y = 4$
Hence, the values of x and y are 2 and 4 respectively.

10. $41x + 53y = 135$
$53x + 41y = 147$
Solution:

Given equations,
$41x + 53y = 135$ ... (1)
$53x + 41y = 147$ ... (2)
On adding (1) and (2), we get
$94x + 94y = 282$
$\Rightarrow x + y = 3$ ... (3)
And, on subtracting (2) from (1) we get
$-12x + 12y = -12$
$-x + y = -1$ … (4)
Now, adding (3) and (4), we get
$-x + y = -1$
$x + y = 3$

$2y = 2$
$\Rightarrow y = 1$
On substituting the value of y in (3), we get
$x + 1 = 3$
$\Rightarrow x = 2$
Hence, the values of x and y are 2 and 1 respectively.

11. If $2x + y = 23$ and $4x - y = 19$; find the values of $x - 3y$ and $5y - 2x$.
Solution:

Given equations,
$2x + y = 23$ ... (1)
$4x - y = 19$ ... (2)
On adding equation (1) and (2), we get
$2x + y = 23$
$4x - y = 19$

$6x = 42 \Rightarrow x = 7$
Now,
On substituting the value of x in (1), we get
$2(7) + y = 23$
$y = 23 - 14$
$\Rightarrow y = 9$
Hence,
$x - 3y = 7 - 3(9)$ and $5y - 2x = 5(9) - 2(7)$
$= 7 - 27$ $\qquad = 45 - 14$
$= -20$ $\qquad = 31$

12. If 10y = 7x - 4 and 12x + 18y = 1; find the values of 4x + 6y and 8y - x.
Solution:

Given equations,
$10y = 7x - 4$
$-7x + 10y = -4$ ... (1)
$12x + 18y = 1$... (2)
Performing (1)×12 + (2)×7, we get
$-84x + 120y = -48$
$84x + 126y = 7$

$246y = -41$
$\Rightarrow y = -41/246$
$y = -1/6$
On substituting the value of y in (1), we get
$-7x + 10 + (-1/6) = -4$
$-7x = -4 + (5/3)$
$\Rightarrow x = 1/3$
Hence, values of $4x + 6y = 4(1/3) + 6(-1/6) = 1/3$
And,
$8y - x = 8(-1/6) – (1/3) = (-5/3)$

13. Solve for x and y:
(i) (y + 7)/5 = (2y – x)/4 + 3x – 5
(7 – 5x)/2 + (3 – 4y)/6 = 5y – 18

(ii) 4x = 17 – (x – y)/8
2y + x = 2 + (5y + 2)/3
Solution:

(i) Given equations,
(y + 7)/5 = (2y - x)/4 + 3x – 5
⇒ 55x + 6y = 128 … (i) [On simplifying]
And,
(7 – 5x)/2 + (3 – 4y)/6 = 5y – 18
⇒ 15x + 34y = 132 … (ii) [On simplifying]

Performing (i)×3 - (ii)×11, we get
165x + 18y = 384
165x + 374y = 1452
(-) (-) (-) [subtracting]

-356y = - 1068
⇒ y = 1068/356
y = 3
Now, on substituting y = 3 in equation (i), we get
55x + 6(3) = 128
55x = 110
x = 2
∴ The solution is x = 2 and y = 3.

(ii) Given equations,
4x = 17 – (x - y)/8
33x – y = 136 … (i) [On simplifying]
2y + x = 2 + (5y + 2)/3
3y + y = 8 … (ii) [On simplifying]

Performing (i) - (ii)×11, we get
33x + 11y = 88
33x - y = 136
(-) (+) (-)

12y = -48
⇒ y = -48/12
y = -4
Now, on substituting y = -4 in equation (i), we get
33x - (-4) = 136
33x = 132
x = 4
∴ The solution is x = 4 and y = -4.

14. Find the value of m, if x = 2, y = 1 is a solution of the equation 2x + 3y = m.
Solution:

Given, x = 2 and y = 1 is the solution of the equation 2x + 3y = m
Then,
2(2) + 3(1) = m
4 + 3 = m
∴ m = 7
Hence, if x = 2 and y = 1 is the solution of the equation 2x + 3y = m, then the value of m = 7.

15. 10% of x + 20% of y = 24
3x - y = 20

Solution:

Given equations,
10% of x + 20% of y = 24
0.1x + 0.2y = 24 …(i) [On simplifying]
3x – y = 20 …(ii)

Performing (ii)×0.2 + (i), we get
0.6x – 0.2y = 4
0.1x + 0.2y = 24

0.7x = 28
So,
x = 28/0/7
x = 40
Now, on substituting x = 40 in (i), we get
0.1(40) + 0.2y = 24
0.2y = 20
y = 100
∴ The solution is x = 40 and y = 100.

16. The value of expression mx - ny is 3 when x = 5 and y = 6. And its value is 8 when x = 6 and y = 5. Find the values of m and n.
Solution:

Given,
The value of expression mx – ny is 3 when x = 5 and y = 6.
⇒ 5m – 6n = 3 … (i)
And,
The value of expression mx – ny is 8 when x = 6 and y = 5
⇒ 6m – 5n = 8 … (ii)
Solving for m and n:

Performing (i)×6 - (ii)×5, we get
30m – 36n = 18
30m – 25n = 40
(-) (+) (-)

-11n = -22
So,
n = 22/11
n = 2
Now, on substituting n = 2 in equation (i), we get
5m – 6(2) = 3
5m = 15
m = 3
∴ The solution is m = 3 and n = 2.

17. Solve:
11(x - 5) + 10(y - 2) + 54 = 0
7(2x - 1) + 9(3y - 1) = 25
Solution:

Given equations,
11(x – 5) + 10(y -2) + 54 = 0
11x – 55 + 10(y-2) + 54 = 0
11x + 10y – 21 = 0
⇒ 11x + 10y = 21 … (1)
And,
7(2x - 1) +9(3y - 1) = 25
14x – 7 + 27y – 9 = 25
14x + 27y – 16 = 25
⇒ 14x + 27y = 41 … (2)

On multiplying equation (1) by 27 and equation (2) by 10, we get
297x + 270y = 567 … (3)
140x + 270y = 410 … (4)
Subtracting equation (4) from equation (3), we get
157x = 157
x = 1
Now, on substituting x = 1 in equation (1), we get
11 × 1 + 10y = 21
10y = 10
y = 1
∴ The solution set is x = 1 and y = 1.

18. Solve:
(7+x)/5 – (2x-y)/4 = 3y – 5

$(5y – 7)/2 + (4x - 3)/6 = 18 – 5x$
Solution:

Given equations,
$(7 + x)/5 – (2x - y)/4 = 3y – 5$
$4(7 + x) – 5(2x - y) = 20(3y - 5)$
$28 + 4x – 10x + 5y = 60y – 100$
$\Rightarrow -6x – 55y = - 128$ … (1)
And,
$(5y – 7)/2 + (4x-3)/6 = 18 – 5x$
$3(5y - 7) + 4x – 3 = 6(18 – 5x)$
$15y – 21 + 4x – 3 = 108 – 30x$
$\Rightarrow 34x + 15y = 132$ … (2)

On multiplying equation (1) by 34 and equation (2) by 6, we get
$-204x – 1870y = - 4352$ … (3)
$204x + 90y = 792$ … (4)
Adding equations (3) and (4), we get
$-1780y = -3560$
$y = 2$
Now, on substituting y = 2 in equation (1), we get
$-6x – 55 \times 2 = -128$
$-6x – 110 = - 128$
$-6x = -18$
$x = 3$
∴ The solution set is x = 3 and y = 2.

19. Solve:
$4x + (x – y)/8 = 17$
$2y + x – (5y+2)/3 = 2$
Solution:

Given equations,
$4x + (x – y)/8 = 17$
$32x + x – y = 136$
$\Rightarrow 33x – y = 136$ … (1)
And,
$2y + x – (5y+2)/3 = 2$
$6y + 3x – 5y – 2 = 6$
$\Rightarrow 3x + y = 8$ … (2)

On adding equations (1) and (2), we get
$36x = 144$
$\Rightarrow x = 4$
Now, on substituting x = 4 in equation (2), we get

$3 \times 4 + y = 8$
$12 + y = 8$
$\Rightarrow y = -4$
$\therefore$ The solution is $x = 4$ and $y = -4$

Exercise 6(C)

Solve, using cross-multiplication:

1. $4x + 3y = 17$
$3x - 4y + 6 = 0$

Solution:

Given equations are $4x + 3y = 17$ and $3x – 4y + 6 = 0$
On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 4$, $b_1 = 3$, $c_1 = -17$ and $a_1 = 3$, $b_1 = -4$, $c_1 = 6$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2-c_2a_1)/(a_1b_2-a_2b_1)$
$x = (3 \times 6 – (-4) \times (-17))/(4 \times (-4) – 3 \times 3)$ and $y = (-17 \times 3 – 6 \times 4)/(4 \times (-4) – 3 \times 3)$
$x = (18 – 68)/(-16 – 9)$ and $y = (-51 – 24)/(-16 – 9)$
$x = (-50/-25)$ and $y = (-75/-25)$
Therefore, $x = 2$ and $y = 3$

2. $3x + 4y = 11$
$2x + 3y = 8$

Solution:

Given equations are $3x + 4y = 11$ and $2x + 3y = 8$
On comparing equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 3$, $b_1 = 4$, $c_1 = -11$ and $a_2 = 2$, $b_2 = 3$, $c_2 = -8$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2-c_2a_1)/(a_1b_2-a_2b_1)$
$x = [4 \times (-8) – 3 \times (-11)]/(3 \times 3 – 2 \times 4)$ and $y = [-11 \times 2 – (-8) \times 3]/(3 \times 3 – 2 \times 4)$
$x = (-32 + 33)/(9 – 8)$ and $y = (-22 + 24)/(9 – 8)$
Therefore, $x = 1$ and $y = 2$

3. $6x + 7y - 11 = 0$
$5x + 2y = 13$

Solution:

Given equations are $6x + 7y – 11 = 0$ and $5x + 2y = 13$
On comparing equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 6$, $b_1 = 7$, $c_1 = -11$ and $a_2 = 5$, $b_2 = 2$, $c_2 = -13$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$
$x = [7 \times (-13) – 2 \times (-11)]/(6 \times 2 – 5 \times 7)$ and $y = [-11 \times 5 – (-13) \times 6]/(6 \times 2 – 5 \times 7)$
$x = (-91 + 22)/(12 – 35)$ and $y = (-55 + 78)/(12 – 35)$
$x = (-69/-23)$ and $y = (23/-23)$
Therefore, $x = 3$ and $y = -1$

4. $5x + 4y + 14 = 0$
$3x = -10 - 4y$

Solution:

Given equations are $5x + 4y + 14 = 0$ and $3x = - 10 – 4y$

On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 5$, $b_1 = 4$, $c_1 = 14$ and $a_2 = 3$, $b_2 = 4$, $c_2 = 10$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$
$x = [4 \times 10 – 4 \times 14]/(5 \times 4 – 3 \times 4)$ and $y = [14 \times 3 – 10 \times 5]/(5 \times 4 – 3 \times 4)$
$x = (-91 + 22)/(12 - 35)$ and $y = (-55 + 78)/(12 - 35)$
$x = (40 - 56)/(20 – 12)$ and $y = (42 – 50)/(20 – 12)$
$x = -16/8$ and $y = -8/8$
Therefore, $x = -2$ and $y = -1$

5. x - y + 2 = 0
7x + 9y = 130
Solution:

Given equations are $x - y + 2 = 0$ and $7x + 9y = 130$
On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 1$, $b_1 = -1$, $c_1 = 2$ and $a_2 = 7$, $b_2 = 9$, $c_2 = -130$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$
$x = [-1\times (-130) – 9 \times 2]/[1 \times 9 – 7 \times (-1)]$ and $y = [2 \times 7 – (130) \times 1]/[1 \times 9 – 7 \times (-1)]$
$x = (130 - 18)/(9 + 7)$ and $y = (14 + 130)/(9 + 7)$
$x = 112/16$ and $y = 144/16$
Therefore, $x = 7$ and $y = 9$

6. 4x - y = 5
5y - 4x = 7
Solution:

Given equations are $4x - y = 5$ and $5y – 4x = 7$
On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 4$, $b_1 = -1$, $c_1 = -5$ and $a_2 = -4$, $b_2 = 5$, $c_2 = -7$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$
$x = [-1\times (-7) – 5 \times (-5)]/[4 \times 5 – (-4) \times (-1)]$ and $y = [(-5) \times (-4) – (-7) \times 4]/[4 \times 5 – (-4) \times (-1)]$
$x = (7 + 25)/(20 - 4)$ and $y = (20 + 28)/(20 - 4)$
$x = 32/16$ and $y = 48/16$
Therefore, $x = 2$ and $y = 3$

7. 4x - 3y = 0
2x + 3y = 18
Solution:

Given equations are $4x - 3y = 0$ and $2x + 3y = 18$
On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 4$, $b_1 = -3$, $c_1 = 0$ and $a_2 = 2$, $b_2 = 3$, $c_2 = -18$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$
$x = [-3 \times (-18) – 3 \times 0]/[4 \times 3 – 2 \times (-3)]$ and $y = [0 \times 2 – (-18) \times 4]/[4 \times 3 – 2 \times (-3)]$
$x = (54 - 0)/(12 + 6)$ and $y = (0 + 72)/(12 + 6)$

$x = 54/18$ and $y = 72/18$
Therefore, $x = 3$ and $y = 4$

8. $8x + 5y = 9$
$3x + 2y = 4$
Solution:

Given equations are $8x + 5y = 9$ and $3x + 2y = 4$
On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 8, b_1 = 5, c_1 = -9$ and $a_2 = 3, b_2 = 2, c_2 = -4$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$
$x = [5 \times (-4) – 2 \times (-9)]/[8 \times 2 – 3 \times 5]$ and $y = [-9 \times 3 – (-4) \times 8]/[8 \times 2 – 3 \times 5]$
$x = (-20 + 18)/(16 - 15)$ and $y = (-27 + 32)/(16 - 15)$
$x = -2/1$ and $y = 5/1$
Therefore, $x = -2$ and $y = 5$

9. $4x - 3y - 11 = 0$
$6x + 7y - 5 = 0$
Solution:

Given equations are $4x - 3y - 11 = 0$ and $6x + 7y – 5 = 0$
On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 4, b_1 = -3, c_1 = 11$ and $a_2 = 6, b_2 = 7, c_2 = -5$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$
$x = [-3\times (-5) – 7 \times (-11)]/[4 \times 7 – 6 \times (-3)]$ and $y = [-11 \times 6 – (-5) \times 4]/[4 \times 7 – 6 \times (-3)]$
$x = (15 + 77)/(28 + 18)$ and $y = (-66 + 20)/(28 + 18)$
$x = (92/46)$ and $y = (-46/46)$
Therefore, $x = 2$ and $y = -1$

10. $4x + 6y = 15$
$3x - 4y = 7$
Solution:

Given equations are $4x + 6y = 15$ and $3x - 4y = 7$
On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 4, b_1 = 6, c_1 = -15$ and $a_2 = 3, b_2 = -4, c_2 = -7$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$
$x = [6\times (-7) – (-4) \times (-15)]/[4 \times (-4) – 3 \times 6]$ and $y = [-15 \times 3 – (-7) \times 4]/[4 \times (-4) – 3 \times 6]$
$x = (-42 - 60)/(-16 - 18)$ and $y = (-45 + 28)/(-16 - 18)$
$x = (-102/-34)$ and $y = (-17/-34)$
Therefore, $x = 3$ and $y = 1/2$

11. $0.4x - 1.5y = 6.5$
$0.3x + 0.2y = 0.9$
Solution:

Given equations are $0.4x – 1.5y = 6.5$ and $0.3x + 0.2y = 0.9$
On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = 0.4$, $b_1 = -1.5$, $c_1 = -6.5$ and $a_2 = 0.3$, $b_2 = 0.2$, $c_2 = -0.9$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$
$x = [(-1.5) \times (-0.9) – (0.2) \times (-6.5)]/[0.4 \times (0.2) – (0.3) \times (-1.5)]$ and $y = [(-6.5) \times (0.3) – (-0.9) \times (0.4)]/[0.4 \times (0.2) – (0.3) \times (-1.5)]$
$x = (1.35 + 1.3)/(0.08 + 0.45)$ and $y = (-1.95 + 0.36)/(0.08 + 0.45)$
$x = (2.65/0.53)$ and $y = (-1.59/0.53)$
Therefore, $x = 5$ and $y = -3$

12. $\sqrt{2}x - \sqrt{3}y = 0$
$\sqrt{5}x + \sqrt{2}y = 0$
Solution:

Given equations are $\sqrt{2}x - \sqrt{3}y = 0$ and $\sqrt{5}x + \sqrt{2}y = 0$
On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have
$a_1 = \sqrt{2}$, $b_1 = -\sqrt{3}$, $c_1 = 0$ and $a_2 = \sqrt{5}$, $b_2 = \sqrt{2}$, $c_2 = 0$
Now, $x = (b_1c_2 - b_2c_1)/(a_1b_2 – a_2b_1)$ and $y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$
$x = [(-\sqrt{3}) \times 0 – \sqrt{2} \times 0]/[\sqrt{2} \times \sqrt{2} – \sqrt{5} \times (-\sqrt{3})]$ and $y = [0 \times \sqrt{5} – 0 \times \sqrt{2}]/[\sqrt{2} \times \sqrt{2} – \sqrt{5} \times (-\sqrt{3})]$
$x = [0/(2 + \sqrt{15})]$ and $y = [0/(2 + \sqrt{15})]$
Therefore, $x = 0$ and $y = 0$

Exercise 6(D)

1. Solve the pairs of equations:
9/x – 4/y = 8
13/x + 7/y = 101
Solution:

Given equations,
9/x – 4/y = 8 … (1)
13/x + 7/y = 101 … (2)

Performing (1)×7 + (2)×4, we get
63/x – 28/y = 56
52/x + 28/y = 202
____
115/x = 460
x = 115/460
⇒ x = 1/4
Now, substituting value of x in (i), we get
9 × (4/1) – (4/y) = 8
-(4/y) = - 28
⇒ y = 1/7
Therefore, the solution is x = ¼ and y = 1/7.

2. Solve the pairs of equations:
(3/x) + (2/y) = 10
(9/x) – (7/y) = 10.5
Solution:

Given equations,
(3/x) + (2/y) = 10 … (i)
(9/x) – (7/y) = 10.5 … (ii)
On multiplying equation (i) by 3, we get
(9/x) + (6/y) = 30 …(iii)
Now, on subtracting (ii) from (iii), we get
13/y = 19.5
y = 13/19.5
⇒ y = 2/3
On substituting the value of y in (i), we get
(3/x) + (2 × 3)/2 = 10
(3/x) + 3 = 10
3/x = 7
⇒ x = 3/7
Therefore, the solution is x = ¼ and y = 1/7.

3. 5x + (8/y) = 19

$3x – (4/y) = 7$
Solution:

Given equations,
$5x + (8/y) = 19$ … (i)
$3x – (4/y) = 7$ … (ii)
On multiplying equation (ii) by 2, we get
$6x – (8/y) = 14$ … (iii)
On adding (i) and (iii), we get
$11x = 33$
$\Rightarrow x = 3$
Substituting $x = 3$ in equation (1), we get
$5(3) + (8/y) = 19$
$(8/y) = 19-15$
$y = (8/4)$
$\Rightarrow y = 2$
Therefore, the solution is $x = 3$ and $y = 2$.

4. Solve: $4x + (6/y) = 15$ and $3x – (4/y) = 7$. Hence, find 'a' if $y = ax - 2$
Solution:

Given equations,
$4x + (6/y) = 15$ … (i)
$3x – (4/y) = 7$ … (ii)
On multiplying (i) by 4 and (ii) by 6
$16x + (24/y) = 60$ … (iii)
$18x – (24/y) = 42$ … (iv)
Now, on adding (iii) and (iv), we get
$34x = 102$
$\Rightarrow x = 3$
On substituting $x = 3$ in (i), we get
$4(3) + (6/y) = 15$
$6/y = 15 – 12$
$y = 6/3$
$\Rightarrow y = 2$
Now, $y = ax – 2$
$2 = a(3) – 2$
$2 = 3a – 2$
$3a = 4$
$\therefore a = 4/3$

5. Solve: $(3/x) – (2/y) = 0$ and $(2/x) + (5/y) = 19$. Hence, find 'a' if $y = ax + 3$.
Solution:

Given equations,
$(3/x) – (2/y) = 0$ … (1)

(2/x) + (5/y) = 19 … (2)
Performing (1)×5 and (2)×2, we get
(15/x) – (10/y) = 0
(4/x) + (10/y) = 38

19/x = 38
x = 38/19
⇒ x = 1/2
Now, substituting the value of x in (1), we get
3(1/2) – (2/y) = 0
y = 1/3
So, y = ax + 3
(1/3) = a(1/2) + 3
a/2 = (-8/3)
∴ a = -16/3

6. Solve:
(i) 20/(x + y) + 3/(x - y) = 7
8/(x - y) – 15/(x + y) = 5
(ii) 34/(3x + 4y) + 15/(3x – 2y) = 5
25/(3x - 2y) – 8.50/(3x + 4y) = 4.5
Solution:

(i) Given equations,
20/(x + y) + 3/(x - y) = 7 … (1)
8/(x + y) – 15(x + y) = 5 … (2)
Performing (1)×8 - (2)×3, we get
160/(x + y) + 24/(x - y) = 56
-45/(x + y) + 24(x - y) = 15
(-) (-) (-)

205/(x + y) = 41
x + y = 205/41
⇒ x + y = 5 … (3)
Using (3) in (1), we get
(20/5) + 3/(x - y) = 7
3/(x - y) = 3
⇒ x – y = 1 … (4)
Now, adding (3) and (4), we get
x + y = 5
x – y = 1

2x = 6
⇒ x = 3
On substituting the value of x in (3), we get
3 + y = 5

⇒ $y = 2$
∴ The solution is $x = 3$ and $y = 2$

(ii) Let's assume $a = 3x + 4y$ and $b = 3x – 2y$
Then, the given equations will become
$(34/a) + (15/b) = 5$ … (i)
$-(8.50/a) + (25/b) = 4.5$ … (ii)
Performing (ii)×4 + (i), we get
$-(34/a) + (100/b) = 18$
$34/a + 15/b = 5$
(+) (+) (+)
___
$115/b = 23$
So, $b = 115/23$
⇒ $b = 5$
Now, we have $3x – 2y = 5$ …(iii)
And, on substituting $b = 5$ in equation (i), we get
$(34/a) + (15/5) = 5$
$2a = 34$
⇒ $a = 17$
So, $3x + 4y = 17$ …(iv)
On subtracting equation (iv) from equation (iii), we get
$3x – 2y = 5$
$3x + 4y = 17$
(-) (-) (-)
___
$-6y = -12$
$y = 2$
Now, on substituting $y = 2$ in equation (iii), we get
$3x – 2(2) = 5$
$3x = 9$
⇒ $x = 3$
∴ The solution is $x = 3$ and $y = 2$.

7. Solve:
(i) $x + y = 2xy$
$x - y = 6xy$
(ii) $x+ y = 7xy$
$2x - 3y = -xy$
Solution:

(i) Given equations,
$x + y = 2xy$ ... (i)
$x – y = 6xy$ … (ii)
___ [Addition]

$2x = 8xy$
$2 = 8y$
$\Rightarrow y = ¼$
On substituting the value of y in (i), we get
$x + ¼ = 2 \times (1/4)$
$1/2x = -1/4$
$\Rightarrow x = -½$
∴ The solution is $x = -½$ and $y = ¼$.

(ii) Given equations,
$x + y = 7xy$ ... (1)
$2x - 3 = -xy$ ... (2)
Performing (1)×3 + (2), we get
$3x + 3y = 21xy$
$2x – 3y = - xy$

$5x = 20xy$
So, $y = 5x/20x$
$y = 1/4$
Now, on substituting the value of y in (i), we get
$x + 1/4 = 7 \times (1/4)$
$¼ = ¾ x$
$\Rightarrow x = 1/3$
∴ The solution is $x = 1/3$ and $y = ¼$.

8. Solve:
$a/x – b/y = 0$
$ab^2/x + a^2b/y = a^2 + b^2$
Solution:

Given equations are $a/x – b/y = 0$ and $ab^2/x + a^2b/y = a^2 + b^2$
On taking $1/x = u$ and $1/y = v$, the above system of equations become
$au – bv + 0 = 0$
$ab^2u + a^2bv – (a^2 + b^2) = 0$
By cross-multiplication, we have
$u/(-b \times (-a^2 + b^2) – a^2b \times 0) = -v/[a \times (-a^2 + b^2) – ab^2 \times 0] = 1/(a \times a^2b - ab^2 \times -b)$
$u/b(a^2 + b^2) = -v/[-a(a^2 + b^2)] = 1/(a^3b + ab^3)$
$u/b(a^2 + b^2) = v/[a(a^2 + b^2)] = 1/[ab(a^2 + b^2)]$
Hence,
$u = [b(a^2 + b^2)]/ [ab(a^2 + b^2)]$ and $v = a(a^2 + b^2)/ [ab(a^2 + b^2)]$
$u = 1/a$ and $v = 1/b$
$1/x = 1/a$ and $1/y = 1/b$
Therefore, the solution is $x = a$ and $y = b$.

9. Solve:

2xy/(x + y) = 3/2
xy/(2x - y) = - 3/10;
x + y ≠ 0 and 2x – y ≠ 0
Solution:

Given equations,
2xy/(x + y) = 3/2
(x + y)/xy = 4/3
⇒ (1/x) + (1/y) = 4/3 … (1)
And, xy/(2x - y) = -3/10
(2x - y)/xy = -10/3
⇒ -(1/x) + (2/y) = -10/3 … (2)
Let's assume 1/x = u and 1/y = v
Then equations (1) and (2) become,
u + v = 4/3 and -u + 2v = -10/3
Now, on multiplying and adding the above equations by 3, we get
3u + 3v = 4
-3u + 6v = -10

9v = -6
⇒ v = -(6/9) = -(2/3)
So, 1/y = -(2/3)
⇒ y = -(3/2)
On substituting y = -(3/2) in (1), we get
1/x – 2/3 = 4/3
1/x = 6/3 = 2
⇒ x = 1/2
Therefore, the solution is x = ½ and y = -3/2.

10. Solve:
3/2x + 2/3y = -1/3
3/4x + 1/2y = -1/8
Solution:

Given equations are 3/2x + 2/3y = -1/3 and 3/4x + 1/2y = -1/8
Let's assume 1/x = u and 1/y = v
Then the system of equations become,
3u/2 + 2v/3 = -1/3 and 3u/4 + v/2 = -1/8
(9u + 4v)/6 = -1/3 and (3u + 2v)/4 = -1/8
⇒ 9u + 4v = -2 and 3u + 2v = -1/2
On multiplying the equations by 3 and 8 respectively, we get
27u + 12v = -6 and 24u + 16v = -4
⇒ 27u + 12v + 6 = 0 and 24u + 16v + 4 = 0
By cross-multiplication method, we have
u/(12 × 4 – 16 × 6) = -v/(27 × 4 – 24 × 6) = 1/ (27 × 16 – 24 × 12)

$u/(48 – 96) = -v/(108 – 144) = 1/(432 – 288)$
$u/-48 = -v/-36 = 1/144$
$u/-48 = v/36 = 1/144$
$\Rightarrow u = -48/144 = -1/3$ and $v = 36/144 = 1/4$
So, $1/x = -1/3$ and $1/y = 1/4$
$\Rightarrow x = -3$ and $y = 4$
Therefore, the solutions is $x = -3$ and $y = 4$.

Exercise 6(E)

1. The ratio of two numbers is 2/3. If 2 is subtracted from the first and 8 from the second, the ratio becomes the reciprocal of the original ratio. Find the numbers.

Solution:

Let's assume the two numbers to be x and y
Then, according to the question, we have
$x/y = 2/3$
$\Rightarrow 3x - 2y = 0$ ... (1)
Also, $(x - 2)/(y - 8) = 3/2$
$\Rightarrow 2x - 3y = -20$ ... (2)
Performing (1)×2 - (2)×3, we get
$6x - 4y = 0$
$6x - 9y = -60$
(-) (+) (+)

$5y = 60$
$\Rightarrow y = 12$
On substituting the value of y in (1), we get
$3x - 2(12) = 0$
$x = 24/3$
$\Rightarrow x = 8$
Therefore, the numbers are 8 and 12.

2. Two numbers are in the ratio 4 : 7. If thrice the larger be added to twice the smaller, the sum is 59. Find the numbers.

Solution:

Let's assume the smaller number to be x and the larger number to be y
Then, according to the question, we have
$x/y = 4/7$
$\Rightarrow 7x - 4y = 0$ ... (1)
And, $3y + 2x = 59$ ... (2)
Performing (1)×3 + (2)×4, we get
$21x - 12y = 0$
$8x + 12y = 236$

$29x = 236$
$\Rightarrow x = 236/29$
On substituting the value of x in (1), we get
$7(236/29) = 4y$
$y = 7(59/29)$
$\Rightarrow y = 413/29$
Therefore, the numbers are 236/29 and 413/29.

3. When the greater of the two numbers increased by 1 divides the sum of the numbers, the result is 3/2. When the difference of these numbers is divided by the smaller, the result 1/2. Find the numbers.
Solution:

Let's consider the two numbers to be a and b respectively such that b > a.
Then, according to given condition, we have
$(a + b)/(b + 1) = 3/2$
$2a + 2b = 3b + 3$
$\Rightarrow 2a – b = 3$ … (i)
And, $(b – a)/a = ½$
$2b – 2a = a$
$\Rightarrow 2b – 3a = 0$ … (ii)
On multiplying (i) by 2, we get
$4a – 2b = 6$ … (iii)
On adding (ii) and (iii), we get
$a = 6$
Now, on substituting a = 6 in (i), we get
$2(6) – b = 3$
$12 – b = 3$
$b = 9$
Therefore, the two numbers are 6 and 9 respectively.

4. The sum of two positive numbers x and y (x > y) is 50 and the difference of their squares is 720. Find the numbers.
Solution:

Let's assume the two numbers to be x and y such that x > y.
Then, according to the question, we have
$x + y = 50$ … (i)
And,
$y^2 - x^2 = 720$
$\Rightarrow (y - x)(y + x) = 720$
$\Rightarrow (y - x)(50) = 720$
$\Rightarrow y - x = 14.4$ … (ii)
On adding (i) and (ii), we get
$2y = 64.4$
$\Rightarrow y = 32.2$
Now, on Substituting the value of y in (i), we have
$x + 32.2 = 50$
$\Rightarrow x = 17.8$
Therefore, the two numbers are 17.8 and 32.2 respectively.

5. The sum of two numbers is 8 and the sum of their reciprocal is 8/15. Find the numbers.

Solution:

Let's consider the two numbers to be x and y respectively
Then, according to the question, we have
$x + y = 8$ … (i)
$\Rightarrow x = 8 - y$
And,
$1/x + 1/y = 8/15$ … (ii)
$\Rightarrow (y + x)/xy = 8/15$
Using x from (i) in (ii), we get
$8/xy = 8/15$
$xy = 15$
$(8 - y)y = 15$
$8y - y^2 = 15$
$y^2 - 8y + 15 = 0$
$y^2 - 3y - 5y + 15 = 0$
$y(y - 3) - 5(y - 3) = 0$
$(y - 3)(y - 5) = 0$
$y = 3$ or $y = 5$
Then, $x = 5$ or $x = 3$
Therefore, the two numbers are 3 and 5 respectively.

6. The difference between two positive numbers x and y (x > y) is 4 and the difference between their reciprocal is 4/21. Find the numbers.
Solution:

Let's assume the two numbers to be x and y respectively such that x > y
Then, according to the question, we have
$x - y = 4$ … (i)
$\Rightarrow x = 4 + y$
And,
$1/y - 1/x = 4/21$ … (ii)
$(x-y)/xy = 4/21$
Now, substituting x from (i) in the above equation, we get
$4/xy = 4/21$
$xy = 21$
$\Rightarrow (4+y)y = 21$
$4y + y^2 = 21$
$y^2 + 4y - 21 = 0$
$y^2 + 7y - 3y - 21 = 0$
$y(y+7) - 3(y+7) = 0$
$(y-3)(y+7) = 0$
$\Rightarrow y = 3$ or $y = -7$
We can neglect $y = -7$ as y taken to be positive.
So, $y = 3$
$x = 4 + y$

x = 4 + 3
⇒ x = 7
Therefore, the two numbers are 7 and 3 respectively.

7. Two numbers are in the ratio 4:5. If 30 is subtracted from each of the numbers, the ratio becomes 1:2. Find the numbers.
Solution:

Let's assume the common multiple between the numbers as x
So, the numbers are 4x and 5x
Then, according to the question, we have
(4x - 30)/(5x - 30) = 1/2
8x – 60 = 5x – 30
3x = 30
⇒ x = 10
So, 4x = 4(10) = 40 and 5x = 5(10) = 50.
Therefore, the numbers are 40 and 50.

8. If the numerator of a fraction is increased by 2 and denominator is decreased by 1, it becomes 2/3. If the numerator is increased by 1 and denominator is increased by 2, it becomes 1/3. Find the fraction.
Solution:

Let's assume the numerator and denominator of the fraction to be x and y respectively
Then, according to the question, we have
(x + 2)/(y - 1) = 2/3
3x – 2y = -8 … (1)
And,
(x + 1)/(y + 2) = 1/3
3x – y = -1 … (2)
Now on subtracting (1) from (2), we get
3x – y = -1
3x – 2y = -8
(-) (+) (+)
____
⇒ y = 7
On substituting the value of y in (1), we get
3x - 2 (7) = -8
3x = -8 + 14
⇒ x = 2
Therefore, the required fraction is 2/7.

9. The sum of the numerator and the denominator of a fraction is equal to 7. Four times the numerator is 8 less than 5 times the denominator. Find the fraction.
Solution:

Let's consider the numerator and denominator of the fraction to be x and y respectively.
So, the fraction will be x/y
Then, according to the question, we have
$x + y = 7$... (1)
$5y - 4x = 8$ ... (2)
Performing (1)×4 + (2), we get
$4x + 4y = 28$
$-4x + 5y = 8$

$9y = 36$
$\Rightarrow y = 4$
On substituting the value of y in (1), we get
$x + 4 = 7$
$\Rightarrow x = 3$
Therefore, the required fraction is ¾.

10. If the numerator of a fraction is by 2 and its denominators increased by 1, it becomes 1. However, if the numerator is increased by 4 and denominator is multiplied by 2, the fraction becomes 1/2. Find the fraction.
Solution:

Let's assume the numerator of the fraction to be x and the denominator to be y
So, the fraction will be x/y
Then, according to the question, we have
$2x/(y+1) = 1$
$2x = y + 1$
$\Rightarrow 2x – y = 1$ … (i)
And, $(x+4)/2y = 1/2$
$2x + 8 = 2y$
$\Rightarrow 2x – 2y = -8$ …(ii)
Subtracting (ii) from (i), we get
$y = 9$
On putting the value of y in (i), we get
$2x – 9 = 1$
$2x = 1 + 9$
$x = 5$
so, the fraction is 5/9

11. A fraction becomes ½ if 5 subtracted from its numerator and 3 is subtracted from its denominator. If the denominator of this fraction is 5 more than its numerator, find the fraction.
Solution:

Let's consider the numerator of the fraction to be x and the denominator of the fraction to be y
So, the fraction will be x/y
Then, according to given conditions, we have

$(x-5)/(y-3) = ½$
$2x – 10 = y – 3$
$\Rightarrow 2x – y = 7$ … (i)
And,
$x + 5 = y$
$\Rightarrow x – y = -5$ … (ii)
Now, on subtracting (ii) from (i), we get
$x = 12$
On substituting the value of x in (ii), we get
$y = x + 5 = 12 + 5 = 17$
Therefore, the required fraction is 12/17.

12. The sum of the digits of the digits of two-digit number is 5. If the digits are reversed, the number is reduced by 27. Find the number.
Solution:

Let's consider the digit at the unit's place to be x and the digit at ten's place to be y.
So, the required number will be 10y + x
If the digit's are reversed,
Reversed number = 10y + x
Then, according to the question, we have
$x + y = 5$ ... (1)
And,
$(10y + x) - (10x + y) = 27$
$9y – 9x = 27$
$\Rightarrow y – x = 3$ … (2)
On adding (1) and (2), we get
$y – x = 3$ … (2)
$y + x = 5$ … (1)
____
$2y = 8$
$\Rightarrow y = 4$
On substituting the value of y in (1), we get
$x + 4 = 5$
$\Rightarrow x = 1$
Thus, the required number is 10 (4) + 1 = 41.

13. The sum of the digits of a two-digit number is 7. If the digits are reversed, the new number decreased by 2, equals twice the original number. Find the number.
Solution:

Let's consider the digit at unit's place to be x and the digit at ten's place to be y
So, the required number will be 10y + x
Now, if the digit's are reversed
Reversed number = 10x + y
Then, according to the question, we have

$x + y = 7$ ... (1)
And,
$10x + y - 2 = 2(10y + x)$.
$8x - 19y = 2$ ... (2)
Performing (1)×19 + (2), we get
$19x + 19y = 133$
$8x - 19y = 2$

$27x = 135$
$\Rightarrow x = 5$
On substituting the value of x in (1), we get
$5 + y = 7$
$\Rightarrow y = 2$
Thus, the required number is 10(2) + 5 = 25.

14. The ten's digit of a two-digit number is three times the unit digit. The sum of the number and the unit digit is 32. Find the number.
Solution:

Let's consider the digit at unit's place to be x and the digit at the ten's place to be y.
So, the required number will be 10y + x
Then, according to the question, we have
$y = 3x$
$\Rightarrow 3x - y = 0$ ... (1)
And, $10y + x + x = 32$
$\Rightarrow 10y + 2x = 32$ ... (2)
Performing (1)×10 + (2)
$30x - 10y = 0$
$2x + 10y = 32$

$32x = 32$
$\Rightarrow x = 1$
Substituting the value of x in (2), we get
$y = 3(1)$
$\Rightarrow y = 3$
Thus, the required number is 10(3) + 1 = 31

15. A two-digit number is such that the ten's digit exceeds twice the unit's digit by 2 and the number obtained by inter-changing the digits is 5 more than the the sum of the digits. Find the two-digit number.
Solution:

Let's assume the digit at unit's place to be x and the digit at ten's place to be y
So, the required number will be 10y + x
Then, according to the question, we have
$y - 2x = 2$

$-2x + y = 2$ ... (1)
And,
$(10x + y) - 3(y + x) = 5$
$7x - 2y = 5$ ... (2)
Performing (1)×2 + (2), we get
$-4x + 2y = 4$
$7x - 2y = 5$
———————
$3x = 9$
$\Rightarrow x = 3$
Now, on substituting the value of x in (1),we get
$-2(3) + y = 2$
$\Rightarrow y = 8$
Thus, the required number is 10(8) + 3 = 83.

16. Four times a certain two-digit number is seven times the number obtained on interchanging its digits. If the difference between the digits is 4; find the number.
Solution:

Let's consider x to be the number at the ten's place and y to be the number at the unit's place
So, the required number will be 10x + y
Now, on interchanging its digit
The reversed = 10y + x
Then, according to the question, we have
$4(10x + y) = 7(10y + x)$
$40x + 4y = 70y + 7x$
$33x – 66y = 0$
$x – 2y = 0$ … (i)
If the difference between the digits is 4, then
$x – y = 4$ … (ii)
On subtracting equation (i) from equation (ii), we get
$x – y = 4$
$x – 2y = 0$
(-) (+) (-)
———————
$y = 4$
On substituting y = 4 in equation (i), we get
$x – 2(4) = 0$
$\Rightarrow x = 8$
Thus, the number is = 10(8) + 4 = 84.

17. The sum of two-digit number and the number obtained by interchanging the digits of the number is 121. If the digits of the number differ by 3, find the number.
Solution:

Let's assume the tens digit of the number to be x and the units digit to be y

So, the required number will be $10x + y$
And, the number obtained by interchanging the digits will be $10y + x$.
Then, according to the question, we have
$10x + y + 10y + x = 121$
$11x + 11y = 121$
$11(x + y) = 121$
$x + y = 11$ … (i)
And,
$x – y = 3$ … (ii)
On adding (i) and (ii), we get
$2x = 14$
$\Rightarrow x = 7$
Now, on substituting value of x in (i), we get
$y = 11 – x$
$= 11 – 7$
$= 4$
Hence, the required number is $10(7) + 4 = 74$.

18. A two-digit number is obtained by multiplying the sum of the digits by 8. Also, it is obtained by multiplying the difference of the digits by 14 and adding 2. Find the number.
Solution:

Let's assume the tens digit of the number to be x and the units digit to be y
So, the required number will be $10x + y$
Then, according to question, we have
$10x + y = 8(x + y)$
$\Rightarrow 2x = 7y$ … (i)
And, $10x + y = 14(x-y) + 2$ or $10x + y = 14(y-x) + 2$
So, we have
$4x – 15y = -2$ … (ii) or $24x – 13y = 2$ … (iii)
By solving (i) and (ii), we get
$y = 2$ and $x = 7$
And, by solving (i) and (iii), we get
$y = 2/71$
This is not possible, since y is a digit and cannot be in fraction form
Thus, the required number will be 72.

Exercise 6(F)

1. Five years ago, A's age was four times the age of B. Five years hence, A's age will be twice the age of B. Find their preset ages.
Solution:

Let the present age of A be taken as x years
And present age of B ne taken as y years
Then according to the question, we have
Five years ago,
$x - 5 = 4(y - 5)$
$x - 4y = -15$ ... (1)
Five years later,
$x + 5 = 2(y + 5)$
$x – 2y = 5$ … (2)
Now, on subtracting (1) from (2), we get
$x – 2y = 5$
$x – 4y = -15$
(-) (+) (+)

$2y = 20$
$\Rightarrow y = 10$
Now, substituting the value of y in (1), we get
$x - 4(10) = -15$
$\Rightarrow x = 25$
Thus, the present ages of A and B are 25 years and 10 years respectively.

2. A is 20 years older than B. 5 years ago, A was 3 times as old as B. Find their present ages.
Solution:

Let A's present age to considered as x years
And B's present age be considered as y years
Then, according to the question, we have
$x = y + 20$
$x - y = 20$ ... (1)
Five years ago,
$x - 5 = 3(y - 5)$
$x – 3y = -10$ … (2)
On subtracting (1) from (2), we get
$x – 3y = -10$ … (2)
$x – y = 20$ … (1)
(-) (+) (-)

$-2y = -30$
$\Rightarrow y = 15$

On substituting the value of y in (1), we get
$x = 15 + 20$
$\Rightarrow x = 35$
Thus, the present ages of A and B are 35 years and 15 years.

3. Four years ago, a mother was four times as old as her daughter. Six years later, the mother will be two and a half times as old as her daughter at that time. Find the present ages of mother and her daughter.
Solution:

Let the present age of the mother be considered as x years and the present age of the daughter be considered as y year
Then, according to the question, we have
$x - 4 = 4(y-4)$
$x - 4 = 4y - 16$
$x - 4y = -12$ … (i)
And,
$x + 6 = 2\frac{1}{2}(y + 6)$
$x + 6 = (5/2)y + 15$
$x - (5/2)y = 9$
$2x - 5y = 18$ … (ii)
On solving (i) and (ii), we get
$y = 14$ and $x = 44$
Thus, the present age of the mother is 44 years and the present age of the daughter is 14 years.

4. The age of a man is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children at that time. Find the present age of the man.
Solution:

Let's consider the present age of the man to be x years
And let the sum of the ages if his two children be taken as y years
Then, according to the question, we have
$x = 2y$ … (i)
And, $x + 20 = y + 40$ … (ii),
On substituting (i) in (ii), we get
$2y + 20 = y + 40$
$\Rightarrow y = 20$
Now, substituting the value of y in (i), we get
$\Rightarrow x = 40$
Thus, the present age of the man is 40 years.

5. The annual incomes of A and B are in the ratio 3 : 4 and their annual expenditure are in the ratio 5 : 7. If each Rs. 5000; find their annual incomes.
Solution:

Let consider A's annual income to be Rs. X and B's annual income to be Rs. y
Then, according to the question, we have
x/y = 3/4
4x - 3y = 0 ... (1)
And, (x – 5000)/(y – 5000) = 5/7
7x - 5y = 10000 ... (2)
Performing (1)×7 - (2)×4, we get
28x – 21y = 0
28x – 20y = 40000
(-) (+) (-)

-y = - 40000
⇒ y = 40000
On substituting the value of y in (1), we get
4x - 3 (40000) = 0
⇒ x = 30000
Thus, A's income in Rs. 30,000 and B's income is Rs. 40,000.

6. In an examination, the ratio of passes to failures was 4 : 1. Had 30 less appeared and 20 less passed, the ratio of passes to failures would have been 5 : 1. Find the number of students who appeared for the examination.
Solution:

Let's assume the number of pass candidates to be x
And, the number of failure candidates to be y
Then, according to the question, we have
x/y = y/1
x - 4y = 0 ...(1)
And,
(x -20)/(y – 20) = 5/1
x - 5y = -30 ...(2)
Performing (1) – (2), we get
x - 4y = 0
x - 5y = -30
(-) (+) (+)

y = 30
On substituting the value of y in (1), we get
x - 4(30) = 0
⇒ x = 120
Thus, total students appeared is (x + y) = 120 + 30
= 150

7. A and B both the have some pencils. If A gives 10 pencils to B, then B will have twice as many as A. And if B gives 10 pencils to A, then they will have the same number of

pencils. How many pencils does each have?
Solution:

Let the number of pencils with A be assumed as x
And the number of pencils with B be assumed as y
If A gives 10 pencils to B, then
$y + 10 = 2(x - 10)$
$2x - y = 30$ ... (1)
If B gives to pencils to A, then
$y - 10 = x + 10$
$x – y = -20$ … (2)
On subtracting (1) from (2), we have
$x – y = -20$
$2x - y = 30$
(-) (+) (-)
____
$-x = -50$
$\Rightarrow x = 50$
Now, substituting the value of x in (1), we get
$2(50) - y = 30$
$\Rightarrow y = 70$
Thus, A has 50 pencils and B has 70 pencils.

8. 1250 persons went to sea a circus-show. Each adult paid Rs. 75 and each child paid Rs. 25 for the admission ticket. Find the number of adults and number of children, if the total collection from them amounts to Rs. 61,250.
Solution:

Let the number of adults be taken as x and the number of children as y
Then according to the question, we have
$x + y = 1250$ ... (1)
And,
$75x + 25y = 61250$
$\Rightarrow 3x + y = 2450$ … (2)
On subtracting (1) from (2), we get
$3x + y = 2450$
$x + y = 1250$
(-) (-) (-)
____
$2x = 1200$
$\Rightarrow x = 6000$
On substituting the value of x in (1), we get
$600 + y = 1250$
$\Rightarrow y = 650$
Thus, number of adults is 600 and the number of children is 650.

9. Two articles A and B are sold for Rs. 1,167 making 5% profit on A and 7% profit on A and 7% profit on B. IF the two articles are sold for Rs. 1,165, a profit of 7% is made on A and a profit of 5% is made on B. Find the cost prices of each article.
Solution:

Let's assume the cost price of article A as Rs. x and the cost price of articles B as Rs. y
Then, according to the question, we have
$(x + 5\% \text{ of } x) + (y + 7\% \text{ of } y) = 1167$
$[x + (5/100)x] + [y + 7/100)y] = 1167$
$(21x/20) + (107y/100) = 1167$
$105x + 107y = 1167$ ... (1)
And,
$(107x/100) + (105y/100) = 1165$
$107x + 105y = 116500$ ... (2)
On adding (1) and (2), we get
$212x + 212y = 233200$
$x + y = 1100$ … (3)
On subtracting (2) from (1), we get
$-2x + 2y = 200$
$\Rightarrow -x + y = 200$ … (4)
Now, subtracting (3) from (4), we get
$-x + y = 100$ … (4)
$-x + y = 1100$ … (3)

$2y = 1200$
$\Rightarrow y = 600$
On substituting the value of y in (3), we get
$x + 600 = 1100$
$\Rightarrow x = 500$
Thus, the cost price of article A is Rs. 500 and that of article B is Rs. 600.

10. Pooja and Ritu can do a piece of work in $17\frac{1}{7}$ days. If one day work of Pooja be three fourth of one day work of Ritu' find in how many days each will do the work alone.
Solution:

Let assume that Pooja's 1 day work = $1/x$ and Ritu's 1 day work = $1/y$
Then, according the question, we have
$(1/x) + (1/y) = 7/120$ … (1)
And, $1/x = (3/4)(1/y)$
$y = (3/4)x$ … (2)
Now, using the value of y from (2) in (1), we get
$(1/x) + (4/3x) = 7/120$
$120(3 + 4) = 7(3x)$
$21x = 120(7)$
$\Rightarrow x = 40$

On substituting the value of x in (2), we get
$y = (\frac{3}{4})(40) = 30$
$y = 30$
Thus, Pooja will complete the work in 40 days and Ritu will complete the work in 30 days.

Exercise 6(G)

1. Rohit says to Ajay, "Give me hundred, I shall then become twice as rich as you." Ajay replies, "if you give me ten, I shall be six times as rich as you." How much does each have originally?

Solution:

Let Rohit have a sum of Rs. x and Ajay have a sum of Rs. y
When Ajay gives Rs. 100 to Rohit, then
$x + 100 = 2(y - 100)$
$x - 2y = -300$ ... (1)
When Rohit gives Rs. 10 to Ajay, then
$6(x - 10) = y + 10$
$6x - y = 70$ ... (2)
Performing (2)×2 – (1), we get

$$\begin{array}{r} 12x - 2y = 140 \\ x - 2y = -300 \\ (-) \quad (+) \quad (+) \\ \hline 11x = 440 \end{array}$$

$\Rightarrow x = 40$
On substituting the value of x in (1), we get
$40 - 2y = -300$
$-2y = -340$
$\Rightarrow y = 170$
Thus, Rohit has Rs. 40 and Ajay has Rs. 170.

2. The sum of a two-digit number and the number obtained by reversing the order of the digits is 99. Find the number, if the digits differ by 3.

Solution:

Let's consider the digits in the tens place as x and the digit in the unit place as y
So, the required number will be $10x + y$
Number on reversing the digits = $10y + x$
And, the difference between the digits = $x - y$ or $y - x$
Then according to the question, we have
$(10x + y) + (10y + x) = 99$
$11x + 11y = 99$
$\Rightarrow x + y = 9$ …(i)
And, $x - y = 3$ …(ii) or $y - x = 3$ …(iii)
Now,
On solving equations (i) and (ii), we get
$2x = 12$
$\Rightarrow x = 6$
So, $y = 3$

On solving equations (i) and (iii), we get
2y = 12
⇒ y = 6
So, x = 3
Hence, the number = 10x + y = 10(6) + 3 = 63 Or 10x + y = 10(3) + 6 = 36
Thus, the required number is either 63 or 36.

3. Seven times a two-digit number is equal to four times the number obtained by reversing the digits. If the difference between the digits is 3 find the number.
Solution:

Let the digit at ten's place be considered as x
And the digit at unit's place be considered as y
So, the required number will be 10x + y
When the digits are interchanged, the reversed number will be 10y + x
Then, according to the question, we have
7(10x + y) = 4(10y + x)
66x = 33y
2x - y = 0 ... (1)
Also,
y – x = 3 … (2)
On adding (1) and (2), we get
x = 3
Now, substituting the value of x in (1), we get
2(3) - y = 0
⇒ y = 6
Thus, the required number is 10(3) + 6 = 36.

4. From Delhi station, if we buy 2 tickets for station A and 3 tickets for station B, the total cost is Rs. 77. But if we buy 3 tickets for station A and 5 tickets for station B, the total cost is Rs. 124. What are the fares from Delhi to station A and to station B?
Solution:

Let's consider the fare of ticket for station A to be Rs. x
and the fare of ticket for station B as Rs. y
Then, according to the question, we have
2x + 3y = 77 ... (1)
And, 3x + 5y = 124 ... (2)
Performing (1)×3 - (2), we get
6x + 9y = 231
6x + 10y = 248
(-) (-) (-)
____
-y = -17
⇒ y = 17
On substituting the value of y in (1), we get

$2x + 3(17) = 77$
$2x = 77 - 51$
$2x = 26$
$\Rightarrow x = 13$
Thus, fare for station A = Rs. 13 and fare for station B = Rs. 17.

5. The sum of digit of a two-digit number is 11. If the digit at ten's place is increased by 5 and the digit at unit place is decreased by 5, the digits of the number are found to be reversed. Find the original number.
Solution:

Let x be the number at the ten's place and y be the number at the unit's place.
So, the number is 10x + y.
Then, given as
The sum of digit of a two-digit number is 11
$\Rightarrow x + y = 11 \quad \ldots(i)$
And,
If the digit at ten's place is increased by 5 and the digit at unit place is decreased by 5, the digits of the number are found to be reversed.
$10(x + 5) + (y - 5) = 10y + x$
$9x - 9y = -45$
$\Rightarrow x - y = -5 \quad \ldots(ii)$
On subtracting equation (i) from equation (ii), we get

$$\begin{array}{r} x - y = -5 \\ x + y = 11 \\ (-)\ (-)\quad (-) \\ \hline -2y = -16 \end{array}$$

$\Rightarrow y = 8$
Now, on substituting y = 8 in equation (i), we get
$x + 8 = 11$
$\Rightarrow x = 3$
Thus, the number is $10x + y = 10(3) + 8 = 38$

6. 90% acid solution (90% pure acid and 10% water) and 97% acid solution are mixed to obtain 21 litres of 95% acid solution. How many litres of each solution are mixed.
Solution:

Let the quantity of 90% acid solution be taken as x litres and
The quantity of 97% acid solution be taken as y litres
Then, according to the question, we have
$x + y = 21 \ldots (1)$
And, 90% of x + 97% of y = 95% of 21
$\Rightarrow 90x + 97y = 1995 \ldots (2)$
Performing (1)×90 - (2), we get
$90x + 90y = 1890$

90x + 90y = 1995
(-) (-) (+)

-7y = -105
⇒ y = 15
On substituting the value of y in (1), we get
x + 15 = 21
⇒ x = 6
Thus, 90% acid solution is 6 litres and 97% acid solution is 15 litres.

7. The class XI students of school wanted to give a farewell party to the outgoing students of class XII. They decided to purchase two kinds of sweets, one costing Rs. 250 per kg and other costing Rs. 350 per kg. They estimated that 40 kg of sweets were needed. If the total budget for the sweets was Rs. 11,800; find how much sweets of each kind were bought?
Solution:

Let's assume x kg of the first kind costing Rs. 250 per kg and y kg of the second kind costing Rs. 350 per kg sweets were bought
It is estimated that 40 kg of sweets were needed
⇒ x + y = 40 … (i)
The total budget for the sweets was Rs. 11,800
⇒ 250x + 350y = 11,800 … (ii)
Performing (i)×250 – (ii), we get
250x + 250y = 10000
250x + 350y = 11,800
(-) (-) (-)

-100y = -1800
⇒ y = 18
On substituting y = 18 in equation (i), we get
x + 18 = 40
⇒ x = 22
Therefore, 22 kgs of the first kind costing Rs. 250 per kg and 18 kgs of the second kind costing rs. 350 per kg sweets were bought.

8. Mr. and Mrs. Abuja weight x kg and y kg respectively. They both take a dieting course, at the end of which Mr. Ahuja loses 5 kg and weights as much as his wife weighed before the course. Mrs. Ahuja loses 4 kg and weighs 7/8 th of what her husband weighed before the course. Form two equations in x and y, find their weights before taking the dieting course.
Solution:

Let's assume the weight of Mr. Ahuja = x kg and weight of Mrs. Ahuja = y kg.
After the dieting,
x - 5 = y

⇒ x - y = 5 ... (1)
And, y – 4 = (7/8)x
⇒ 7x - 8y = -32 ... (2)
Performing (1)×7 – (2), we get
7x – 7y = 35
7x – 8y = -32
(-) (+) (+)
___
y = 67
Now, on substituting the value of y in (1), we get
x - 67 = 5
⇒ x = 72
Thus, weight of Mr. Ahuja = 72 kg and that of Mr. Anuja = 67 kg.

9. A part of monthly expenses of a family is constants and the remaining vary with the number of members in the family. For a family of 4 person, the total monthly expenses are Rs. 10,400 whereas for a family of 7 persons, the total monthly expenses are Rs. 15,800. Find the constant expenses per month and the monthly expenses of each member of a family.
Solution:

Let's assume x to be the constant expense per month of the family and y to be the expense per month for a single member of the family.
Then,
For a family of 4 people, the total monthly expense is Rs. 10,400
x + 4y = 10,400 … (i)
And, for a family of 7 people, the total monthly expense is Rs. 15,800
x + 7y = 15,800 … (ii)

On subtracting equation (i) from equation (ii), we get
x + 7y = 15800
x + 4y = 10400
(-) (-) (-)
___
3y = 5400
⇒ y = 1800
Now, on substituting y = 1800 in equation (i), we get
x + 4(1800) = 10,400
⇒ x = 3200
Thus, the constant expense is Rs. 3,200 per month and the monthly expense of each member of a family is Rs. 1,800.

10. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs. 315 and for a journey of 15 km, the charge paid is Rs. 465. What are the fixed charges and the charge per kilometer? How much does a person have to pay for travelling a distance of 32 km?

Solution:

Let assume the fixed charge to be Rs. x and the charger per kilometer be Rs. y.
Then,
The charges for 10km = Rs.10y
The charges for 15km = Rs.15y
Now, according to the question, we have
$x + 10y = 315$ … (i)
$x + 15y = 465$ … (ii)
On solving the equations, we get
$-5y = -150$
$\Rightarrow y = 30$
And,
$x = 315 – 10y$
$= 315 – 10(30)$
$= 15$
Hence, the fixed charge is Rs.15 and the charges per kilometer is Rs.30.
Thus, to travel 32km, a person has to pay Rs.15 + Rs.30(32) = Rs.15 + Rs.960
= Rs.975

11. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Geeta paid Rs. 27 for a book kept for seven days, while Mohit paid Rs. 21 for the book he kept for five days. Find the fixed charges and the charge for each extra day.
Solution:

Let's assume the fixed charge to be Rs. x and the charge for each extra day to be Rs. y
Then, according to the question, we have
$x + 4y = 27$ … (i)
And, $x + 2y = 21$ … (ii)
Subtracting (ii) from (i), we get
$2y = 6$
$\Rightarrow y = 3$
and $x = 21 – 2y$
$= 21 – 2(3)$
$= 15$
Thus, the fixed charge is Rs.15 and the charge for each extra day is Rs. 3

12. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. However, if the length of this rectangle increases by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.
Solution:

Let's assume the length of the rectangle to be x units and the breadth of the rectangle to be y units

We know that, area of rectangle = length × breadth = xy
Then, according to the question, we have
$xy - 9 = (x - 5)(y + 3)$
$xy - 9 = xy + 3x - 5y - 15$
$3x - 5y = 6$ … (i)
And,
$xy + 67 = (x + 3)(y + 2)$
$xy + 67 = xy + 2x + 3y + 6$
$2x + 3y = 61$ … (ii)
Performing (i)×2 + (ii)×3, we get
$-19y = -171$
$\Rightarrow y = 9$
On substituting the value of y in (i), we get
$3x - 5(9) = 6$
$3x = 6 + 45$
$x = 51/3$
$\Rightarrow x = 17$
Thus, the length of the rectangle is 17 units and the breadth of the rectangle is 9 units.

13. It takes 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter is used for 9 hours, only half of the pool is filled. How long would each pipe take to fill the swimming pool?
Solution:

Let the pipe with larger diameter and smaller diameter be considered as pipes A and B respectively
Also, let's assume that pipe A works at rate of x hours/unit and pipe B works at a rate of y hours/unit
Then, according to the question, we have
$x + y = 1/12$
$\Rightarrow 12x + 12y = 1$ … (i)
And, $4x + 9y = 1/2$
$\Rightarrow 8x + 18y = 1$ … (ii)
Performing (i)×2 - (ii)×3, we get
$24x + 24y = 2$
$24x + 54y = 3$
(-)__(-)___(-)__
$-30y = -1$
$\Rightarrow y = 1/30$
On substituting the value of y in (i), we get
$x = 1/20$
Thus, the pipe with larger diameter will take 20 hours to fill the swimming pool and the pipe with smaller diameter will take 30 hours to fill the swimming pool.