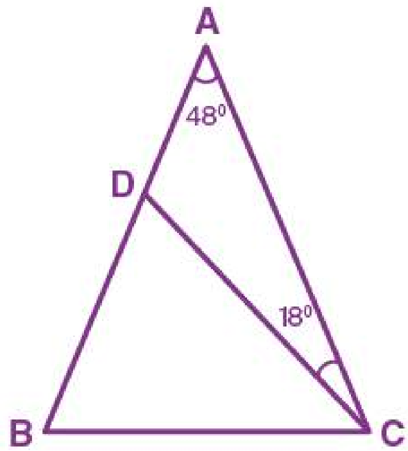
Exercise 10(A)

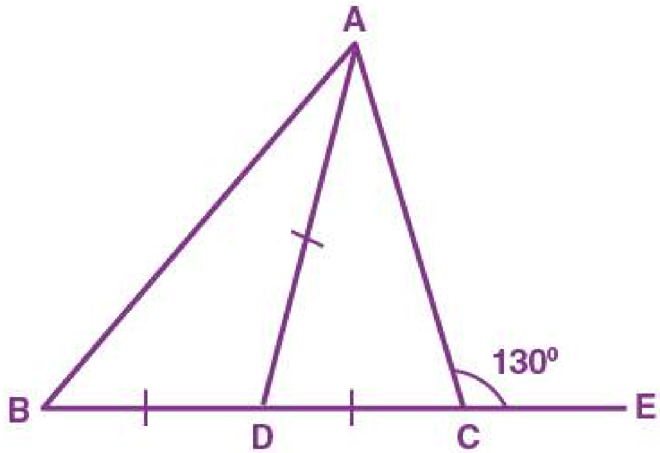
1. In the figure alongside,



AB = AC
∠A = 48º and
∠ACD = 18º.
Show that BC = CD.
Solution:

In ΔABC, we have
$\angle BAC + \angle ACB + \angle ABC = 180^0$
$48^0 + \angle ACB + \angle ABC = 180^0$
But, $\angle ACB = \angle ABC$ [Given, AB = AC]
$2\angle ABC = 180^0 - 48^0$
$2\angle ABC = 132^0$
$\angle ABC = 66^0 = \angle ACB$ ……(i)
$\angle ACB = 66^0$
$\angle ACD + \angle DCB = 66^0$
$18^0 + \angle DCB = 66^0$
$\angle DCB = 48^0$ ………(ii)
Now, In ΔDCB,
$\angle DBC = 66^0$ [From (i), Since $\angle ABC = \angle DBC$]
$\angle DCB = 48^0$ [From (ii)]
$\angle BDC = 180^0 - 48^0 - 66^0$
$\angle BDC = 66^0$
Since $\angle BDC = \angle DBC$
Therefore, BC = CD
Equal angles have equal sides opposite to them.

2. Calculate:
(i) ∠ADC
(ii) ∠ABC
(iii) ∠BAC



Solution:

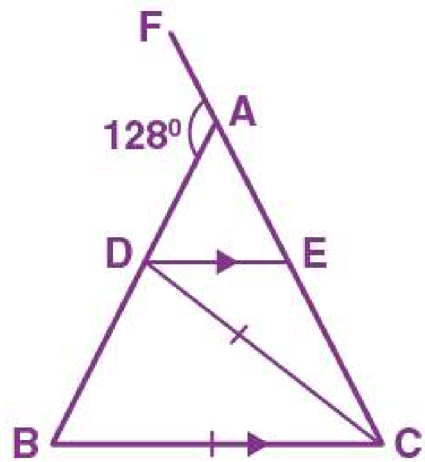
Given: ∠ACE = 130^0; AD = BD = CD
Proof:
(i) ∠ACD + ∠ACE = 180° [DCE is a straight line]
∠ACD = 180° – 130°
∠ACD = 50°
Now,
CD = AD
∠ACD = ∠DAC = 50° … (i) [Since angles opposite to equal sides are equal]
In △ADC,
∠ACD = ∠DAC = 50°
∠ACD + ∠DAC + ∠ADC = 180°
50° + 50° + ∠ADC = 180°
∠ADC = 180° – 100°
∠ADC = 80°

(ii) ∠ADC = ∠ABD + ∠DAB [Exterior angle is equal to sum of opposite interior angles]
But, AD = BD
∴ ∠DAB=∠ABD
80° = ∠ABD + ∠ABD
2∠BD = 80°
∠ABD = 40° = ∠DAB … (ii)

(iii) We have,
∠BAC = ∠DAB + ∠DAC
Substituting the values from (i) and (ii),
∠BAC = 40° + 50°
Hence, ∠BAC = 90°

3. In the following figure, AB = AC; BC = CD and DE is parallel to BC. Calculate:
(i) ∠CDE

(ii) ∠DCE



Solution:

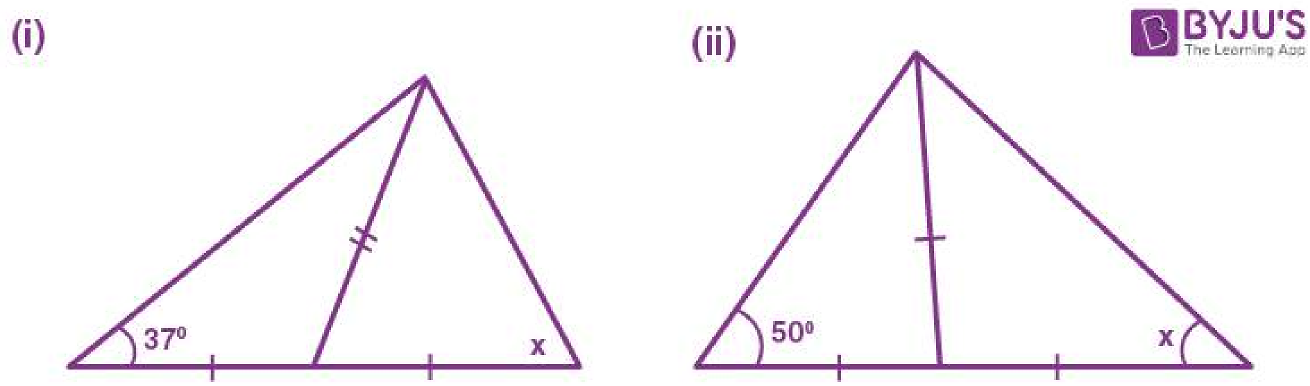
Given, $\angle FAB = 128^{O}$
$\angle BAC + \angle FAB = 180^{O}$ [As FAC is a straight line]
$\angle BAC = 180^{O} – 128^{O}$
$\angle BAC = 52^{O}$

In $\triangle ABC$, we have
$\angle A = 52^{O}$
$\angle B = \angle C$ [Given AB = AC and angles opposite to equal sides are equal]
Now, by angle sum property
$\angle A + \angle B + \angle C =180^{O}$
$\angle A + \angle B + \angle B = 180^{O}$
$52^{O}+ 2\angle B = 180^{O}$
$2\angle B = 128^{O}$
$\angle B = 64^{O} = \angle C$… (i)
$\angle B = \angle ADE$ [Given DE ll BC]

(i) Now, $\angle ADE + \angle CDE + \angle B = 180^{O}$ [As ADB is a straight line]
$64^{O} + \angle CDE + 64^{O}= 180^{O}$
$\angle CDE = 180^{O}+ 128^{O}$
$\angle CDE = 52^{O}$

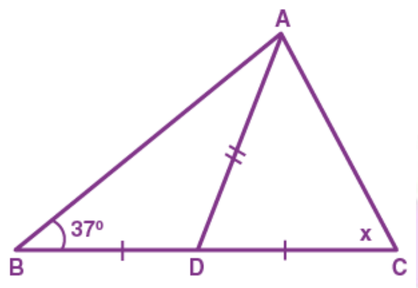
(ii) Given DE ll BC and DC is the transversal
$\angle CDE = \angle DCB = 52^{o}$… (ii)
Also, $\angle ECB = 64^{o}$… [From (i)]
But,
$\angle ECB = \angle DCE +\angle DCB$
$64^{o} = \angle DCE + 52^{o}$
$\angle DCE = 64^{o} - 52^{o}$
$\angle DCE = 12^{o}$

4. Calculate x:



Solution:

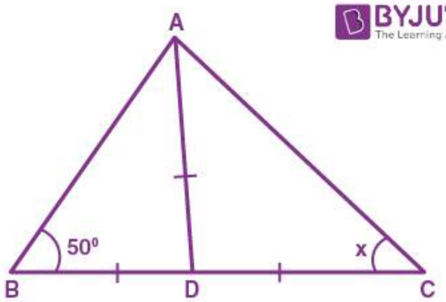
(i) Let the triangle be ABC and the altitude be AD.



In ∆ABD, we have
∠DBA = ∠DAB = 37° [Given BD = AD and angles opposite to equal sides are equal]
Now,
∠CDA = ∠DBA + ∠DAB [Exterior angle is equal to the sum of opposite interior angles]
∠CDA = 37° + 37°
∴ ∠CDA = 74°

Now, in ∆ADC, we have
∠CDA = ∠CAD = 74° [Given CD = AC and angels opposite to equal sides are equal]
Now, by angle sum property
∠CAD + ∠CDA + ∠ACD = 180°
74° + 74° + x = 180°
x = 180° - 148°
x = 32°

(ii) Let triangle be ABC and altitude be AD.



In $\triangle ABD$, we have

$\angle DBA = \angle DAB = 50^\circ$ [Given BD = AD and angles opposite to equal sides are equal]

Now,

$\angle CDA = \angle DBA + \angle DAB$ [Exterior angle is equal to the sum of opposite interior angles]

$\angle CDA = 50^\circ + 50^\circ$

$\therefore \angle CDA = 100^\circ$

In $\triangle ADC$, we have

$\angle DAC = \angle DCA = x$ [Given AD = DC and angels opposite to equal sides are equal]

So, by angle sum property

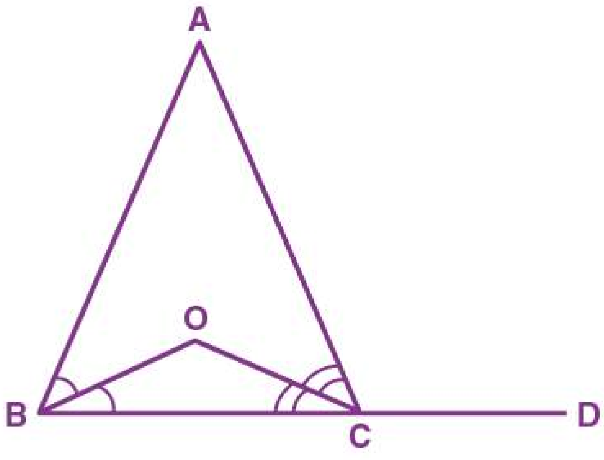
$\angle DAC + \angle DCA + \angle ADC = 180^\circ$

$x + x + 100^\circ = 180^\circ$

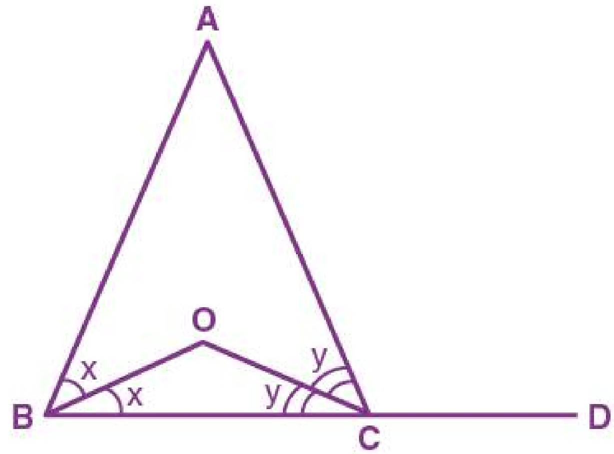
$2x = 80^\circ$

$x = 40^\circ$

5. In the figure, given below, AB = AC. Prove that: $\angle BOC = \angle ACD$.



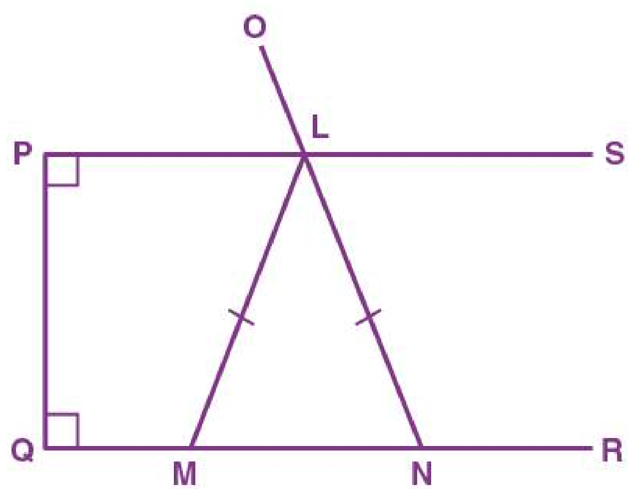
Solution:



Let's assume ∠ABO = ∠OBC = x and ∠ACO = ∠OCB = y
In ABC, we have
∠BAC = 180º - 2x - 2y…(i)
As, ∠B = ∠C [Since, AB = AC]
½ ∠B = ½ ∠C
⇒ x = y
Now,
∠ACD = 2x + ∠BAC [Exterior angle is equal to sum of opposite interior angle]
= 2x + 180º - 2x - 2y [From (i)]
∠ACD = 180º - 2y… (ii)
In ∆OBC, we have
∠BOC = 180º - x - y
∠BOC = 180º - y – y [Since x = y]
∠BOC = 180º - 2y… (iii)

Thus, from (ii) and (iii) we get
∠BOC = ∠ACD

6. In the figure given below, LM = LN; ∠PLN = 110º. Calculate:

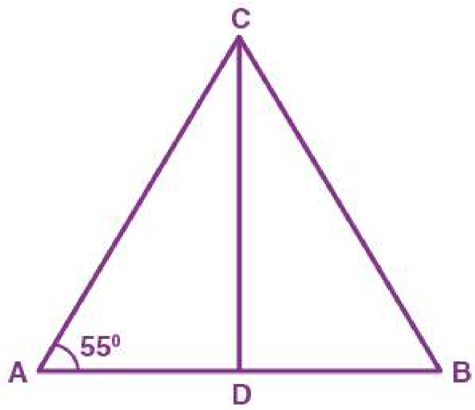


(i) ∠LMN
(ii) ∠MLN
Solution:

Given, LM = LN and ∠PLN = 110°
(i) We know that the sum of the measure of all the angles of a quadrilateral is 360°.
In quad. PQNL,
∠QPL + ∠PLN +LNQ + ∠NQP = 360°
90° + 110° + ∠LNQ + 90° = 360°
∠LNQ = 360° – 290°
∠LNQ = 70°
∠LNM = 70°… (i)
In ΔLMN, we have
LM = LN [Given]
⇒ ∠LNM = ∠LMN [Angles opposite to equal sides are equal]
∠LMN = 70°…(ii) [From (i)]

(ii) In ΔLMN, we have
∠LMN + ∠LNM + ∠MLN = 180°
But, ∠LNM = ∠LMN = 70° [From (i) and (ii)]
⇒ 70° + 70° + ∠MLN = 180°
∠MLN = 180° - 140°
∴ ∠MLN = 40°

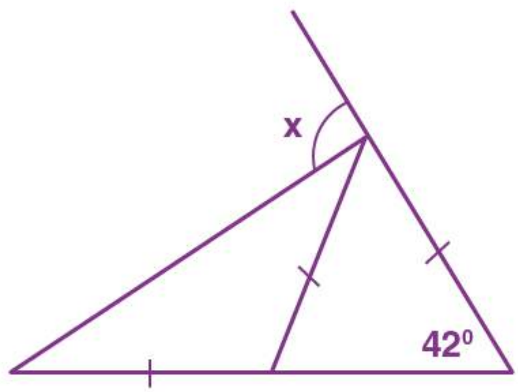
7. An isosceles triangle ABC has AC = BC. CD bisects AB at D and ∠CAB = 55°.
Find: (i) ∠DCB (ii) ∠CBD.
Solution:



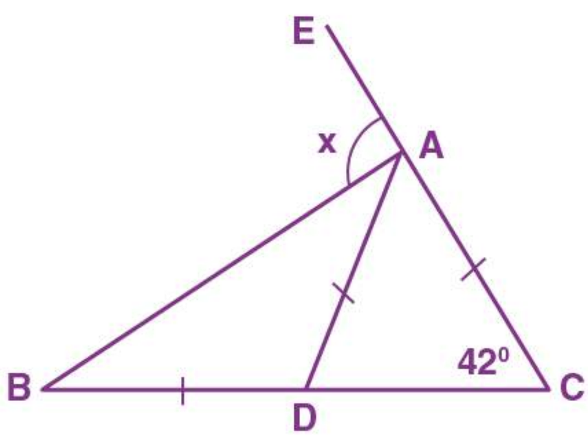
In ΔABC, we have
AC = BC [Given]
So, ∠CAB = ∠CBD [Angles opposite to equal sides are equal]
⇒ ∠CBD = 55°
In ΔABC, we have
∠CBA + ∠CAB + ∠ACB = 180°

But, ∠CAB = ∠CBA = 55^o
$55^o + 55^o$ + ∠ACB = 180^o
∠ACB = $180^o - 110^o$
∠ACB = 70^o
Now,
In △ACD and △BCD, we have
AC = BC [Given]
CD = CD [Common]
AD = BD [Given that CD bisects AB]
∴ △ACD ≅ △BCD
So, By CPCT
∠DCA = ∠DCB
∠DCB = ∠ACB/2 = $70^o/2$
Thus, ∠DCB = 35^o

8. Find x:



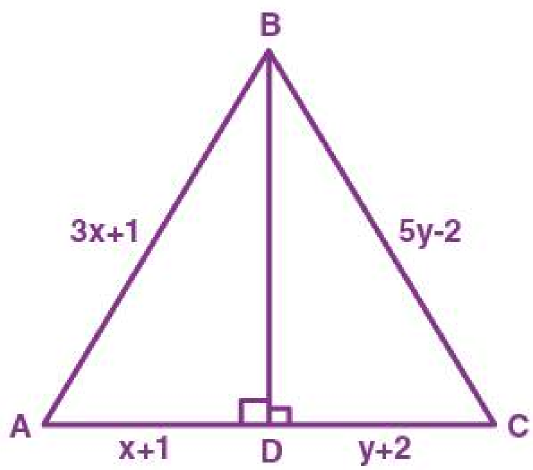
Solution:
Let's put markings to the figure as following:



In △ABC, we have
AD = AC [Given]
∴ ∠ADC = ∠ACD [Angles opposite to equal sides are equal]
So, ∠ADC = 42^o

Now,
∠ADC = ∠DAB + DBA [Exterior angle is equal to the sum of opposite interior angles]
But,
∠DAB = ∠DBA [Given: BD = DA]
∴ ∠ADC = 2∠DBA
2∠DBA = 42°
∠DBA = 21°
To find x:
x = ∠CBA + ∠BCA [Exterior angle is equal to the sum of opposite interior angles]
We know that,
∠CBA = 21°
∠BCA = 42°
⇒ x = 21° + 42°
∴ x = 63°

9. In the triangle ABC, BD bisects angle B and is perpendicular to AC. If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the values of x and y.

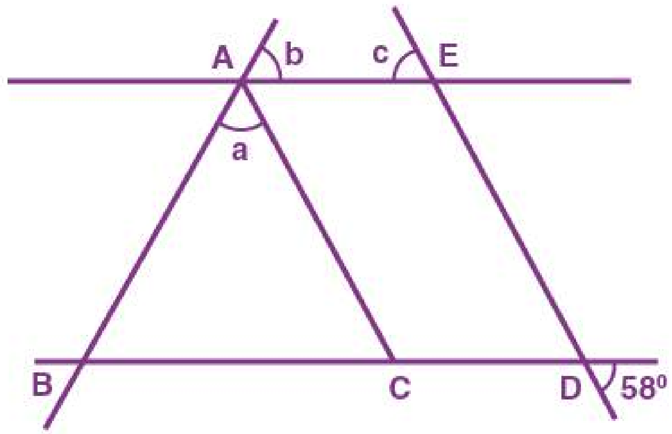


Solution:

In ∆ABC and ∆DBC, we have
BD = BD [Common]
∠BDA = ∠BDC [Each equal to 90°]
∠ABD = ∠DBC [BD bisects ∠ABC]
∴ ∆ABD ≅ ∆DBC [ASA criterion]
Therefore, by CPCT
AD = DC
x + 1 = y + 2
x = y + 1… (i)
And, AB = BC
3x + 1 = 5y - 2

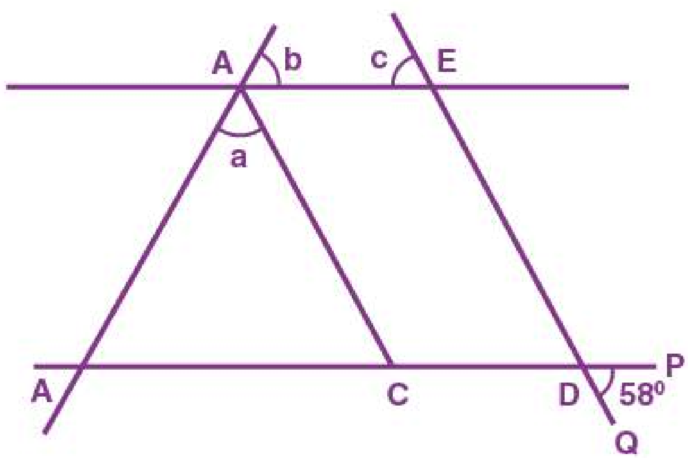
Substituting the value of x from (i), we get
$3(y+1) + 1 = 5y - 2$
$3y + 3 + 1 = 5y - 2$
$3y + 4 = 5y - 2$
$2y = 6$
$y = 3$
Putting $y = 3$ in (i), we get
$x = 3 + 1$
$\therefore x = 4$

10. In the given figure; AE // BD, AC // ED and AB = AC. Find ∠a, ∠b and ∠c.



Solution:

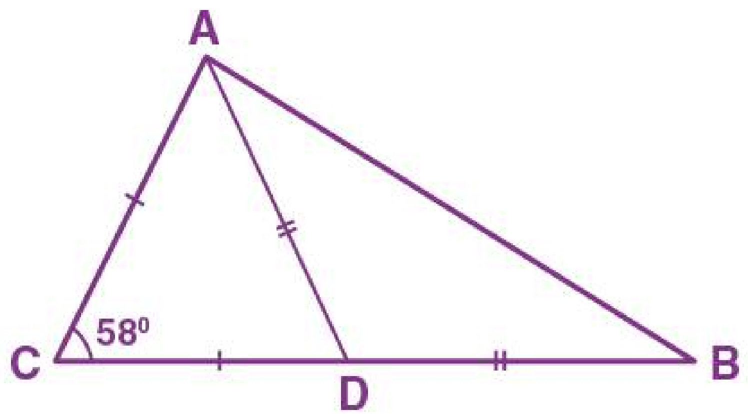
Let's assume points P and Q as shown below:



Given, $\angle PDQ = 58^o$
$\angle PDQ = \angle EDC = 58^o$ [Vertically opposite angles]
$\angle EDC = \angle ACB = 58^o$ [Corresponding angles $\because$ AC ll ED]
In $\triangle ABC$, we have
AB = AC [Given]
$\therefore \angle ACB = \angle ABC = 58^o$ [Angles opposite to equal sides are equal]

Now,
$\angle ACB + \angle ABC + \angle BAC = 180^o$
$58^o + 58^o + a = 180^o$
$\angle a = 180^o - 116^o$
$\angle a = 64^o$
Since, AE ll BD and AC is the transversal
$\angle ABC = \angle b$ [Corrosponding angles]
$\therefore \angle b = 58^o$
Also, since AE ll BD and ED is the transversal
$\angle EDC = \angle c$ [Corrosponding angles]
$\therefore \angle c = 58^o$

11. In the following figure; AC = CD, AD = BD and ∠C = 58º.

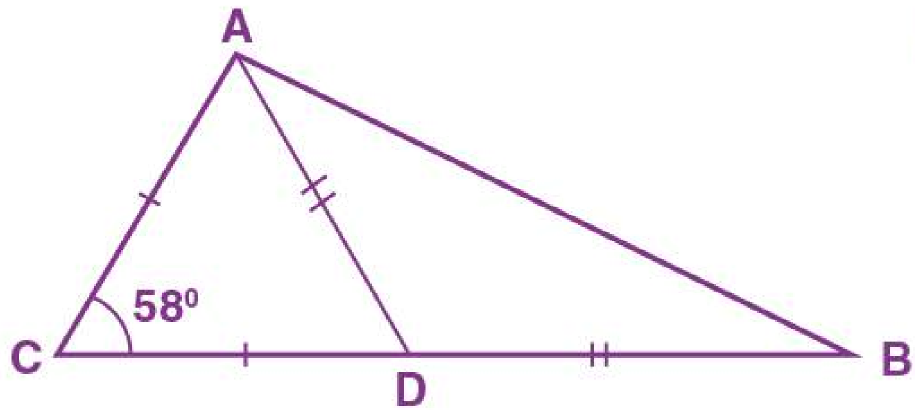


Find ∠CAB.
Solution:

In $\triangle ACD$, we have
AC = CD [Given]
$\therefore \angle CAD = \angle CDA$ [Angles opposite to equal sides are equal]
And,
$\angle ACD = 58^o$ [Given]
By angle sum property, we have
$\angle ACD + \angle CDA + \angle CAD = 180^o$
$58^o + 2\angle CAD = 180^o$
$2\angle CAD = 122^o$
$\angle CAD = \angle CDA = 61^o$… (i)
Now,
$\angle CDA = \angle DAB + \angle DBA$ [Exterior angles is equal to sum of opposite interior angles]
But,
$\angle DAB = \angle DBA$ [Given, AD = DB]
So, $\angle DAB + \angle DAB = \angle CDA$
$2\angle DAB = 61^o$
$\angle DAB = 30.5^o$… (ii)

In $\triangle$ABC, we have
$\angle CAB = \angle CAD + \angle DAB$
$\angle CAB = 61^o + 30.5^o$ [From (i) and (ii)]
$\therefore \angle CAB = 91.5^o$

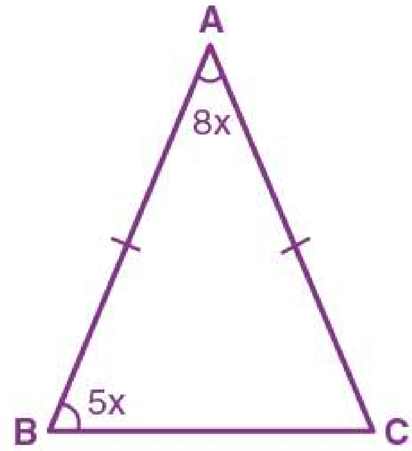
12. In the figure of Q.11 is given above, if AC = AD = CD = BD; find angle ABC.



Solution:

In $\triangle$ACD, we have
AC = AD = CD [Given]
Hence, ACD is an equilateral triangle
$\therefore \angle ACD = \angle CDA = \angle CAD = 60^o$
Now,
$\angle CDA = \angle DAB + \angle ABD$ [Exterior angle is equal to sum of opposite interior angles]
But,
$\angle DAB = \angle ABD$ [Given, AD = DB]
So, $\angle ABD + \angle ABD = \angle CDA$
$2\angle ABD = 60^o$
$\therefore \angle ABD = \angle ABC = 30^o$

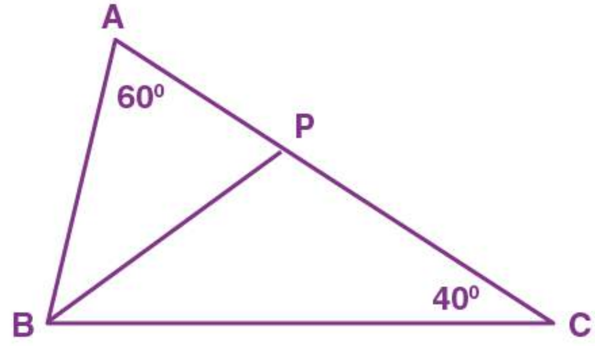
13. In $\triangle$ABC; AB = AC and $\angle A: \angle B = 8: 5$; find $\angle A$.
Solution:



Let, ∠A = 8x and ∠B = 5x

Given, In △ABC
AB = AC
So, ∠B = ∠C = 5x [Angles opp. to equal sides are equal]
Now, by angle sum property
∠A + ∠B +C = 180^o
8x + 5x + 5x = 180^o
18x = 180^o
x = 10^o
Thus, as ∠A = 8x
∠A = 8 × 10^o
∴ ∠A = 80^o

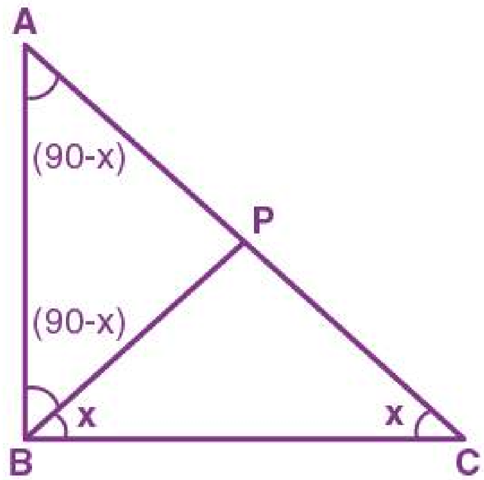
14. In triangle ABC; ∠A = 60^o, ∠C = 40^o, and bisector of angle ABC meets side AC at point P. Show that BP = CP.
Solution:



In △ABC, we have
∠A = 60^o
∠C = 40^o
∴ ∠B = 180^o - 60^o - 40^o [By angle sum property]
∠B = 80^o
Now, as BP is the bisector of ∠ABC

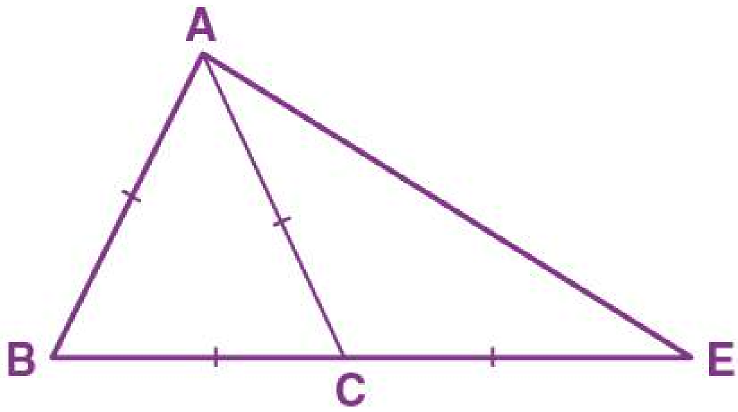
∴ ∠PBC = ∠ABC/2
∠PBC = 40°
In ∆PBC, we have
∠PBC = ∠PCB = 40°
∴ BP = CP [Sides opposite to equal angles are equal]

15. In triangle ABC; angle ABC = 90° and P is a point on AC such that ∠PBC = ∠PCB. Show that: PA = PB.
Solution:



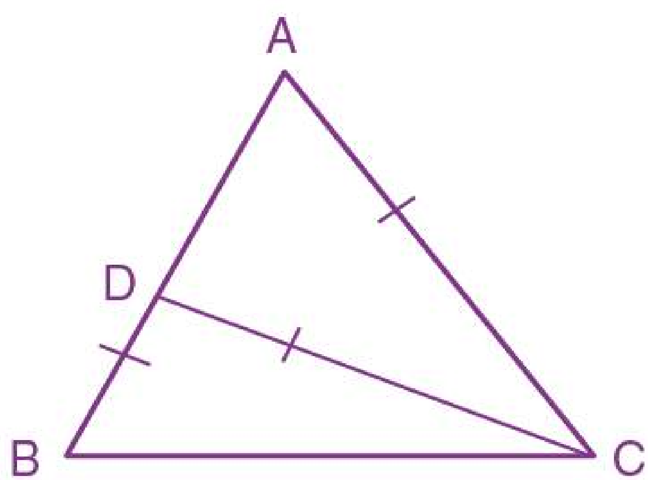
Let's assume ∠PBC = ∠PCB = x
In the right-angled triangle ABC,
∠ABC = 90°
∠ACB = x
∠BAC = 180° - (90° + x) [By angle sum property]
∠BAC = (90° - x) …(i)
And
∠ABP = ∠ABC - ∠PBC
∠ABP = 90° – x …(ii)
Thus, in the ∆ABP from (i) and (ii), we have
∠BAP = ∠ABP
Therefore, PA = PB [sides opp. to equal angles are equal]

16. ABC is an equilateral triangle. Its side BC is produced upto point E such that C is mid-point of BE. Calculate the measure of angles ACE and AEC.
Solution:



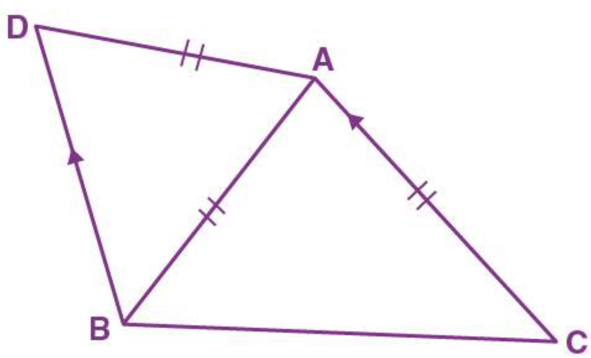
Given, ΔABC is an equilateral triangle
So, AB = BC = AC
$\angle ABC = \angle CAB = \angle ACB = 60^o$
Now, as sum of two non-adjacent interior angles of a triangle is equal to the exterior angle
$\angle CAB + \angle CBA = \angle ACE$
$60^o + 60^o = \angle ACE$
$\angle ACE = 120^o$
Now,
ΔACE is an isosceles triangle with AC = CF
$\angle EAC = \angle AEC$
By angle sum property, we have
$\angle EAC + \angle AEC + \angle ACE = 180^o$
$2\angle AEC + 120^o = 180^o$
$2\angle AEC = 180^o - 120^o$
$\angle AEC = 30^o$

17. In triangle ABC, D is a point in AB such that AC = CD = DB. If ∠B = 28°, find the angle ACD.
Solution:



From given, we get
ΔDBC is an isosceles triangle
$\Rightarrow CD = DB$
$\angle DBC = \angle DCB$ [If two sides of a triangle are equal, then angles opposites to them are equal]
And, $\angle B = \angle DBC = \angle DCB = 28^o$
By angle sum property, we have
$\angle DCB + \angle DBC + \angle BCD = 180^o$
$28^o + 28^o + \angle BCD = 180^o$
$\angle BCD = 180^o - 56^o$
$\angle BCD = 124^o$
As sum of two non-adjacent interior angles of a triangle is equal to the exterior angle, we have
$\angle DBC + \angle DCB = \angle DAC$
$28^o + 28^o = 56^o$
$\angle DAC = 56^o$
Now,
ΔACD is an isosceles triangle with AC = DC
$\Rightarrow \angle ADC = \angle DAC = 56^o$
$\angle ADC + \angle DAC + \angle DCA = 180^o$ [By angle sum property]
$56^o + 56^o + \angle DCA = 180^o$
$\angle DCA = 180^o - 112^o$
$\angle DCA = 64^o$
Thus, $\angle ACD = 64^o$

18. In the given figure, AD = AB = AC, BD is parallel to CA and ∠ACB = 65°. Find ∠DAC.



Solution:

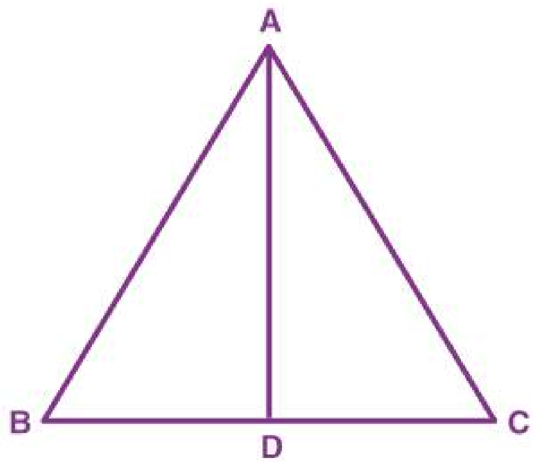
From figure, it's seen that
ΔABC is an isosceles triangle with AB = AC
$\Rightarrow \angle ACB = \angle ABC$
As $\angle ACB = 65^o$ [Given]
$\therefore \angle ABC = 65^o$
By angle sum property, we have
$\angle ACB + \angle CAB + \angle ABC = 180^o$
$65^o + 65^o + \angle CAB = 180^o$
$\angle CAB = 180^o - 130^o$

∠CAB = 50º
As BD is parallel to CA, we have
∠CAB = ∠DBA as they are alternate angles
⇒ ∠CAB = ∠DBA = 50º

Again, from figure, it's seen that
∆ADB is an isosceles triangle with AD = AB.
⇒ ∠ADB = ∠DBA = 50º
By angle sum property, we have
∠ADB + ∠DAB + ∠DBA = 180º
50º + ∠DAB + 50º = 180º
∠DAB = 180º - 100º
∠DAB = 80º
Now,
∠DAC = ∠CAB + ∠DAB
∠DAC = 50º - 80º
∠DAC = 130º

19. Prove that a triangle ABC is isosceles, if:
(i) altitude AD bisects angles BAC, or
(ii) bisector of angle BAC is perpendicular to base BC.
Solution:

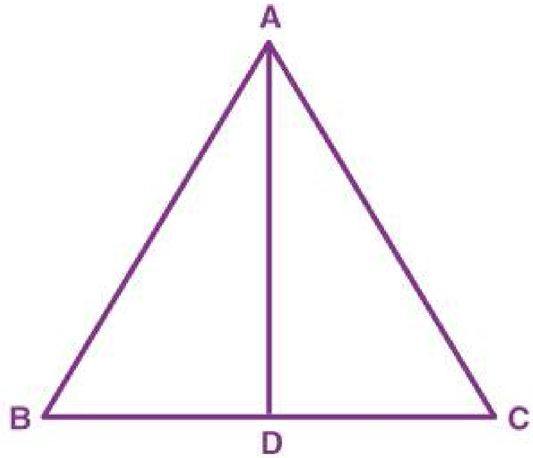
(i) In ΔABC, if the altitude AD bisect ∠BAC.
Then, to prove: ΔABC is isosceles.



In ΔADB and ΔADC, we have
∠BAD = ∠CAD (AD is bisector of ∠BAC)
AD = AD (Common)
∠ADB = ∠ADC (Each equal to 90°)
Therefore, ΔADB ≅ ΔADC by ASA congruence criterion
So, by CPCT

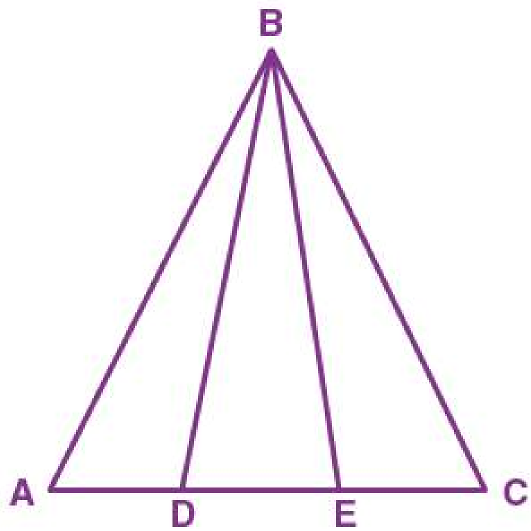
AB = AC
Hence, ΔABC is an isosceles.

(ii) In Δ ABC, the bisector of ∠BAC is perpendicular to the base BC.
Then, to prove: ΔABC is isosceles.



In ΔADB and ΔADC,
∠BAD = ∠CAD (AD is bisector of ∠BAC)
AD = AD (Common)
∠ADB = ∠ADC (Each equal to 90°)
Therefore, ΔADB ≅ ΔADC by ASA congruence criterion
Thus, by CPCT
AB = AC
Hence, ΔABC is an isosceles.

20. In the given figure; AB = BC and AD = EC.
Prove that: BD = BE.

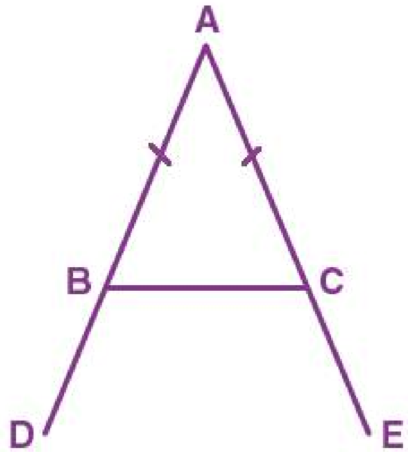


Solution:

In ΔABC, we have
AB = BC (Given)
So, ∠BCA = ∠BAC (Angles opposite to equal sides are equal)
⇒ ∠BCD = ∠BAE ….(i)
Given, AD = EC
AD + DE = EC + DE (Adding DE on both sides)
⇒ AE = CD ….(ii)
Now, in ΔABE and ΔCBD, we have
AB = BC (Given)
∠BAE = ∠BCD [From (i)]
AE = CD [From (ii)]
Therefore, ΔABE ≅ ΔCBD by SAS congruence criterion
So, by CPCT
BE = BD

Exercise 10(B)

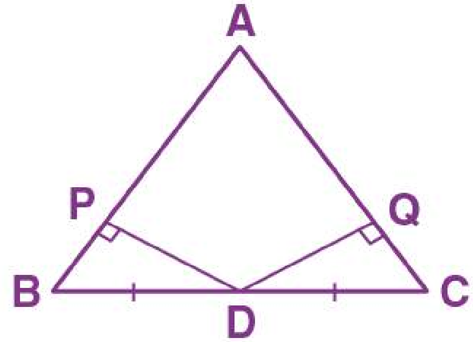
1. If the equal sides of an isosceles triangle are produced, prove that the exterior angles so formed are obtuse and equal.
Solution:



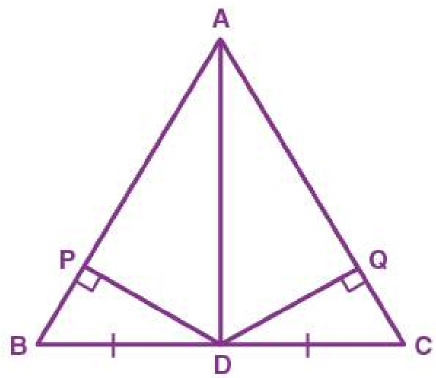
Construction: AB is produced to D and AC is produced to E so that exterior angles ∠DBC and ∠ECB are formed.

In △ABC, we have
AB = AC [Given]
∴ ∠C = ∠B …(i) [Angles opposite to equal sides are equal]
Since, ∠B and ∠C are acute they cannot be right angles or obtuse angles
Now,
∠ABC + ∠DBC = 180^0 [ABD is a straight line]
∠DBC = 180^0 - ∠ABC
∠DBC = 180^0 - ∠B …(ii)
Similarly,
∠ACB + ECB = 180^0 [ABD is a straight line]
∠ECB = 180^0 - ∠ACB
∠ECB = 180^0 - ∠C …(iii)
∠ECB = 180^0 - ∠B …(iv) [from (i) and (iii)]
∠DBC = ∠ECB [from (ii) and (iv)]
Now,
∠DBC = 180^0 - ∠B
But, ∠B is an acute angle
⇒ ∠DBC = 180^0 – (acute angle) = obtuse angle
Similarly,
∠ECB = 180^0 - ∠C
But, ∠C is an acute angle
⇒ ∠ECB = 180^0 – (acute angle) = obtuse angle
Therefore, exterior angles formed are obtuse and equal.

2. In the given figure, AB = AC. Prove that:



(i) DP = DQ
(ii) AP = AQ
(iii) AD bisects ∠A
Solution:



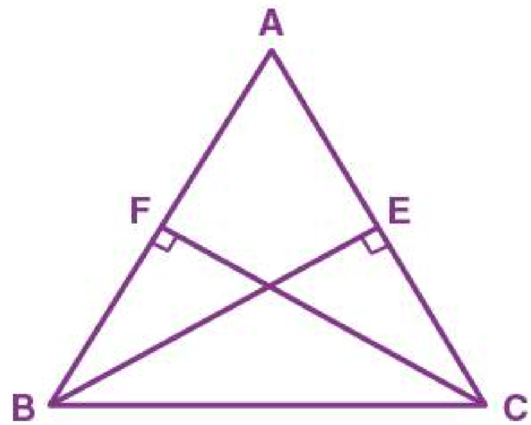
Construction: Join AD.
In $\triangle$ABC, we have
AB = AC [Given]
∴ ∠C = ∠B …(i) [Angles opposite to equal sides are equal]
(i) In $\triangle$BPD and $\triangle$CQD, we have
∠BPD = ∠CQD [Each = 90º]
∠B = ∠C [Proved]
BD = DC [Given]
Thus, $\triangle$BPD ≅ $\triangle$CQD by AAS congruence criterion
∴ DP = DQ by CPCT

(ii) Since, $\triangle$BPD ≅ $\triangle$CQD
Therefore, BP = CQ [CPCT]
Now,
AB = AC [Given]
AB - BP = AC - CQ
AP = AQ

(iii) In $\triangle$APD and $\triangle$AQD, we have

DP = DQ [Proved]
AD = AD [Common]
AP = AQ [Proved]
Thus, △APD ≅ △AQD by SSS congruence criterion
∠PAD = ∠QAD by CPCT
Hence, AD bisects angle A.

3. In triangle ABC, AB = AC; BE ⊥ AC and CF ⊥ AB. Prove that:

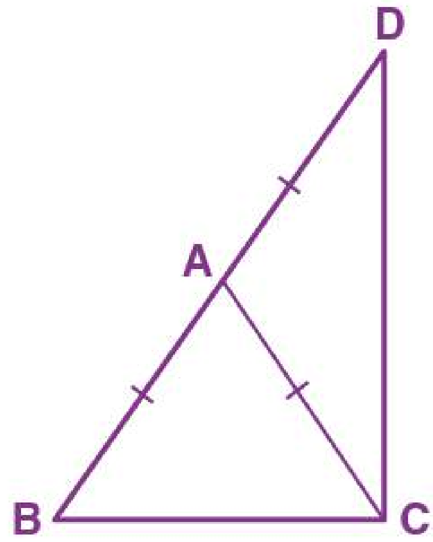


(i) BE = CF
(ii) AF = AE
Solution:

(i) In △AEB and △AFC, we have
∠A = ∠A [Common]
∠AEB = ∠AFC = 90º [Given : BE ⊥ AC and CE ⊥ AB]
AB = AC [Given]
Thus, △AEB ≅ △AFC by AAS congruence criterion
∴ BE = CF by CPCT

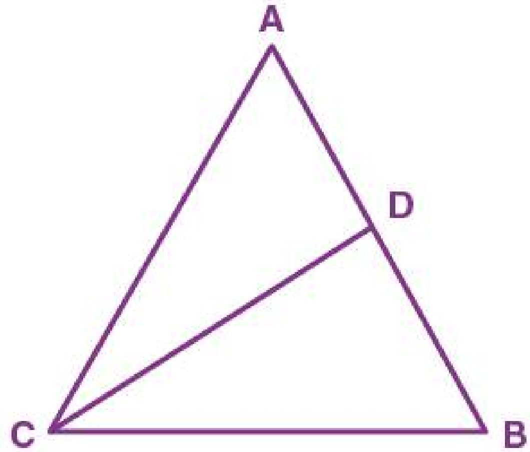
(ii) Since, △AEB ≅ △AFC
∠ABE = ∠AFC
∴ AF= AE by CPCT

4. In isosceles triangle ABC, AB = AC. The side BA is produced to D such that BA = AD. Prove that: ∠BCD = 90º
Solution:



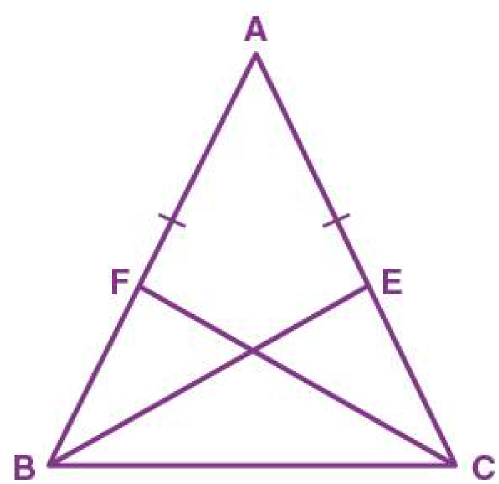
Construction: Join CD.
In $\triangle ABC$, we have
AB = AC [Given]
$\therefore \angle C = \angle B$ … (i) [Angles opposite to equal sides are equal]
In $\triangle ACD$, we have
AC = AD [Given]
$\therefore \angle ADC = \angle ACD$ … (ii)
Adding (i) and (ii), we get
$\angle B + \angle ADC = \angle C + \angle ACD$
$\angle B + \angle ADC = \angle BCD$ … (iii)
In $\triangle BCD$, we have
$\angle B + \angle ADC + \angle BCD = 180^o$
$\angle BCD + \angle BCD = 180^o$ [From (iii)]
$2\angle BCD = 180^o$
$\therefore \angle BCD = 90^o$

5. (i) In $\triangle ABC$, AB = AC and $\angle A = 36°$. If the internal bisector of $\angle C$ meets AB at point D, prove that AD = BC.
(ii) If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.
Solution:



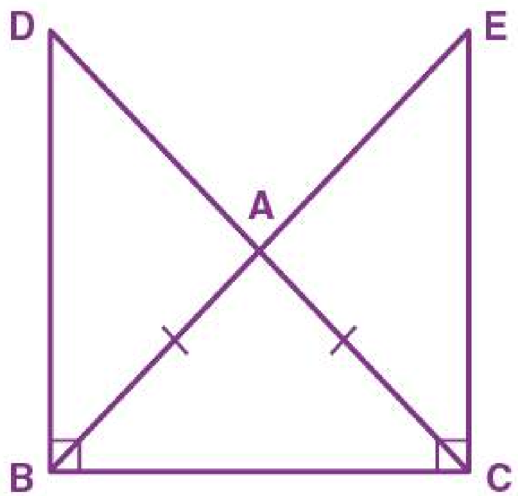
Given, AB = AC and ∠A = 36°
So, ∆ABC is an isosceles triangle.
∠B = ∠C = (180° – 36°)/2 = 72°
∠ACD = ∠BCD = 36° [∵ CD is the angle bisector of ∠C]
Now, ∆ADC is an isosceles triangle as ∠DAC = ∠DCA = 36°
∴ AD = CD …(i)
In ∆DCB, by angle sum property we have
∠CDB = 180° – (∠DCB + ∠DBC)
= 180° – (36° + 72°)
= 180° - 108°
= 72°
Now, ∆DCB is an isosceles triangle as ∠CDB = ∠CBD = 72°
∴ DC = BC …(ii)
From (i) and (ii), we get
AD = BC
- Hence Proved.

6. Prove that the bisectors of the base angles of an isosceles triangle are equal.
Solution:



In ∆ABC, we have
AB = AC [Given]
∴ ∠C = ∠B …(i) [Angles opposite to equal sides are equal]
½∠C = ½∠B
⇒ ∠BCF = ∠CBE …(ii)
Now, in ∆BCE and ∆CBF, we have
∠C = ∠B [From (i)]
∠BCF = ∠CBE [From (ii)]
BC = BC [Common]
∴ ∆BCE ≅ ∆CBF by AAS congruence criterion
Thus, BE = CF by CPCT

7. In the given figure, AB = AC and ∠DBC = ∠ECB = 90º

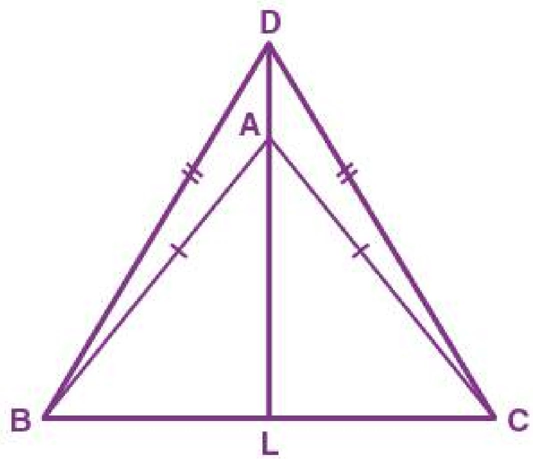


Prove that:
(i) BD = CE
(ii) AD = AE
Solution:

In ∆ABC, we have
AB = AC [Given]
∴ ∠ACB = ∠ABC [Angles opposite to equal sides are equal]
⇒ ∠ABC = ∠ACB … (i)
∠DBC = ∠ECB = 90º [Given]
⇒ ∠DBC = ∠ECB …(ii)
Subtracting (i) from (ii), we get
∠DCB - ∠ABC = ∠ECB - ∠ACB
∠DBA = ∠ECA … (iii)
Now,
In ΔDBA and ΔECA, we have
∠DBA = ∠ECA [From (iii)]
∠DAB = ∠EAC [Vertically opposite angles]
AB = AC [Given]

∴ ΔDBA ≅ ΔECA by ASA congruence criterion
Thus, by CPCT
BD = CE
And, also
AD = AE

8. ABC and DBC are two isosceles triangles on the same side of BC. Prove that:
(i) DA (or AD) produced bisects BC at right angle.
(ii) ∠BDA = ∠CDA.
Solution:



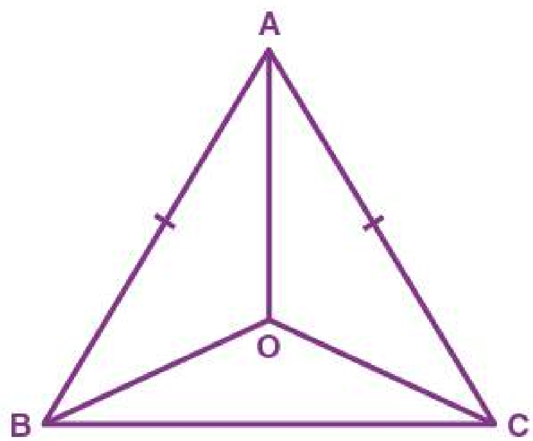
DA is produced to meet BC in L
In ΔABC, we have
AB = AC [Given]
∴ ∠ACB = ∠ABC … (i) [Angles opposite to equal sides are equal]
In ΔDBC, we have
DB = DC [Given]
∴ ∠DCB = ∠DBC … (ii) [Angles opposite to equal sides are equal]
Subtracting (i) from (ii), we get
∠DCB - ∠ACB = ∠DBC - ∠ABC
∠DCA = ∠DBA …(iii)

Now,
In ΔDBA and ΔDCA, we have
DB = DC [Given]
∠DBA = ∠DCA [From (iii)]
AB = AC [Given]
∴ ΔDBA ≅ ΔDCA by SAS congruence criterion
∠BDA = ∠CDA …(iv) [By CPCT]
In ΔDBA, we have
∠BAL = ∠DBA + ∠BDA …(v) [Exterior angle = sum of opposite interior angles]
From (iii), (iv) and (v), we get

$\angle BAL = \angle DCA + \angle CDA$ …(vi) [Exterior angle = sum of opposite interior angles]
In $\triangle DCA$, we have
$\angle CAL = \angle DCA + \angle CDA$ …(vi)
From (vi) and (vii)
$\angle BAL = \angle CAL$ …(viii)

In $\triangle BAL$ and $\triangle CAL$,
$\angle BAL = \angle CAL$ [From (viii)]
$\angle ABL = \angle ACL$ [From (i)
$AB = AC$ [Given]
$\therefore \triangle BAL \cong \triangle CAL$ by ASA congruence criterion
So, by CPCT
$\angle ALB = \angle ALC$
And, $BL = LC$ …(ix)
Now,
$\angle ALB + \angle ALC = 180^o$
$\angle ALB + \angle ALB = 180^o$ [Using (ix)]
$2\angle ALB = 180^o$
$\angle ALB = 90^o$
$\therefore AL \perp BC$
Or $DL \perp BC$ and $BL \perp LC$
Therefore, DA produced bisects BC at right angle.

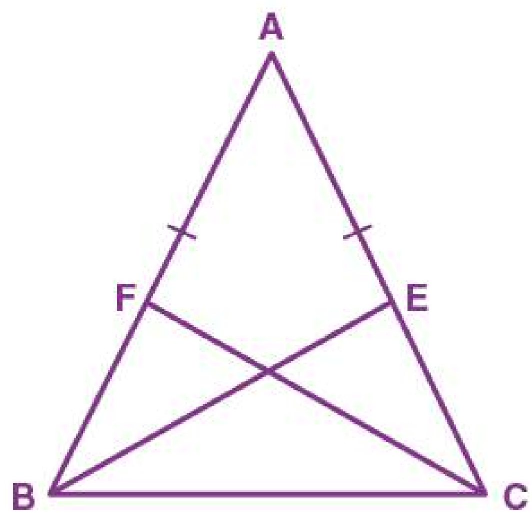
9. The bisectors of the equal angles B and C of an isosceles triangle ABC meet at O. Prove that AO bisects angle A.
Solution:



In $\triangle ABC$, we have $AB = AC$
$\angle B = \angle C$ [Angles opposite to equal sides are equal]
$\frac{1}{2}\angle B = \frac{1}{2}\angle C$
$\angle OBC = \angle OCB$ …(i)
$\Rightarrow OB = OC$ …(ii) [Sides opposite to equal angles are equal]

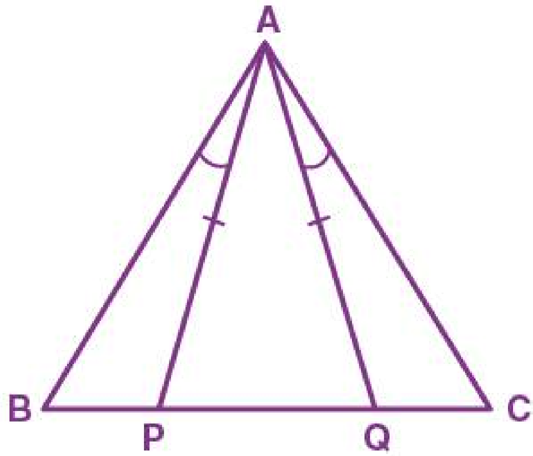
Now,
In ∆ABO and ∆ACO, we have
AB = AC [Given]
∠OBC = ∠OCB [From (i)]
OB = OC [From (ii)]
Thus, ∆ABO ≅ ∆ACO by SAS congruence criterion
So, by CPCT
∠BAO = ∠CAO
Therefore, AO bisects ∠BAC.

10. Prove that the medians corresponding to equal sides of an isosceles triangle are equal.
Solution:



In ∆ABC, we have
AB = AC [Given]
∠C = ∠B … (i) [Angles opposite to equal sides are equal]
Now,
½ AB = ½ AC
BF = CE … (ii)
In ∆BCE and ∆CBF, we have
∠C = ∠B [From (i)]
BF = CE [From (ii)]
BC = BC [Common]
∴ ∆BCE ≅ ∆CBF by SAS congruence criterion
So, CPCT
BE = CF

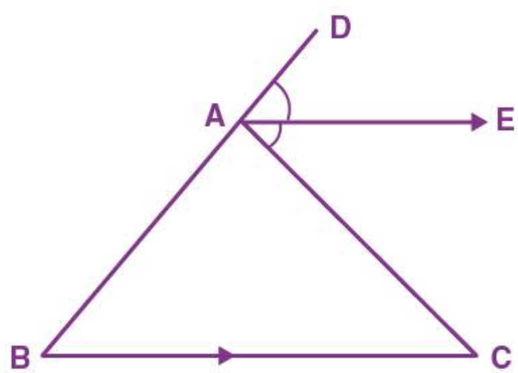
11. Use the given figure to prove that, AB = AC.



Solution:

In $\triangle APQ$, we have
AP = AQ [Given]
$\therefore \angle APQ = \angle AQP$ …(i) [Angles opposite to equal sides are equal]
In $\triangle ABP$, we have
$\angle APQ = \angle BAP + \angle ABP$ …(ii) [Exterior angle is equal to sum of opposite interior angles]
In $\triangle AQC$, we have
$\angle AQP = \angle CAQ + \angle ACQ$ …(iii) [Exterior angle is equal to sum of opposite interior angles]
From (i), (ii) and (iii), we get
$\angle BAP + \angle ABP = \angle CAQ + \angle ACQ$
But, $\angle BAP = \angle CAQ$ [Given]
$\angle CAQ + ABP = \angle CAQ + \angle ACQ$
$\angle ABP = \angle CAQ + \angle ACQ - \angle CAQ$
$\angle ABP = \angle ACQ$
$\angle B = \angle C$
So, in $\triangle ABC$, we have
$\angle B = \angle C$
$\Rightarrow$ AB = AC [Sides opposite to equal angles are equal]

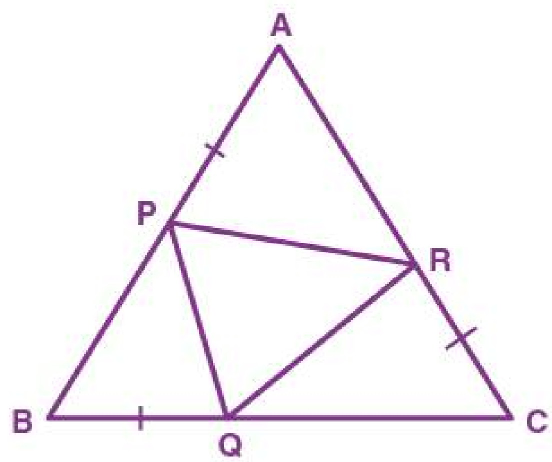
12. In the given figure; AE bisects exterior angle CAD and AE is parallel to BC. Prove that: AB = AC.



Solution:

Since, AE || BC and DAB is the transversal
∴ ∠DAE = ∠ABC = ∠B [Corresponding angles]
Since, AE || BC and AC is the transversal
∠CAE = ∠ACB = ∠C [Alternate angles]
But, AE bisects ∠CAD
∴ ∠DAE = ∠CAE
∠B = ∠C
⇒ AB = AC [Sides opposite to equal angles are equal]

13. In an equilateral triangle ABC; points P, Q and R are taken on the sides AB, BC and CA respectively such that AP = BQ = CR. Prove that triangle PQR is equilateral.
Solution:

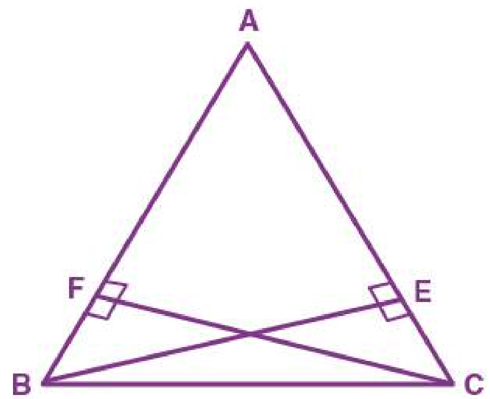


Given, AB = BC = CA (Since, ABC is an equilateral triangle) …(i)
and AP = BQ = CR …(ii)
Subtracting (ii) from (i), we get
AB - AP = BC - BQ = CA - CR
BP = CQ = AR …(iii)
∴ ∠A = ∠B = ∠C …(iv) [Angles opposite to equal sides are equal]
In ΔBPQ and ΔCQR, we have
BP = CQ [From (iii)]
∠B = ∠C [From (iv)]
BQ = CR [Given]
∴ ΔBPQ ≅ ΔCQR by SAS congruence criterion
So, PQ = QR [by CPCT] … (v)

In ΔCQR and ΔAPR, we have
CQ = AR [From (iii)]
∠C = ∠A [From (iv)]

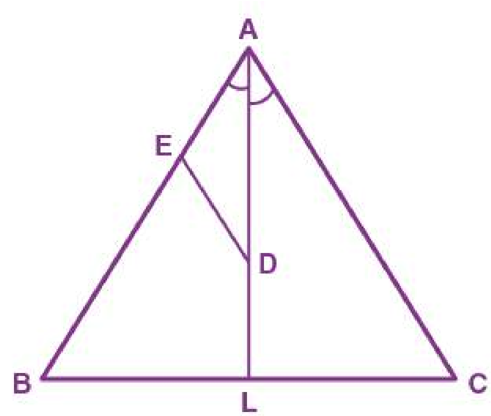
CR = AP [Given]
∴ ΔCQR ≅ ΔAPR by SAS congruence criterion
So, QR = PR [By CPCT] … (vi)
From (v) and (vi), we get
PQ = QR = PR
Therefore, PQR is an equilateral triangle.

14. In triangle ABC, altitudes BE and CF are equal. Prove that the triangle is isosceles.
Solution:



In ΔABE and ΔACF, we have
∠A = ∠A [Common]
∠AEB = ∠AFC = 90^0 [Given: BE ⊥ AC and CF ⊥ AB]
BE = CF [Given]
∴ ΔABE ≅ ΔACF by AAS congruence criterion
So, by CPCT
AB = AC
Therefore, ABC is an isosceles triangle.

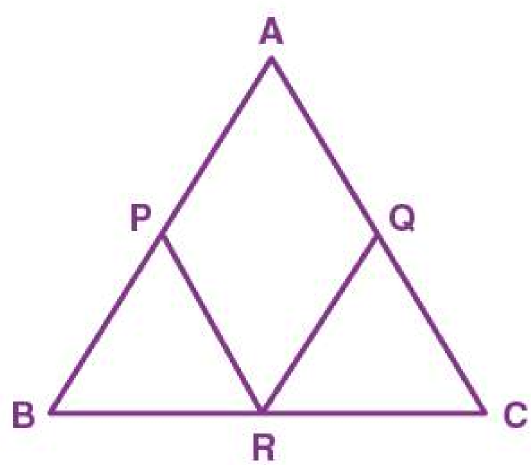
15. Through any point in the bisector of angle, a straight line is drawn parallel to either arm of the angle. Prove that the triangle so formed is isosceles.
Solution:



Let's consider △ABC, AL is bisector of ∠A.
Let D is any point on AL.
From D, a straight-line DE is drawn parallel to AC.
DE || AC [Given]
So, ∠ADE = ∠DAC ...(i) [Alternate angles]
∠DAC = ∠DAE … (ii) [AL is bisector of ∠A]
From (i) and (ii), we get
∠ADE = ∠DAE
∴ AE = ED [Sides opposite to equal angles are equal]
Therefore, AED is an isosceles triangle.

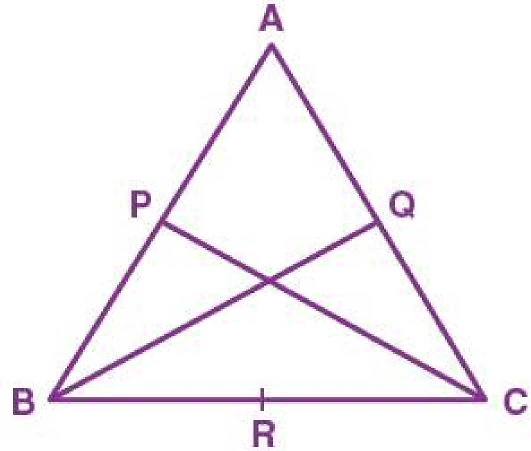
16. In triangle ABC; AB = AC. P, Q and R are mid-points of sides AB, AC and BC respectively. Prove that:
(i) PR = QR (ii) BQ = CP
Solution:

(i)



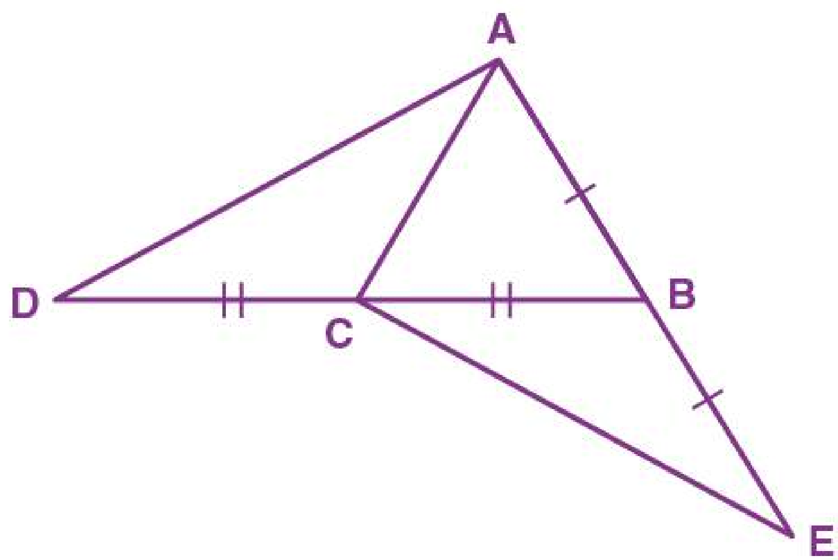
In △ABC, we have
AB = AC
½ AB = ½ AC
AP = AQ ... (i) [Since P and Q are mid - points]
In △BCA, we have
PR = ½ AC [PR is line joining the mid - points of AB and BC]
PR = AQ … (ii)
In △CAB, we have
QR = ½ AB [QR is line joining the mid - points of AC and BC]
QR = AP …(iii)
From (i), (ii) and (iii), we get
PR = QR

(ii)



Given, AB = AC
⇒ ∠B = ∠C
Also,
½ AB = ½ AC
BP = CQ [P and Q are mid – points of AB and AC]
Now, in △BPC and △CQB, we have
BP = CQ
∠B = ∠C
BC = BC (Common)
Therefore, ΔBPC ≅ CQB by SAS congruence criterion
∴ BP = CP by CPCT

17. From the following figure, prove that:
(i) ∠ACD = ∠CBE
(ii) AD = CE



Solution:

(i) In $\triangle ACB$, we have

$AC = AC$ [Given]

$\therefore \angle ABC = \angle ACB$ …(i) [Angles opposite to equal sides are equal]

$\angle ACD + \angle ACB = 180^0$ … (ii) [Since, DCB is a straight line]

$\angle ABC + \angle CBE = 180^0$ …(iii) [Since, ABE is a straight line]

Equating (ii) and (iii), we get

$\angle ACD + \angle ACB = \angle ABC + \angle CBE$

$\angle ACD + \angle ACB = \angle ACB + \angle CBE$ [From (i)]

$\Rightarrow \angle ACD = \angle CBE$

(ii) In $\triangle ACD$ and $\triangle CBE$, we have

$DC = CB$ [Given]

$AC = BE$ [Given]

$\angle ACD = \angle CBE$ [Proved above]

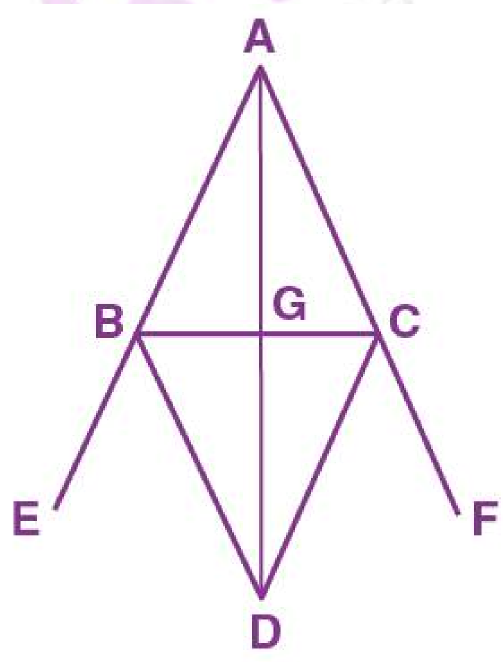
$\therefore \triangle ACD \cong \triangle CBE$ by SAS congruence criterion

Hence, by CPCT

$AD = CE$

18. Equal sides AB and AC of an isosceles triangle ABC are produced. The bisectors of the exterior angle so formed meet at D. Prove that AD bisects angle A.

Solution:



AB is produced to E and AC is produced to F. BD is bisector of angle CBE and CD is bisector of angle BCF. BD and CD meet at D.

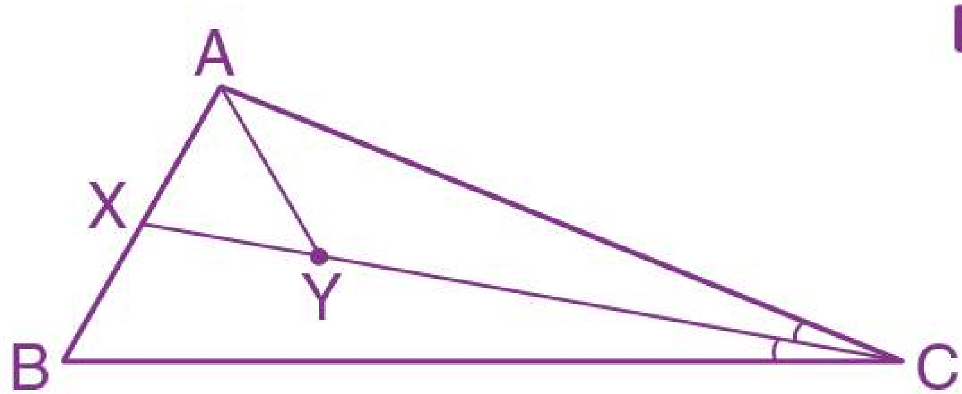
In $\triangle ABC$, we have

$AB = AC$ [Given]

$\therefore \angle C = \angle B$ [angles opposite to equal sides are equal]

$\angle CBE = 180^0 - \angle B$ [ABE is a straight line]
$\angle CBD = (180^o - \angle B)/ 2$ [BD is bisector of $\angle CBE$]
$\angle CBD = 90^o - \angle B/ 2$ …(i)
Similarly,
$\angle BCF = 180^0 - \angle C$ [ACF is a straight line]
$\angle BCD = (180^o - \angle C)/ 2$ [CD is bisector of $\angle BCF$]
$\angle BCD = 90^o - \angle C/2$ …(ii)
Now,
$\angle CBD = 90^o - \angle C/2$ [$\because \angle B = \angle C$]
$\angle CBD = \angle BCD$
In $\triangle BCD$, we have
$\angle CBD = \angle BCD$
$\therefore$ BD = CD
In $\triangle ABD$ and $\triangle ACD$, we have
AB = AC [Given]
AD = AD [Common]
BD = CD [Proved]
$\therefore \triangle ABD \cong \triangle ACD$ by SSS congruence criterion
So, $\angle BAD = \angle CAD$ [By CPCT]
Therefore, AD bisects $\angle A$.

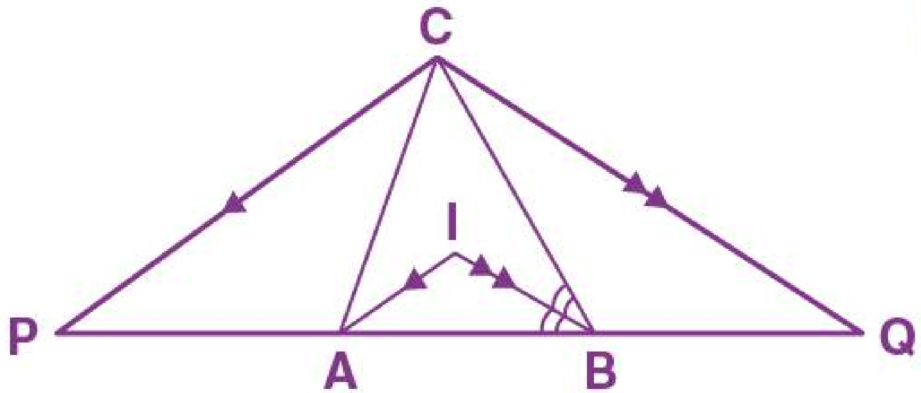
19. ABC is a triangle. The bisector of the angle BCA meets AB in X. A point Y lies on CX such that AX = AY. Prove that $\angle CAY = \angle ABC$.
Solution:



In $\triangle ABC$, we have
CX is the angle bisector of $\angle C$
So, $\angle ACY = \angle BCX$ …(i)
In $\angle AXY$, we have
AX = AY [Given]
$\angle AXY = \angle AYX$ ...(ii) [Angles opposite to equal sides are equal]
Now, $\angle XYC = \angle AXB = 180°$ [Straight line angle]
$\angle AYX + \angle AYC = \angle AXY + \angle BXY$

∠AYC = ∠BXY… (iii)   [From (ii)]
In △AYC and △BXC, we have
∠AYC + ∠ACY + ∠CAY = ∠BXC + ∠BCX + ∠XBC = 180°
∠CAY = ∠XBC   [From (i) and (iii)]
Thus, ∠CAY = ∠ABC

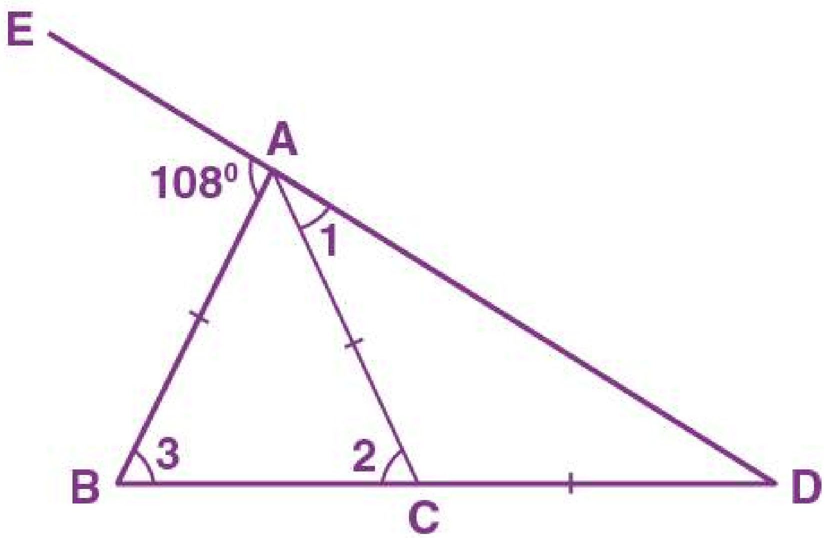
20. In the following figure; IA and IB are bisectors of angles CAB and CBA respectively. CP is parallel to IA and CQ is parallel to IB.



Prove that:
PQ = The perimeter of the △ABC.
Solution:

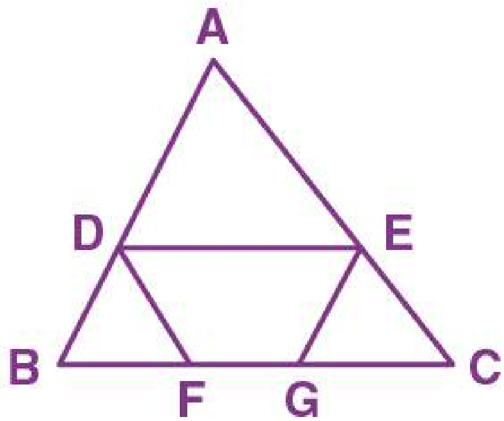
Since IA || CP and CA is a transversal
We have, ∠CAI = ∠PCA   [Alternate angles]
Also, IA || CP and AP is a transversal
We have, ∠IAB = ∠APC   [Corresponding angles]
But ∴ ∠CAI = ∠IAB   [Given]
∴ ∠PCA = ∠APC
AC = AP
Similarly, BC = BQ
Now,
PQ = AP + AB + BQ
= AC + AB + BC
= Perimeter of △ABC

21. Sides AB and AC of a triangle ABC are equal. BC is produced through C upto a point D such that AC = CD. D and A are joined and produced upto point E. If angle BAE = 108º; find angle ADB.
Solution:



In $\triangle ABD$, we have
$\angle BAE = \angle 3 + \angle ADB$
$108^0 = \angle 3 + \angle ADB$
But, AB = AC
$\angle 3 = \angle 2$
$108^0 = \angle 2 + \angle ADB$ … (i)
Now,
In $\triangle ACD$, we have
$\angle 2 = \angle 1 + \angle ADB$
But, AC = CD
$\angle 1 = \angle ADB$
$\angle 2 = \angle ADB + \angle ADB$
$\angle 2 = 2\angle ADB$
Putting this value in (i), we get
$108^0 = 2\angle ADB + \angle ADB$
$3\angle ADB = 108^0$
$\therefore \angle ADB = 36^0$

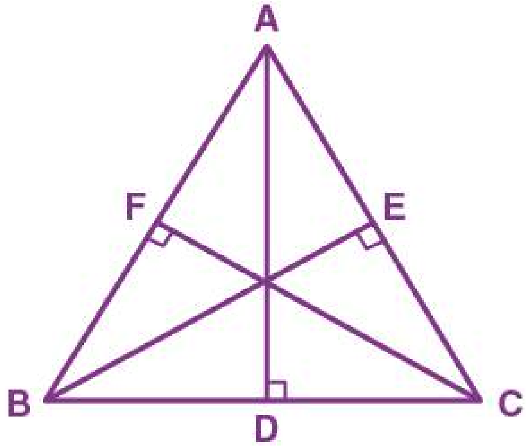
22. The given figure shows an equilateral triangle ABC with each side 15 cm. Also, DE || BC, DF || AC and EG || AB. If DE + DF + EG = 20 cm, find FG.



Solution:

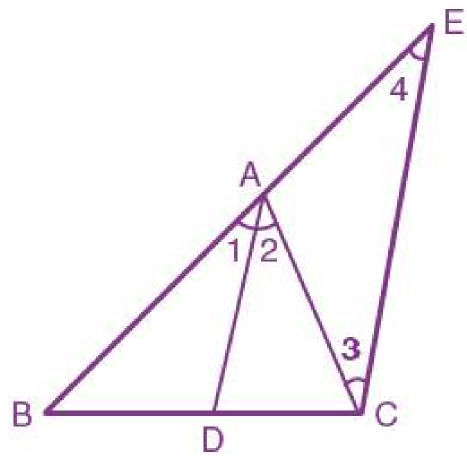
Given, ABC is an equilateral triangle.
AB = BC = AC = 15 cm
$\angle A = \angle B = \angle C = 60^o$
In $\triangle ADE$, we have DE ll BC
$\angle AED = 60^o$ [$\because \angle ACB = 60^o$]
$\angle ADE = 60^o$ [$\because \angle ABC = 60^o$]
$\angle DAE = 180^o - (60^o + 60^o)$
$= 60^o$
Thus, $\triangle ADE$ is an equilateral triangle
Similarly, $\triangle BDF$ and $\triangle GEC$ are equilateral triangles
Now,
Let AD = x, AE = x and DE = x [$\because \triangle ADE$ is an equilateral triangle]
Let BD = y, FD = y and FB = y [$\because \triangle BDF$ is an equilateral triangle]
Let EC = z, GC = z and GE = z [$\because \triangle GEC$ is an equilateral triangle]
Now, AD + DB = 15
x + y = 15 … (i)
AE + EC = 15
x + z = 15 … (ii)
Given, DE + DF + EG = 20
x + y + z = 20
15 + z = 20 [From (i)]
z = 5
From (ii), we get, x = 10
$\therefore$ y = 5
Also, BC = 15
BF + FG + GC = 15
y + FG + z = 15
$\therefore$ FG = 5

23. If all the three altitudes of a triangle are equal, the triangle is equilateral. Prove it.
Solution:



In right ∆BEC and ∆BFC, we have
BE = CF [Given]
BC = BC [Common]
∠BEC = ∠BFC [Each = 90^0]
∴ ∆BEC ≅ ∆BFC by RHS congruence criterion
By CPCT, we get
∠B = ∠C
Similarly,
∠A = ∠B
Hence, ∠A = ∠B = ∠C
⇒ AB = BC = AC
Therefore, ABC is an equilateral triangle.

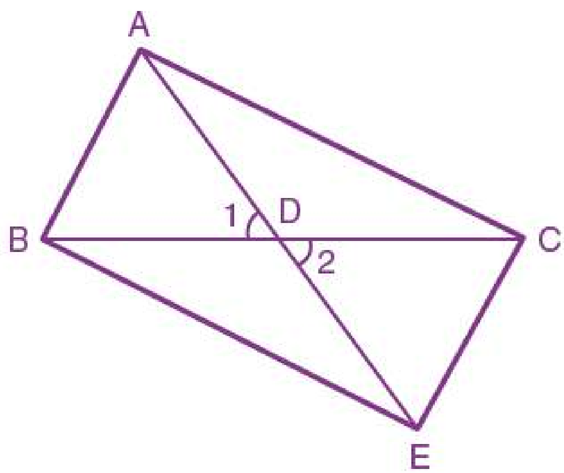
24. In a ∆ABC, the internal bisector of angle A meets opposite side BC at point D. Through vertex C, line CE is drawn parallel to DA which meets BA produced at point E. Show that ∆ACE is isosceles.
Solution:



Given, DA || CE
∠1 = ∠4 … (i) [Corresponding angles]
∠2 = ∠3 … (ii) [Alternate angles]

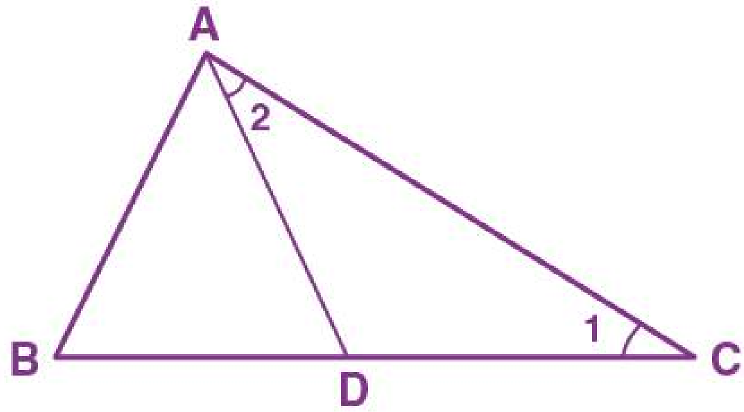
But ∠1 = ∠2 …(iii) [As AD is the bisector of ∠A]
From (i), (ii) and (iii), we get
∠3 = ∠4
⇒AC = AE
Therefore, ∆ACE is an isosceles triangle.

25. In triangle ABC, bisector of angle BAC meets opposite side BC at point D. If BD = CD, prove that ∆ABC is isosceles.
Solution:



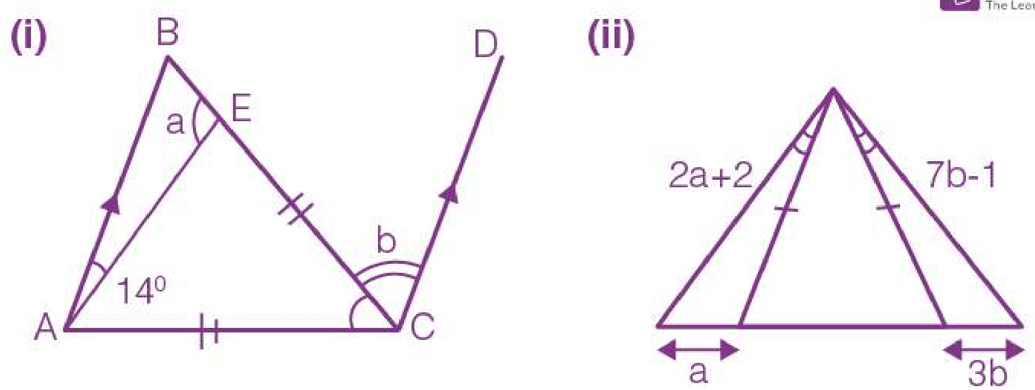
Let's produce AD up to E such that AD = DE.
In ∆ABD and ∆EDC, we have
AD = DE [By construction]
BD = CD [Given]
∠1 = ∠2 [Vertically opposite angles]
∴ ∆ABD ≅ ∆EDC by SAS congruence criterion
So, by CPCT,
AB = CE …(i)
And, ∠BAD = ∠CED
But, ∠BAD = ∠CAD [AD is bisector of ∠BAC]
∴ ∠CED = ∠CAD
AC = CE …(ii)
From (i) and (ii), we get
AB = AC
Hence, ABC is an isosceles triangle.

26. In ∆ABC, D is point on BC such that AB = AD = BD = DC. Show that: ∠ADC: ∠C = 4: 1.
Solution:



As, AB = AD = BD, we have
ΔABD is an equilateral triangle.
$\therefore \angle ADB = 60^o$
Now,
$\angle ADC = 180^o - \angle ADB$
$= 180^o - 60^o$
$= 120^o$
Again in ΔADC, we have
AD = DC
$\therefore \angle 1 = \angle 2$
But,
$\angle 1 + \angle 2 + \angle ADC = 180^o$ [By angle sum property]
$2\angle 1 + 120^o = 180^o$
$2\angle 1 = 60^o$
$\angle 1 = 30^o$
$\angle C = 30^o$
$\Rightarrow \angle ADC : \angle C = 120^o : 30^o$
Therefore, $\angle ADC : \angle C = 4 : 1$

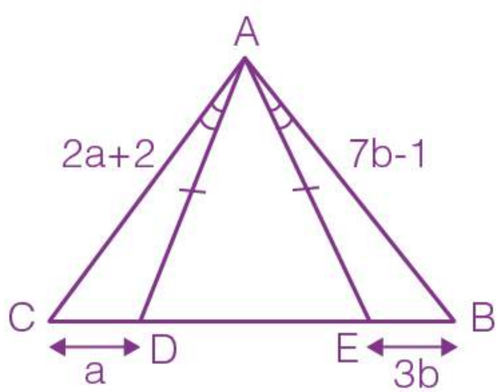
27. Using the information given in each of the following figures, find the values of a and b. [Given: CE = AC]



Solution:

(i) In $\triangle CAE$, we have
$\angle CAE = \angle AEC$ [$\because$ CE = AC]
$= (180^o - 60^o)/2$
$= 56^o$
In $\angle BEA$, we have
$a = 180^o - 56^o = 124^o$
In $\triangle ABE$, we have
$\angle ABE = 180^o - (124^o + 14^o)$
$= 180^o - 138^o$
$= 42^o$

(ii)



In $\triangle AEB$ and $\triangle CAD$, we have
$\angle EAB = \angle CAD$ [Given]
$\angle ADC = \angle AEB$ [$\because \angle ADE = \angle AED$, since, AE = AD
$180^o - \angle ADE = 180^o - \angle AED$
$\angle ADC = \angle AEB$]
AE = AD [Given]
$\therefore \triangle AEB \cong \triangle CAD$ by ASA congruence criterion
Thus, AC = AB by CPCT
$2a + 2 = 7b - 1$
$2a - 7b = - 3$ … (i)
CD = EB
$a = 3b$ … (ii)
Solving (i) and (ii) we get,
$a = 9$ and $b = 3$