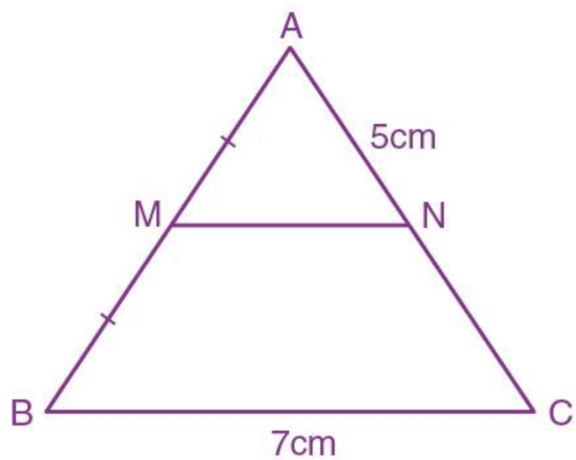
Exercise 12(A)

1. In triangle ABC, M is mid-point of AB and a straight line through M and parallel to BC cuts AC in N. Find the lengths of AN and MN if BC = 7 cm and AC = 5 cm.
Solution:

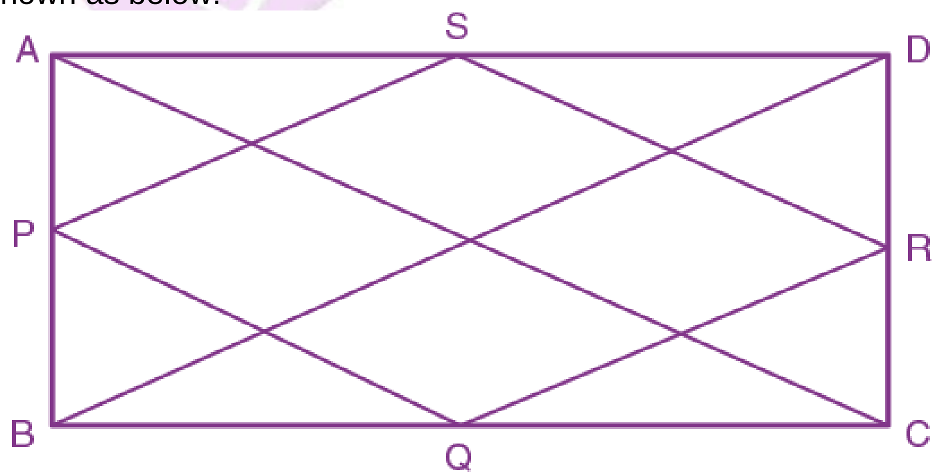
The triangle is shown as below:



Since M is the midpoint of AB and MN || BC
Then, by mid-point theorem N is the midpoint of AC.
Therefore,
MN = ½ BC = ½ × 7 = 3.5cm
And, AN = ½ AC = ½ × 5 = 2.5cm

2. Prove that the figure obtained by joining the mid-points of the adjacent sides of a rectangle is a rhombus.
Solution:

The figure is shown as below:

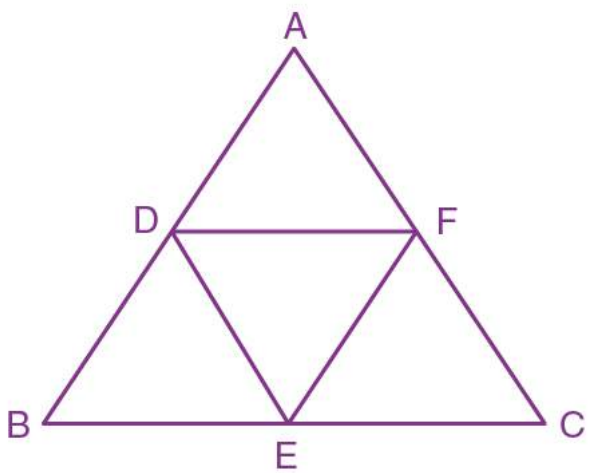


Let ABCD be a rectangle where P, Q, R, S are the midpoint of AB, BC, CD, DA. Then, we need to show that PQRS is a rhombus.

Let's draw two diagonals BD and AC as shown in figure
And, BD = AC [Since diagonals of rectangle are equal]
Proof:
From ∆ABD and ∆BCD, we have
PS = ½ BD = QR and PS || BD || QR
2PS = 2QR = BD and PS || QR … (1)
Similarly,
2PQ = 2SR = AC and PQ || SR … (2)
From (1) and (2) we get
PQ = QR = RS = PS
Therefore, PQRS is a rhombus.
- Hence proved

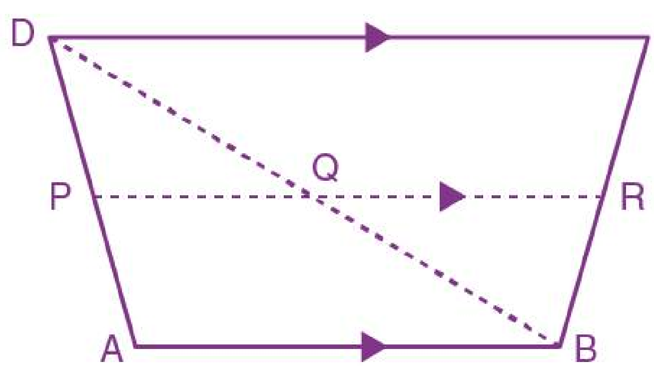
3. D, E and F are the mid-points of the sides AB, BC and CA of an isosceles ∆ABC in which AB = BC. Prove that ∆DEF is also isosceles.
Solution:

The figure is shown as below:



Given, ABC is an isosceles triangle and AB = AC
Since D, E and F are midpoints of AB, BC and CA respectively
Therefore, by mid-point theorem
2DE = AC and 2EF = AB
⇒ DE = EF
Therefore, DEF is an isosceles triangle where DE = EF.
- Hence proved

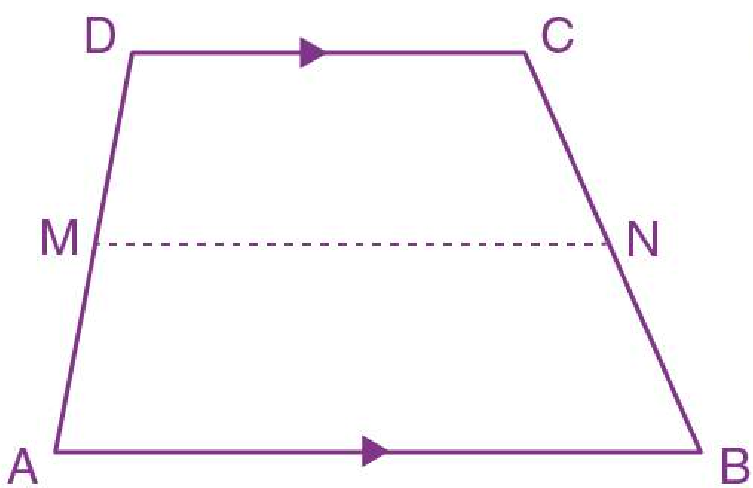
4. The following figure shows a trapezium ABCD in which AB // DC. P is the mid-point of AD and PR // AB. Prove that:
PR = ½ (AB + CD)



Solution:

Given,
In ∆ABD, P is the midpoint of AD and PR || AB
Therefore, Q is the midpoint of BD [By mid-point theorem]
Similarly, R is the midpoint of BC as PR || CD || AB
Now, from ∆ABD
2PQ = AB … (1)
And, from ∆BCD
2QR = CD … (2)
Adding (1) and (2), we get
2(PQ + QR) = AB + CD
2PR = AB + CD
PR = ½ (AB + CD)
- Hence proved.

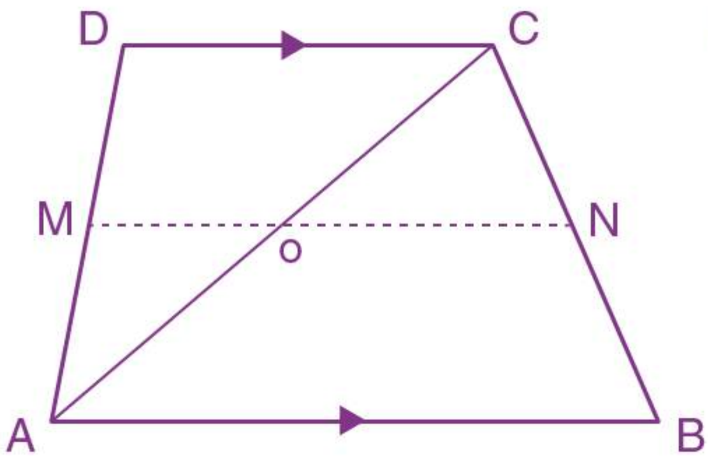
5. The figure, given below, shows a trapezium ABCD. M and N are the mid-point of the non-parallel sides AD and BC respectively. Find:



(i) MN, if AB = 11 cm and DC = 8 cm.
(ii) AB, if DC = 20 cm and MN = 27 cm.

(iii) DC, if MN = 15 cm and AB = 23 cm.
Solution:

Let's draw a diagonal AC as shown in the figure below,



(i) Given, AB = 11cm and CD = 8cm
From ΔABC, we have
ON = ½ AB = ½ × 11 = 5.5cm
From ΔACD, we have
OM = ½ CD = ½ × 8 = 4cm
Hence, MN = OM +ON
= (4 + 5.5)
= 9.5cm

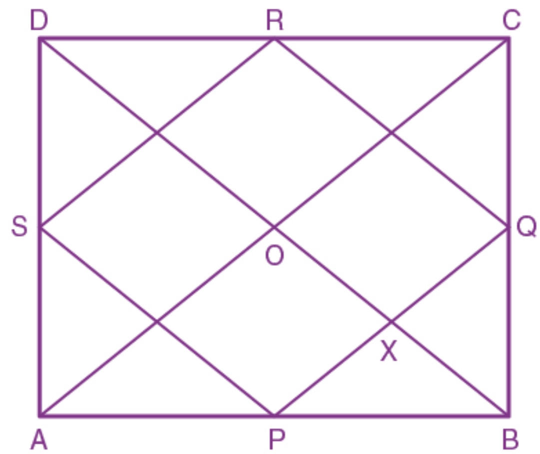
(ii) Given, CD = 20cm and MN = 27cm
From ΔACD, we have
OM = ½ CD = ½ × 20 = 10cm
Therefore, ON = 27 – 10 = 17cm
Then from ΔABC, we have
AB = 2 ON
= 2 × 17
= 34cm

(iii) Given, AB = 23cm and MN = 15cm
From ΔABC, we have
ON = ½ AB = ½ × 23 = 11.5cm
Therefore, OM = 15 - 11.5 = 3.5cm
Then from ΔACD, we have
CD = 2 OM
= 2 × 3.5
= 7cm

6. The diagonals of a quadrilateral intersect at right angles. Prove that the figure obtained by joining the mid-points of the adjacent sides of the quadrilateral is rectangle.

Solution:

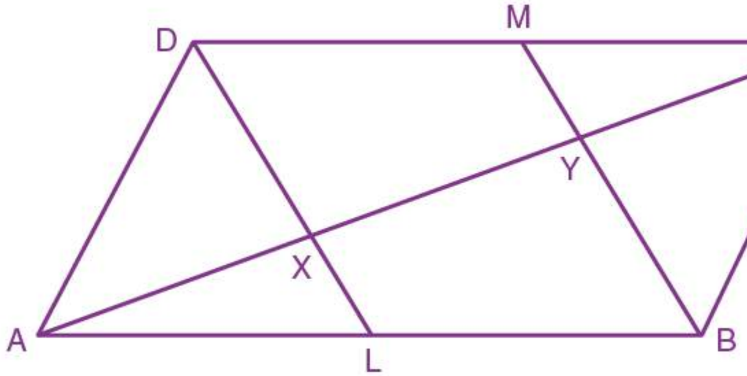
The figure is shown as below:



Let ABCD be a quadrilateral where P, Q, R, S are the midpoints of AB, BC, CD, DA.
Diagonals AC and BD intersects at right angles at point O.
Required to prove: PQRS is a rectangle
Proof:
From $\triangle$ABC and $\triangle$ADC, we have
2PQ = AC and PQ || AC … (1)
2RS = AC and RS || AC … (2)
From (1) and (2) we get,
PQ = RS and PQ || RS
Similarly,
PS = RQ and PS||RQ
Therefore, PQRS is a parallelogram.
Now as PQ||AC, we have
$\angle AOD = \angle PXO = 90^o$ [Corresponding angles]
Again, as BD||RQ, we have
$\angle PXO = \angle RQX = 90^o$ [Corresponding angle]
Similarly,
$\angle QRS = \angle RSP = \angle SPQ = 90^o$
Therefore, PQRS is a rectangle.
- Hence proved

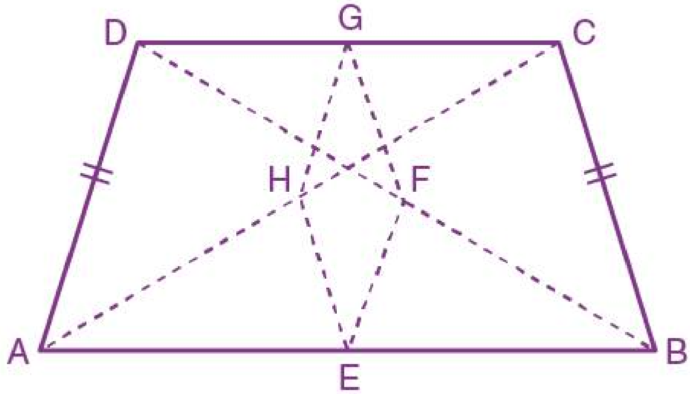
7. L and M are the mid-point of sides AB and DC respectively of parallelogram ABCD. Prove that segments DL and BM trisect diagonal AC.
Solution:

The required figure is shown as below:



We have,
BL = DM and BL || DM and BLMD is a parallelogram
⇒ BM || DL
Now, in ∆ABY, we have
L is the midpoint of AB and XL || BY,
Therefore, x is the midpoint of AY
⇒ AX = XY ... (1)
Similarly for triangle CDX
⇒ CY = XY … (2)
From (1) and (2), we get
AX = XY = CY and AC = AX + XY + CY
Thus, segments DL and BM trisect the diagonal AC of parallelogram ABCD
- Hence proved

8. ABCD is a quadrilateral in which AD = BC. E, F, G and H are the mid-points of AB, BD, CD and Ac respectively. Prove that EFGH is a rhombus.

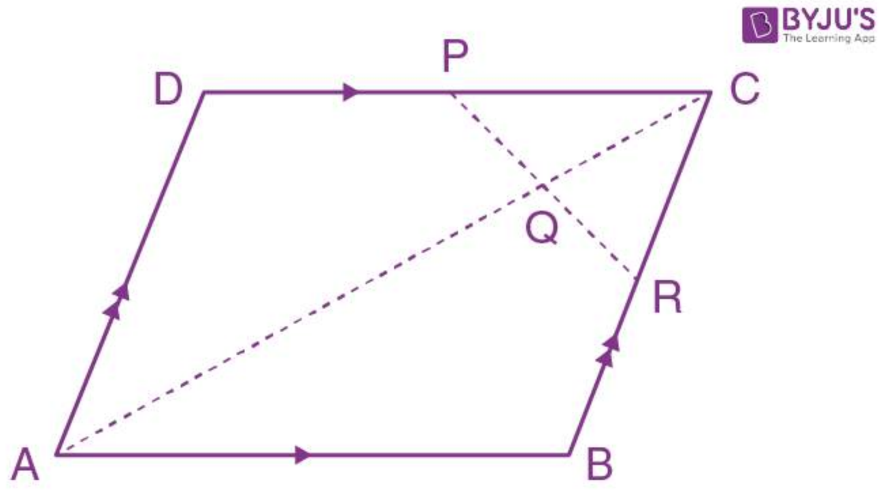


Solution:

Given, AD = BC ... (1)
From the figure,
In ∆ADC and ∆ABD, we have

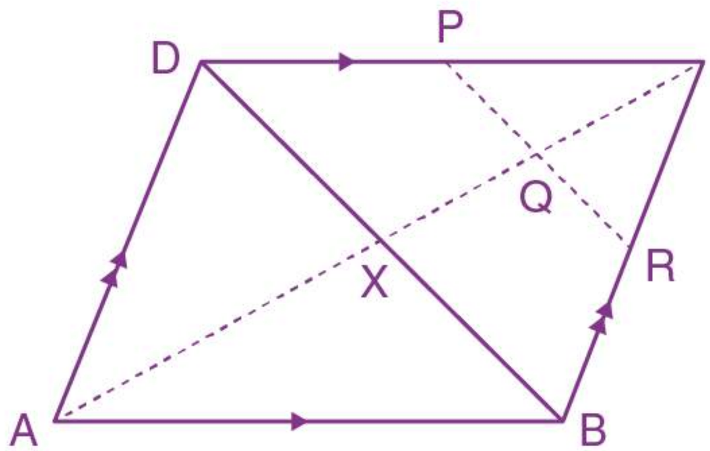
2GH = AD and 2EF = AD,
⇒ 2GH = 2EF = AD ... (2)
Now, in ∆BCD and ∆ABC, we have
2GF = BC and 2EH = BC
⇒ 2GF = 2EH = BC ... (3)
From (1), (2), (3) we get
2GH = 2EF = 2GF = 2EH
⇒ GH = EF = GF = EH
Therefore, EFGH is a rhombus.
- Hence proved

9. A parallelogram ABCD has P the mid-point of DC and Q a point of AC such that CQ = ¼ AC. PQ produced meets BC at R.



Prove that:
(i) R is the midpoint of BC
(ii) PR = ½ DB

Solution:
Let's draw the diagonal BD as shown below.



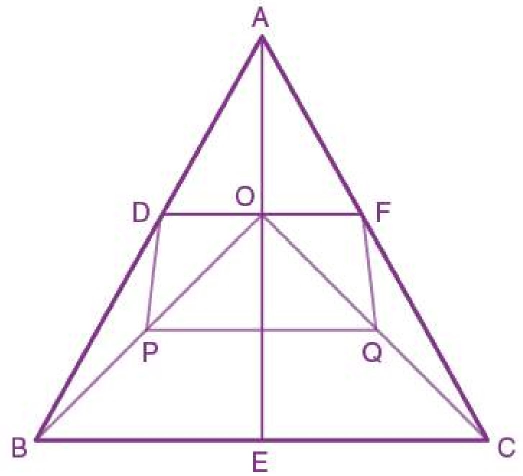
The diagonal AC and BD cuts at point X.
We know that the diagonals of a parallelogram bisect each other.
Therefore, AX = CX and BX = DX
Given,
CQ = ¼ AC
CQ = ¼ × 2CX
CQ = ½ CX
Therefore, Q is the midpoint of CX.

(i) For △CDX, PQ || DX or PR || BD
And in △CBX, Q is the midpoint of CX and QR||BX
Therefore, R is the midpoint of BC

(ii) In △BCD,
As P and R are the midpoint of CD and B, we have
Thus, PR = ½ DB

10. D, E and F are the mid-points of the sides AB, BC and CA respectively of △ABC. AE meets DF at O. P and Q are the mid-points of OB and OC respectively. Prove that DPQF is a parallelogram.
Solution:

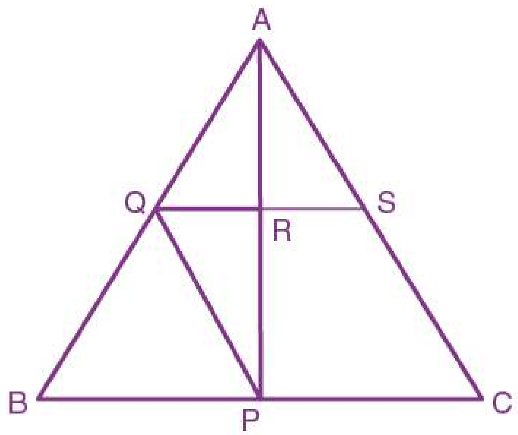
The required figure is shown as below:



In $\triangle ABC$ and $\triangle OBC$, we have
2DE = BC and 2PQ = BC,
Therefore, DE = PQ ... (1)
In $\triangle ABO$ and $\triangle ACO$, we have
2PD = AO and 2FQ = AO,
Therefore, PD = FQ ... (2)
From (1) and (2), we get that PQFD is a parallelogram.
- Hence proved.

11. In triangle ABC, P is the mid-point of side BC. A line through P and parallel to CA meets AB at point Q; and a line through Q and parallel to BC meets median AP at point R. Prove that:
(i) AP = 2AR
(ii) BC = 4QR
Solution:

The required figure is shown as below:

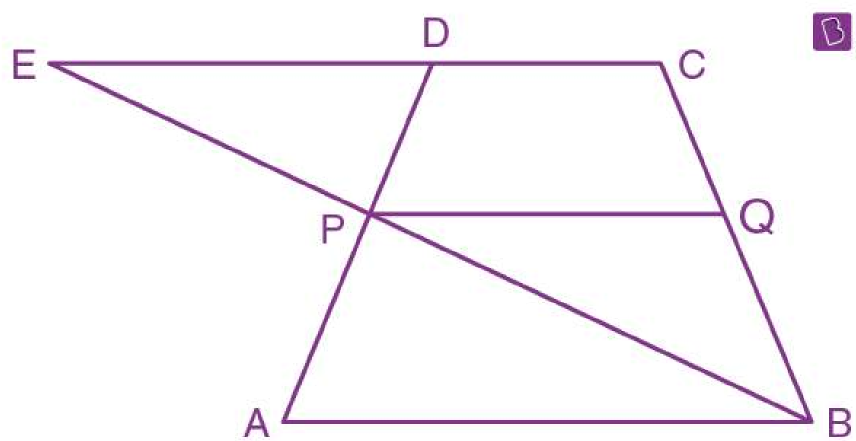


It's seen that P is the midpoint of BC, PQ || AC and QR || BC
Therefore, Q is the midpoint of AB and R is the midpoint of AP
(i) Thus, AP = 2AR

(ii) Let's extend QR such that it cuts AC at S as shown in the figure.
Now, in ∆PQR and ∆ARS, we have
∠PQR = ∠ARS (Opposite angles)
PR = AR
PQ = AS (Since, PQ = AS = ½ AC)
Thus, ∆PQR ≅ ∆ARS by SAS congruence criterion
Therefore, by CPCT
QR = RS
Now,
BC = 2QS
BC = 2 × 2QR
BC = 4QR
- Hence proved

12. In trapezium ABCD, AB is parallel to DC; P and Q are the mid-points of AD and BC respectively. BP produced meets CD produced at point E. Prove that:
(i) Point P bisects BE,
(ii) PQ is parallel to AB.
Solution:

The required figure is shown as below:



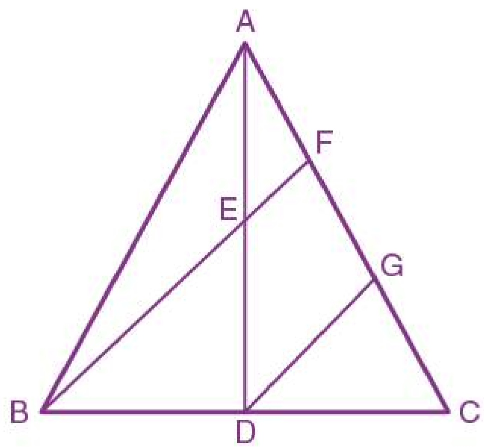
(i) In ∆PED and ∆ABP, we have
PD = AP [Since, P is the mid-point of AD]
∠DPE = ∠APB [Opposite angles]
∠PED = ∠PBA [Alternate angles as AB || CE]
∴ ∆PED ≅ ∆ABP by ASA congruence postulate
Thus, by CPCT
EP = BP

(ii) For ∆ECB, we have PQ||CE
Also, CE||AB
Therefore, PQ||AB

- Hence proved

13. In a triangle ABC, AD is a median and E is mid-point of median AD. A line through B and E meets AC at point F. Prove that AC = 3AF.
Solution:

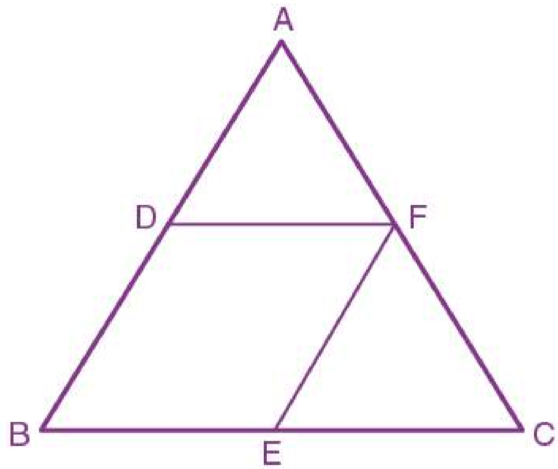
The required figure is shown as below:



Let's draw a line DG || BF
Now,
In ∆ADG, we have
DG || BF and E is the midpoint of AD
Therefore, F is the midpoint of AG
⇒ AF = GF ... (1)
And, in ∆BCF, we have
DG || BF and D is the midpoint of BC
Therefore, G is the midpoint of CF
⇒ GF=CF … (2)
AC = AF + GF + CF [From figure]
AC = 3AF [From (1) and (2)]
- Hence proved.

14. D and F are mid-points of sides AB and AC of a triangle ABC. A line through F and parallel to AB meets BC at point E.
(i) Prove that BDFE is parallelogram
(ii) Find AB, if EF = 4.8 cm.
Solution:

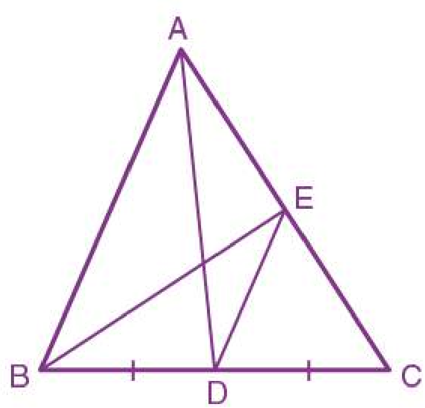
The required figure is shown as below:



(i) Since F is the midpoint and EF || AB
Therefore, E is the midpoint of BC
So, BE = ½ BC and EF = ½ AB … (1)
And,
Since D and F are the midpoint of AB and AC
Therefore, DE||BC
So, DF = ½ BC and DB = ½ AB … (2)
From (1) and (2), we get
BE = DF and BD = EF
Hence, BDEF is a parallelogram.

(ii) Now, AB = 2EF
= 2 × 4.8
= 9.6cm

15. In triangle ABC, AD is the median and DE, drawn parallel to side BA, meets AC at point E. Show that BE is also a median.
Solution:



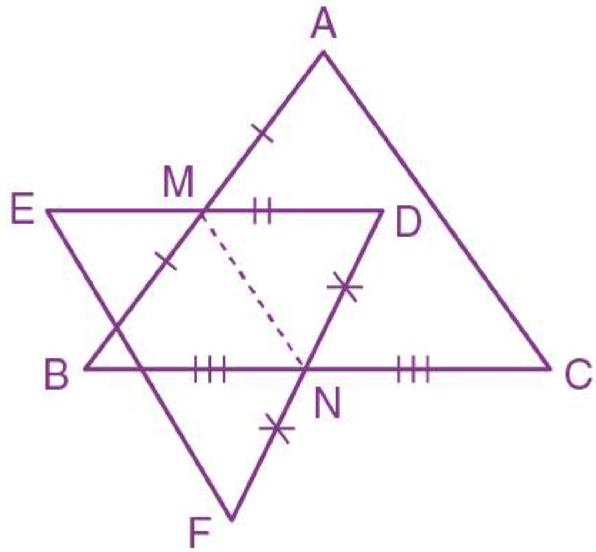
In $\triangle$ABC, we have
AD is the median of BC

D is the mid-point of BC
Given that DE || BA
So, by the converse of the mid-point theorem,
DE bisects AC
⇒ E is the mid-Point of AC
And, BE is the median of AC
Hence, BE is also a median.

16. In △ABC, E is mid-point of the median AD and BE produced meets side AC at point Q. Show that BE: EQ = 3:1.
Solution:

Construction: Draw DY || BQ
In △BCQ and △DCY, we have
∠BCQ = DCY [Common]
∠BQC = ∠DYC [Corresponding angles]
So, △BCQ ~ △DCY by AA similarity criterion
Thus,
BQ/DY = BC/DC = CQ/CY [Corresponding sides are proportional]
BQ/DY = 2 … (i)
Similarly, △AEQ ~ △ADY
EQ/DY = AE/ED = ½ [E is the mid-point of AD]
⇒ EQ/DY = ½ … (ii)
On dividing (i) by(ii), we get
BQ/EQ = 4
BQ = 4 EQ
BE + EQ = 4EQ
BE = 3EQ
Therefore, BE/EQ = 3/1

17. In the given figure, M is mid-point of AB and DE, whereas N is mid-point of BC and DF. Show that: EF = AC.

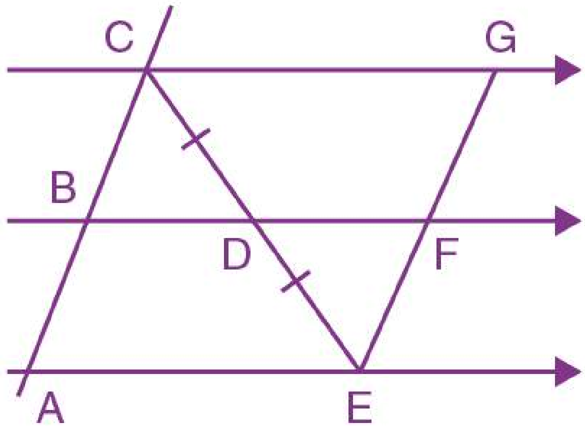


Solution:

In △EDF, we have
M is the mid-point of AB and N is the mid-point of DE
So, MN = ½ EF (By mid-point theorem)
EF = 2MN … (i)
In △ABC, we have
M is the mid-point of AB and N is the mid-point of BC
MN = ½ AC (By mid-point theorem)
AC = 2MN … (ii)
From (i) and (ii), we get
EF = AC

Exercise 12(B)

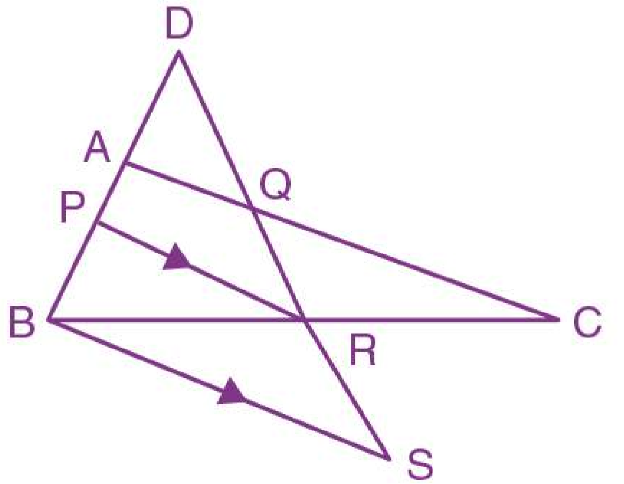
1. Use the following figure to find:
(i) BC, if AB = 7.2 cm.
(ii) GE, if FE = 4 cm.
(iii) AE, if BD = 4.1 cm
(iv) DF, if CG = 11 cm.



Solution:

According to equal intercept theorem, as CD = DE
⇒ AB = BC and EF = GF
Thus,
(i) BC = AB = 7.2cm
(ii) GE = EF + GF
= 2EF
= 2 × 4
= 8 cm
Since B, D and F are the midpoints and AE || BF || CG
Therefore, AE = 2BD and CG = 2DF
(iii) AE = 2BD
= 2 × 4.1
= 8.2 cm
(iv) DF = ½ CG
= ½ × 11
= 5.5 cm

2. In the figure, give below, 2AD = AB, P is mid-point of AB, Q is mid-point of DR and PR // BS. Prove that:
(i) AQ // BS
(ii) DS = 3 Rs.



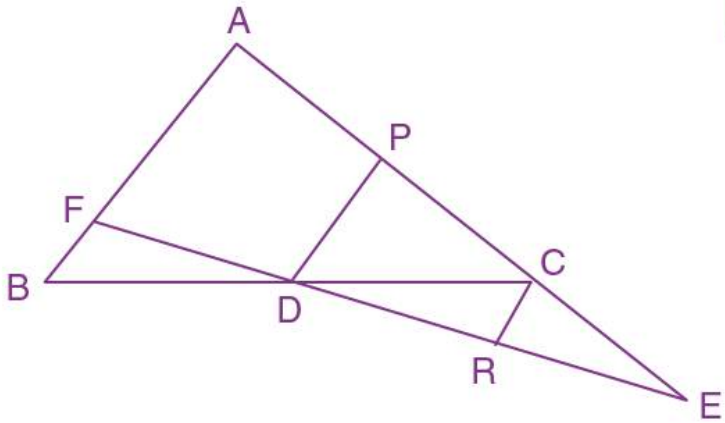
Solution:

Given, AD = AP = PB as 2AD = AB and P is the midpoint of AB
(i) In ∆DPR, we have
A and Q are the midpoints of DP and DR
Therefore, AQ || PR
Now, as PR || BS
∴ AQ || BS

(ii) In ∆ABC, P is the midpoint and PR || BS
Therefore, R is the midpoint of BC
Now, in ∆BRS and ∆QRC, we have
∠BRS = ∠QRC
BR = RC
∠RBS = ∠RCQ
∴ ∆BRS ≅ ∆QRC by SAS Congruence criterion
Hence, by CPCT
QR = RS
Thus, DS = DQ + QR + RS
= QR + QR + RS
= 3RS

3. The side AC of a triangle ABC is produced to point E so that CE = ½ AC. D is the mid-point of BC and ED produced meets AB at F. Lines through D and C are drawn parallel to AB which meet AC at point P and EF at point R respectively. Prove that:
(i) 3DF = EF (ii) 4CR = AB.
Solution:

Let’s consider the figure as below:

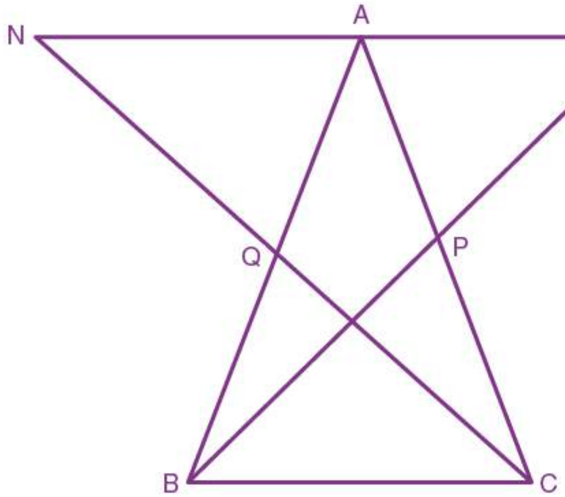


Here D is the midpoint of *BC* and *DP* is parallel to *AB*,
Therefore, P is the midpoint of AC and PD = ½ AB
(i) Again, in ΔAEF we have AE || PD || CR and AP = 1/3 AE
Therefore, DF = 1/3 EF
⇒ 3DF = EF
- Hence proved

(ii) In ΔPED, we have PD || CR and C is the midpoint of PE
So, CR = ½ PD
Now,
PD = ½ AB
½ PD = ¼ AB
CR = ¼ AB
4CR = AB
- Hence proved

4. In triangle ABC, the medians BP and CQ are produced upto points M and N respectively such that BP = PM and CQ = QN. Prove that:
(i) M, A and N are collinear.
(ii) A is the mid-point of MN.
Solution:

The figure is shown as below:

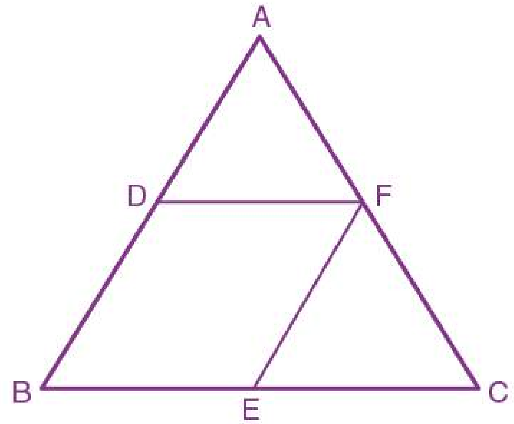


(i) In ΔBPC and ΔMPA, we have
$\angle$BPC = $\angle$APN [Vertically opposite angle]
BP = MP
PC = PA
$\therefore$ ΔBPC $\cong$ ΔMPA by SAS congruence postulate
Thus, by CPCT
$\angle$PCB = $\angle$PAM … (1)
And, BC = AM … (2)
Similarly,
Considering ΔCQB and ΔNQA, we have
$\angle$QBC = $\angle$QAN … (3)
And, BC = AN … (4)
Now, by angle sum property of ΔABC
$\angle ABC + \angle ACB + \angle BAC = 180^o$
$\Rightarrow \angle QBC + \angle PCB + \angle BAC = 180^o$
$\angle QAN + \angle PAM + \angle BAC = 180^o$ [From (1) and (3)]
Therefore, it's a straight angle and M, A, N must be collinear.

(ii) Now, from (2) and (4) we have
AM = AN
Hence, A is the midpoint of MN.

5. In triangle ABC, angle B is obtuse. D and E are mid-points of sides AB and BC respectively and F is a point on side AC such that EF is parallel to AB. Show that BEFD is a parallelogram.
Solution:

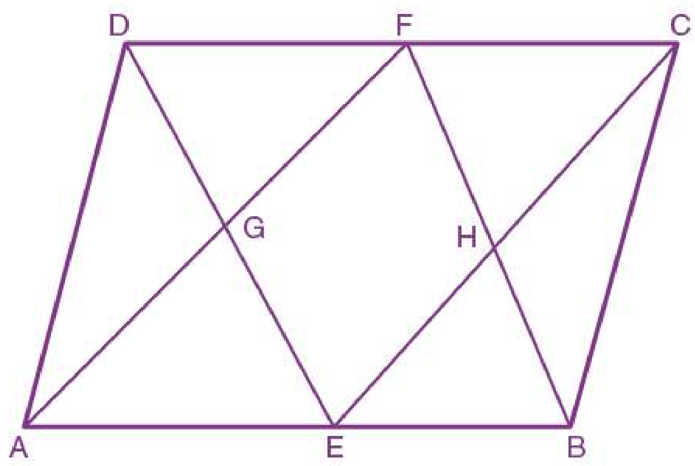
The figure is shown as below:



We have,
EF||AB and E is the midpoint of BC
Therefore, F is the midpoint of AC
And,
EF = BD as D is the midpoint of AB
Now, as BE || DF
BE = DF as E is the midpoint of BC.
Therefore, BEFD is a parallelogram.

6. In parallelogram ABCD, E and F are mid-points of the sides AB and CD respectively. The line segments AF and BF meet the line segments ED and EC at points G and H respectively. Prove that:
(i) Triangles HEB and FHC are congruent
(ii) GEHF is a parallelogram.
Solution:

The figure is shown as below:



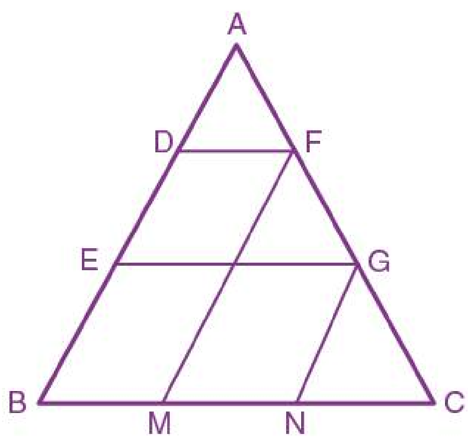
(i) In ∆HEB and ∆HCF, we have
BE = FC [Given]
∠EHB = ∠FHC [Vertically opposite angles]

∠HBE = ∠HFC [Alternate angles]
∴ ∆HEB ≅ ∆HCF by ASA congruence criterion
∴ EH = CH, BH = FH

(ii) Similarly, AG = GF and EG = DG ... (1)
In ∆ECD, we have
F and H are the midpoints of CD and EC respectively
Therefore, HF || DE and HF = ½ DE ... (2)
From (1) and (2), we get
HF = EG and HF || EG
Similarly, we can show
EH = GF and EH || GF
Therefore, GEHF is a parallelogram.

7. In triangle ABC, D and E are points on side AB such that AD = DE = EB. Through D and E, lines are drawn parallel to BC which meet side AC at points F and G respectively. Through F and G, lines are drawn parallel to AB which meet side BC at points M and N respectively. Prove that: BM = MN = NC.
Solution:

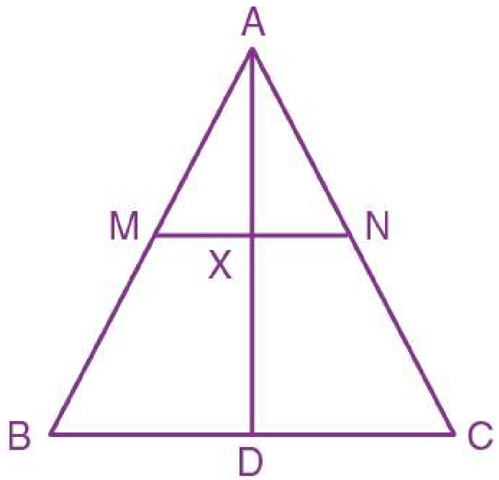
The figure is shown as below:



In ∆AEG, we have
D is the midpoint of AE and DF || EG || BC
Therefore, F is the midpoint of AG
⇒ AF = GF ... (1)
Again, we have DF || EG || BC and DE = BE
Therefore, GF = GC … (2)
From (1) and (2), we get
AF = GF = GC
Similarly, as GN || FM || AB and AF = GF
Therefore, BM = MN = NC
- Hence proved.

8. In triangle ABC; M is mid-point of AB, N is mid-point of AC and D is any point in base BC. Use intercept theorem to show that MN bisects AD.
Solution:

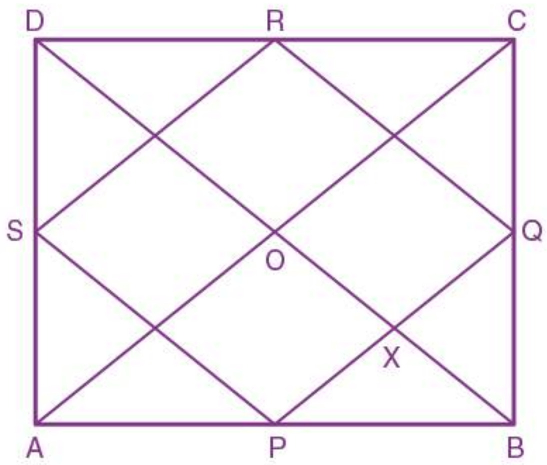
The figure is shown as below



As M and N are the midpoint of AB and AC respectively and MN || BC
Then according to intercept theorem, we have
AM = BM,
And therefore, AX = DX.
- Hence proved

9. If the quadrilateral formed by joining the mid-points of the adjacent sides of quadrilateral ABCD is a rectangle, show that the diagonals AC and BD intersect at right angle.
Solution:

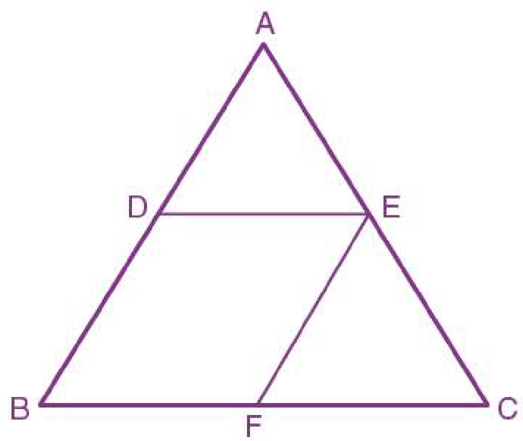
The figure is shown as below:



Let ABCD be a quadrilateral where P, Q, R and S are the midpoints of AB, BC, CD and DA.
And, PQRS is a rectangle
Diagonal AC and BD intersect at point O.
Required to show: AC and BD intersect at right angle.
Proof:
As PQ || AC,
⇒ ∠AOD = ∠PXO [Corresponding angles] ... (1)
Again, as BD||RQ,
⇒ ∠PXO = ∠RQX = 90º [Corresponding angle and angle of rectangle] ... (2)
From (1) and (2), we get
∠AOD = 90º
Similarly,
∠AOB = ∠BOC = ∠DOC = 90º
Therefore, diagonals AC and BD intersect at right angle
- Hence proved.

10. In triangle ABC; D and E are mid-points of the sides AB and AC respectively. Through E, a straight line is drawn parallel to AB to meet BC at F. Prove that BDEF is a parallelogram. If AB = 16 cm, AC = 12 cm and BC = 18 cm, find the perimeter of the parallelogram BDEF.
Solution:

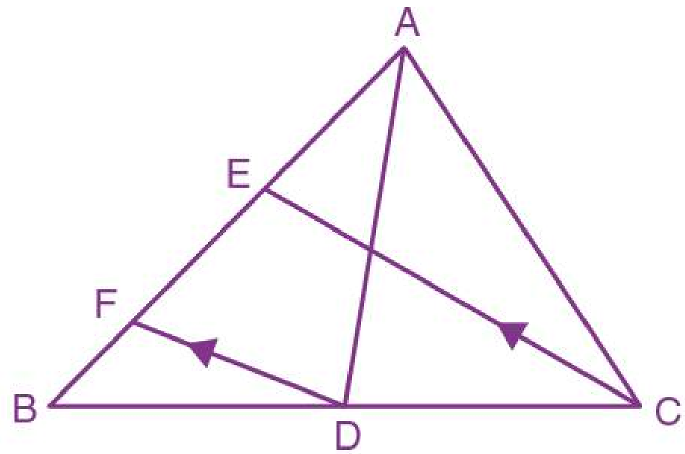
The figure is shown below as:



As E is the midpoint of AC and EF||AB, we have
Thus, F is the midpoint of BC and
2DE = BC or DE = BF
Again, as D and E are midpoints, we have
DE||BF and EF = BD
Hence, BDEF is a parallelogram.
Now,
BD = EF = ½ AB
= ½ ×16
= 8cm

BF = DE = ½ BC
= ½ ×18
= 9cm
Therefore, perimeter of BDEF = 2(BF + EF) = 2(9 + 8) = 34cm

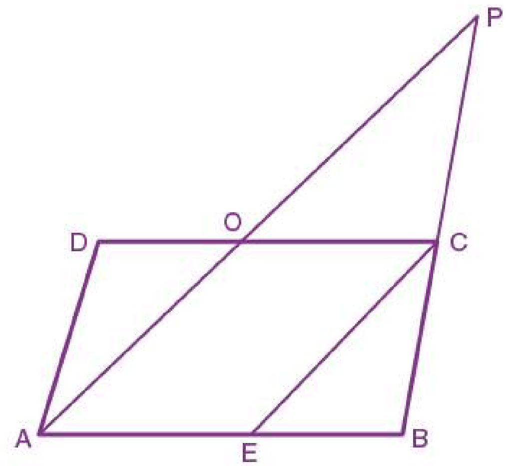
11. In the given figure, AD and CE are medians and DF||CE. Prove that: FB = ¼ AB



Solution:

Given, AD and CE are medians and DF || CE
We know that from the midpoint theorem,
If two lines are parallel and the starting point of segment is at the midpoint on one side, then the other point meets at the midpoint of the other side.
Consider ∆BEC,
Given that DF || CE and D is midpoint of BC
So, F must be the midpoint of BE
⇒ FB = ½ BE … (i)
But, BE = ½ AB
On substituting the value of BE in (i), we get
FB = ¼ AB
- Hence Proved

12. In parallelogram ABCD, E is the mid-point of AB and AP is parallel to EC which meets DC at point O and BC produced at P.
Prove that:
(i) BP = 2AD
(ii) O is the mid-point of AP.



Solution:

Given ABCD is parallelogram,
⇒ AD = BC and AB = CD

(i) Now, in ∆APB
Given, EC is parallel to AP and E is midpoint of side AB
So, by midpoint theorem,
C is the midpoint of BP
So, BP = 2BC
But,
BC = AD as ABCD is a parallelogram.
Hence, BP = 2AD

(ii) In ∆APB, we have
AB || OC as ABCD is a parallelogram
So, by midpoint theorem
O is the midpoint of AP
Hence Proved.

13. In trapezium ABCD, sides AB and DC are parallel to each other. E is mid-point of AD and F is mid-point of BC. Prove that: AB + DC = 2EF.
Solution:

In trapezium ABCD, we have
E and F are the midpoints of sides AD and BC respectively

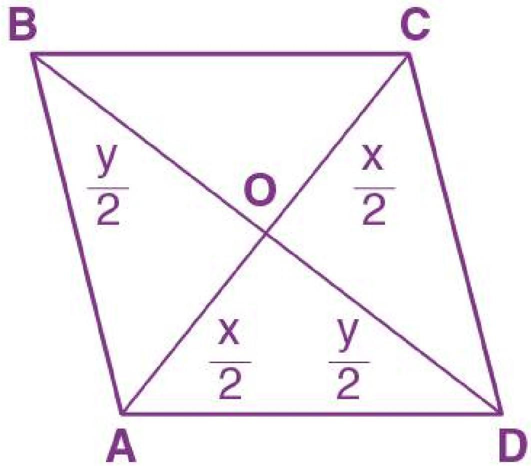
We know that, AB = GH = IJ
From midpoint theorem, we have
EG = ½ DI and
HF = ½ JC
Now, consider the L.H.S, we have
AB + CD = AB + CJ + JI + ID
= AB + 2HF + AB + 2EG
So, AB + CD = 2(AB + HF + EG)
= 2(EG + GH + HF)
= 2EF
∴ AB + CD = 2EF
- Hence Proved.

14. In ΔABC, AD is the median and DE is parallel to BA, where E is a point in AC. Prove that BE is also a median.
Solution:

Given, ΔABC and AD is the median
So, D is the midpoint of side BC
Also, given DE || AB
By the midpoint theorem,
E has to be midpoint of AC
So, line joining the vertex and midpoint of the opposite side is always the median.
Thus, BE is also median of ΔABC

15. Adjacent sides of a parallelogram are equal and one of the diagonals is equal to any one of the sides of this parallelogram. Show that its diagonals are in the ratio √3:1.
Solution:

If adjacent sides of a parallelogram are equal, then it is rhombus.
Now, the diagonals of a rhombus bisect each other and are perpendicular to each other.
Let's consider the lengths of the diagonals to be x and y
Diagonal of length y be equal to the sides of rhombus.
Thus, each side of rhombus = y



Now, in right angled ΔBOC

By Pythagoras theorem,

$OB^2 + OC^2 = BC^2$

$(y/2)^2 + (x/2)^2 = y^2$

$x^2/4 = y^2 - y^2/4$

$x^2/4 = (4y^2 - y^2)/4$

$x^2/4 = 3y^2/4$

$x^2 = 3y^2$

$x^2/y^2 = 3/1$

$x/y = \sqrt{3}/1$

Thus, the diagonals are in the ratio $\sqrt{3} : 1$