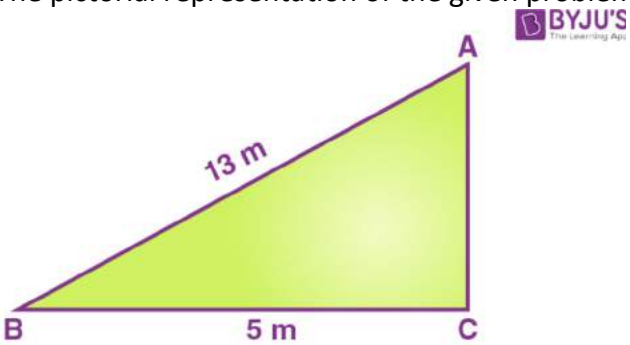
EXERCISE 13A PAGE: 158

1. A ladder 13 m long rests against a vertical wall. If the foot of the ladder is 5 m from the foot of the wall, find the distance of the other end of the ladder from the ground.

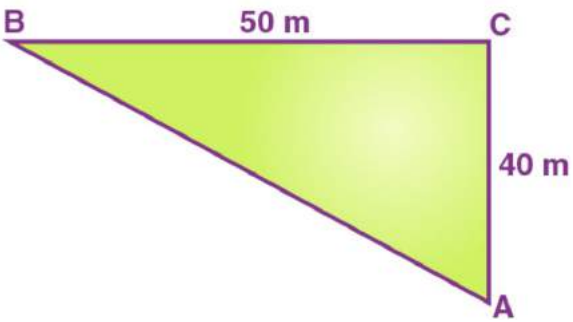
Solution:
The pictorial representation of the given problem is given below,



Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
(i)Here, AB is the hypotenuse. Therefore, applying the Pythagoras theorem, we get,
$AB^2 = BC^2 + CA^2$
$13^2 = 5^2 + CA^2$
$CA^2 = 13^2 – 5^2$
$CA^2 = 144$
CA = 12m
Therefore, the distance of the other end of the ladder from the ground is 12m

2. A man goes 40 m due north and then 50 m due west. Find his distance from the starting point.

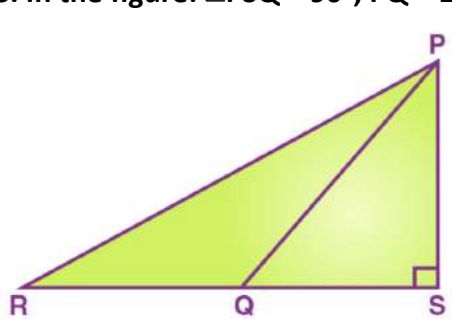
Solution:
Here, we need to measure the distance AB as shown in the figure below,



Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Therefore, in this case

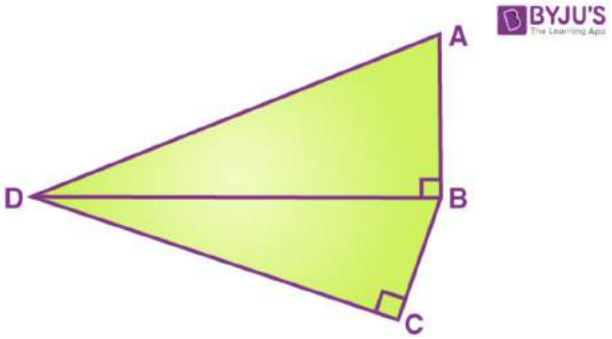
$AB^2 = BC^2 + CA^2$
$AB^2 = 50^2 + 40^2$
$AB^2 = 2500 + 1600$
$AB^2 = 4100$
AB = 64.03
Therefore, the required distance is 64.03 m.

3. In the figure: ∠PSQ = 90°, PQ = 10 cm, QS = 6 cm and RQ = 9 cm. Calculate the length of PR.



Solution:
Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
First, we consider the triangle PQS and applying Pythagoras theorem we get,
$PQ^2 = PS^2 + QS^2$
$10^2 = PS^2 + 6^2$
$PS^2 = 100 – 36$
$PS^2 = 64$
PS = 8
Now, we consider the triangle PRS and applying Pythagoras theorem we get,
$PR^2 = RS^2 + PS^2$
$PR^2 = 15^2 + 8^2$
PR = 17
Therefore, the length of PR = 17cm

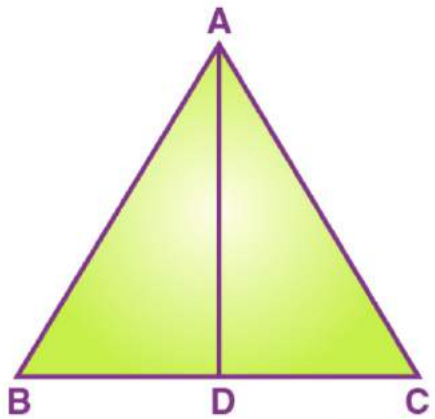
4. The given figure shows a quadrilateral ABCD in which AD = 13 cm, DC = 12 cm, BC = 3 cm and ∠ABD = ∠BCD = 90°. Calculate the length of AB.



Solution:
Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
First, we consider the triangle BDC and applying Pythagoras theorem we get,
$DB^2 = DC^2 + BC^2$
$DB^2 = 12^2 + 3^2$
$= 144 + 9$
$= 153$
Now, we consider the triangle ABD and applying Pythagoras theorem we get,
$DA^2 = DB^2 + BA^2$
$13^2 = 153 + BA^2$
$BA^2 = 169 – 153$
$BA = 4$
The length of AB is 4 cm.

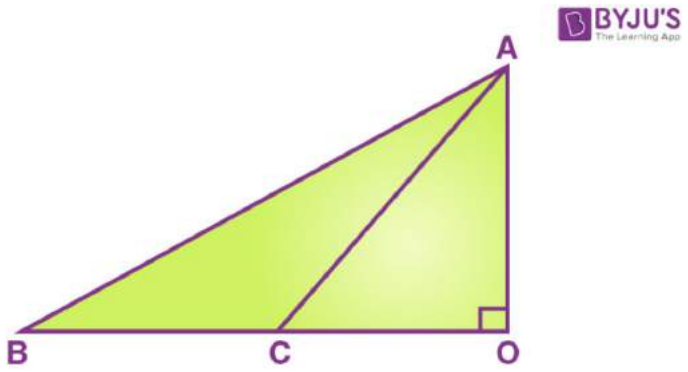
5. AD is drawn perpendicular to base BC of an equilateral triangle ABC. Given BC = 10 cm, find the length of AD, correct to 1 place of decimal.

Solution:
Since ABC is an equilateral triangle therefore, all the sides of the triangle are of same measure and the perpendicular AD will divide BC in two equal parts.



Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Here, we consider the triangle ABD and applying Pythagoras theorem we get,
$AB^2 = AD^2 + BD^2$
$AD^2 = 100^2 – 5^2$
$AD^2 = 100 – 25$
$AD^2 = 75$
$= 8.7$
Therefore, the length of AD is 8.7 cm

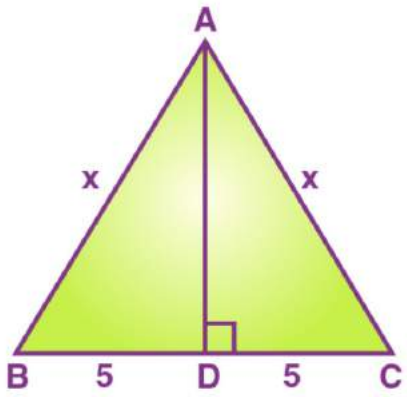
6. In triangle ABC, given below, AB = 8 cm, BC = 6 cm and AC= 3 cm. Calculate the length of OC.



Solution:
We have Pythagoras theorem which states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
First, we consider the triangle ABO and applying Pythagoras theorem we get,
$AB^2 = AO^2 + OB^2$
$AO^2 = AB^2 – OB^2$
$AO^2 = AB^2 – (BC + OC)^2$
Let OC = x
$AO^2 = AB^2 – (BC + x)^2$ …… (1)
First, we consider the triangle ACO and applying Pythagoras theorem we get
$AC^2 = AO^2 + x^2$
$AO^2 = AC^2 – x^2$ …… (2)
From 1 and 2
$AB^2 – (BC + x)^2 = AC^2 – x^2$
$8^2 – (6 + x)^2 = 3^2 – x^2$
X = 1 7/12 cm
Therefore, the length of OC will be 19/12 cm

7. In triangle ABC, AB = AC = x, BC = 10 cm and the area of the triangle is 60 cm^2. Find x.

Solution:
Here, the diagram will be,



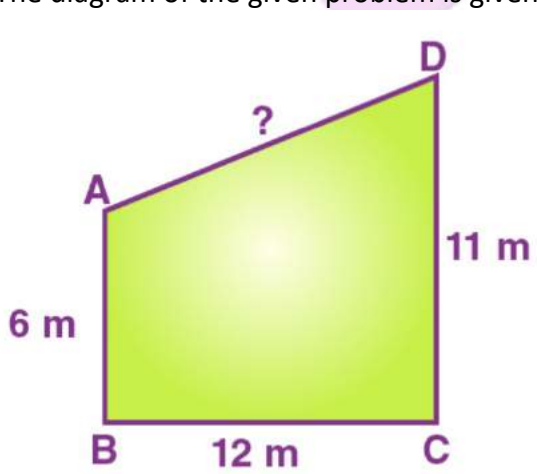
We have Pythagoras theorem which states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Since, ABC is an isosceles triangle, therefore perpendicular from vertex will cut the base in two equal segments.
First, we consider the triangle ABD and applying Pythagoras theorem we get,
$AB^2 = AD^2 + BD^2$
$AD^2 = x^2 – 5^2$
$AD^2 = x^2 – 25$
$AD = \sqrt{(x^2 – 25)}$ ……. (1)
Now,
Area = 60
½ (10) AD = 60
$½ (10) [\sqrt{(x^2 – 25)}] = 60$
x = 13
Therefore x = 13 cm

8. If the sides of triangle are in the ratio 1 :2: 1, show that is a right-angled triangle.

Solution:
Let, the sides of the triangle be, $x^2 + x^2 = 2x^2 = (\sqrt{2}x)^2$ …… (1)
Here, in (i) it is shown that, square of one side of the given triangle is equal to the addition of square of other two sides. This is nothing but Pythagoras theorem which states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Therefore, the given triangle is right angled triangle.

9. Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m; find the distance between their tips.

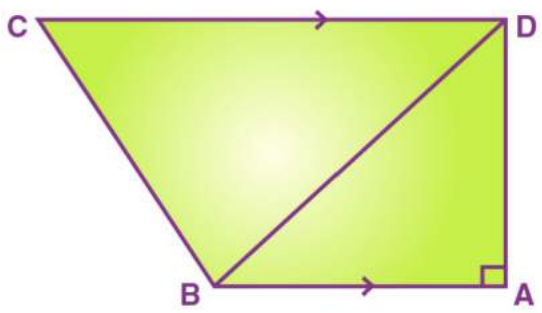
Solution:
The diagram of the given problem is given below,



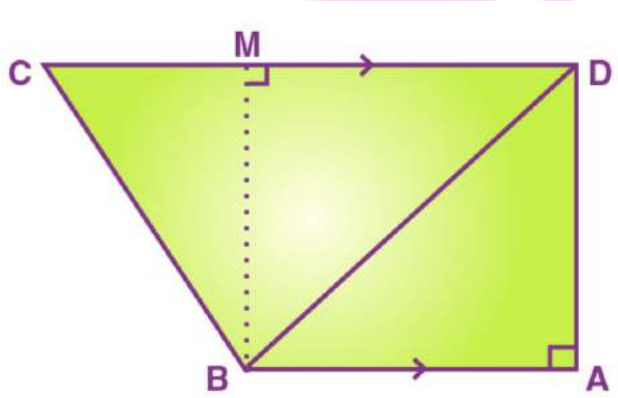
We have Pythagoras theorem which states that in a right-angled triangle, the square on the

hypotenuse is equal to the sum of the squares on the remaining two sides.
Here 11 – 6 = 5m
Base = 12 m
Applying Pythagoras theorem, we get
$h^2 = 5^2 + 12^2$
= 25 + 144
= 169
h = 13
therefore, the distance between the tips will be 13m

10. In the given figure, AB//CD, AB = 7 cm, BD = 25 cm and CD = 17 cm; find the length of side BC.



Take M be the point on CD such that AB = DM.
So, DM = 7cm and MC = 10 cm
Join points B and M to form the line segment BM.
So BM || AD also BM = AD.

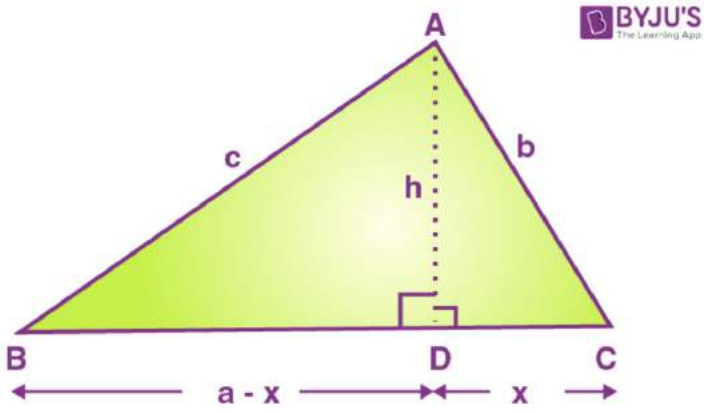


In triangle BAD
$BD^2 = AD^2 + BA^2$
$25^2 = AD^2 + 7^2$
$AD^2 = 576$
AD = 24
In triangle CMB
$CB^2 = CM^2 + MB^2$
$CB^2 = 10^2 + 24^2$

$CB^2 = 676$
CB = 26 cm

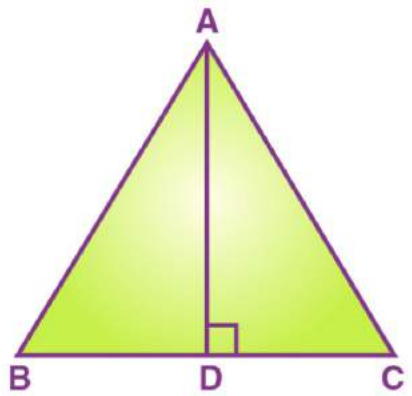
EXERCISE 13B PAGE: 163

1. In the figure, given below, AD parallel to BC. Prove that: $c^2 = a^2 + b^2 - 2ax$



Solution:
Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
First, we consider the triangle ABD and applying Pythagoras theorem we get,
$AB^2 = AD^2 + BD^2$
$c^2 = h^2 + (a – x)^2$
$h^2 = c^2 – (a - x)^2$ …… (1)
First, we consider the triangle ACD and applying Pythagoras theorem we get
$AC^2 = AD^2 + CD^2$
$b^2 = h^2 + x^2$
$h^2 = b^2 – x^2$ ……. (2)
from 1 and 2
$c^2 – (a – x)^2 = b^2 – x^2$
$c^2 – a^2 – x^2 + 2ax = b^2 – 2ax$
$c^2 = a^2 + b^2 – 2ax$
hence the proof.

2. In equilateral Δ ABC, AD parallel to BC and BC = x cm. Find, in terms of x, the length of AD.

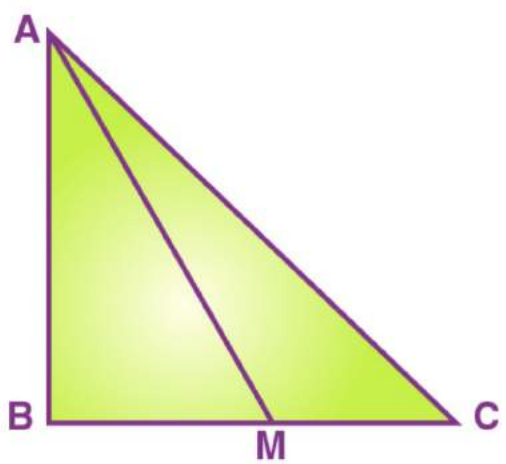


Solution:

In equilateral Δ ABC, AD parallel to BC.
Therefore, BD = DC = x/2 cm.
Applying Pythagoras theorem, we get
In right angled triangle ADC
$AC^2 = AD^2 + DC^2$
$x^2 = AD^2 + (x/2)^2$
$AD^2 = (x)^2 – (x/2)^2$
$AD^2 = (x/2)^2$
AD = (x/2) cm

3. ABC is a triangle, right-angled at B. M is a point on BC. Prove that:
$AM^2 + BC^2 = AC^2 + BM^2$.

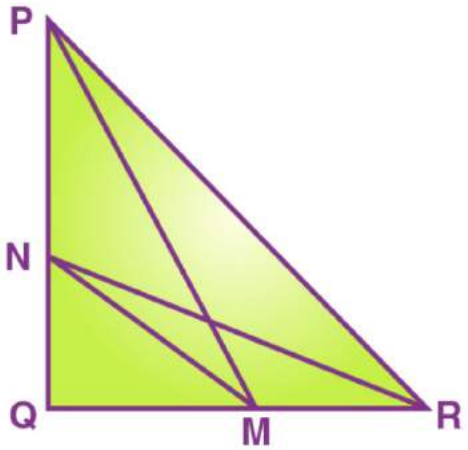
Solution:
The pictorial form of the given problem is as follows,
Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
First, we consider the triangle ABM and applying Pythagoras theorem we get,



$AM^2 = AB^2 + BM^2$

$AB^2 = AM^2 – BM^2$ …….. (1)
Now we consider the triangle ABC and applying Pythagoras theorem we get
$AC^2 = AB^2 + BC^2$
$AB^2 = AC^2 – BC^2$ …… (2)
From 1 and 2 we get
$AM^2 – BM^2 = AC^2 + BM^2$
$AM^2 + BC^2 = AC^2 + BM^2$
Hence the proof.

4. M and N are the mid-points of the sides QR and PQ respectively of a triangle PQR, right-angled at Q. Prove that:
(i) $PM^2 + RN^2 = 5\ MN^2$
(ii) $4\ PM^2 = 4\ PQ^2 + QR^2$
(iii) $4\ RN^2 = PQ^2 + 4\ QR^2$
(iv) $4\ (PM^2 + RN^2) = 5\ PR^2$



Solution:

Draw, PM, MN, NR
Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Since, M and N are the mid-points of the sides QR and PQ respectively, therefore, PN = NQ, QM = RM
(i)
First, we consider the triangle PQM and applying Pythagoras theorem we get,
$PM^2 = PQ^2 + MQ^2$
$= (PN + NQ)^2 + MQ^2$
$= PN^2 + NQ^2 + 2\ PN.\ NQ + MQ^2$
$= MN^2 + PN^2 + 2\ PN.\ NQ$ [we know $MN^2 = NQ^2 + MQ^2$] …….. (1)
Now we consider the triangle RNQ and applying Pythagoras theorem,
$RN^2 = NQ^2 + RQ^2$
$= NQ^2 + (QM + RM)^2$
$= NQ^2 + QM^2 + 2\ QM.\ RM + RM^2$ …….. (2)

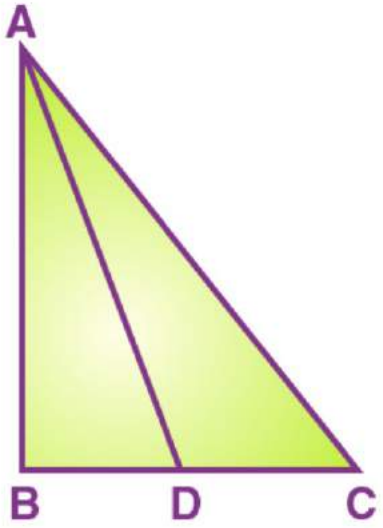
Adding 1 and 2 we get
$PM^2 + RN^2 = MN^2 + PN^2 + 2PN. NQ + MN^2 + RM^2 + 2QM. RM$
$PM^2 + RN^2 = 2MN^2 + PN^2 + RM^2 + 2PN. NQ + 2QM. RM$
$PM^2 + RN^2 = 2MN^2 + NQ^2 + QM^2 + 2(QN)^2 + 2 (QM)^2$
$PM^2 + RN^2 = 2MN^2 + MN^2 + 2MN^2$
$PM^2 + RN^2 = 5MN^2$
Hence the proof.

(ii) Now consider the triangle PQM and apply Pythagoras theorem we get
$PM^2 = PQ^2 + MQ^2$
$4PM^2 = 4PQ^2 + 4 MQ^2$ [multiplying both sides by 4]
$4PM^2 = 4PQ^2 + 4 (½ QR^2)$ [MQ = ½ QR]
$4PM^2 = 4PQ^2 + QR^2$
Hence the proof.

(iii) now consider triangle RQN and apply Pythagoras theorem we get
$RN^2 = NQ^2 + RQ^2$
$4RN^2 = 4NQ^2 + 4 QR^2$ [multiplying both sides by 4]
$4RN^2 = 4QR^2 + 4 (½ PQ^2)$ [NQ = ½ PQ]
$4RN^2 = PQ^2 + 4QR^2$
Hence the proof.

(iv) now consider the triangle PQM and apply Pythagoras theorem,
$PM^2 = PQ^2 + MQ^2$
$= (PN + NQ)^2 + MQ^2$
$= PN^2 + NQ^2 + 2 PN. NQ + MQ^2$
$= MN^2 + PN^2 + 2 PN. NQ$ [we know $MN^2 = NQ^2 + MQ^2$] ……. (1)
Now we consider the triangle RNQ and applying Pythagoras theorem,
$RN^2 = NQ^2 + RQ^2$
$= NQ^2 + (QM + RM)^2$
$= NQ^2 + QM^2 + 2 QM. RM + RM^2$
$= MN^2 + RM^2 + 2 QM. RM$ ……. (2)
Adding 1 and 2 we get
$PM^2 + RN^2 = MN^2 + PN^2 + 2PN. NQ + MN^2 + RM^2 + 2QM. RM$
$PM^2 + RN^2 = 2MN^2 + PN^2 + RM^2 + 2PN. NQ + 2QM. RM$
$PM^2 + RN^2 = 2MN^2 + NQ^2 + QM^2 + 2(QN)^2 + 2 (QM)^2$
$PM^2 + RN^2 = 2MN^2 + MN^2 + 2MN^2$
$PM^2 + RN^2 = 5MN^2$
$4 (PM^2 + RN^2) = 4. 5 (NQ^2 + MQ^2]$
$4 (PM^2 + RN^2) = 4. 5 [(½ PQ)^2 + (½ QR)^2]$
$4 (PM^2 + RN^2) = 5PR^2$
Hence the proof.

5. In triangle ABC, ∠B = 90° and D is the mid-point of BC. Prove that: $AC^2 = AD^2 + 3CD^2$.

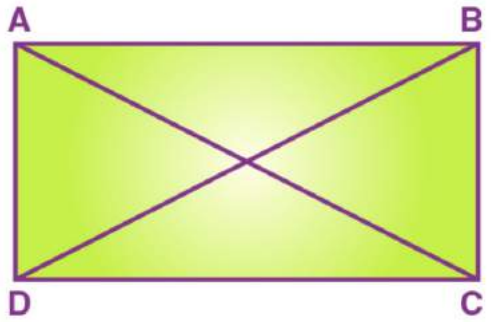


Solution:

Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
In triangle ABC, ∠B = 90° and D is the mid-point of BC. Join AD. Therefore, BD=DC
First, we consider the triangle ADB and applying Pythagoras theorem we get,
$AD^2 = AB^2 + BD^2$
$AB^2 = AD^2 – BD^2$ …. (1)
Similarly, we get from rt. angle triangles ABC we get,
$AC^2 = AB^2 + BC^2$
$AB^2 = AC^2 – BC^2$ …. (1)
From 1 and 2 we get
$AC^2 – BC^2 = AD^2 – BD^2$
$AC^2 = AD^2 – BD^2 + BC^2$
$AC^2 = AD^2 – CD^2 + 4CD^2$ [BD = CD = ½ BC]
$AC^2 = AD^2 + 3CD^2$
Hence the proof.

6. In a rectangle ABCD, prove that: $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$.

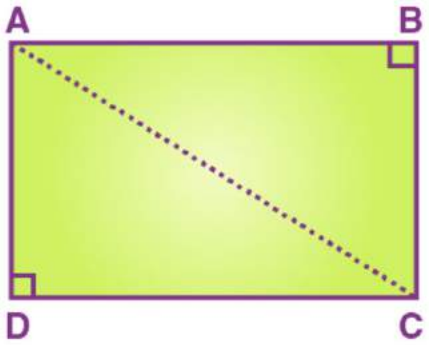
Solution:



Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Since, ABCD is a rectangle angles A, B, C and D are rt. angles.
First, we consider the triangle ACD and applying Pythagoras theorem we get,
$AC^2 = DA^2 + CD^2$ ……. (1)
Similarly, we get from rt. angle triangle BDC we get,
$BD^2 = BC^2 + CD^2$
$= BC^2 + AB^2$ [In a rectangle opposite sides are equal CD = AB]
Adding (i) and (ii),
$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$
Hence the proof.

7. In a quadrilateral ABCD, $\angle B = 90^0$ and $\angle D = 90^0$. Prove that: $2AC^2 - AB^2 = BC^2 + CD^2 + DA^2$

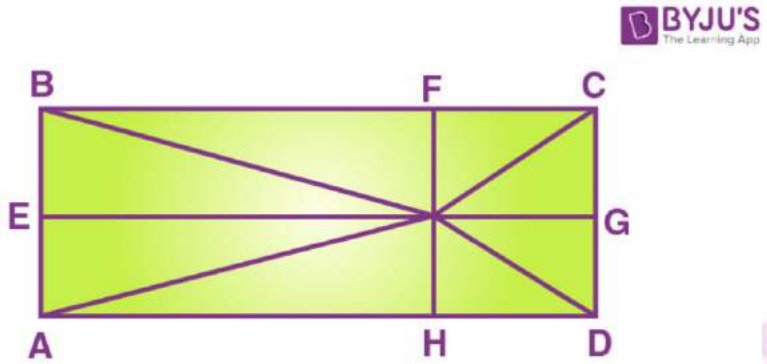
Solution:



In quadrilateral ABCD $\angle B = 90^0$ and $\angle D = 90^0$
So triangle ABC and triangle ADC are right angles.
For triangle ABC, apply Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$AB^2 = AC^2 – BC^2$ …….. (i)
For triangle ADC, apply Pythagoras theorem,
$AC^2 = AD^2 + DC^2$ …….. (ii)
LHS = $2AC^2 – AB^2$
$= 2AC^2 – (AC^2 – BC^2)$ from 1

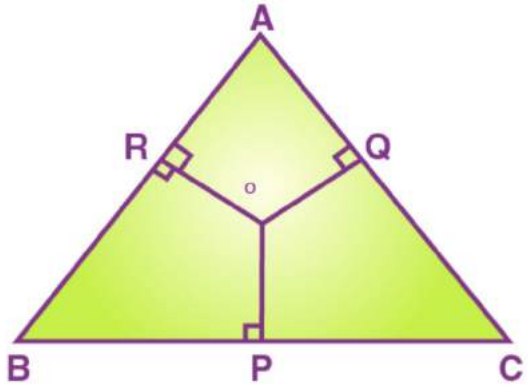
$= 2AC^2 – AC^2 + BC^2$
$= AC^2 + BC^2$
$= AD^2 + DC^2 + BC^2$ from 2
= RHS

8. O is any point inside a rectangle ABCD. Prove that: $OB^2 + OD^2 = OC^2 + OA^2$.
Solution:

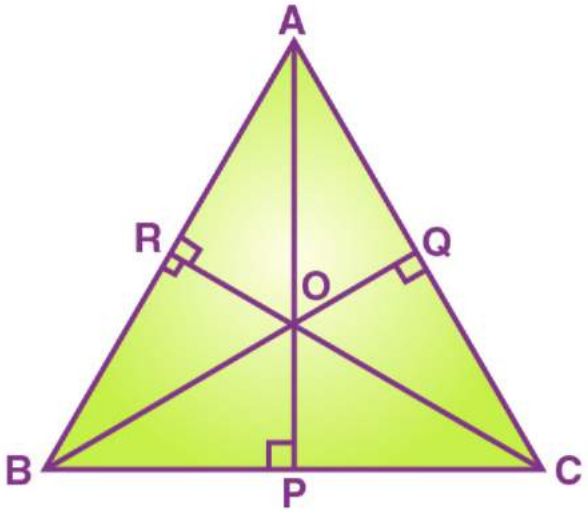


Draw rectangle ABCD with arbitrary point O within it, and then draw lines OA, OB, OC, OD. Then draw lines from point O perpendicular to the sides: OE, OF, OG, OH.
Pythagoras theorem states that in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Using Pythagorean theorem, we have from the above diagram:
$OA^2 = AH^2 + OH^2 = AH^2 + AE^2$
$OC^2 = CG^2 + OG^2 = EB^2 + HD^2$
$OB^2 = EO^2 + BE^2 = AH^2 + BE^2$
$OD^2 = HD^2 + OH^2 = HD^2 + AE^2$
Adding these equalities, we get:
$OA^2 + OC^2 = AH^2 + HD^2 + AE^2 + EB^2$
$OB^2 + OD^2 = AH^2 + HD^2 + AE^2 + EB^2$
From which we prove that for any point within the rectangle there is the relation
$OA^2 + OC^2 = OB^2 + OD^2$
Hence Proved.

9. In the following figure, OP, OQ and OR are drawn perpendiculars to the sides BC, CA and AB respectively of triangle ABC. Prove that: $AR^2 + BP^2 + CQ^2 = AQ^2 + CP^2 + BR^2$



Solution:
Here, we first need to join OA, OB, and OC after which the figure becomes as follows,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides. First, we consider the ΔARO and applying Pythagoras theorem we get,

$AO^2 = AR^2 + OR^2$

$AR^2 = AO^2 – OR^2$ ……. (1)

Similarly, from triangles, BPO, COQ, AOQ, CPO and BRO we get the following results,

$BP^2 = BO^2 – OP^2$ ……. (2)

$CQ^2 = OC^2 – OQ^2$ ……. (3)

$AQ^2 = AO^2 – OQ^2$ ……. (4)

$CP^2 = OC^2 – OP^2$ ……. (5)

$BR^2 = OB^2 – OR^2$ ……. (6)

Adding 1, 2 and 3 we get

$AR^2 + BP^2 + CQ^2 = AO^2 – OR^2 + BO^2 – OP^2 + OC^2 – OQ^2$ ……. (7)

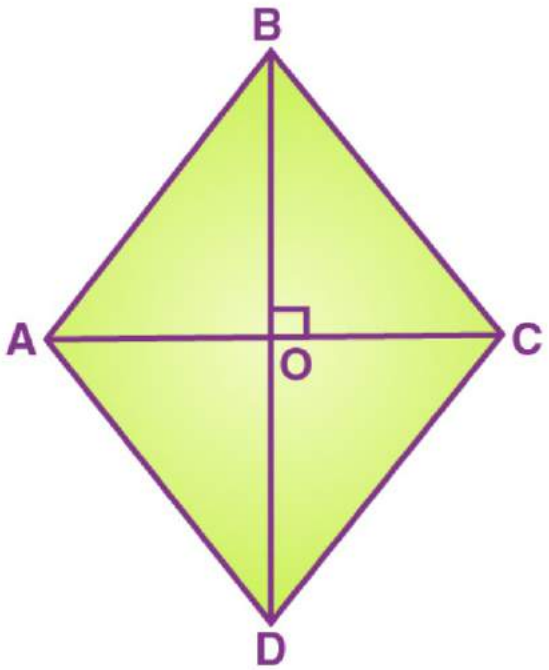
Adding 4, 5 and 6 we get

$AQ^2 + CP^2 + BR^2 = AO^2 – OQ^2 + OC^2 – OP^2 + OB^2 – OR^2$ …… (8)

From 7 and 8, we get,

$AR^2 + BP^2 + CQ^2 = AQ^2 + CP^2 + BR^2$
Hence proved.

10. Diagonals of rhombus ABCD intersect each other at point O. Prove that: $OA^2 + OC^2 = 2AD^2 – BD^2/2$
Solution:



We know diagonals of the rhombus are perpendicular to each other.
In quadrilateral ABCD, $\angle AOD = \angle COD = 90^o$
We know triangle AOD and COD are right angle triangle.
In triangle AOD, apply Pythagoras theorem,
$AD^2 = OA^2 + OD^2$
$OA^2 = AD^2 – OD^2$ …… (1)
In triangle COD, apply Pythagoras theorem,
$CD^2 = OC^2 + OD^2$
$OC^2 = CD^2 – OD^2$ …… (2)
LHS = $OA^2 + OC^2$
= $AD^2 – OD^2 + CD^2 – OD^2$ from 1 and 2
= $AD^2 – AD^2 – 2(BD/2)^2$ [AD = CD and OD = BD/2]
= $2AD^2 – BD^2/2$
= RHS