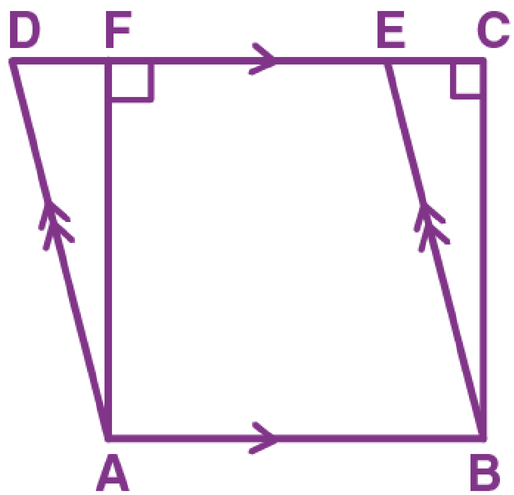
Exercise 16(A)

1. In the given figure, if area of triangle ADE is 60 cm^2, state, given reason, the area of:
(i) Parallelogram ABED
(ii) Rectangle ABCF
(iii) Triangle ABE



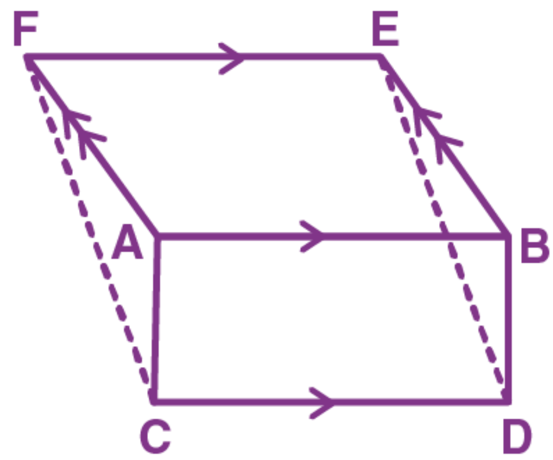
Solution:

(i) As ΔADE and parallelogram ABED are on the same base AB and between the same parallels DE || AB, the area of the ΔADE will be half the area of parallelogram ABED.
So,
Area of parallelogram ABED = 2 x Area of ΔADE
= 2 x 60 cm^2
= 120 cm^2

(ii) Area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e., between the same parallels
So,
Area of rectangle ABCF = Area of parallelogram ABED = 120 cm^2

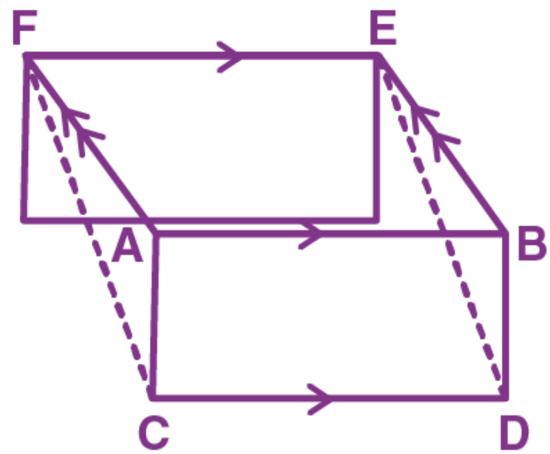
(iii) We know that, area of triangles on the same base and between same parallel lines are equal
So,
Area of ΔABE = Area of ΔADE = 60 cm^2

2. The given figure shows a rectangle ABDC and a parallelogram ABEF; drawn on opposite sides of AB. Prove that:
(i) Quadrilateral CDEF is a parallelogram.
(ii) Area of quad. CDEF = Area of rect. ABDC + Area of || gm. ABEF.



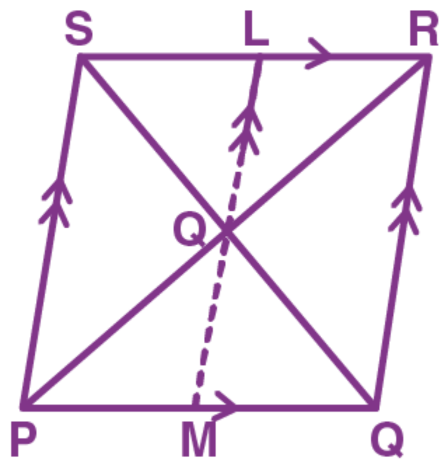
Solution:

After drawing the opposite sides of AB, we get



It's seen from the figure that CD|| FE. Therefore, FC must parallel to DE.
Thus, it is proved that the quadrilateral CDEF is a parallelogram.
We know that,
Area of parallelograms on same base and between same parallel lines is always equal. Also, area of parallelogram is equal to the area of rectangle on the same base and of the same altitude i.e., between same parallel lines.
So,
Area of CDEF = Area of ABDC + Area of ABEF
- Hence Proved.

3. In the given figure, diagonals PR and QS of the parallelogram PQRS intersect at point O and LM is parallel to PS. Show that:



(i) 2 Area (∆POS) = Area (||gm PMLS)
(ii) Area (∆POS) + Area (∆QOR) = ½ Area (||gm PQRS)
(iii) Area (∆POS) + Area (∆QOR) = Area (∆POQ) + Area (∆SOR).
Solution:

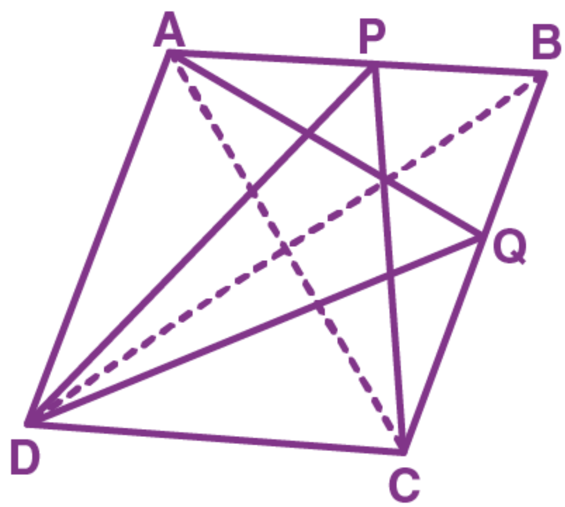
(i) As POS and parallelogram PMLS are on the same base PS and between the same parallels i.e., SP || LM
Since O is the center of LM and ratio of areas of triangles with same vertex and bases along the same line is equal to ratio of their respective bases
Also, the area of the parallelogram is twice the area of the triangle if they lie on the same base and in between the same parallels
Thus, 2 Area (∆PSO) = Area(PMLS)

(ii) Taking LHS, we have
LM is parallel to PS and PS is parallel to RQ, therefore, LM is parallel to RQ
And, since ∆POS lie on the base PS and in between the parallels PS and LM, we have
Area (∆POS) = ½ Area (PSLM)
Also, since ∆QOR lie on the base QR and in between the parallels LM and RQ, we have
Area (∆QOR) = ½ Area (LMQR)
Now,
Area(∆POS) + Area(∆QOR) = ½ Area(PSLM) + ½ Area(LMQR)
= ½ [Area(PSLM) + Area(LMQR)]
= ½ Area(PQRS)

(iii) We know that, the diagonals in a parallelogram bisect each other.
So, OS = OQ
In ∆PQS, as OS = OQ
OP is the median of the ∆PQS.
We know that median of a triangle divides it into two triangles of equal area.
Therefore,
Area (∆POS) = Area (∆POQ) … (1)
Similarly, as OR is the median of the triangle QRS, we have

Area (∆QOR) = Area (∆SOR) … (2)
Now, adding (1) and (2) we get
Area (∆POS) + area (∆QOR) = Area (∆POQ) + Area (∆SOR)
- Hence Proved.

4. In parallelogram ABCD, P is a point on side AB and Q is a point on side BC.
Prove that:
(i) ∆CPD and ∆AQD are equal in area.
(ii) Area (∆AQD) = Area (∆APD) + Area (∆CPB)
Solution:



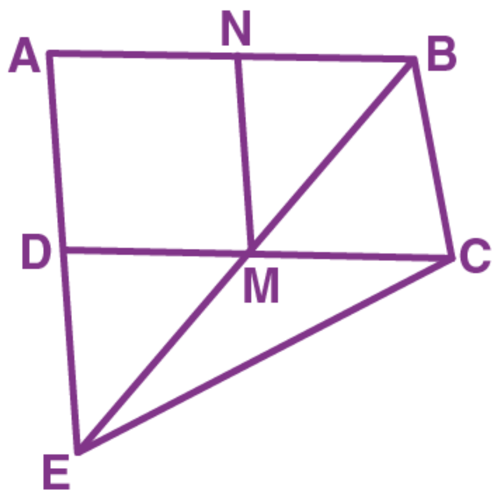
Given, ABCD is a parallelogram.
P and Q are any points on the sides AB and BC respectively.
Join diagonals AC and BD.

Proof:
(i) As triangles with same base and between same set of parallel lines have equal areas
Area (∆CPD) = Area (∆BCD) …… (1)
Again, diagonals of the parallelogram bisect area in two equal parts
Area (∆BCD) = ½ area of parallelogram ABCD …… (2)
From (1) and (2)
Area (∆CPD) = ½ Area (ABCD) …… (3)
Similarly,
Area (∆AQD) = Area (∆ABD) = ½ Area (ABCD) …… (4)
From (3) and (4), we get
Area (∆CPD) = Area (∆AQD)
- Hence proved.

(ii) We know that, area of triangles on the same base and between same parallel lines are equal
So,

Area of ∆AQD = Area of ∆ACD = Area of ∆PDC = Area of ∆BDC = Area of ∆ABC = Area of ∆APD + ∆Area of BPC
- Hence Proved

5. In the given figure, M and N are the mid-points of the sides DC and AB respectively of the parallelogram ABCD.

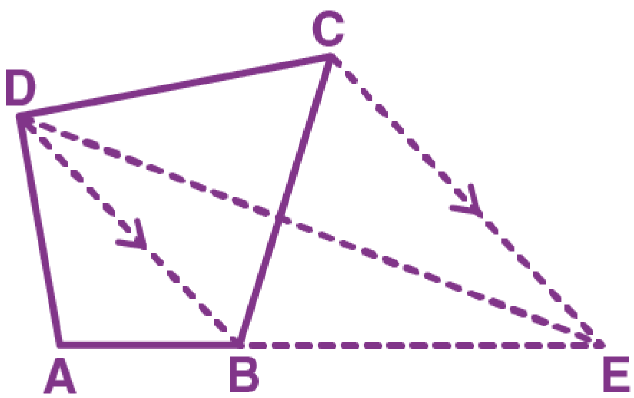


If the area of parallelogram ABCD is 48 cm^2
(i) State the area of the triangle BEC.
(ii) Name the parallelogram which is equal in area to the triangle BEC.
Solution:

(i) As ∆BEC and parallelogram ABCD are on the same base BC and between the same parallels i.e., BC || AD.
Area (∆BEC) = ½ x Area (ABCD) = ½ x 48 = 24 cm^2

(ii) Area (ANMD) = Area (BNMC)
= ½ x Area (ABCD)
= ½ x 2 x Area (∆BEC)
= Area (∆BEC)
Therefore, the parallelograms ANMD and NBCM have areas equal to ∆BEC.

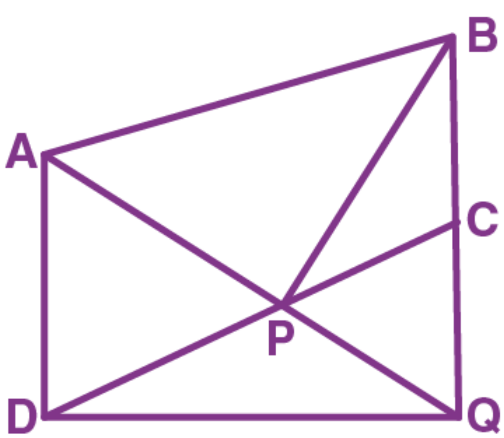
6. In the following figure, CE is drawn parallel to diagonals DB of the quadrilateral ABCD which meets AB produced at point E.
Prove that ∆ADE and quadrilateral ABCD are equal in area.



Solution:

As, ∆DCB and ∆DEB are on the same base DB and between the same parallels i.e., DB || CE,
We have,
Area (∆DCB) = Area (∆DEB)
Area (∆DCB + ∆ADB) = Area (∆DEB + ∆ADB)
Area (ABCD) = Area (∆ADE)
- Hence proved

7. ABCD is a parallelogram a line through A cuts DC at point P and BC produced at Q. Prove that triangle BCP is equal in area to triangle DPQ.

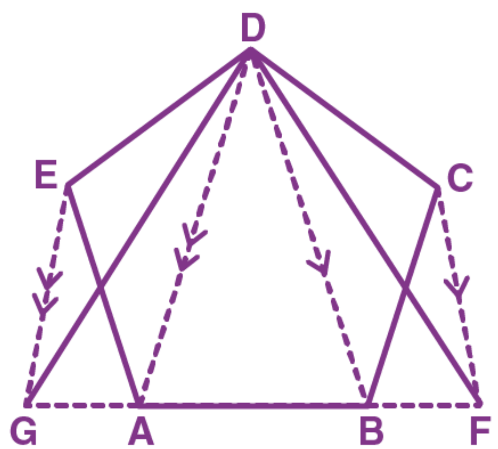


Solution:

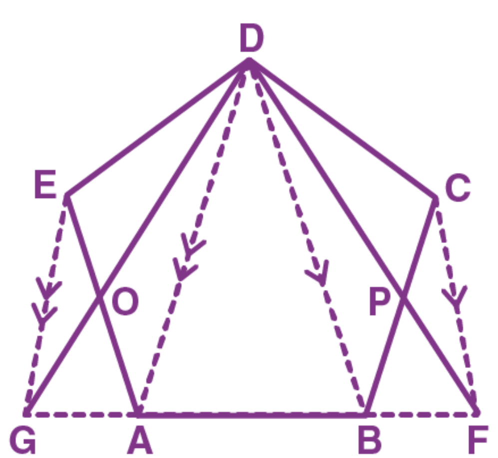
Its seen that, ∆APB and parallelogram ABCD are on the same base AB and between the same parallel lines AB and CD.
So,
Ar(∆APB) = ½ Ar(||gm ABCD) … (i)
Now,
∆ADQ and parallelogram ABCD are on the same base AD and between the same parallel lines

AD and BQ.
So,
Ar(∆ADQ) = ½ Ar(||gm ABCD) … (ii)
On adding equation (i) and (ii), we get
Ar(∆APB) + Ar(∆ADQ) = ½ Ar(||gm ABCD) + ½ Ar(||gm ABCD) = Ar(||gm ABCD)
Ar(quad. ADQB) – Ar(∆BPQ) = Ar(||gm ABCD)
Ar(quad. ADQB) – Ar(∆BPQ) = Ar(quad. ADQB) – Ar(∆DCQ)
Ar(∆BPQ) = Ar(∆DCQ)
Subtracting Ar(∆PCQ) from both the sides, we get
Ar(∆BPQ) - Ar(∆PCQ) = Ar(∆DCQ) - Ar(∆PCQ)
Ar(∆BCP) = Ar(∆DPQ)
- Hence proved.

8. The given figure shows a pentagon ABCDE. EG drawn parallel to DA meets BA produced at G and CF draw parallel to DB meets AB produced at F.
Prove that the area of pentagon ABCDE is equal to the area of triangle GDF.



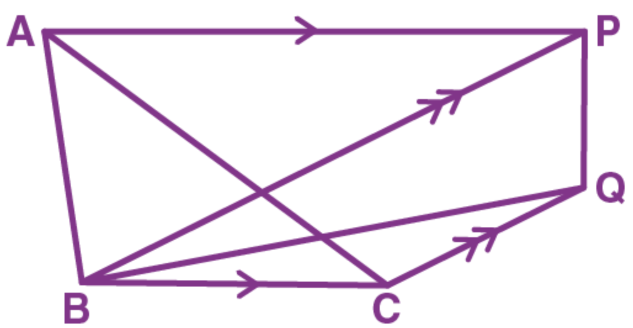
Solution:



It’s seen that triangles EDG and EGA are on the same base EG and between the same parallel lines EG and DA, so
Ar(∆EDG) = Ar(∆EGA)

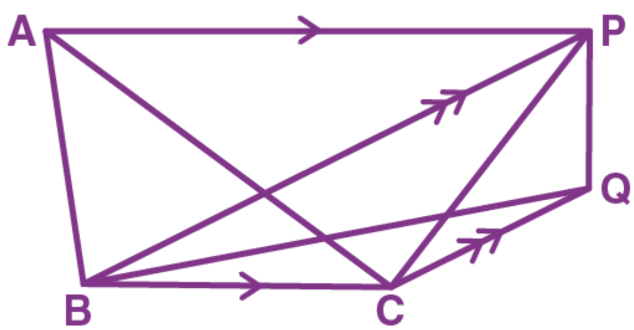
On subtracting ΔEOG from both sides, we have
Ar(ΔEDG) – Ar(ΔEOG) = Ar(ΔEGA) – Ar(ΔEOG)
Ar(ΔEOD) = Ar(ΔGOA) … (i)
Similarly,
Ar(ΔDPC) = Ar(ΔBPF) … (ii)
Now,
Ar(GDF) = Ar(GOA) + Ar(BPF) + Ar(pen. ABPDO)
= Ar(EOD) + Ar(DPC) + Ar(pen. ABPDO)
= Ar(pen. ABCDE)
- Hence proved

9. In the given figure, AP is parallel to BC, BP is parallel to CQ. Prove that the area of triangles ABC and BQP are equal.



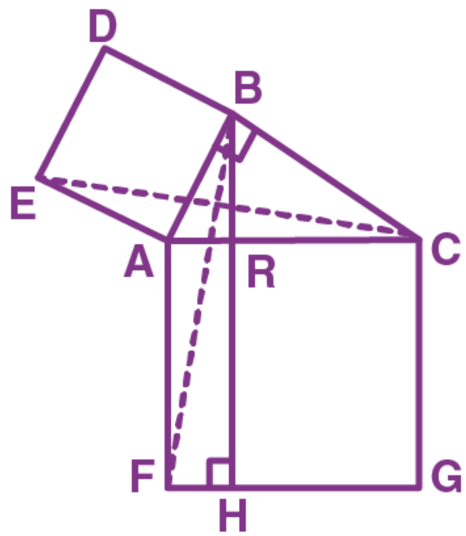
Solution:

On joining PC, we see that



ΔABC and ΔBPC are on the same base BC and between the same parallel lines AP and BC.
Ar(ΔABC) = Ar(ΔBPC) … (i)
And, ΔBPC and ΔBQP are on the same base BP and between the same parallel lines BP and CQ.
Ar(ΔBPC) = Ar(BQP) … (ii)
From (i) and (ii), we get
Ar(ΔABC) = Ar(ΔBQP)
- Hence proved.

10. In the figure given alongside, squares ABDE and AFGC are drawn on the side AB and the hypotenuse AC of the right triangle ABC.



If BH is perpendicular to FG prove that:
(i) $\Delta EAC \cong \Delta BAF$.
(ii) Area of the square ABDE = Area of the rectangle ARHF.
Solution:

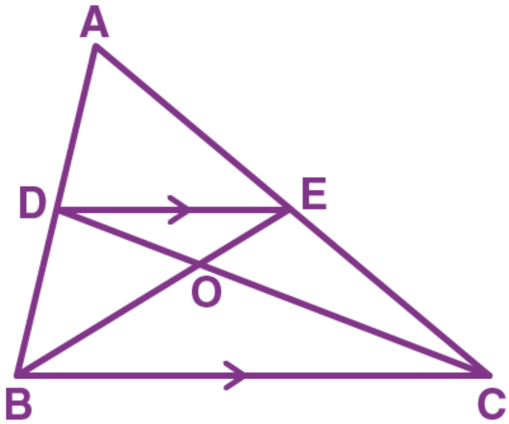
(i) From figure,
$\angle EAC = \angle EAB + \angle BAC$
$= 90^o + \angle BAC$ … (i)
$\angle BAF = \angle FAC + \angle BAC$
$= 90^o + \angle BAC$ … (ii)
From (i) and (ii), we get
$\angle EAC = \angle BAF$
In ΔEAC and ΔBAF, we have,
EA = AB
$\angle EAC = \angle BAF$ (proved above)
AC = AF
Therefore, $\Delta EAC \cong \Delta BAF$ (SAS axiom of congruency)

(ii) As ΔABC is a right triangle, we have
$AC^2 = AB^2 + BC^2$ [Using Pythagoras theorem in ΔABC]
$AB^2 = AC^2 – BC^2$
$AB^2 = (AB^2 + BC^2) – [BR^2 + RC^2]$ [Since $AC^2 = AR^2 + RC^2$and using Pythagoras Theorem in ΔBRC]
$AB^2 = AR^2 + 2AR \times RC + RC^2 - (BR^2 + RC^2)$ [Using the identity]
$AB^2 = AR^2 + 2AR \times RC + RC^2 - (AB^2 - AR^2 + RC^2)$ [Using Pythagoras Theorem in ΔABR]
$2AB^2 = 2AR^2 + 2AR \times RC$
$AB^2 = AR (AR + RC)$

$AB^2 = AR \times AF$

∴ Area (ABDE) = Area (rectangle ARHF)

11. In the following figure, DE is parallel to BC. Show that:
(i) Area (∆ADC) = Area(∆AEB).
(ii) Area (∆BOD) = Area (∆COE).

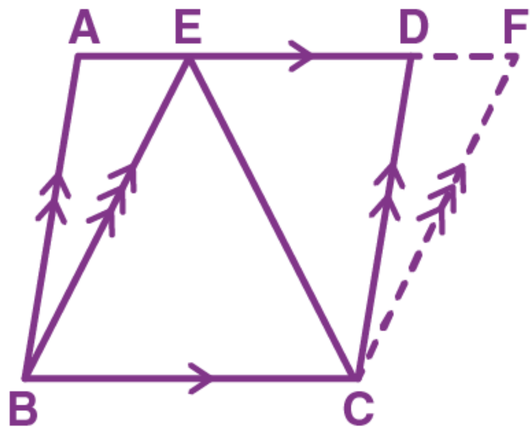


Solution:

(i) In ∆ABC, D is midpoint of AB and E is the midpoint of AC
And, DE is parallel to BC
So, by mid-point theorem, we have
AD/AB = AE/AC
Ar(∆ADC) = Ar(∆BDC) = ½ Ar(∆ABC) … (i)
Again,
Ar(∆AEB) = Ar(∆BEC) = ½ Ar(∆ABC) … (ii)
From equations (i) and (ii), we have
Area (∆ADC) = Area(∆AEB).
- Hence Proved

(ii) We know that, area of triangles on the same base and between same parallel lines are equal
So, Area(∆DBC) = Area(∆ BCE)
Area(∆DOB) + Area(∆BOC) = Area(∆BOC) + Area(∆COE)
Thus, Area(∆DOB) = Area(∆COE) [On subtracting Area(∆BOC) on both sides]

12. ABCD and BCFE are parallelograms. If area of triangle EBC = 480 cm^2; AB = 30 cm and BC = 40 cm; Calculate:
(i) Area of parallelogram ABCD
(ii) Area of the parallelogram BCFE
(iii) Length of altitude from A on CD
(iv) Area of triangle ECF



Solution:

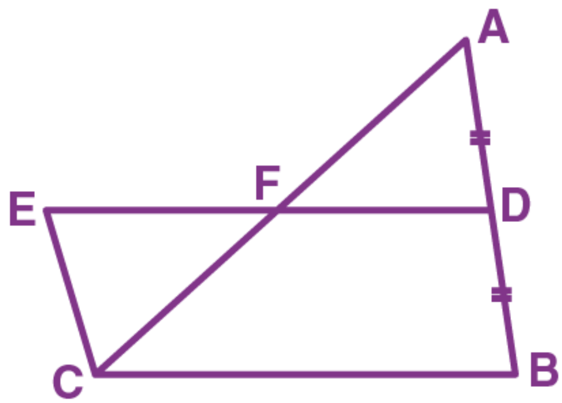
(i) As ΔEBC and parallelogram ABCD are on the same base BC and between the same parallels i.e., BC||AD.
So, Area(ΔEBC) = ½ Area(||gm ABCD)
Area(||gm ABCD) = 2 x Ar(ΔEBC)
= 2 x 480 cm^2
= 960 cm^2

(ii) We know that, parallelograms on same base and between same parallels are equal in area
So, Area of BCFE = Area of ABCD = 960 cm^2

(iii) Area of ΔACD = 960/2 = 480
= (1/2) x 30 x Altitude
Thus,
Altitude = 480/15
= 32 cm

(iv) We know that, the area of a triangle is half that of a parallelogram on the same base and between the same parallels
Therefore,
Area(ΔECF) = ½ Area(||gm CBEF)
Similarly,
Area(ΔBCE) = ½ Area(||gm CBEF)
Thus,
Area(ΔECF) = Area(ΔBCE) = 960/2 = 480 cm^2

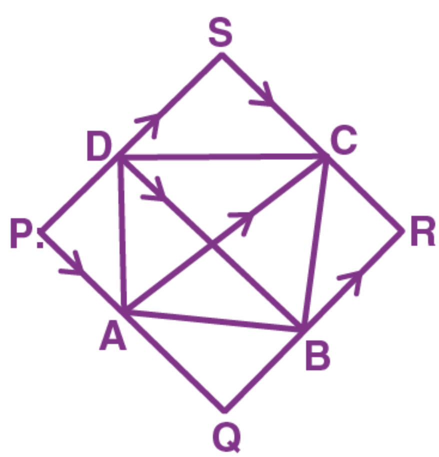
13. In the given figure, D is mid-point of side AB of ΔABC and BDEC is a parallelogram.



Prove that: Area of ∆ABC = Area of ||gm BDEC.
Solution:

Here, AD = DB and EC = DB (Given)
So, EC = AD
It’s seen that EFC = AFD (Vertically opposite angles)
And, as ED and CB are parallel lines with AC cutting these lines, we have
∠ECF = ∠FAD (Alternate interior angles)
From the above conditions, we have
∆EFC ≅ ∆AFD by AAS Congruency criterion
So, Area (∆EFC) = Area (∆AFD)
Now, adding quadrilateral CBDF in both sides, we get
Area of || gm BDEC = Area of ∆ABC
- Hence proved

14. In the following, AC || PS || QR and PQ || DB || SR.
Prove that:
Area of quadrilateral PQRS = 2 × Area of quad. ABCD.

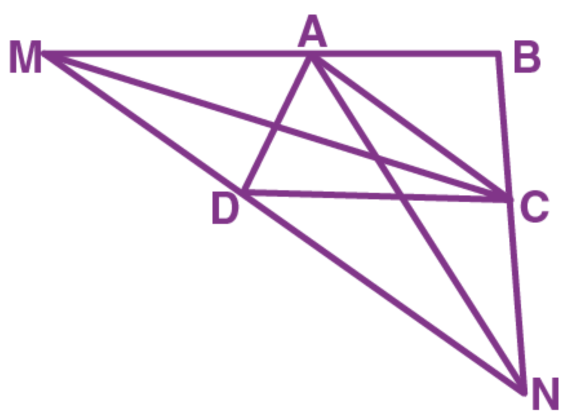


Solution:

In parallelogram PQRS, AC || PS || QR and PQ || DB || SR
Similarly, AQRC and APSC are also parallelograms.
As, ΔABC and parallelogram AQRC are on the same base AC and between the same parallels, we have
Ar(ΔABC) = ½ Ar(AQRC)......(i)
Similarly,
Ar(ΔADC) = ½ Ar(APSC).......(ii)
On adding (i) and (ii), we get
Ar(ΔABC) + Ar(ΔADC) = ½ Ar(AQRC) + ½ Ar(APSC)
Area (quad. ABCD) = ½ Area (quad. PQRS)
Therefore,
Area of quad. PQRS = 2 × Area of quad. ABCD

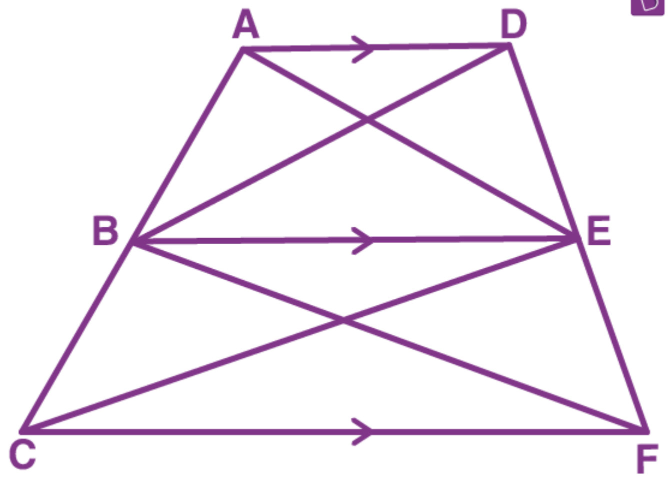
15. ABCD is trapezium with AB || DC. A line parallel to AC intersects AB at point M and BC at point N. Prove that: area of ΔADM = area of ΔACN.
Solution:

Given: ABCD is a trapezium
AB || CD, MN || AC



Let's join C and M
We know that, area of triangles on the same base and between same parallel lines are equal.
So, Area of ΔAMD = Area of ΔAMC … (i)
Similarly, considering quad. AMNC where MN || AC, we have
ΔACM and ΔACN are on the same base and between the same parallel lines
Thus, their areas should be equal.
i.e., Area of ΔACM = Area of ΔCAN … (ii)
From equations (i) and (ii), we get
Area of ΔADM = Area of ΔCAN
- Hence Proved.

16. In the given figure, AD || BE || CF. Prove that area (ΔAEC) = area (ΔDBF)

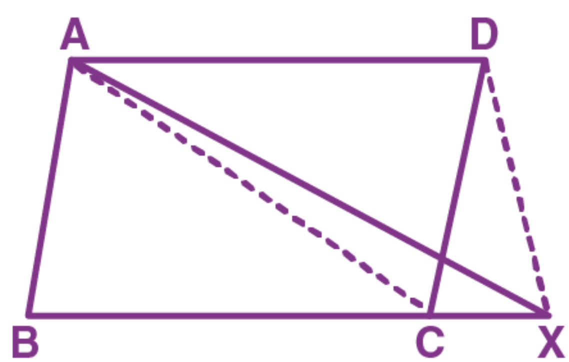


Solution:

We know that,
Area of triangles on the same base and between same parallel lines are equal.
So, in ABED quadrilateral and AD||BE
With common base, BE and between AD and BE parallel lines, we have
Area of ΔABE = Area of ΔBDE … (i)

Similarly, in BEFC quadrilateral and BE||CF
With common base BC and between BE and CF parallel lines, we have
Area of ΔBEC = Area of ΔBEF … (ii)
On adding equations (i) and (ii), we get
Area of ΔABE + Area of ΔBEC = Area of ΔBEF + Area of ΔBDE
Thus,
Area of ΔAEC = Area of ΔDBF
- Hence Proved

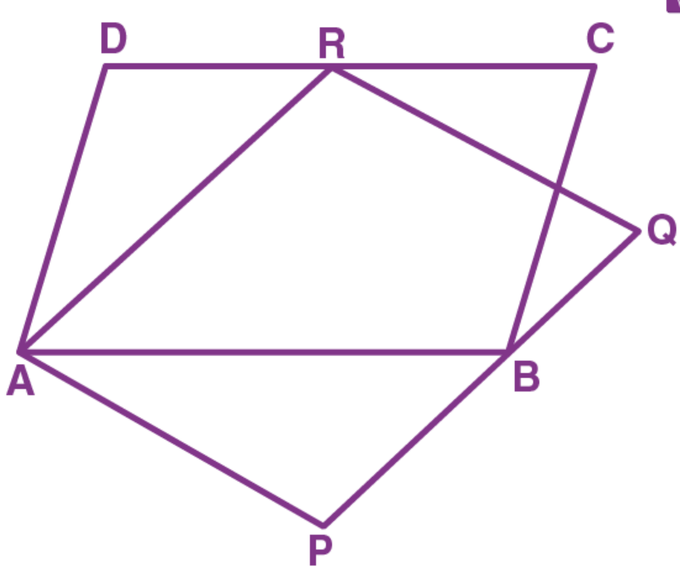
17. In the given figure, ABCD is a parallelogram; BC is produced to point X. Prove that: area (Δ ABX) = area (quad. ACXD).



Solution:

Given: ABCD is a parallelogram.
We know that,
Area of ΔABC = Area of ΔACD (Diagonal divides a ||gm into 2 triangles of equal area)
Now, consider ΔABX
Area of ΔABX = Area of ΔABC + Area of ΔACX
We also know that, area of triangles on the same base and between same parallel lines are equal.
So, Area of ΔACX = Area of ΔCXD
From above equations, we have
Area of ΔABX = Area of ΔABC + Area of ΔACX
= Area of ΔACD+ Area of ΔCXD
= Area of quadrilateral ACXD
- Hence Proved

18. The given figure shows parallelograms ABCD and APQR. Show that these parallelograms are equal in area.



Solution:

Let's join B and R and also P and R.
We know that, the area of the parallelogram is equal to twice the area of the triangle, if the triangle and the parallelogram are on the same base and between the parallels
Taking ABCD parallelogram:
As ||gm ABCD and ΔABR lie on AB and between the parallels AB and DC, we have
Area (||gm ABCD) = 2 x Area (ΔABR) ... (i)

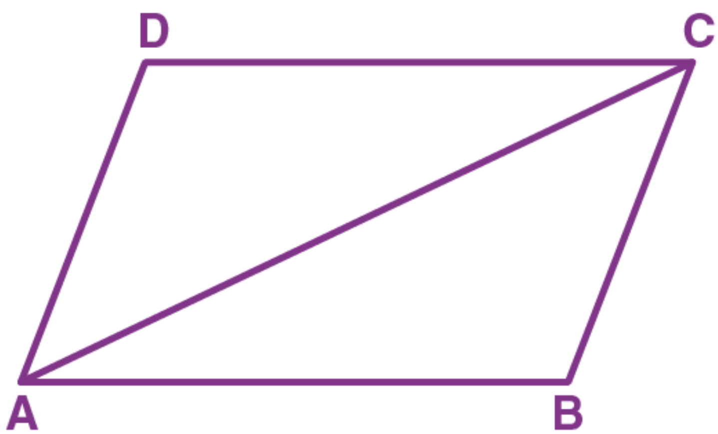
Also, the area of triangles with same base and between the same parallel lines are equal.
As the triangles ABR and APR lie on the same base AR and between the parallels AR and QP, we have,
Area (ΔABR) = Area (ΔAPR) ... (ii)
From equations (i) and (ii), we have

Area (||gm ABCD) = 2 x Area (ΔAPR) … (iii)
Also, its seen that APR and ||gm ARQP lie on the same base AR and between the same parallels AR and QP
So, Area (ΔAPR) = ½ Area (||gm ARQP) … (iv)
Using (iv) in (iii), we get
Area (||gm ABCD) = 2 x ½ x Area (||gm ARQP)
Area (||gm ABCD) = Area (||gm ARQP)
- Hence proved

Exercise 16(B)

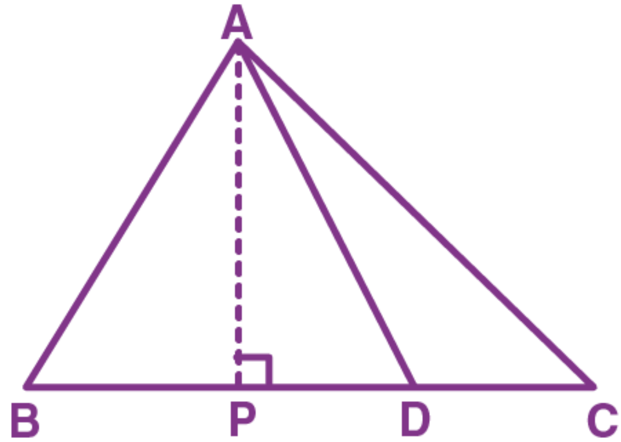
1. Show that:
(i) A diagonal divides a parallelogram into two triangles of equal area.
(ii) The ratio of the areas of two triangles of the same height is equal to the ratio of their bases.
(iii) The ratio of the areas of two triangles on the same base is equal to the ratio of their heights.
Solution:

(i) Let ABCD be a parallelogram (Given)



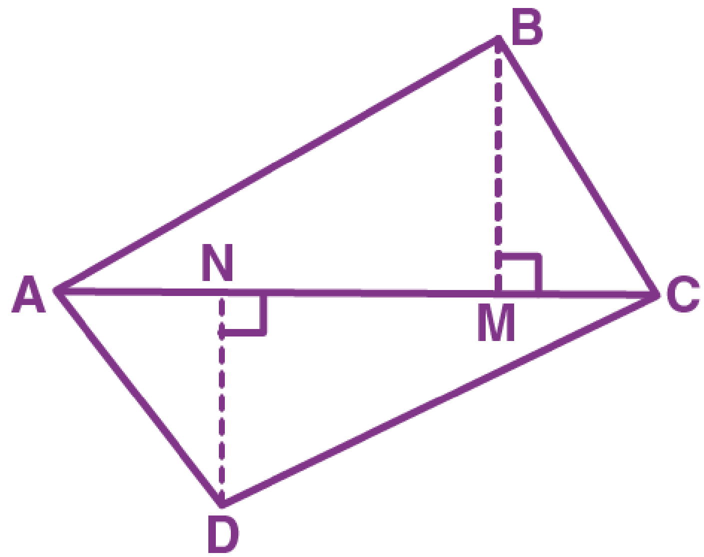
Considering the triangles ABC and ADC, we have
AB = CD (opposite sides of ||m)
AD = BC (opposite sides of ||gm)
AD = AD (Common)
Thus, ΔABC ≅ ΔADC by SSS congruence criterion
Area of congruent triangles are equal.
Therefore, Area (ΔABC) = Area (ΔADC)

(ii) Consider the following figure:



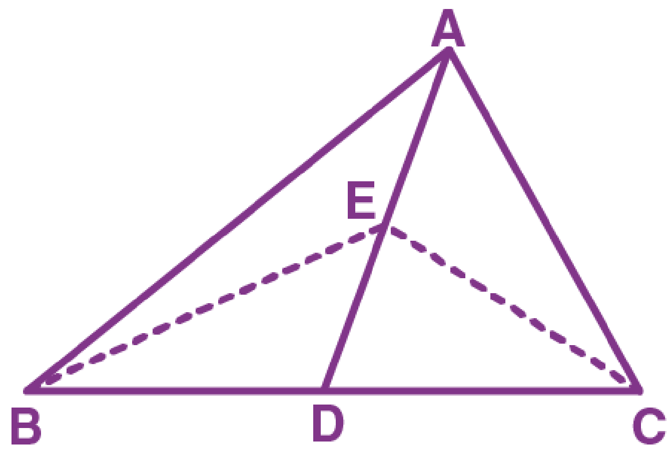
Here AP ⊥ BC,
We have,
Ar.(ΔABD) = ½ BD x AP
And, Ar.(ΔADC) = ½ DC x AP
Thus,
Area(ΔABD)/Area(ΔADC) = (½ × BD × AP)/ (½ × DC × AP)
= BD/DC
- Hence proved

(iii) Consider the following figure:



Here,
Ar.(ΔABC) = ½ BM x AC
And, Ar.(ΔADC) = ½ DN x AC
Thus,
Area(ΔABC)/Area(ΔADC) = (½ × BM × AC)/ (½ × DN × AC)
= BM/DN
- Hence proved

2. In the given figure; AD is median of ΔABC and E is any point on median AD. Prove that Area (ΔABE) = Area (ΔACE).



Solution:

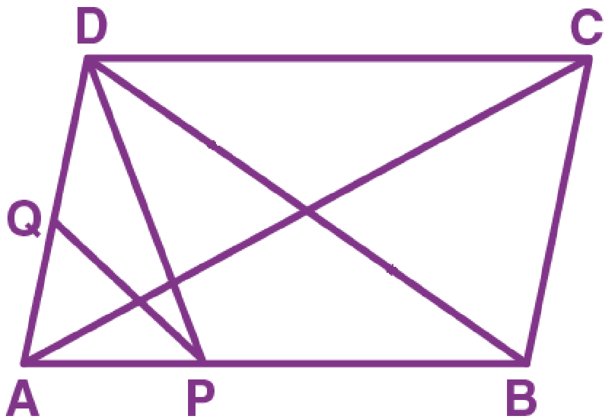
As AD is the median of ΔABC, it will divide ΔABC into two triangles of equal areas.
So, Area (ΔABD) = Area (ΔACD) … (i)
Also, since ED is the median of ΔEBC
So, Area (ΔEBD) = Area (ΔECD) … (ii)
On subtracting equation (ii) from (i), we have
Area (ΔABD) - Area(ΔEBD) = Area(ΔACD) - Area(ΔECD)
Therefore,
Area (ΔABE) = Area (ΔACE)
- Hence proved

3. In the figure of question 2, if E is the midpoint of median AD, then prove that: Area (ΔABE) = ¼ Area (ΔABC).
Solution:

As AD is the median of ΔABC, it will divide ΔABC into two triangles of equal areas.
Hence, Area (ΔABD) = Area (ΔACD)
Area (ΔABD) = ½ Area (ΔABC) … (i)
In ΔABD, E is the mid-point of AD. So, BE is the median.
Thus,
Area (ΔBED) = Area (ΔABE)
Area (ΔBED) = ½ Area (ΔABD)
Area (ΔBED) = ½ x ½ Area (ΔABC) … [From equation (i)]
Therefore,
Area (ΔBED) = ¼ Area (ΔABC)

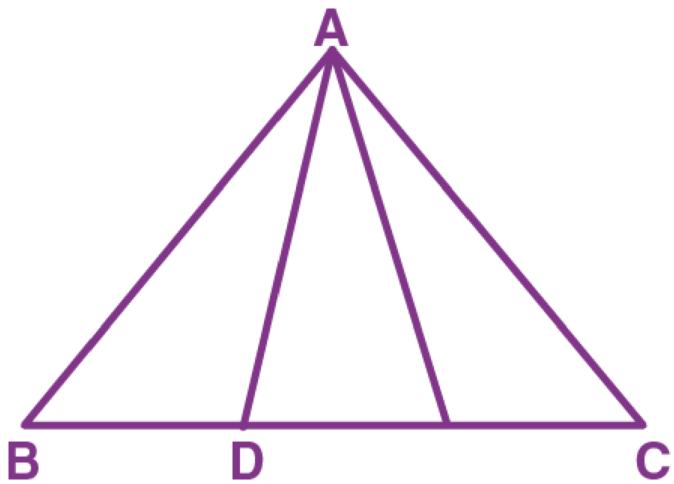
4. ABCD is a parallelogram. P and Q are the mid-points of sides AB and AD respectively. Prove that area of triangle APQ = 1/8 of the area of parallelogram ABCD.
Solution:

Let's join PD and BD.



BD is the diagonal of the parallelogram ABCD. Thus, it divides the parallelogram into two equal parts of area.
So, Area (ΔABD) = Area (ΔDBC)
= ½ Area (parallelogram ABCD) … (i)
Now,
DP is the median of ΔABD. Thus, it will divide ΔABD into two triangles of equal areas.
So, Area(ΔAPD) = Area (ΔDPB)
= ½ Area (ΔABD)
= ½ x (½ x Area (parallelogram ABCD)) [from equation (i)]
= ¼ Area (parallelogram ABCD) … (ii)
Similarly,
In ΔAPD, Q is the mid-point of AD. Hence, PQ is the median.
So, Area (ΔAPQ) = Area(ΔDPQ)
= ½ Area (ΔAPD)
= ½ × (¼ Area (parallelogram ABCD)) [Using equation (ii)]
Therefore,
Area (ΔAPQ) = 1/8 Area (parallelogram ABCD)
- Hence proved.

5. The base BC of ΔABC is divided at D so that BD = ½ DC.
Prove that area of ΔABD = 1/3rd of the area of △ABC.
Solution:



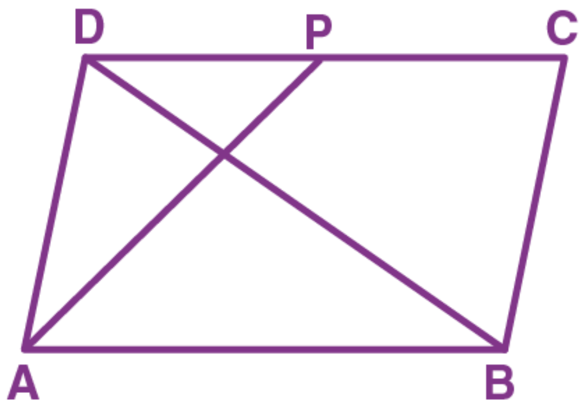
In ΔABC, As BD = ½ DC, we have
BD/DC = ½
So, Ar(ΔABD): Ar(ΔADC) = 1: 2
But, Ar(ΔABD) + Ar(ΔADC) = Ar(ΔABC)
Ar(ΔABD) + 2 Ar(ΔABD) = Ar(ΔABC)
3 Ar(ΔABD) = Ar(ΔABC)
Thus,
Ar(ΔABD) = 1/3 Ar(ΔABC)

6. In a parallelogram ABCD, point P lies in DC such that DP: PC = 3: 2. If area of ΔDPB = 30 sq. cm, find the area of the parallelogram ABCD.
Solution:

We know that,
Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases.
So, we have
Area (ΔDPB)/Area (ΔPCB) = DP/PC = 3/2

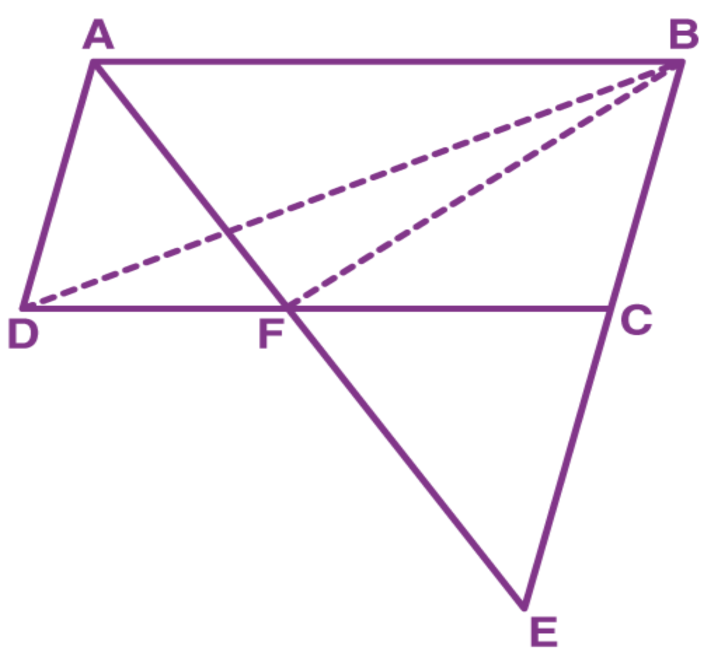
Given, area of ΔDPB = 30 sq. cm
Let 'x' be the area of ΔPCB,
Then,
30/x = 3/2
x = 30/3 x 2 = 20 sq. cm.
Therefore, area of ΔPCB = 20 sq. cm

Now, consider the following figure.



It is seen clearly from the diagram,
Area (ΔCDB) = Area (ΔDPB) + Area (ΔCPB)
= 30 + 20
= 50 sq. cm
Diagonal of the parallelogram divides it into two triangles ΔADB and ΔCDB of equal area.
Therefore,
Area (||gm ABCD) = 2 x ΔCDB
= 2 x 50
= 100 sq. cm

7. ABCD is a parallelogram in which BC is produced to E such that CE = BC and AE intersects CD at F. If ar. (ΔDFB) = 30 cm^2; find the area of parallelogram.

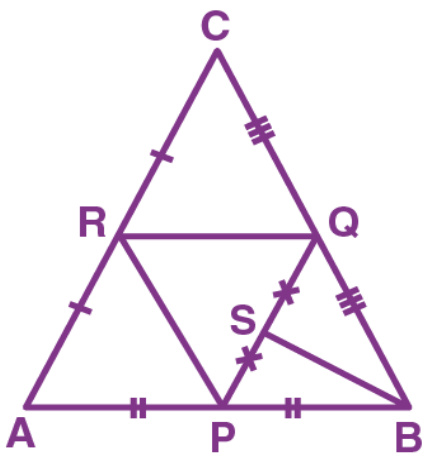


Solution:

Given, BC = CE
Also, in ||gm ABCD we have
BC = AD
Hence, AD = CE
In ΔADF and ΔECF, we have
AD = CE
∠ADF = ∠ECF (Alternate angles)
∠DAF = ∠CEF (Alternate angles)
Thus, ΔADF ≅ ΔECF by ASA congruence criterion
So, area (ΔADF) = area (ΔECF) … (i)

Also,
In ΔFBE, FC is the median (As BC = CE)
So, Area (ΔBCF) = Area (ΔECF) … (ii)
From (i) and (ii), we have
Area (ΔADF) = Area (ΔBCF) … (iii)
Again,
As ADF and BDF are on the same base DF and between the same parallels DF and AB
Area (ΔBDF) = Area (ΔADF) … (iv)
From (iii) and (iv), we have
Area (ΔBDF) = Area (ΔBCF)
Given, Area (ΔDFB) = 30 cm^2
So, Area (ΔBCF) = 30 cm^2
Area (ΔBCD) = Area (ΔBDF) + Area (ΔBCF)
= (30 + 30) cm^2
= 60 cm^2
Hence,
Area of ||gm ABCD = 2 x Area(ΔBCD)
= 2 x 60
= 120 cm^2

8. The following figure shows a triangle ABC in which P, Q and R are mid-points of sides AB, BC and CA respectively. S is mid-point of PQ.
Prove that: ar.(△ABC) = 8 × ar.(△QSB)



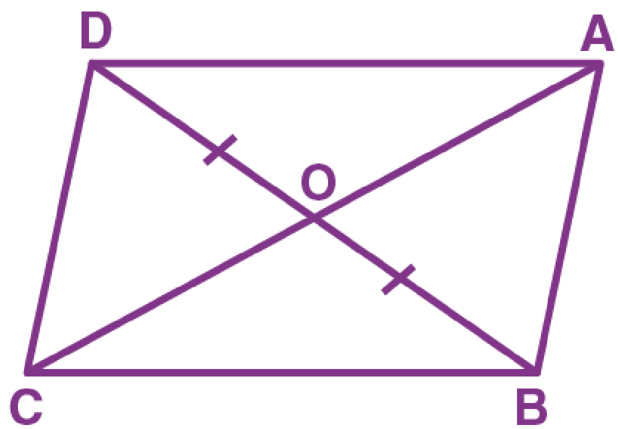
Solution:

In △ABC, R and Q are the mid-points of AC and BC respectively
So, by mid-point theorem RQ || AB
⇒ RQ || PB
So, area(△PBQ) area(△APR) …(i) [Since AP = PB and triangles on the same base and between the same parallels are equal in area]
Now,
Since P and R are the mid-points of AB and AC respectively.
By mid-point theorem, PR || BC
⇒ PR || BQ
So, quadrilateral PMQR is a parallelogram.
Also, area(ΔPBQ) = area(ΔPQR) …(ii) [diagonal of a parallelogram divide the parallelogram in two triangles with equal area]
From (i) and (ii), we have
Area (ΔPQR) = area(ΔPBQ) = area(ΔAPR) … (iii)
Similarly, P and Q the mid-points of AB and BC respectively.
By mid-point theorem,
PQ || AC ⇒ PQ || RC
So, quadrilateral PQRC is a parallelogram.
Also, area(ΔRQC) = area(ΔPQR) …(iv) [Diagonal of parallelogram divide the parallelogram in two triangles with equal area]
From (iii) and (iv),
Area(ΔPQR) = area(ΔPBQ) = area(ΔRQC) = area(ΔAPR)

So, area (ΔPBQ) = ¼ area (ΔABC) …(v)
Also, since S is the mid-point of PQ
BS is the median of ΔPBQ
So, area (ΔQSB) = ½ area(ΔPBQ)
Now from (v), we have
Area(ΔQSB) = ½ × ¼ area (ΔABC)
Area(ΔABC) = 8 × area(ΔQSB)

Exercise 16(C)

1. In the given figure, the diagonals AC and BD intersect at point O. If OB = OD and AB//DC, show that:
(i) Area (Δ DOC) = Area (Δ AOB).
(ii) Area (Δ DCB) = Area (Δ ACB).
(iii) ABCD is a parallelogram.



Solution:

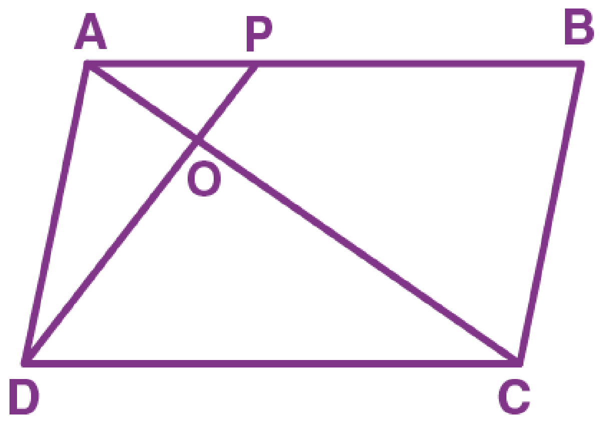
(i) Ratio of area of triangles with same vertex and bases along the same line is equal to the ratio of their respective bases.
So, we have
Area of ΔDOC/Area of ΔBOC = DO/BO = 1 … (i)
Similarly,
Area of ΔDOA/Area of ΔBOA = DO/BO = 1 … (ii)
We know that area of triangles on the same base and between same parallel lines are equal.
Area of Δ ACD = Area of Δ BCD
Area of Δ AOD + Area of Δ DOC = Area of Δ DOC + Area of Δ BOC
⇒ Area of Δ AOD = Area of Δ BOC … (iii)
From (i), (ii) and (iii) we have
Area (Δ DOC) = Area (Δ AOB)
- Hence Proved.

(ii) Similarly, from 1, 2 and 3, we also have
Area of Δ DCB = Area of Δ DOC + Area of Δ BOC = Area of Δ AOB + Area of Δ BOC = Area of Δ ABC
So, Area of Δ DCB = Area of Δ ABC
- Hence Proved.

(iii) We know that area of triangles on the same base and between same parallel lines are equal.
Given: triangles are equal in area on the common base, so it indicates AD|| BC.

So, ABCD is a parallelogram.
- Hence Proved

2. The given figure shows a parallelogram ABCD with area 324 sq. cm. P is a point in AB such that AP: PB = 1:2 Find The area of ΔAPD.



Solution:

Ratio of area of triangles with the same vertex and bases along the same line is equal to the ratio of their respective bases.
So, we have
Area of ΔAPD/Area of ΔBPD = AP/BP = ½
Area of parallelogram ABCD = 324 sq.cm

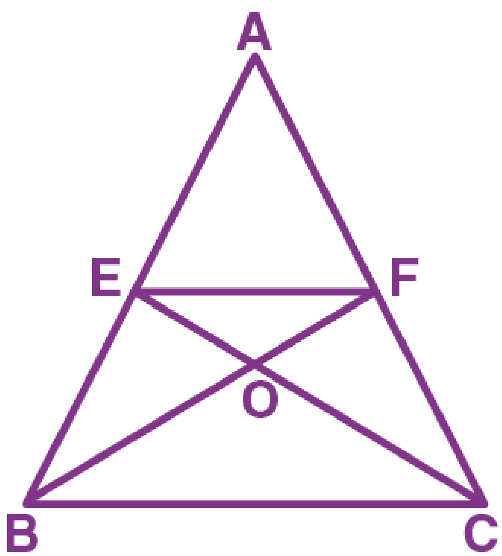
Area of the triangles with the same base and between the same parallels are equal.
We know that area of the triangle is half the area of the parallelogram if they lie on the same base and between the parallels.
Therefore, we have,
Area(ΔABD) = ½ × Area(||gm ABCD)
= 324/2
= 162 sq. cm
Also, from the diagram it is clear that
Area(ΔABD) = Area(ΔAPD) + Area(ΔBPD)
162 = Area(ΔAPD) + 2 × Area(ΔAPD)
162 = 3 × Area(ΔAPD)
Area(ΔAPD) = 162/3
= 54 sq. cm

(ii) Consider ΔAOP and ΔCOD
∠AOP = ∠COD [Vertically opposite angles]
∠CDO = ∠APD [AB || DC and DP is the transversal, alternate interior angles are equal]
Thus, ΔAOP ~ ΔCOD by AA similarity

Hence the corresponding sides are proportional.
AP/CD = OP/OD
= AP/AB
= AP/(AP+PB)
= AP/3AP
= 1/3
Therefore, OP: OD = 1: 3

3. In △ABC, E and F are mid-points of sides AB and AC respectively. If BF and CE intersect each other at point O, prove that the △OBC and quadrilateral AEOF are equal in area.
Solution:

Given, E and F are the midpoints of the sides AB and AC
Let's consider the following figure.



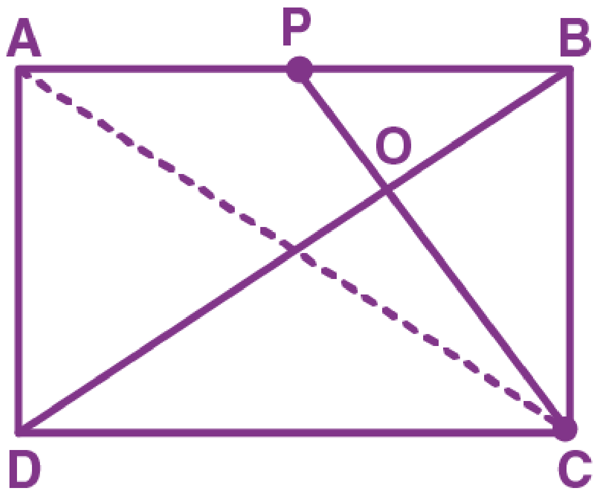
Therefore, by midpoint theorem
We have, EF || BC
Triangles BEF and CEF lie on the common base EF and between the parallels, EF and BC
Therefore, Ar(ΔBEF) = Ar(ΔCEF)
⇒ Ar(ΔBOE) + Ar(ΔEOF) = Ar(ΔEOF) + Ar(ΔCOF)
Ar(ΔBOE) = Ar(ΔCOF)

Now, BF and CE are the medians of the triangle ABC
And, median of the triangle divide it into two equal areas of triangles.
Thus, Ar(ΔABF) = Ar(ΔCBF)
Now, subtracting Ar(ΔBOE) on the both the sides, we get
Ar(ΔABF) – Ar(ΔBOE) = Ar(ΔCBF) – Ar(ΔBOE)
Since, Ar(ΔBOE) = Ar(ΔCOF)
⇒ Ar(ΔABF) – Ar(ΔBOE) = Ar(ΔCBF) – Ar(ΔCOF)
Ar(quad. AEOF) = Ar(ΔOBC)

- Hence proved

4. In parallelogram ABCD, P is mid-point of AB. CP and BD intersect each other at point O. If area of ∆POB = 40 cm^2, and OP: OC = 1: 2, find:
(i) Areas of ∆BOC and ∆PBC
(ii) Areas of ∆ABC and parallelogram ABCD.
Solution:

(i) On joining AC, we have the following figure as below



Let's consider ∆POB and ∆COD
∠POB = ∠DOC [Vertically opposite angles]
∠OPB = ∠ODC [Since, AB || DC; CP and BD are the transversal, alternate interior angles are equal]
Therefore, ∆POB ~ ∆COD by AA similarity criterion
As, P is the midpoint
AP = BP and AB = CD, we have CD = 2BP
Therefore, we have
BP/CD = OP/OC = OB/OD = ½
OP: OC = 1: 2

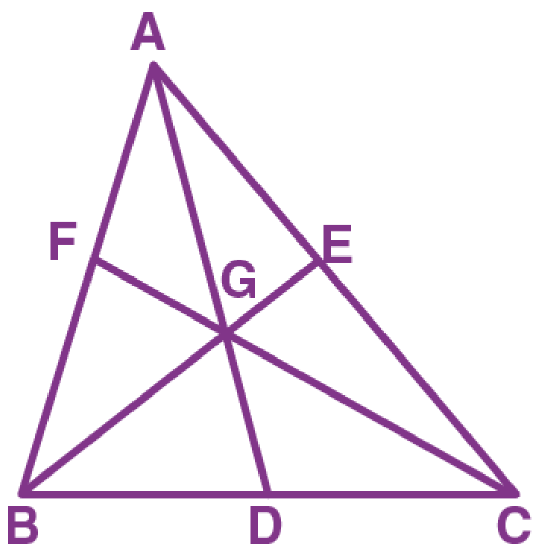
(ii) From (i), we have
BP/CD = OP/OC = OB/OD = ½,
Ratio between the areas of two similar triangles is equal to the ratio between the square of the corresponding sides
Here, ∆DOC and ∆POB are similar triangles.
Thus, we have
Ar(∆DOC)/Ar(∆POB) = DC^2/PB^2
Ar(∆DOC)/Ar(∆POB) = $(2PB)^2/PB^2$
Ar(∆DOC)/Ar(∆POB) = $4PB^2/PB^2$
Ar(∆DOC)/Ar(∆POB) = 4
Ar(∆DOC) = 4 × Ar(∆POB)

$= 4 \times 40$
$= 160\ cm^2$
Now, consider Ar(∆DBC) = Ar(∆DOC) + Ar(BOC)
$= 160 + 80$
$= 240\ cm^2$
Two triangles are equal in area if they are on the equal bases and between the same parallels
Therefore,
Ar(∆DBC) = Ar(∆ABC) = 240 cm^2
We know that,
Median divides the triangle into areas of two equal triangles
Thus, CP is the median of the triangle ABC.
Hence, Ar(∆ABC) = 2 × Ar(∆PBC)
Ar(∆PBC) = Ar(∆ABC)/2
$= 240/2$
$= 120 cm^2$

(iii) From (ii), we have
Ar(∆ABC) = 2 × Ar(∆PBC) 240 cm^2
Area of a triangle is half the area of the parallelogram
If both are on equal bases and between the same parallels
Thus, Ar(∆ABC) = ½ Ar(||gm ABCD)
⇒ Ar(llgm ABCD) = 2 Ar(∆ABC)
$= 2 \times 240$
$= 480\ cm^2$

5. The medians of a triangle ABC intersect each other at point G. If one of its medians is AD, prove that:
(i) Area (∆ABD) = 3 × Area (∆BGD)
(ii) Area (∆ACD) = 3 × Area (∆CGD)
(iii) Area (∆BGC) = 1/3 × Area (∆ABC)
Solution:

The figure is as shown below



We know that, medians interest at the centroid
Given that G is the point of intersect of medians
So, G is the centroid of the triangle ABC.
Now, the centroid divides the medians in the ratio 2:1

(i) We have, AG:GD = 2:1
So, Area(ΔAGB)/Area(ΔBGD) = 2/1
Area(ΔAGB) = 2Area(ΔBGD)
Now, from the figure, it is clearly seen that
Area(ΔABD) = Area(ΔAGB) + Area(ΔBGD)
Area(ΔABD) = 2 × Area(ΔBGD) + Area(ΔBGD)
Thus, Area(ΔABD) = 3 × Area(ΔBGD) … (1)

(ii) Similarly, CG divides AD in the ratio 2:1
So, Area(ΔAGC) / Area(ΔCGD) = 2/1
Area(ΔAGC) = 2 × Area(ΔCGD)
Now, from the figure, it is clearly seen that
Area(ΔACD) = Area(ΔAGC) + Area(ΔCGD)
Area(ΔACD) = 2 × Area(ΔCGD) + Area(ΔCGD)
Thus, Area(ΔACD) = 3 × Area(ΔCGD) … (2)

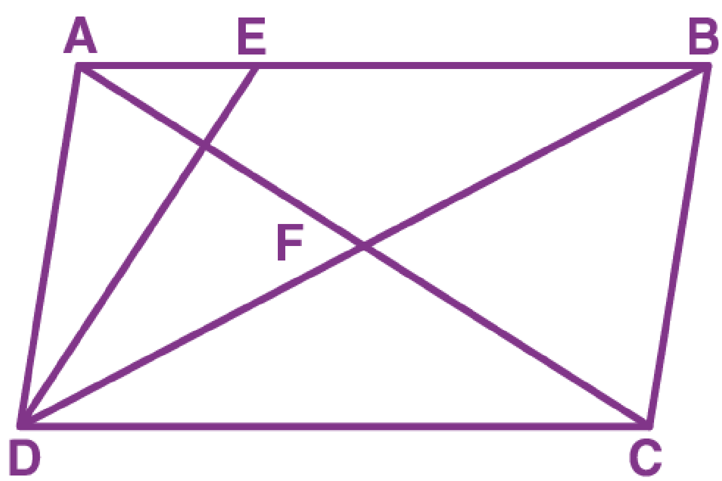
(iii) Adding equation (1) and (2), we have
Area(ΔABD) + Area(ΔACD) = 3Area(ΔBGD) + 3Area(ΔCGD)
Area(ΔABC) = 3 × [Area(ΔBGD0 + Area(ΔCGD)]
Area(ΔABC) = 3 × Area(ΔBGC)
Area(ΔABC)/3 = Area(ΔBGC)
Thus, Area(ΔBGC) = 1/3 × Area(ΔABC)

6. The perimeter of a triangle ABC is 37 cm and the ratio between the lengths of its altitudes be 6 : 5 : 4. Find the lengths of its sides.

Solution:

Let's consider the sides of ΔABC to be x cm, y cm and (37 – x – y) cm
Also, let the lengths of its altitudes be 6a cm, 5a cm and 4a cm
We know that,
Area of a triangle= ½ × base × altitude
⇒ ½ × 6a = ½ × y × 5a = ½ × (37 – x – y) × 4a
6x = 5y = 148 – 4x – 4y
6x = 5y and 6x = 148 – 4x – 4y
6x - 5y = 0 and 10x + 4y – 148 = 0
Now, by solving both the equations, we have
x = 10 cm and y = 12 cm
And, (37 – x – y) = 15cm

7. In parallelogram ABCD, E is a point in AB and DE meets diagonal AC at point F. If DF:FE = 5:3 and area of ΔADF is 60 cm^2; find
(i) Area of ΔADE
(ii) If AE:EB = 4:5, find the area of ΔADB
(iii) Also, find area of parallelogram ABCD
Solution:



Triangles ADF and AFE have the same vertex A and their bases are on the same straight line DE
Hence, Ar(ΔADF)/Ar(ΔAFE) = DF/FE
60/Ar(ΔAFE) = 5/3
Ar(ΔAFE) = (60 × 3)/5
= 36 cm^2
Now, Ar(ΔADE) = Ar(ΔADF) + Ar(ΔAFE)
= 60 cm^2 + 36 cm^2
= 96 cm^2

ΔADE and ΔEDB have their bases are on the same straight line AB

$\therefore$ Ar(ΔADE)/Ar(ΔEDB) = AE/EB

96/Ar(ΔEDB) = 4/5

Ar(ΔEDB) = (96 × 5)/4

= 120cm^2

Now, Ar(ΔADB) = Ar(ΔADE) + Ar(ΔEDB)

= 96 cm^2 + 120 cm^2

= 216 cm^2

Now,

ΔADB and $||^m$ ABCD are on the same base AB and between the same parallels AB and DC

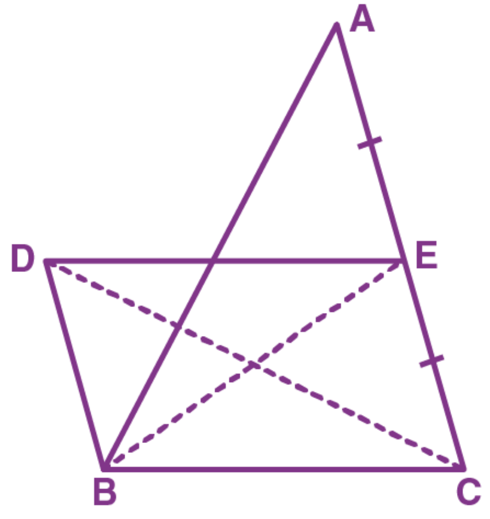
$\therefore$ Ar(ADB) = ½ Ar($||^m$ ABCD)

216 = ½ Ar($||^m$ ABCD)

Thus, Ar($||^m$ ABCD) = 2 × 216

= 432 cm^2

8. In the following figure, BD is parallel to CA, E is mid-point of CA and BD = ½ CA. Prove that: Ar (ABC) = 2 x Ar (DBC)



Solution:

Here, BCED is a parallelogram, since BD = CE and BD || CE.

Now,

Ar(ΔDBC) = Ar(ΔEBC) … (i) [Since they have the same base and area between the same parallels]

In ΔABC, we have

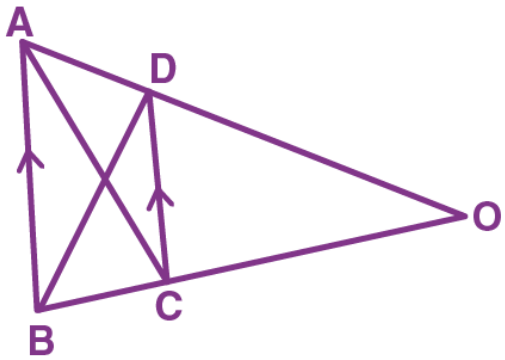
BE as the median

So, Ar(ΔEBC) = ½ [Ar(ΔEBC) + Ar(ΔABE)] = ½ Ar(ΔABC)

$\Rightarrow$ Ar(ΔABC) = 2Ar(ΔEBC)

Therefore, Ar(ΔABC) = 2Ar(ΔDBC)   [From (i)]

9. In the following figure, OAB is a triangle and AB || DC.

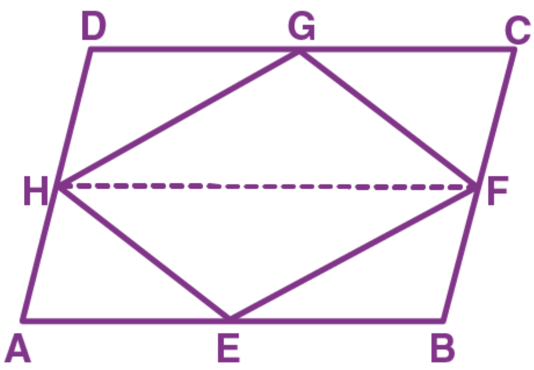


If the area of $\triangle CAD = 140\ cm^2$ and the area of $\triangle ODC = 172\ cm^2$, find
(i) the area of $\triangle DBC$
(ii) the area of $\triangle OAC$
(iii) the area of $\triangle ODB$.
Solution:

Given: $\triangle CAD = 140cm^2$, $\triangle ODC = 172cm^2$ and AB || CD
As triangles DBC and CAD have the same base CD and between the same parallel lines
Hence,
$Ar(\triangle DBC) = Ar(\triangle CAD) = 140cm^2$
$Ar(\triangle OAC) = Ar(\triangle CAD) + Ar(\triangle ODC)$
$= 140cm^2 + 172cm^2$
$= 312cm^2$
$Ar(\triangle ODB) = Ar(\triangle DBC) + Ar(\triangle ODC)$
$= 140cm^2 + 172cm^2$
$= 312cm^2$

10. E, F, G and H are the mid- points of the sides of a parallelogram ABCD. Show that area of quadrilateral EFGH is half of the area of parallelogram ABCD.
Solution:

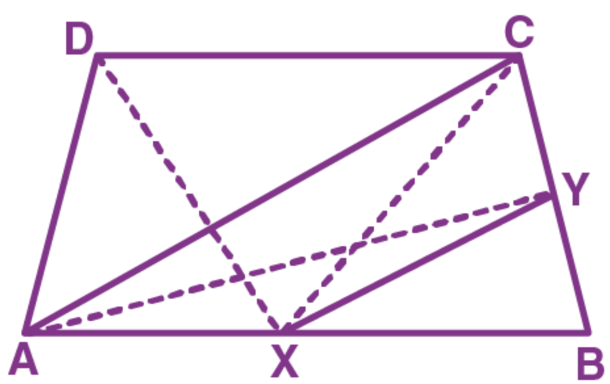
Let's join HF.



Since H and F are mid-points of AD and BC respectively, we have
AH = ½ AD and BF = ½ BC
Now, ABCD is a parallelogram
AD = BC and AD ∥ BC
⇒ ½ AD = ½ BC and AD ∥ BC
So, AH = BF and AH ∥ BF
Thus, ABFH is a parallelogram
Now,
Since parallelogram FHAB and ΔFHE are on the same base FH and between the same parallels HF and AB, we have
Ar(ΔFHE) = ½ A(∥^{m}FHAB) …(i)
Similarly,
Ar(ΔFHG) = ½ Ar(∥^{m}FHDC) …(ii)
Adding (i) and (ii), we get
Ar(ΔFHE) + Ar(ΔFHG) = ½ Ar(∥^{m}FHAB) + ½ Ar(∥^{m}FHDC)
Ar(EFGH) = ½ [Ar(∥^{m}FHAB) + ½ Ar(∥^{m}FHDC)]
∴ Ar(EFGH) = ½ Ar(∥^{m}ABCD)

11. ABCD is a trapezium with AB parallel to DC. A line parallel to AC intersects AB at X and BC at Y. Prove that area of △ADX = area of △ACY.
Solution:

Let's join CX, DX and AY.



Now, triangles ADX and ACX are on the same base AX and between the parallels AB and DC.
∴ Ar(ΔADX) = Ar(ΔACX) ….(i)
Also, triangles ACX and ACY are on the same base AC and between the parallels AC and XY.
∴ Ar(ΔACX) = Ar(ΔACY) ….(ii)
From (i) and (ii), we get
Ar(ΔADX) = Ar(ΔACY)
- Hence proved