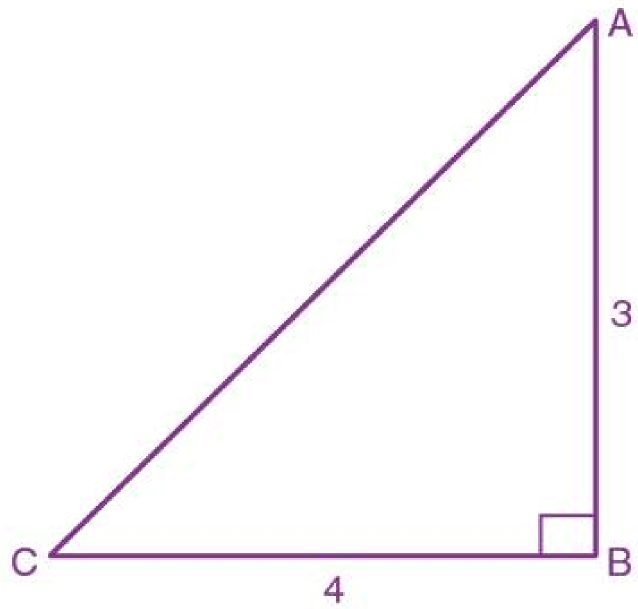
Exercise 22(A)

1. From the following figure, find the values of:
(i) sin A
(ii) cos A
(iii) cot A
(iv) sec C
(v) cosec C
(vi) tan C



Solution:

Given, $\angle ABC = 90^{\circ}$
$AC^2 = AB^2 + BC^2$ (AC is hypotenuse)
$AC^2 = 3^2 + 4^2$
$= 9 + 16$
$= 25$
Taking square root on both sides, we get
AC = 5 cm

(i) sin A = perpendicular/hypotenuse
= BC/AC
= 4/5

(ii) cos A = base/hypotenuse
= AB/AC
= 3/5

(iii) cot A = base/perpendicular

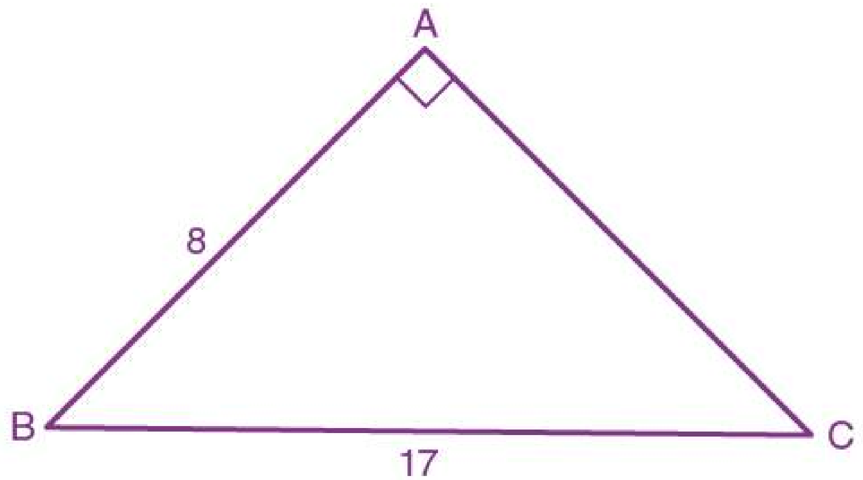
= AB/BC
= 3/4

(iv) sec C = hypotenuse/base
= AC/BC
= 5/4

(v) cosec C = hypotenuse/perpendicular
= AC/AB
= 5/3

(vi) tan C = perpendicular/base
= AB/BC
= 3/4

2. Form the following figure, find the values of:
(i) cos B
(ii) tan C
(iii) $\sin^2 B + \cos^2 B$
(iv) sin B. cos C + cos B. sin C



Solution:

Given, ∠BAC = 90^o
$BC^2 = AB^2 + AC^2$ (As BC is the hypotenuse)
$17^2 = 8^2 + AC^2$
$AC^2 = 289 – 64$
$= 225$
Taking square root on both sides, we get
AC = 15 cm

(i) cos B = base/hypotenuse
= AB/BC

= 8/17

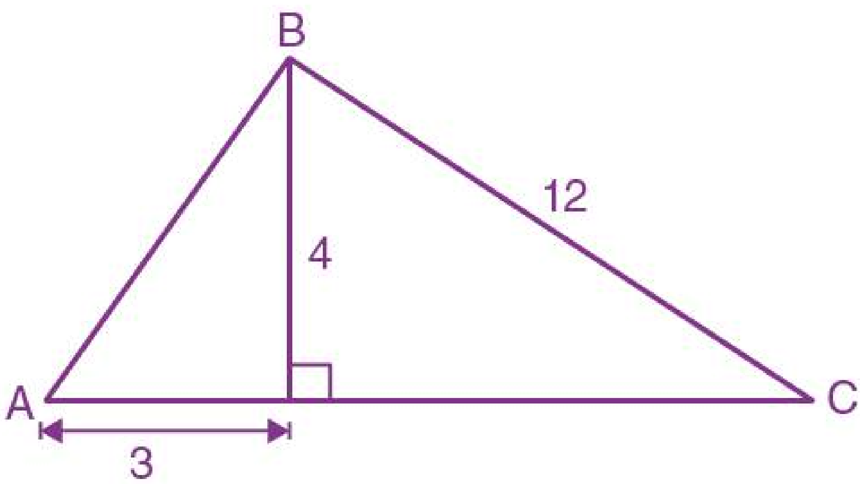
(ii) tan C = perpendicular/base
= AB/AC
= 8/15

(iii) sin B = perpendicular/hypotenuse
= AC/BC
= 15/17
cos B = base/hypotenuse
= AB/BC
= 8/17
Now,
$\sin^2 B + \cos^2 B = (15/17)^2 + (8/17)^2$
= (225 + 64)/289
= 289/289
= 1

(iv) sin B = perpendicular/hypotenuse
= AC/BC
= 15/17
cos B = base/hypotenuse
= AB/BC
= 8/17
sin C = perpendicular/hypotenuse
= AB/BC
= 8/17
cos C = base/hypotenuse
= AC/BC
= 15/17

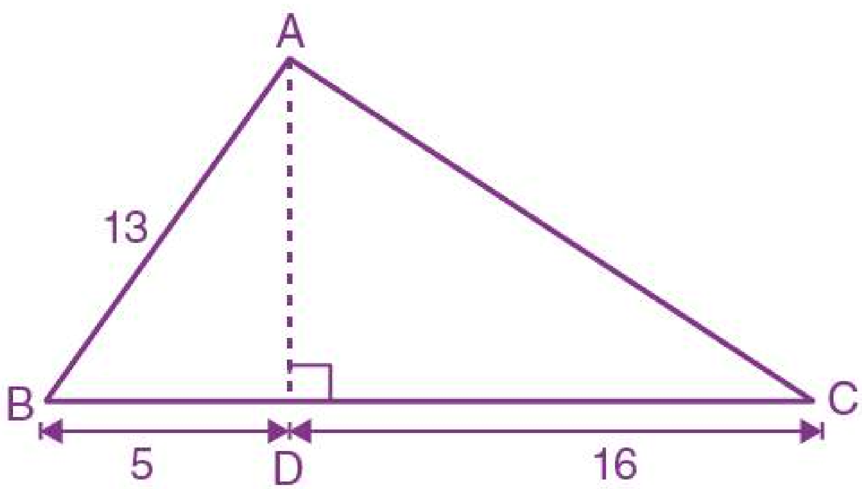
Now,
sin B. cos C + cos B. sin C = 15/17 x 15/17 + 8/17 x 8/17
= (225 + 64)/289
= 289/289
= 1

3. From the following figure, find the values of:
(i) cos A
(ii) cosec A
(iii) $\tan^2 A - \sec^2 A$
(iv) sin C
(v) sec C
(vi) $\cot^2 C - 1/\sin^2 C$



Solution:

Considering the given diagram, we have



$\angle ADB = 90^o$ and $\angle BDC = 90^o$
So, by Pythagoras theorem
$AB^2 = AD^2 + BD^2$ (As AB is the hypotenuse in ΔABD)
$= 3^2 + 4^2$
$= 9 + 16$
$= 25$
Taking square root on both sides, we get
AB = 5
Also,
$BC^2 = BD^2 + DC^2$ (As BC is the hypotenuse in ΔBDC)
$DC^2 = BC^2 - BD^2$
$= 12^2 - 4^2$
$= 144 – 16$
$= 128$

Taking square root on both sides, we get
DC = $8\sqrt{2}$

Now,
(i) cos A = base/hypotenuse
= AD/AB
= 3/5

(ii) cosec A = hypotenuse/perpendicular
= AB/BD
= 5/4

(iii) tan A = perpendicular/base
= BD/AD
= 4/3
sec A = hypotenuse/base
= AB/AD
= 5/3
$\tan^2 A - \sec^2 A = (4/3)^2 - (5/3)^2$
= 16/9 – 25/9
= -9/9
= -1

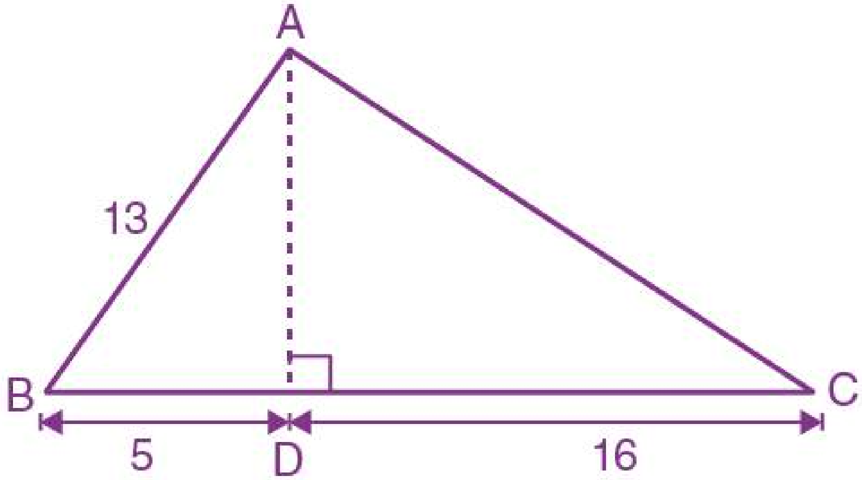
(iv) sin C = perpendicular/hypotenuse
= BD/BC
= 4/12
= 1/3

(v) sec C = hypotenuse/base
= BC/DC
= $12/8\sqrt{2}$
= $3/2\sqrt{2}$
= $3\sqrt{2}/4$

(vi) cot C = base/perpendicular
= DC/BD
= $8\sqrt{2}/4$
= $2\sqrt{2}$
sin C = perpendicular/hypotenuse
= BD/BC
= 4/12
= 1/3
Now,
$\cot^2 C - 1/\sin^2 C = (2\sqrt{2})^2 - 1/(1/3)^2$
= 8 – 1/(1/9)
= 8 – 9

= -1

4. From the following figure, find the values of:
(i) sin B
(ii) tan C
(iii) $\sec^2 B - \tan^2 B$
(iv) $\sin^2 C + \cos^2 C$



Solution:

From the figure, we have
∠ADB = 90º and ∠ADC = 90º
So, by Pythagoras theorem
$AB^2 = AD^2 + BD^2$ (As AB is the hypotenuse in ∆ABD)
$13^2 = AD^2 + 5^2$
$AD^2 = 13^2 - 5^2$
= 169 – 25
= 144
Taking square root on both sides, we get
AD = 12
Also,
$AC^2 = AD^2 + DC^2$ (As AC is the hypotenuse in ∆ADC)
$AC^2 = 12^2 + 16^2$
= 144 + 256
= 400
Taking square root on both sides, we get
AC = 20

Now,
(i) sin B = perpendicular/hypotenuse
= AD/AB
= 12/13

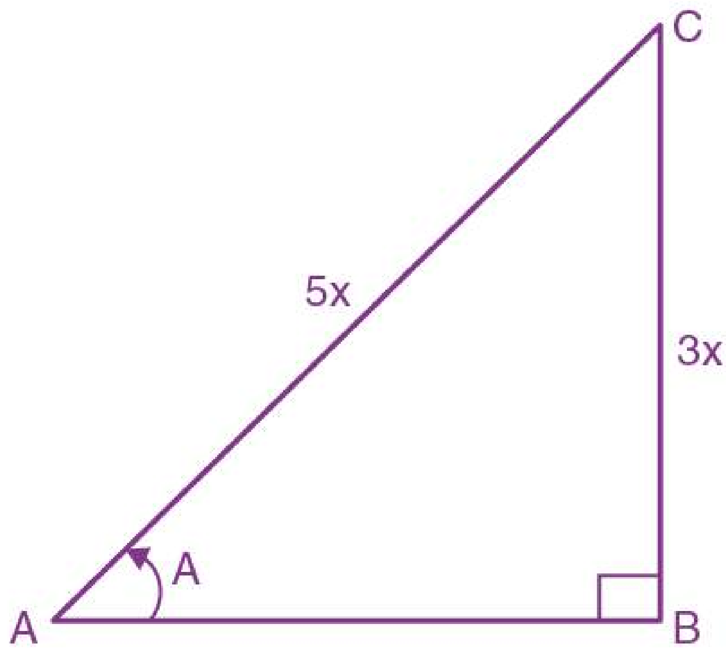
(ii) tan C = perpendicular/base
= 12/16
= ¾

(iii) sec B = hypotenuse/base
= AB/BD
= 13/5
tan B = perpendicular/base
= AD/BD
= 12/5
Hence,
$\sec^2 B - \tan^2 B = (13/5)^2 - (12/5)^2$
= (169 - 144)/25
= 25/25
= 1

(iv) sin C = perpendicular/hypotenuse
= AD/AC
= 12/20
= 3/5
cos C = base/hypotenuse
= DC/AC
= 16/20
= 4/5
Hence,
$\sin^2 C + \cos^2 C = (3/5)^2 + (4/5)^2$
= (9 + 16)/25
= 25/25
= 1

5. Given: sin A = 3/5, find:
(i) tan A
(ii) cos A
Solution:

Let's consider the diagram below:

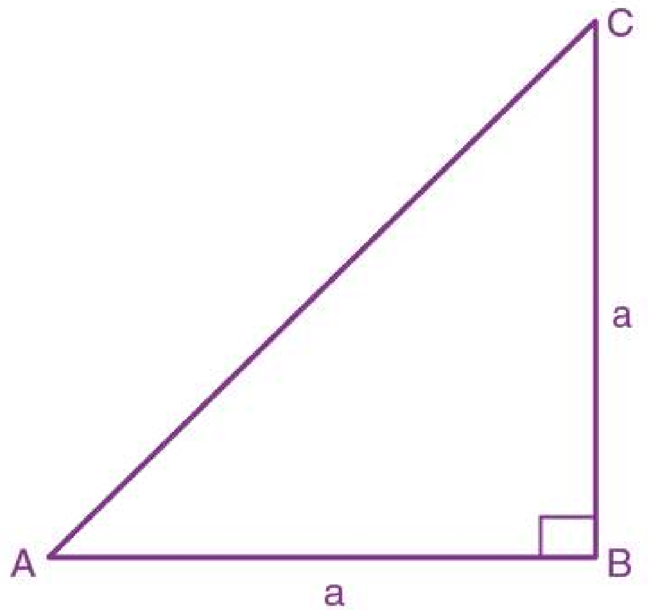


Given, sin A = 3/5
⇒ perpendicular/hypotenuse = 3/5
BC/AC = 3/5
Hence,
If the length of BC is 3x, the length of AC is 5x
We have,
$AB^2 + BC^2 = AC^2$ [By Pythagoras Theorem]
$AB^2 + (3x)^2 = (5x)^2$
$AB^2 = 25x^2 - 9x^2$
$= 16x^2$
Taking square root on both sides, we get
AB = 4x, which is the base

Now,
(i) tan A = perpendicular/base
= 3x/4x
= 3/4

(ii) cos A = base/hypotenuse
= 4x/5x
= 4/5

6. From the following figure, find the values of:
(i) sin A
(ii) sec A
(iii) $\cos^2 A + \sin^2 A$



Solution:

From the given figure, we have
∠ABC = 90º and AC is the hypotenuse ∆ABC
So, by Pythagoras Theorem
$AC^2 = AB^2 + BC^2$
$= a^2 + a^2$
$= 2a^2$
Taking square root on both sides, we get
AC = √2a

Now,
(i) sin A = perpendicular/hypotenuse
= BC/AB
= a/√2a
= 1/√2

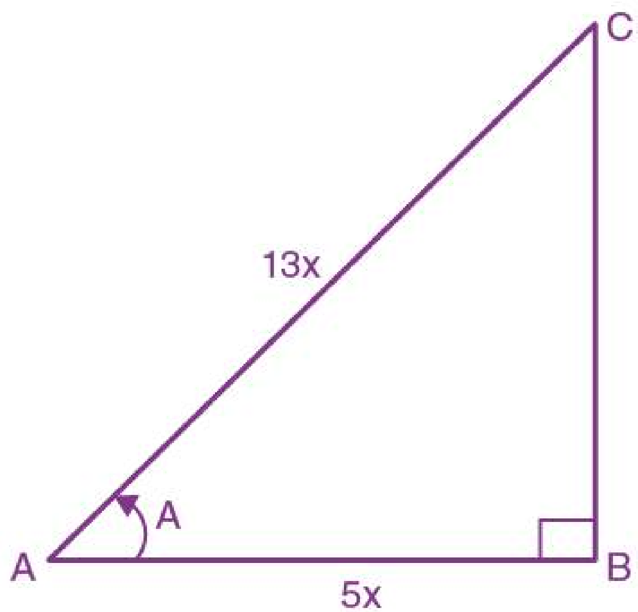
(ii) sec A = hypotenuse/base
= AC/AB
= √2a/a
= √2

(iii) sin A = perpendicular/hypotenuse
= BC/AC
= a/√2a
= 1/√2
cos A = base/hypotenuse

= AB/AC
= a/√2a
= 1/√2
Hence,
$\cos^2 A + \sin^2 A = (1/\sqrt{2})^2 + (1/\sqrt{2})^2$
= ½ + ½
= 1

7. Given: cos A = 5/13
Evaluate: (i) (sin A – cot A)/2tan A (ii) cot A + 1/cos A
Solution:

Let's consider the following diagram:



Given, cos A = 5/13
⇒ base/hypotenuse = 5/13
AB/AC = 5/13
Hence,
If length of AB = 5x, the length of AC = 13x
So, by Pythagoras Theorem
$AB^2 + BC^2 = AC^2$
$(5x)^2 + BC^2 = (13x)^2$
$BC^2 = 169x^2 – 25x^2$
$= 144x^2$
Taking square root on both sides, we get
BC = 12x, which is the perpendicular

Now,

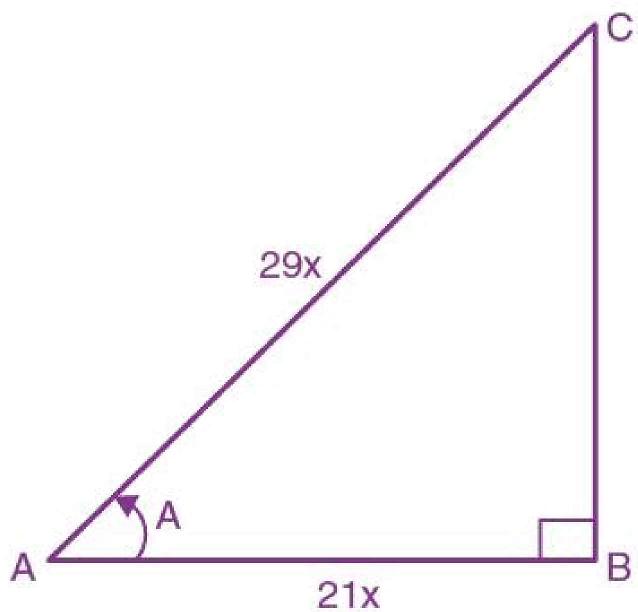
tan A = perpendicular/base
= 12x/5x
= 12/5
sin A = perpendicular/base
= 12x/13x
= 12/13
cot A = base/perpendicular
= 5x/12x
= 5/12

(i) (sin A – cot A)/2tan A = [(12/13) – (5/12)]/ 2(12/5)
= 79/156 x 5/24
= 395/3744

(ii) cot A + 1/cos A = 5/12 + 1/(5/13)
= 5/12 + 13/5
= 181/60

8. Given: sec A = 29/21, evaluate: sin A – 1/tan A
Solution:

Let's consider the diagram below:



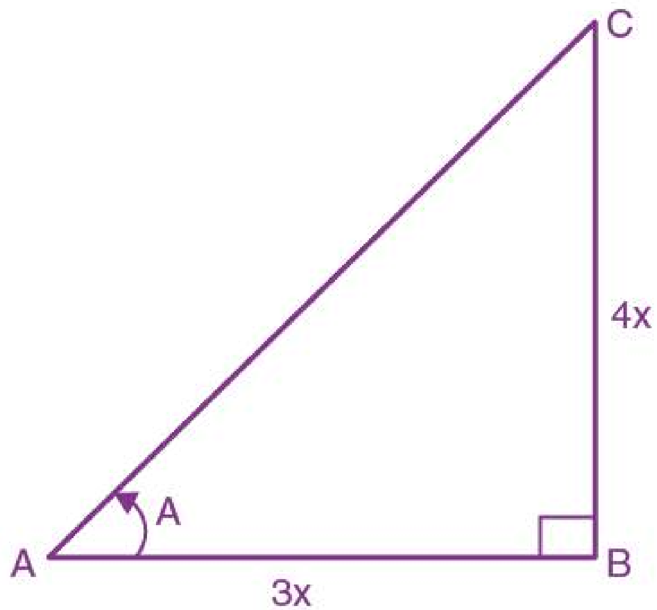
Given, sec A = 29/21
⇒ hypotenuse/base = 29/21
AC/AB = 29/21
Hence,
If length of AB = 21x, the length of AC = 29x

So, by Pythagoras Theorem
$AB^2 + BC^2 = AC^2$
$(21x)^2 + BC^2 = (29x)^2$
$BC^2 = 841x^2 – 441x^2$
$= 400x^2$
Taking square root on both sides, we get
BC = 20x, which is the perpendicular

Now,
sin A = perpendicular/hypotenuse
= 20x/29x
= 20/29
tan A = perpendicular/base
= 20x/21x
= 20/21
Therefore,
sin A – 1/tan A = 20/29 – 1/(20/21)
= 20/29 – 21/20
= - 209/580

9. Given: tan A = 4/3, find: cosec A/(cot A – sec A)
Solution:

Let's consider the diagram below:



Given, tan A = 4/3
⇒ perpendicular/base = 4/3
BC/AB = 4/3

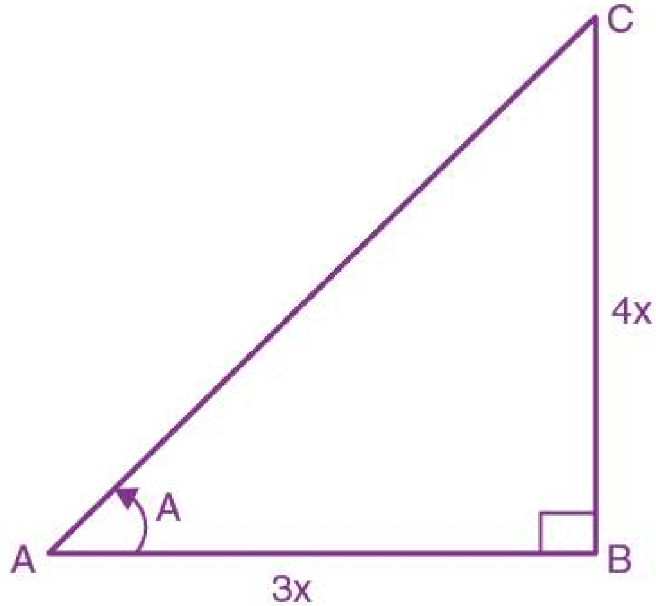
Hence,
If length of AB = 3x, the length of BC = 4x
So, by Pythagoras Theorem
$AB^2 + BC^2 = AC^2$
$(3x)^2 + (4x)^2 = AC^2$
$AC^2 = 9x^2 + 16x^2$
$= 25x^2$
Taking square root on both sides, we get
AC = 5x, which is the hypotenuse

Now,
sec A = hypotenuse/base
= AC/AB
= 5x/3x
= 5/3
cot A = base/perpendicular
= AB/BC
= 3x/4x
= ¾
cosec A = hypotenuse/perpendicular
= AC/BC
= 5x/4x
= 5/x

Therefore,
cosec A/(cot A – sec A) = (5/4)/(3/4 – 5/3)
= (5/4)/(-11/12)
= - 60/44
= - 15/11

10. Given: 4 cot A = 3, find;
(i) sin A
(ii) sec A
(iii) $cosec^2$ A - cot^2A.
Solution:

Let's consider the diagram below:



Given, 4 cot A = 3
cot A = 3/4
⇒ base/perpendicular = 4/3
AB/BC = 3/4
Hence,
If length of AB = 3x, the length of BC = 4x
So, by Pythagoras Theorem
$AB^2 + BC^2 = AC^2$
$(3x)^2 + (4x)^2 = AC^2$
$AC^2 = 9x^2 + 16x^2$
$= 25x^2$
Taking square root on both sides, we get
AC = 5x, which is the hypotenuse

Now,
(i) sin A = perpendicular/hypotenuse
= 4x/5x
= 4/5

(ii) sec A = hypotenuse/base
= AC/AB
= 5x/3x
= 5/3

(iii) cosec A = hypotenuse/perpendicular
= AC/BC
= 5x/4x

$= 5/4$

$\cot A = 3/4$

Hence,

$\text{cosec}^2 A - \cot^2 A = (5/4)^2 - (3/4)^2$

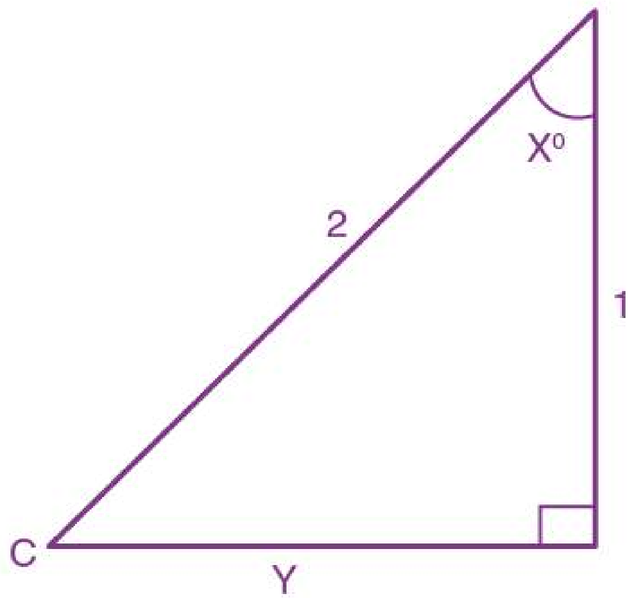
$= (25 - 9)/16$

$= 16/16$

$= 1$

Exercise 22(B)

1. From the following figure, find:
(i) y
(ii) sin x^o
(iii) (sec x^o - tan x^o) (sec x^o + tan x^o)



Solution:

In the given figure,
(i) As it's a right-angled triangle, so using Pythagorean Theorem
$2^2 = y^2 + 1^2$
$y^2 = 2^2 - 1^2$
$= 4 - 1$
$= 3$
Taking square root on both sides, we get
$y = \sqrt{3}$

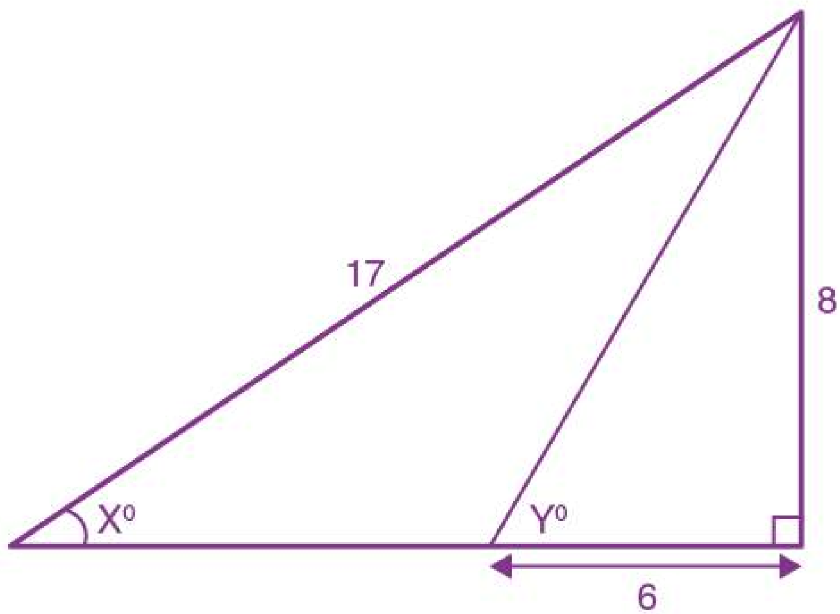
(ii) sin x^o = perpendicular/hypotenuse
$= \sqrt{3}/2$

(iii) tan x^o = perpendicular/base
$= \sqrt{3}$
sec x^o = hypotenuse/base
$= 2$

Therefore,
(sec x^o – tan x^o) (sec x^o + tan x^o) = $(2 - \sqrt{3})(2 + \sqrt{3})$
$= 4 - \sqrt{3}$

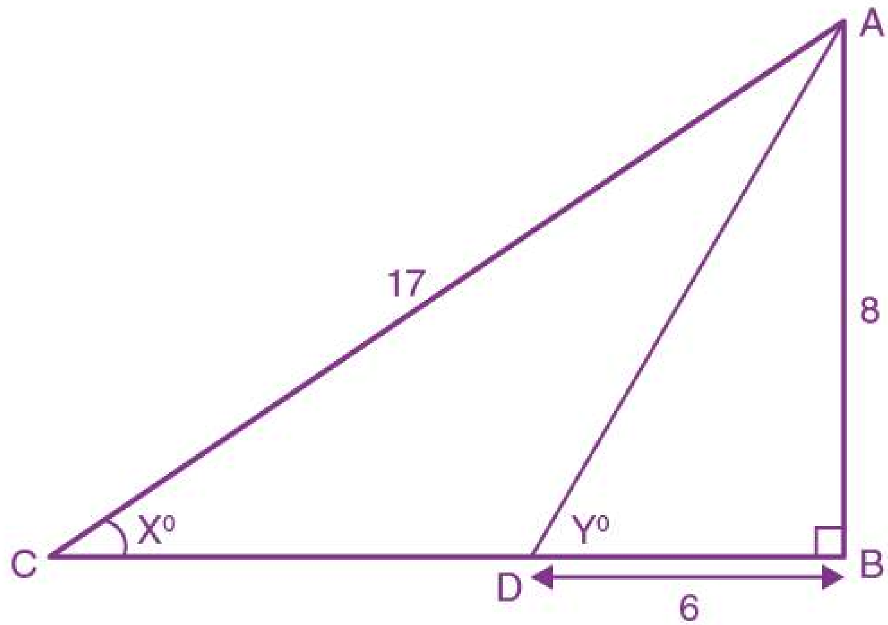
= 1

2. Use the given figure to find:
(i) sin x°
(ii) cos y°
(iii) 3 tan x° - 2 sin y° + 4 cos y°



Solution:

Let's consider the given figure,



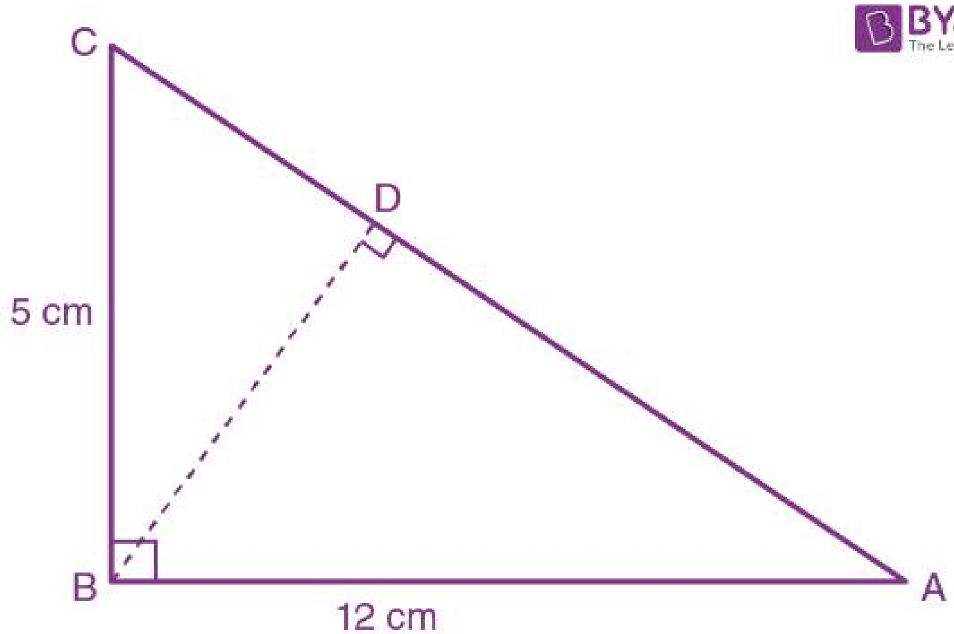
As the triangle is a right-angled triangle, so using Pythagorean Theorem
$AD^2 = 8^2 + 6^2$
$= 64 + 36$
$= 100$
Taking square root on both sides, we get
AD = 10
Also, by Pythagorean Theorem
$BC^2 = AC^2 – AB^2$
$= 17^2 - 8^2$
$= 289 – 64$
$= 225$
Taking square root on both sides, we get
BC = 15

Now,
(i) sin x^o = perpendicular/hypotenuse
= 8/17

(ii) cos y^o = base/hypotenuse
= 6/10
= 3/5

(iii) sin y^o = perpendicular/base
= AB/AD
= 8/10
= 4/5
And,
cos y^o = 6/10
= 3/5
So,
tan x^o = perpendicular/base
= AB/BC
= 8/15
Therefore,
3 tan x^o – 2 sin y^o + 4 cos y^o
= 3(8/15) – 2(4/5) + 4(3/5)
= 8/5 – 8/5 + 12/5
= 12/5

3. In the diagram, given below, triangle ABC is right-angled at B and BD is perpendicular to AC. Find:
(i) cos ∠DBC (ii) cot ∠DBA



Solution:

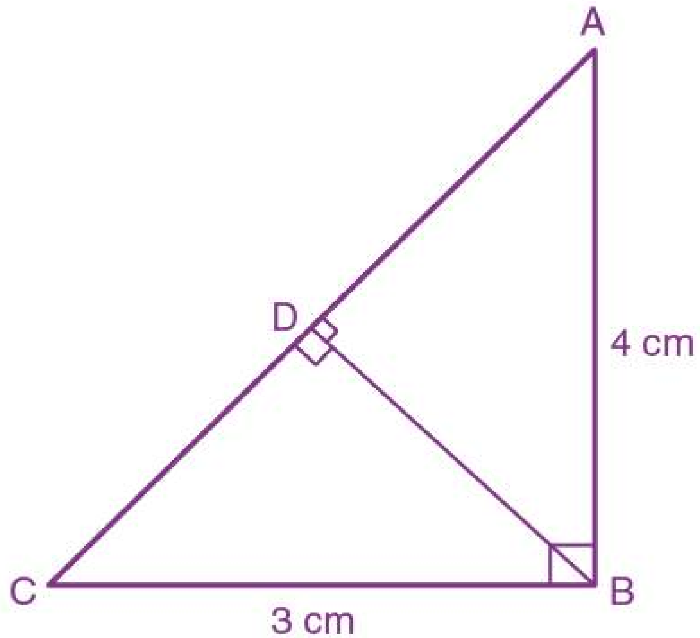
Let's consider the given figure,
As the triangle is a right-angled triangle, so using Pythagorean Theorem
$AC^2 = 5^2 + 12^2$
$= 25 + 144$
$= 169$
Taking square root on both sides, we get
AC = 13
In ΔCBD and ΔCBA,
$\angle$C is common to both the triangles
$\angle CDB = \angle CBA = 90^o$
Hence, $\angle CBD = \angle CAB$
Thus, ΔCBD and ΔCBA are similar triangles according to AAA criterion
So, we have
AC/BC = AB/BD
13/5 = 12/BD
BD = 60/13

Now,
(i) cos $\angle$DBC = base/hypotenuse
= BD/BC
= (60/13)/5
= 12/13

(ii) cot $\angle$DBA = base/perpendicular
= BD/AB

= (60/13)/12
= 5/13

4. In the given figure, triangle ABC is right-angled at B. D is the foot of the perpendicular from B to AC. Given that BC = 3 cm and AB = 4 cm. Find:
(i) tan ∠DBC
(ii) sin ∠DBA



Solution:

Considering the given figure, we have
A right-angled triangle ABC, so by using Pythagorean Theorem we have
$AC^2 = BC^2 + AB^2$
$= 4^2 + 3^2$
= 16 + 9
= 25
Taking square root on both sides, we get
AC = 5

In ∆CBD and ∆CAB, we have
∠BCD = ∠ACB (Common)
∠CDB = ∠CBA = 90^o
Hence, ∆CBD ~ ∆CAB by AA similarity criterion
So,
AC/BC = AB/BD
5/3 = 4/BD
BD = 12/5

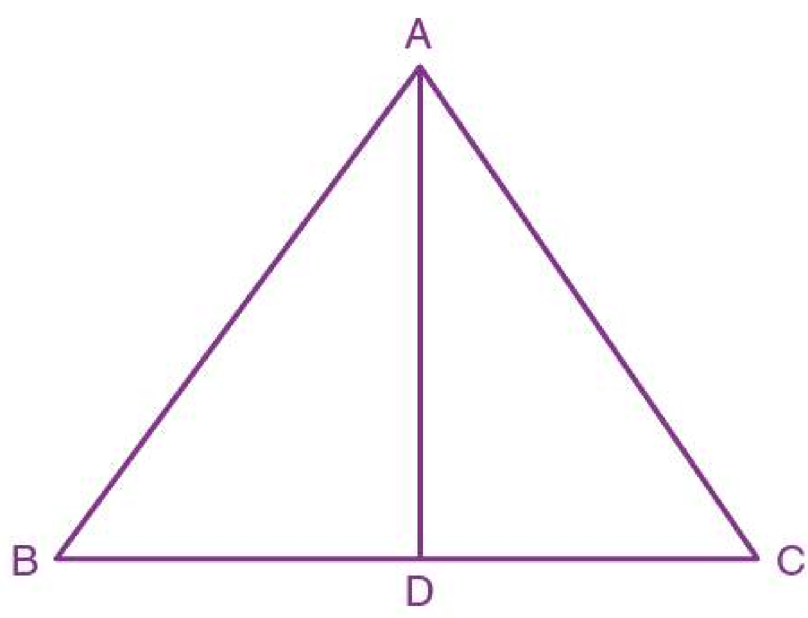
Now, using Pythagorean Theorem in ∆BDC
$DC^2 = BC^2 – BD^2$
$= 3^2 – (12/5)^2$
= 9 – 144/25
= (225 – 144)/25
= 81/25
Taking square root on both sides, we get
DC = 9/5
Therefore,
AD = AC – DC
= 5 – 9/5
= 16/5

Now,
(i) tan ∠DBC = perpendicular/ base
= DC/BD
= (9/5)/(12/5)
= 3/4

(ii) sin ∠DBA = AD/AB
= (16/5)/4
= 4/5

5. In triangle ABC, AB = AC = 15 cm and BC = 18 cm, find cos ∠ABC.
Solution:

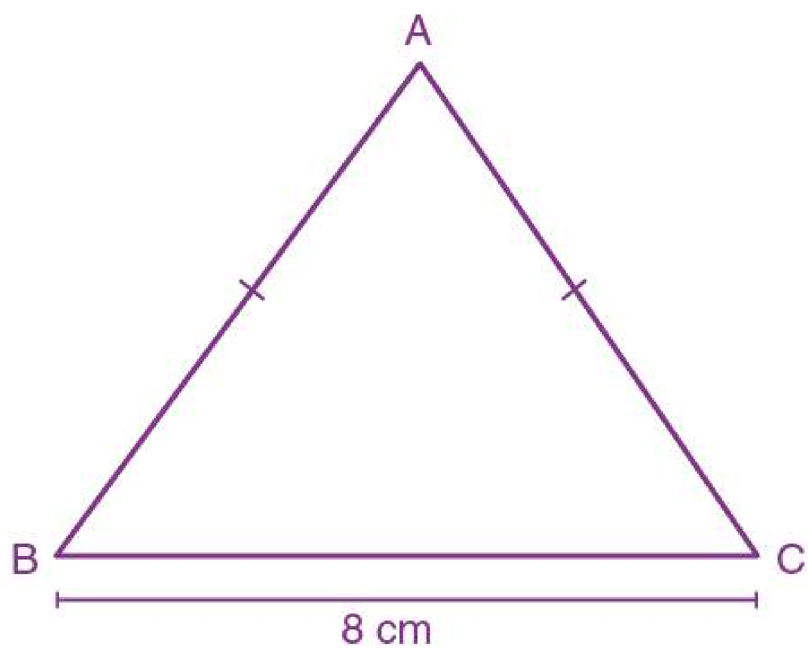
Let's consider the figure below:



In the isosceles ∆ABC, we have

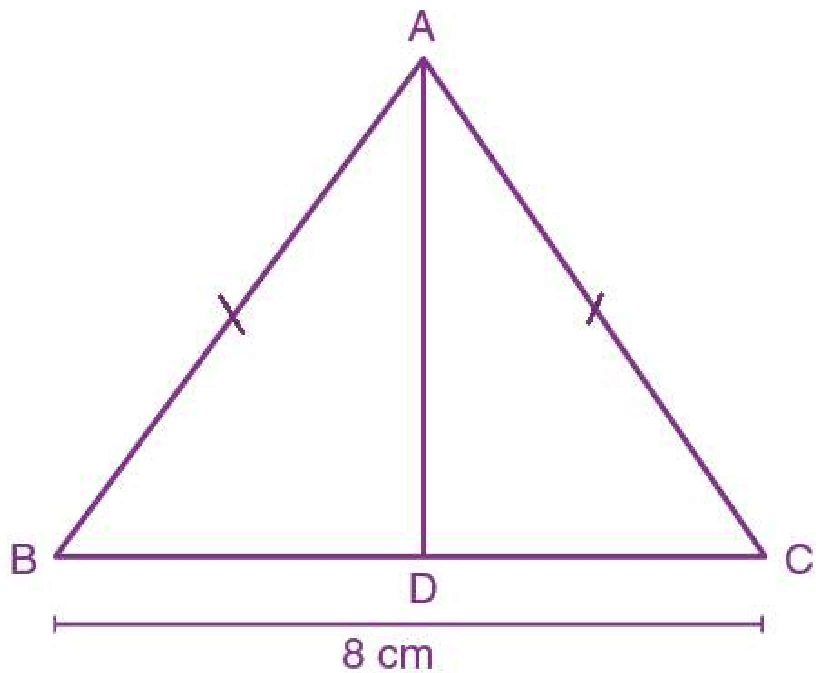
AB = AC = 15 cm
BC = 18 cm
Now, the perpendicular drawn from angle A to its opposite BC divides its into two equal parts
i.e., BD = DC = 9cm
Hence,
cos ∠ABC = base/hypotenuse
= BD/AB
= 9/15
= 3/5

6. In the figure given below, ABC is an isosceles triangle with BC = 8 cm and AB = AC = 5 cm. Find:
(i) sin B
(ii) tan C
(iii) $\sin^2 B + \cos^2 B$
(iv) tan C - cot B



Solution:

Let's consider the figure below:



In the isosceles ΔABC, we have
AB = AC = 5 cm
BC = 8 cm
Now, the perpendicular drawn from angle A to its opposite BC divides its into two equal parts
i.e., BD = DC = 4cm

As, ∠ADB = 90^o in ΔABD, we have
$AB^2 = AD^2 + BD^2$
$AD^2 = AB^2 - BD^2$
$= 5^2 - 4^2$
$= 25 – 16$
$= 9$
Taking square root on both sides, we get
AD = 3

Now,
(i) sin B = AD/AB
= 3/5

(ii) tan C = AD/DC
= 3/4

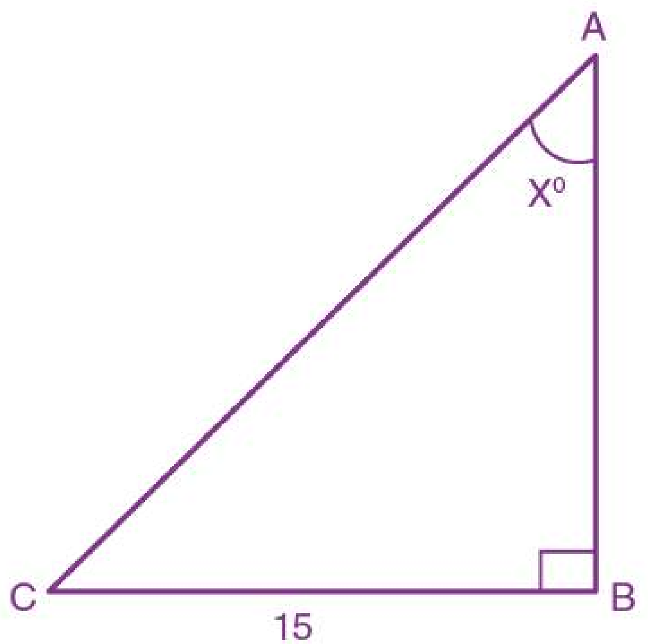
(iii) sin B = AD/AB
= 3/5
cos B = BD/AB
= 4/5
Hence,

$\sin^2 B + \cos^2 B = (3/5)^2 + (4/5)^2$
$= 9/25 + 16/25$
$= 25/25$
$= 1$

(iv) tan C = AD/DC
= ¾
cot B = BD/AD
= 4/3
Hence,
tan C – cot B = ¾ - 4/3
= (9 - 16)/12
= -7/12

7. In triangle ABC; ∠ABC = 90º, ∠CAB = xº, tan xº = ¾ and BC = 15 cm. Find the measures of AB and AC.
Solution:

Let's consider the figure below:



Given, tan xº = ¾
⇒ perpendicular/base = ¾
BC/AB = ¾
Hence,
If length of base AB = 4x, the length of perpendicular BC = 3x
So, by Pythagoras Theorem
$BC^2 + AB^2 = AC^2$
$(3x)^2 + (4x)^2 = AC^2$

$AC^2 = 9x^2 + 16x^2$

$= 25x^2$

Taking square root on both sides, we get

AC = 5x, which is the hypotenuse

Now, we have

BC = 15

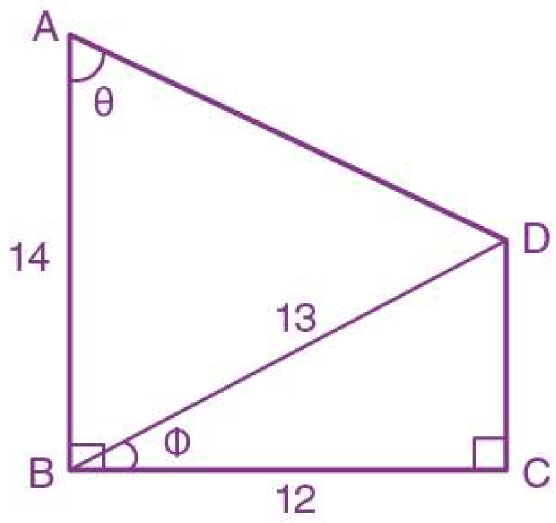
⇒ 3x = 15

x = 5

Therefore, AB = 4x = 4(5) = 20 cm

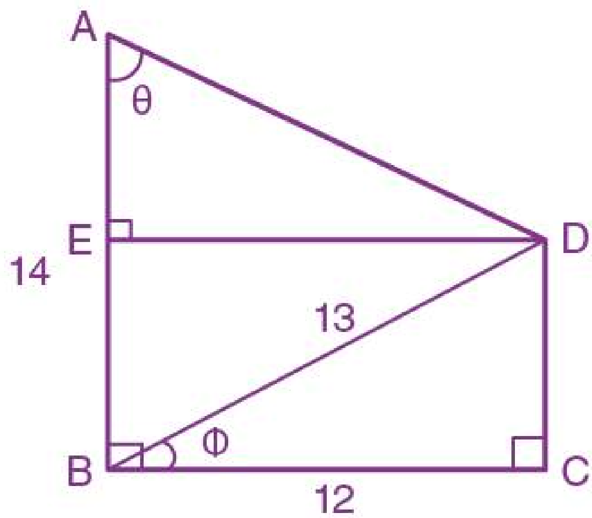
And, AC = 5x = 5 × 5 = 25 cm

8. Using the measurements given in the following figure:
(i) Find the value of sin Ø and tan θ
(ii) Write an expression for AD in terms of θ



Solution:

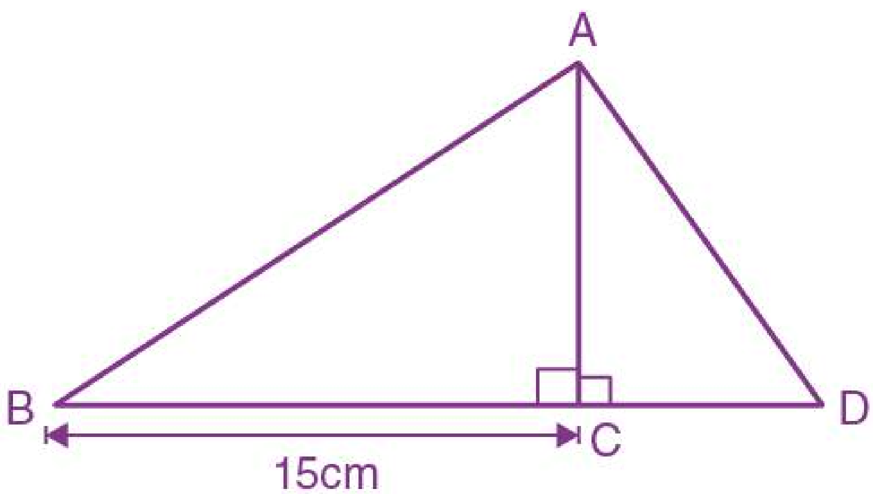
Let's consider the figure below:



Constructing a perpendicular from D to the side AB at point E which makes BCDE a rectangle.
Now, in right angled ∆BCD using Pythagorean Theorem, we have
$BD^2 = BC^2 + CD^2$ [As AB is the hypotenuse]
$CD^2 = BD^2 - BC^2$
$CD^2 = 13^2 - 12^2$
= 169 – 144
= 25
Taking square root on both sides, we get
CD = 5
As BCDE is a rectangle,
ED = 12 cm, EB = 5 cm and AE = (14 – 5) cm = 9 cm
Now,
(i) sin Ø = CD/BD
= 5/13
tan θ = ED/AE
= 12/9
= 4/3

(ii) sec θ = AD/AE
= AD/9
AD = 9 sec θ
Or
cosec θ = AD/ED
= AD/12
AD = 12cosec θ

9. In the given figure:
BC = 15 cm and sin B = 4/5



(i) Calculate the measure of AB and AC.
(ii) Now, if tan ∠ADC = 1; calculate the measures of CD and AD.
Also, show that: $\tan^2 B - 1/\cos^2 B = -1$
Solution:

Given, BC = 15 cm and sin B = 4/5
⇒ Perpendicular/hypotenuse = AC/AB
= 4/5
Hence, if the length of perpendicular is 4x, the length of hypotenuse will be 5x
In right triangle ABC, we have
$BC^2 + AC^2 = AB^2$ [By Pythagoras Theorem]
$BC^2 = AB^2 - AC^2$
$= (5x)^2 - (4x)^2$
$= 25x^2 - 16x^2$
$= 9x^2$
Taking square root on both sides, we get
BC = 3x
Now, as BC = 15 (given)
3x = 15
x = 15/3
x = 5

(i) AC = 4x
= 4(5)
= 20 cm
And,
AB = 5x
= 5(5)
= 25 cm

(ii) Given,

tan ∠ADC = 1
perpendicular/base = AC/CD
= 1/1
Hence,
If length of perpendicular is x, then the length of hypotenuse will be x
And, we have
$AC^2 + CD^2 = AD^2$ [Using Pythagoras Theorem]
$x^2 + x^2 = AD^2$
$AD^2 = 2x^2$
Taking square root on both sides, we get
AD = $\sqrt{2}x$

Now,
AC = 20 ⇒x = 20
So,
AD = $\sqrt{2}x = \sqrt{2}(20) = 20\sqrt{2}$ cm
And,
CD = 20 cm
Hence,
tan B = AC/BC
= 20/15
= 4/3
cos B = BC/AB
= 15/25
= 3/5
Thus,
$\tan^2 B – 1/\cos^2 B = (4/3)^2 – 1/(3/5)^2$
= 16/9 – 1/(9/25)
= 16/9 – 25/9
= - 9/9
= -1

10. If sin A + cosec A = 2;
Find the value of $\sin^2 A + \text{cosec}^2 A$.
Solution:

Given, sin A + cosec A = 2
On squaring on both sides, we have
$(\sin A + \text{cosec } A)^2 = 2^2$
$\sin^2 A + \text{cosec}^2 A$ + 2sinA. cosec A = 4
$\sin^2 A + \text{cosec}^2 A + 2 = 4$ [As sin A. cosec A = sin A x 1/sin A = 1]
$\sin^2 A + \text{cosec}^2 A = 4 – 2 = 2$
Hence, the value of $(\sin^2 A + \text{cosec}^2 A)$ is 2