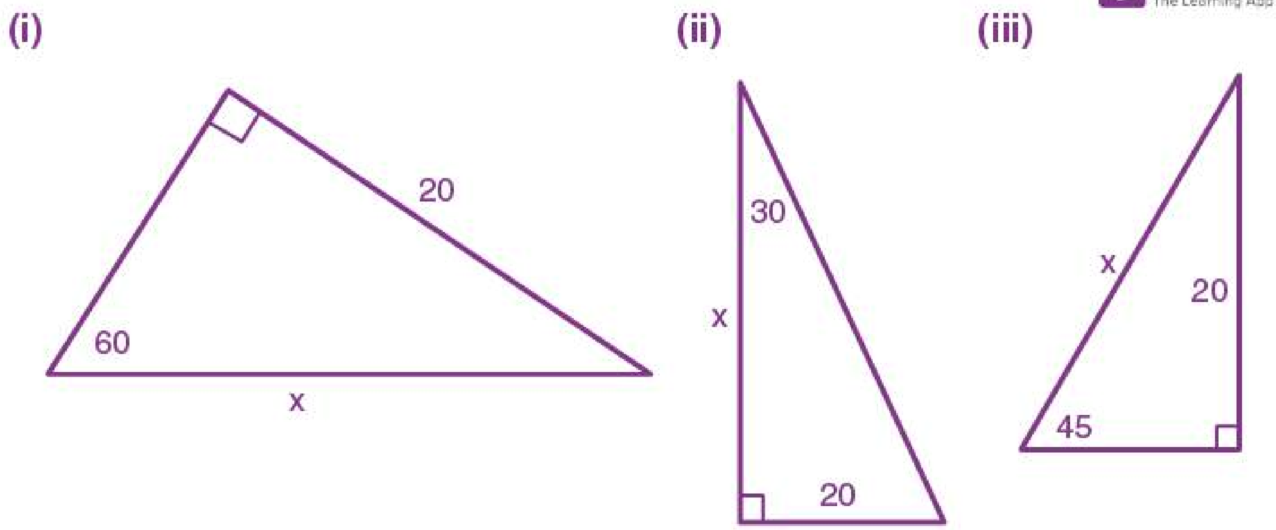
Exercise 24

1. Find 'x' if:



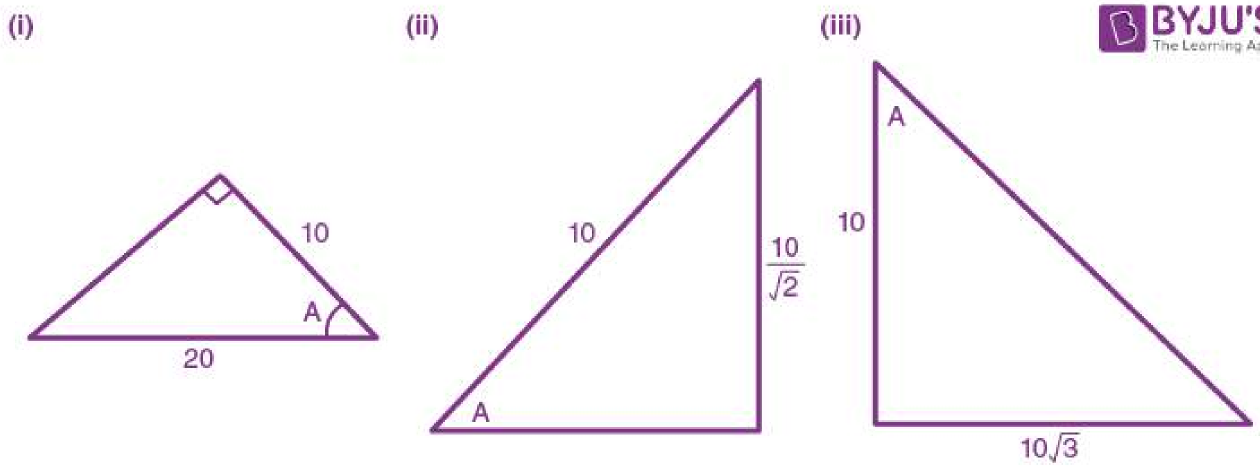
Solution:

(i) From the figure, we have
sin 60º = 20/x
√3/2 = 20/x
∴ x = 40/√3

(ii) From the figure, we have
tan 30º = 20/x
1/√3 = 20/x
∴ x = 20√3

(iii) From the figure, we have
sin 45º = 20/x
1/√2 = 20/x
∴ x = 20√2

2. Find angle 'A' if:



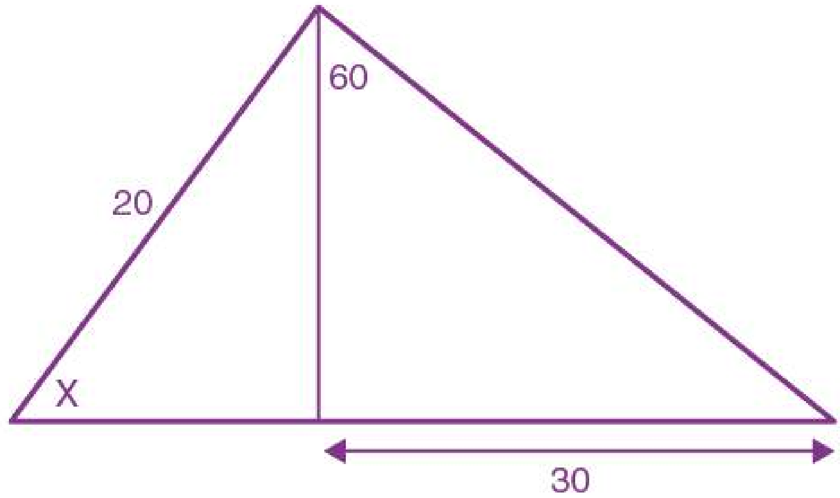
Solution:

(i) From the figure, we have
cos A = 10/20
= ½
cos A = cos 60°
Hence,
A = 60°

(ii) From the figure, we have
sin A = (10/√2)/10
= 1/√2
sin A = sin 45°
Hence,
A = 45°

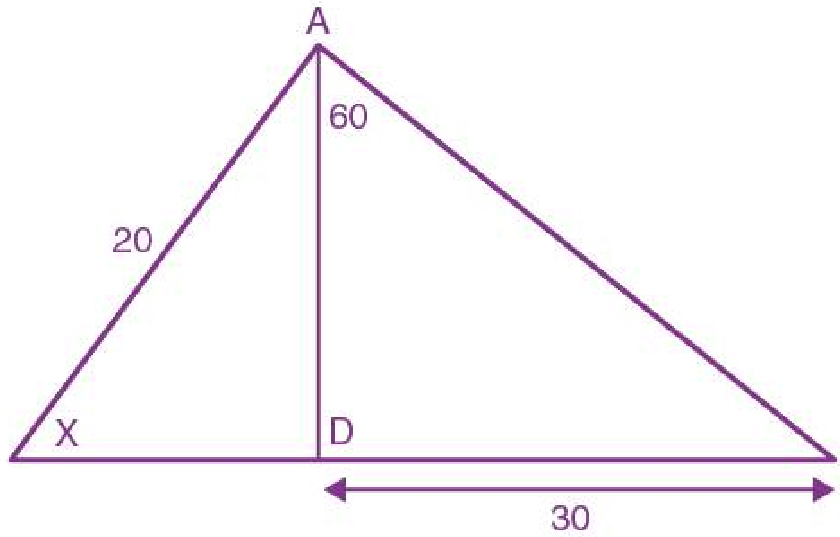
(iii) From the figure, we have
tan A = (10√3)/10
= √3
tan A = tan 60°
Hence,
A = 60°

3. Find angle 'x' if:



Solution:

The given figure is drawn as follows:



We have,

$\tan 60^o = 30/AD$

$\sqrt{3} = 30/AD$

$AD = 30/\sqrt{3}$

Again,

$\sin x = AD/20$

$AD = 20 \sin x$

Now,

$20 \sin x = 30/\sqrt{3}$

$\sin x = 30/20\sqrt{3}$

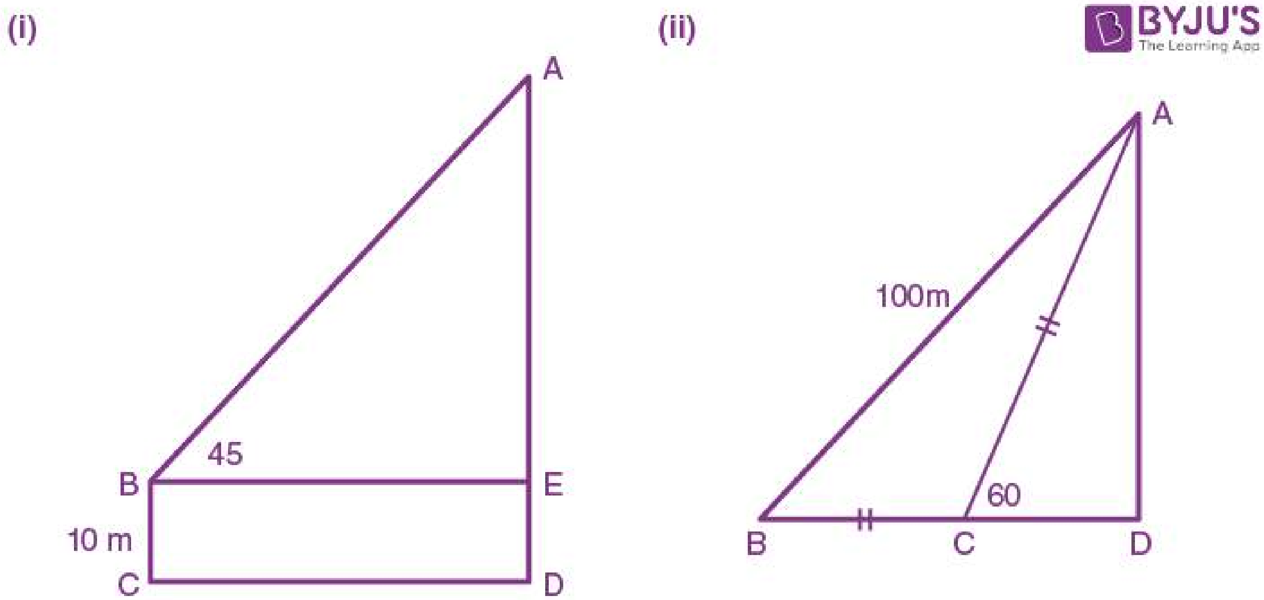
$\sin x = 3/2\sqrt{3}$

$\sin x = \sqrt{3}/2$

$\sin x = \sin 60^o$

⇒ $x = 60^o$

4. Find AD, if:

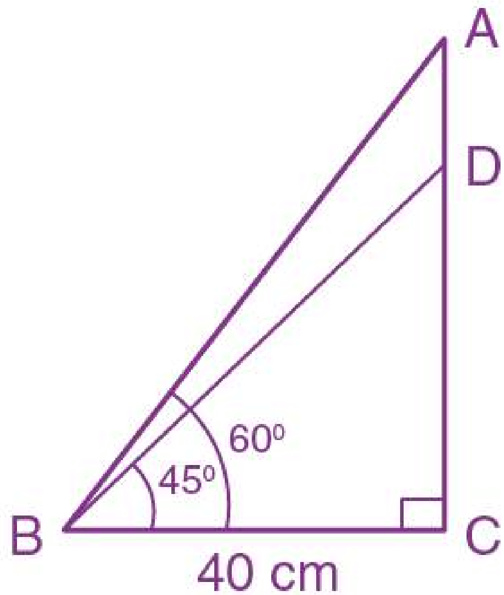


Solution:

(i) In right ∆ABE, we have
$\tan 45^o = AE/BE$
$1 = AE/BE$
$AE = BE$
Thus, AE = BE = 50 m
Now,
In rectangle BCDE, we have
DE = BC = 10 m
Thus, the length of AD is given by
AD = AE + DE
= 50 + 10
= 60 m

(ii) In right ∆ABD, we have
sin B = AD/AB
$\sin 30^o = AD/100$ [As ∠ACD is the exterior angle of ∆ABC]
½ = AD/100
⇒ AD = 50 m

5. Find the length of AD.
Given: ∠ABC = 60º, ∠DBC = 45º and BC = 40 cm

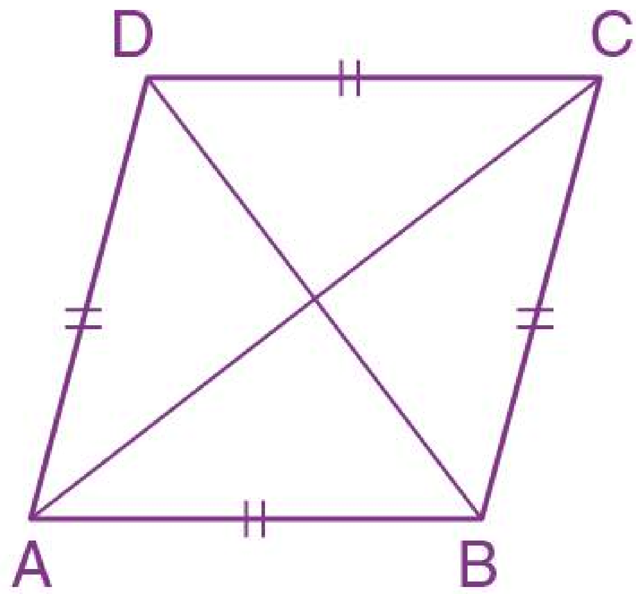


Solution:

In right ΔABC, we have
tan 60° = AC/BC
$\sqrt{3}$ = AC/40
AC = 40$\sqrt{3}$ cm
Next,
In right ΔBDC, we have
tan 45° = DC/BC
1 = DC/40
DC = 40 cm

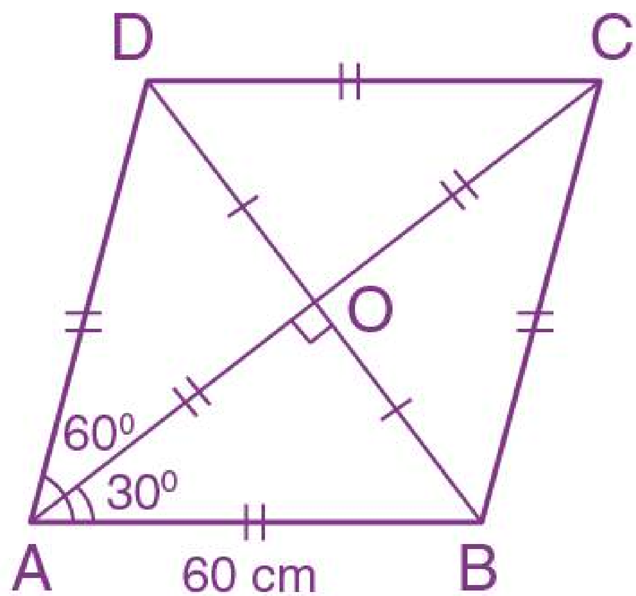
Now, from the figure it's clearly seen that
AD = AC – DC
= 40$\sqrt{3}$ - 40
= 40($\sqrt{3}$ - 1)
Hence, the length of AD is 29.28 cm

6. Find the lengths of diagonals AC and BD. Given AB = 60 cm and ∠BAD = 60°.



Solution:

We know that, diagonals of a rhombus bisect each other at right angles and also bisect the angle of vertex.
Considering the figure as shown below:

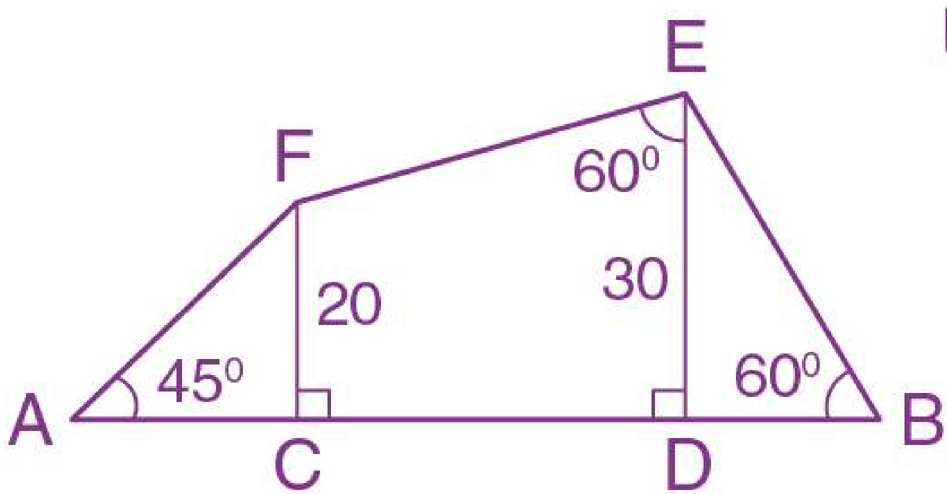


Now, we have
OA = OC = ½ AC,
OB = OD = ½ BD
∠AOB = 90° and
∠OAB = 60°/2 = 30°
Also given that AB = 60 cm

Now,
In right ΔAOB, we have
sin 30° = OB/AB
½ = OB/60
OB = 30
Also,
cos 30° = OA/AB
√3/2 = OA/60
OA = 51.96 cm

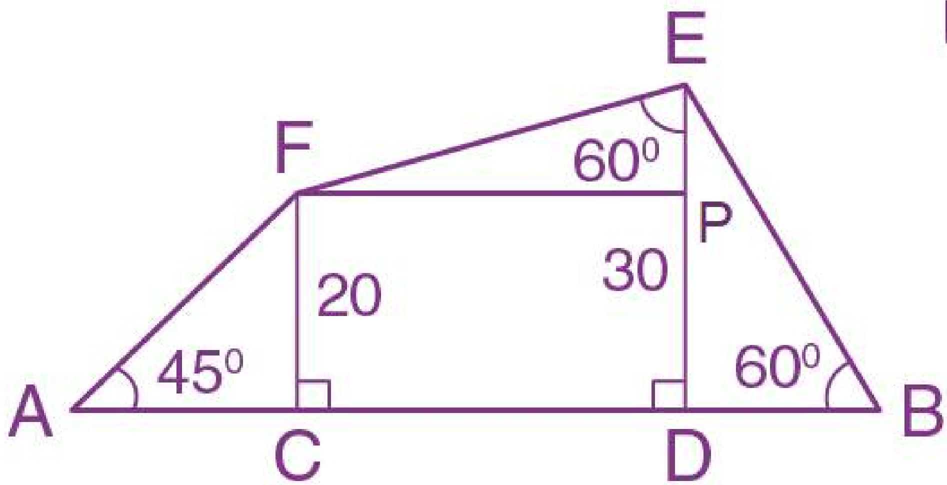
Therefore,
Length of diagonal AC = 2 x OA
= 2 x 51.96
= 103.92 cm
Length of diagonal BD = 2 x OB
= 2 x 30
= 60 cm

7. Find AB.



Solution:

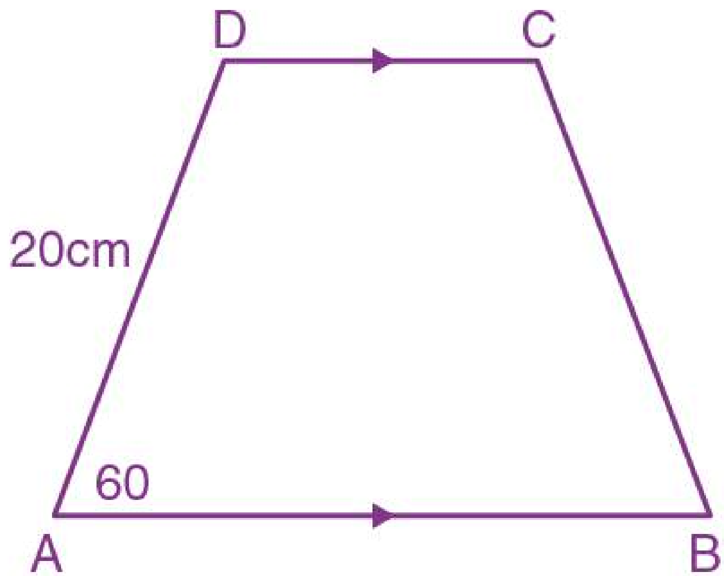
Considering the given figure, let's construct FP ⊥ ED



Now,
In right ΔACF, we have
tan 45º = 20/AC
1 = 20/AC
AC = 20
Next,
In right ΔDEB, we have
tan 60º = 30/BD
√3 = 30/BD
BD = 30/√3
= 17.32 cm
Also, given FC = 20 and ED = 30
So, EP = 10 cm
Thus,
tan 60º = FP/EP
√3 = FP/10
FP = 10√3
= 17.32 cm
And, FP = CD

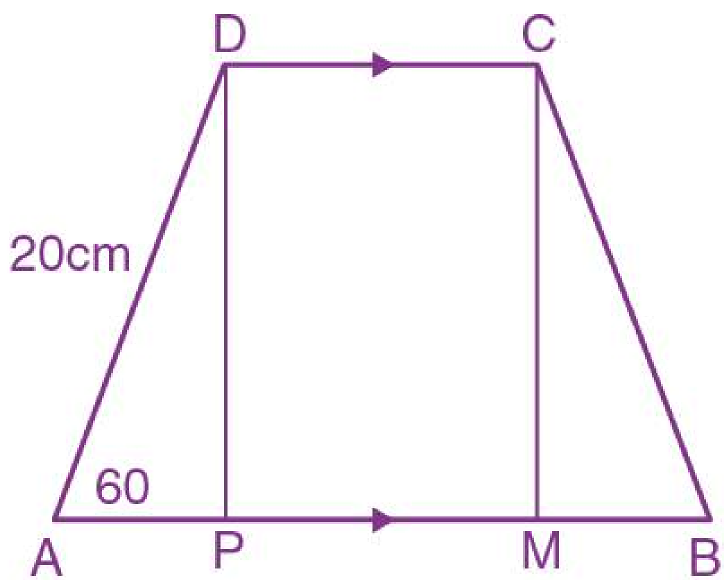
Therefore, AB = AC + CD + BD
= 20 + 17.32 + 17.32
= 54.64 cm

8. In trapezium ABCD, as shown, AB || DC, AD = DC = BC = 20 cm and ∠A = 60º. Find:
(i) length of AB
(ii) distance between AB and DC



Solution:

Constructing two perpendiculars to AB from the point D and C respectively.
Now, since AB||CD we have PMCD as a rectangle
Considering the figure,

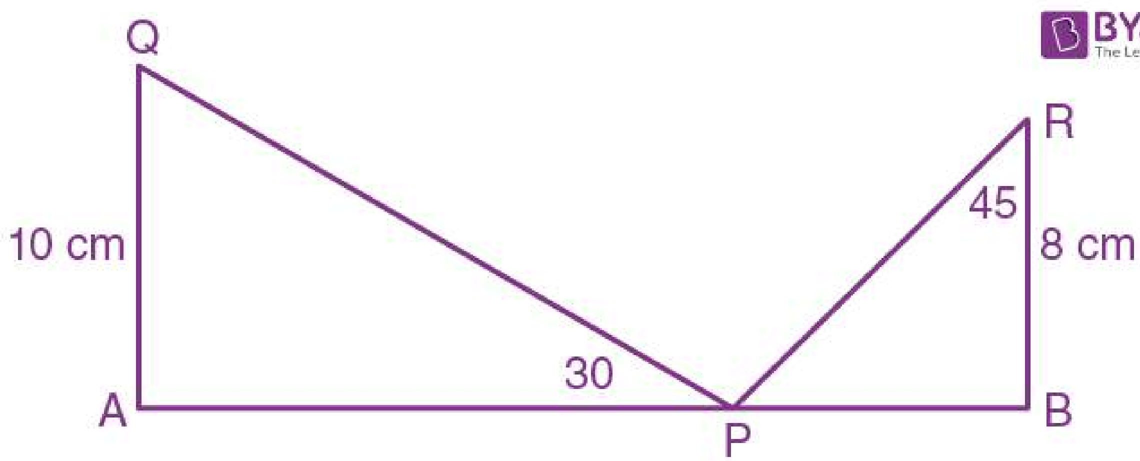


(i) From right ΔADP, we have
$\cos 60^o = AP/AD$
$½ = AP/20$
AP = 10
Similarly,
In right ΔBMC, we have
BM = 10 cm

Now, from the rectangle PMCD we have
CD = PM = 20 cm
Therefore,
AB = AP + PM + MB
= 10 + 20 + 10
= 40 cm

(ii) Again, from the right ∆APD, we have
sin 60^o = PD/20
√3/2 = PD/20
PD = 10√3
Hence, the distance between AB and CD is 10√3 cm

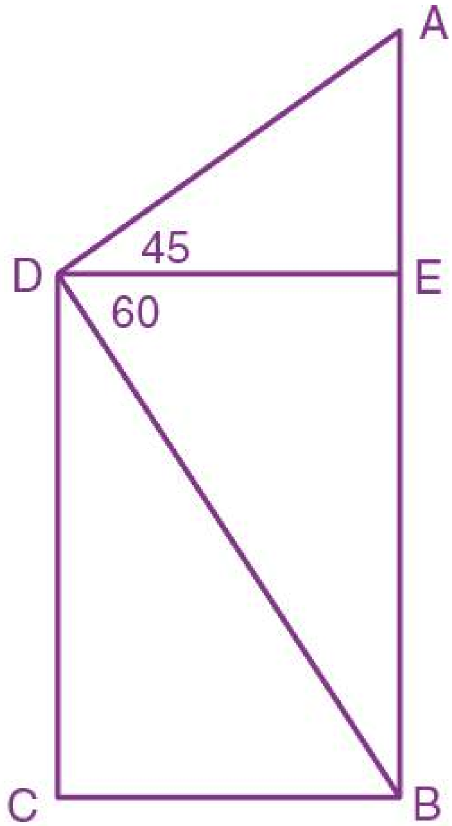
9. Use the information given to find the length of AB.



Solution:

In right ∆AQP, we have
tan 30^o = AQ/AP
1/√3 = 10/AP
AP = 10√3
Also,
In right ∆PBR, we have
tan 45^o = PB/BR
1 = PB/8
PB = 8
Therefore, AB = AP + PB
= 10√3 + 8

10. Find the length of AB.



Solution:

In right ΔADE, we have

tan 45° = AE/DE

1 = AE/30

AE = 30 cm

Also, in right ΔDBE, we have

tan 60° = BE/DE

√3 = BE/30

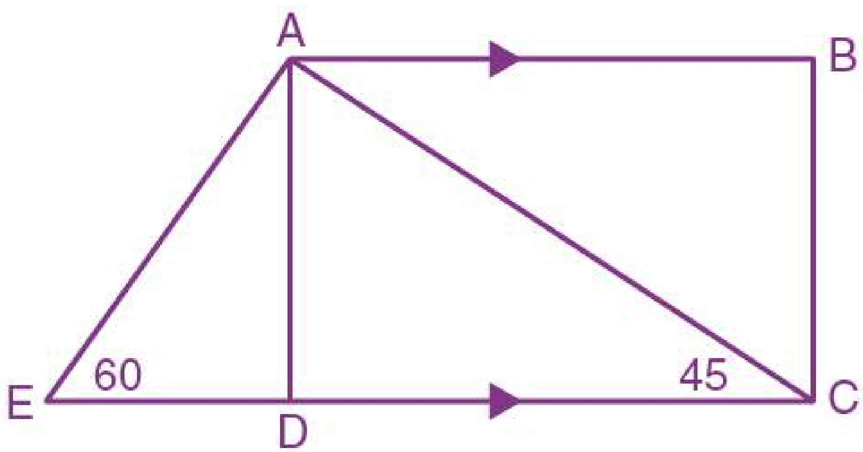
BE = 30√3 cm

Therefore, AB = AE + BE

= 30 + 30√3

= 30(1 + √3) cm

11. In the given figure, AB and EC are parallel to each other. Sides AD and BC are 2 cm each and are perpendicular to AB.



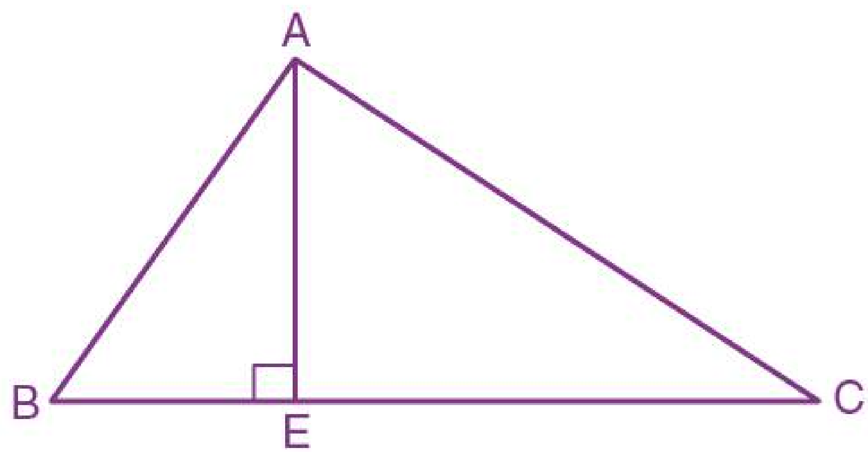
Given that ∠AED = 60º and ∠ACD = 45º. Calculate:
(i) AB (ii) AC (iii) AE
Solution:

(i) In right ΔADC, we have
tan 45º = AD/DC
1 = 2/DC
DC = 2 cm
And, as AD || DC and AD ⊥ EC, ABCD is a parallelogram
So, opposite sides are equal
Hence, AB = DC = 2 cm

(ii) Again, in right ΔADC
sin 45º = AD/AC
1/√2 = AD/AC
AC = 2√2 cm

(iii) In right ΔADE, we have
sin 60º = AD/AE
√3/2 = 2/AE
AE = 4/√3

12. In the given figure, ∠B = 60˚, AB = 16 cm and BC = 23 cm,
Calculate:
(i) BE (ii) AC



Solution:

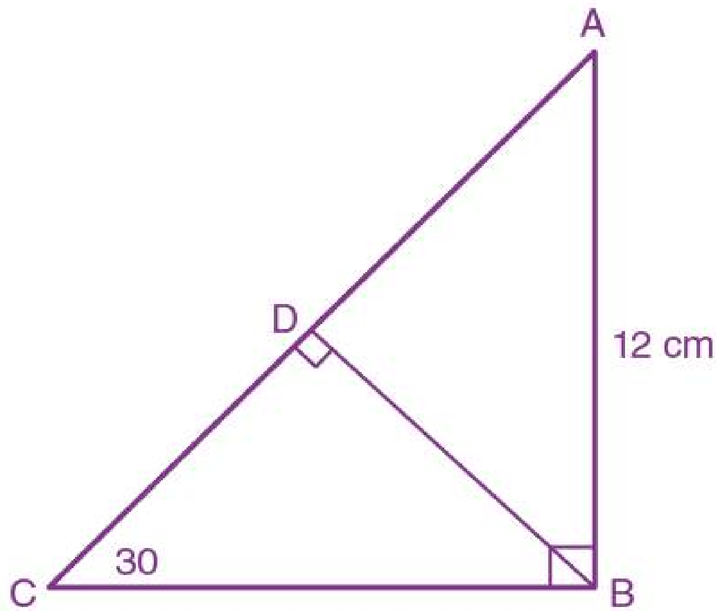
In $\triangle ABE$, we have
sin 60^o = AE/AB
$\sqrt{3}/2$ = AE/16
AE = $\sqrt{3}/2 \times 16$
= $8\sqrt{3}$ cm

(i) In $\triangle ABE$, we have $\angle AEB = 90^\circ$
So, by Pythagoras Theorem, we get
$BE^2 = AB^2 - AE^2$
$BE^2 = (16)^2 - (8\sqrt{3})^2$
$BE^2 = 256 - 192$
$BE^2 = 64$
Taking square root on both sides, we get
BE = 8 cm

(ii) EC = BC - BE
= 23 - 8
= 15
In $\triangle AEC$, we have
$\angle AEC = 90^\circ$
So, by Pythagoras Theorem, we get
$AC^2 = AE^2 + EC^2$
$AC^2 = (8\sqrt{3})^2 + (15)^2$
$AC^2 = 192 + 225$
$AC^2 = 147$
Taking square root on both sides, we get
AC = 20.42 cm

13. Find
(i) BC

(ii) AD
(iii) AC



Solution:

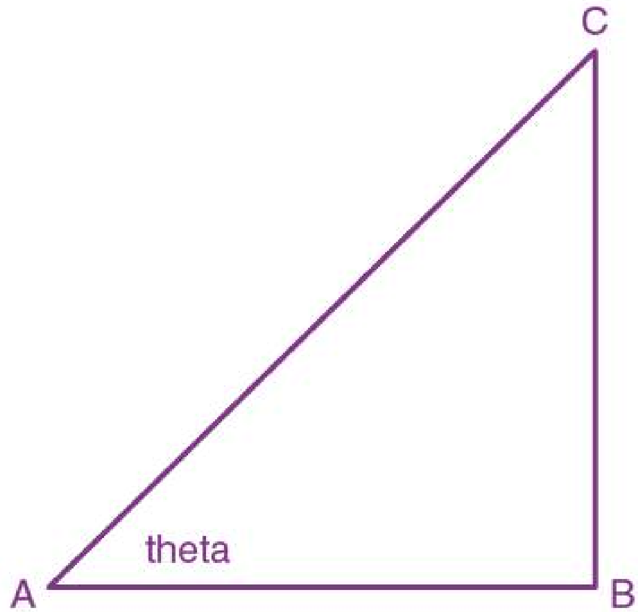
(i) In right ∆AEC, we have
tan 30º = AB/BC
1/√3 = 12/BC
BC = 12√3 cm

(ii) In right ∆ABD, we have
cos A = AD/AB
cos 60º = AD/AB
½ = AD/12
AD = 12/2
AD = 6 cm

(iii) In right ∆ABC, we have
sin B = AB/AC
sin 30º = AB/AC
½ = 12/AC
AC = 12 x 2
AC = 24 cm

14. In right-angled triangle ABC; ∠B = 90º. Find the magnitude of angle A, if:
(i) AB is √3 times of BC
(ii) BC is √3 times of AB
Solution:

Considering the figure below:

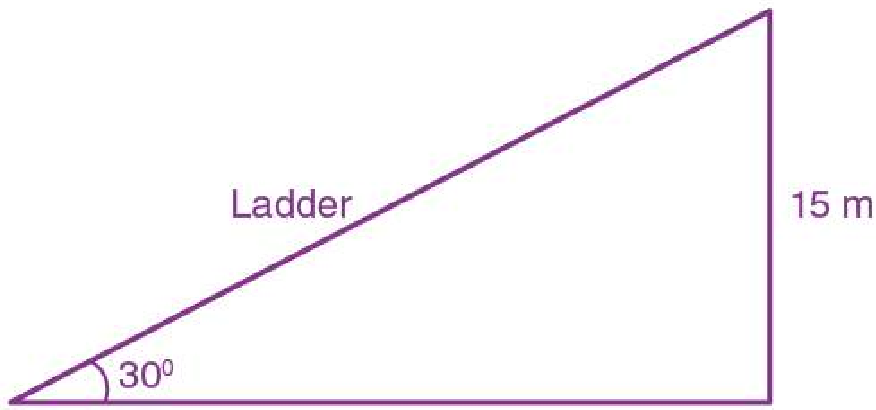


(i) We have, AB is √3 times of BC
AB/BC = √3
cot A = cot 30º
A = 30º
Hence, the magnitude of angle A is 30º

(ii) Again, from the figure
BC/AB = √3
tan A = √3
tan A = tan 60º
A = 60º
Hence, the magnitude of angle A is 60º

15. A ladder is placed against a vertical tower. If the ladder makes an angle of 30º with the ground and reaches upto a height of 15 m of the tower; find length of the ladder.
Solution:

Given that the ladder makes an angle of 30º with the ground and reaches upto a height of 15 m of the tower
Let's consider the figure shown below:

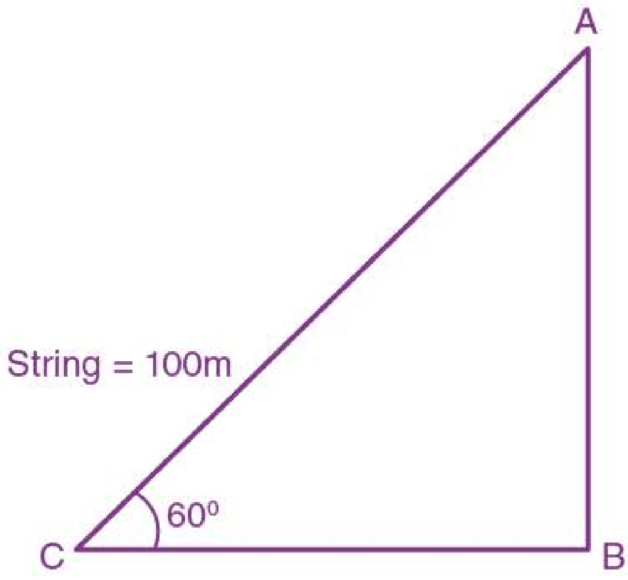


Let's assume the length of the ladder is x metre
Now, from the figure we have
15/x = sin 30° [As perpendicular/hypotenuse = sine]
15/x = ½
x = 30 m

Hence, the length of the ladder is 30 m.

16. A kite is attached to a 100 m long string. Find the greatest height reached by the kite when its string makes an angles of 60° with the level ground.
Solution:

Given that the kite is attached to a 100 m long string and it makes an angle of 60° with the ground level
Let's consider the figure shown below:

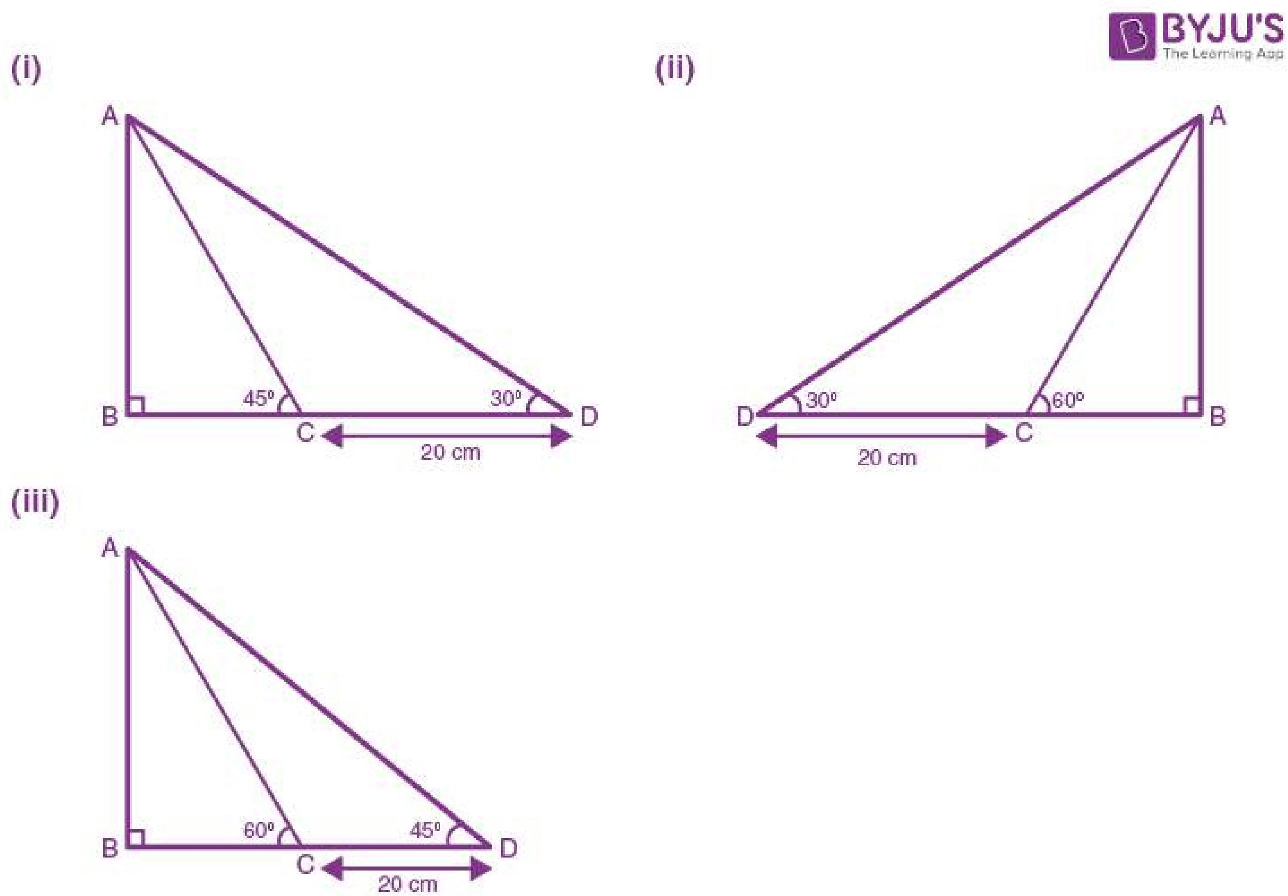


Let's

assume the greatest height to be *x* metre
Now, from the figure we have
x/100 = sin 60º [As perpendicular/hypotenuse = sine]
x/100 = √3/2
x = 100 x (√3/2)
= 86.6 m

Hence, the greatest height reached by the kite is 86.6 m.

17. Find AB and BC, if:



Solution:

(i) Let assume BC to be x cm
BD = BC + CD = (x + 20) cm
Now,
In ∆ABD, we have
tan 30º = AB/BD
1/√3 = AB/(x + 20)
x + 20 = √3AB ... (1)
And,

In ΔABC, we have
tan 45º = AB/BC
1 = AB/x
AB = x ... (2)

Now, using (1) in (2) we get
AB + 20 = √3AB
AB(√3 - 1) = 20
AB = 20/(√3 - 1)
= 20/(√3 - 1) x [(√3 + 1)/(√3 + 1)]
= 20(√3 + 1)/ (3 – 1)
= 27.32 cm

Hence from (2), we have
AB = BC = x = 27.32 cm
Therefore, AB = 27.32cm, BC = 27.32cm

(ii) Let's assume BC to be x m
BD = BC + CD
= (x + 20) cm
In ΔABD, we have
tan 30º = AB/BD
1/√3 = AB/(x + 20)
x + 20 = √3 AB … (1)

In ΔABC, we have
tan 60º = AB/BC
√3 = AB/x
x = AB/√3 … (2)

Now, from (1) we have
AB/√3 + 20 = √3AB
AB + 20√3 = 3AB
2AB = 20√3
AB = 20√3/2
= 10√3
= 17.32 cm
And, from (2) we get
x = AB/√3
= 17.32/√3
= 10 cm

Hence, BC = x = 10cm
Therefore,
AB = 17.32 cm and BC = 10cm

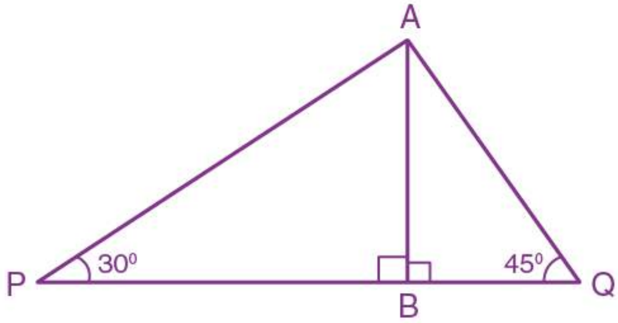
(iii) Let's assume BC to be x cm
BD = BC + CD
= (x + 20) cm
In ΔABD, we have
tan 45º = AB/BD
1 = AB/(x + 20)
x + 20 = AB … (1)
Also,
In ΔABD, we have
tan 60º = AB/BC
$\sqrt{3}$ = AB/x
x = AB/$\sqrt{3}$ … (2)

Now, from (1) we have
AB/$\sqrt{3}$ + 20 = AB
AB + 20$\sqrt{3}$ = $\sqrt{3}$AB
AB ($\sqrt{3}$ - 1) = 20$\sqrt{3}$
AB = 20$\sqrt{3}$/($\sqrt{3}$ - 1)
On rationalizing, we get
AB = 20$\sqrt{3}$($\sqrt{3}$ + 1)/(3 -1)
= 10$\sqrt{3}$($\sqrt{3}$ + 1)
= 47.32 cm
And, from (2) we get
x = AB/$\sqrt{3}$
= 47.32/$\sqrt{3}$
= 27.32 cm
Hence, BC = x = 27.32 cm

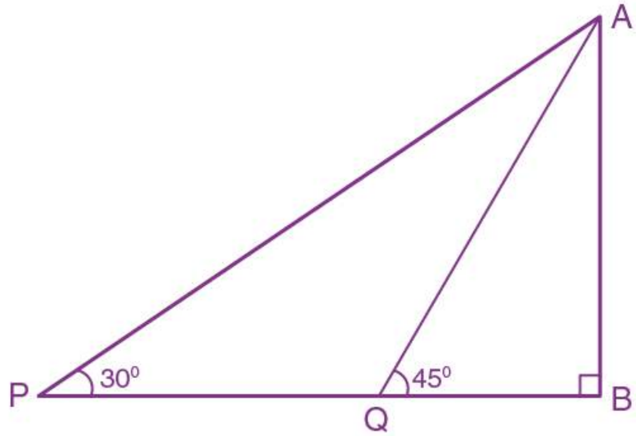
Therefore,
AB = 47.32 cm and BC = 27.32 cm

18. Find PQ, if AB = 150 m, ∠P = 30º and ∠Q = 45º.

(i)



(ii)



Solution:

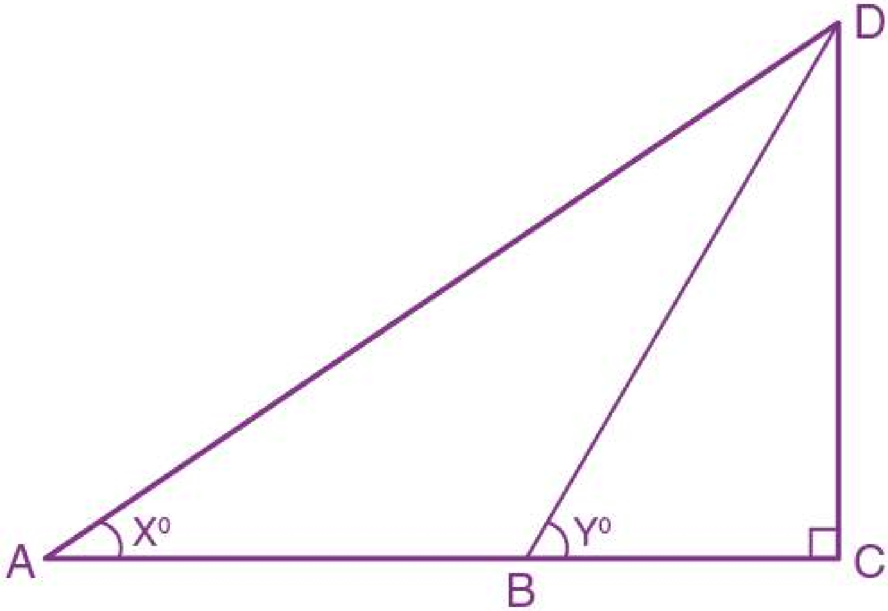
(i) In ∆APB, we have
tan 30º = AB/PB
1/√3 = 150/PB
PB = 150√3
= 259.80 m
Also, in ∆ABQ
tan 45º = AB/BQ
1 = 150/BQ
BQ = 150 m
Therefore,
PQ = PB + BQ
= 259.80 + 150
= 409.80 m

(ii) In ∆APB, we have
tan 30º = AB/PB
1/√3 = 150/PB
PB = 150√3
= 259.80 m
Also, in in ∆APB, we have

$\tan 45^o = AB/BQ$
$1 = 150/BQ$
$BQ = 150$ cm

Therefore, $PQ = PB - BQ$
$= 259.80 - 150$
$= 109.80$ m

19. If $\tan x^o = 5/12$, $\tan y^o = 3/4$ and AB = 48 m; find the length of CD.



Solution:

Given,
$\tan x^o = 5/12$, $\tan y^o = 3/4$ and AB = 48 m;
Let's consider the length of BC to x metre
In ΔADC, we have
$\tan x^o = DC/AC$
$5/12 = DC/(48 + x)$
$5(48 + x) = 12DC$
$240 + 5x = 12DC$ … (1)

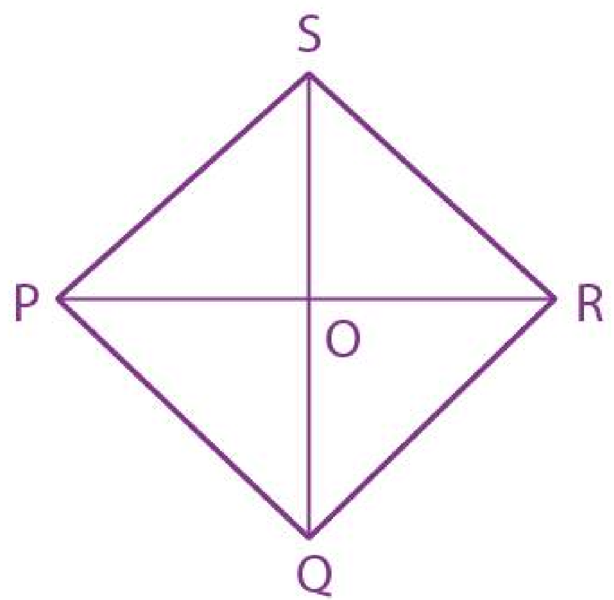
Also, in ΔBDC we have
$\tan y^o = CD/BC$
$3/4 = CD/x$
$x = 4CD/3$ … (2)
Also, from (1) we get
$240 + 5(4CD/3) = 12CD$
$240 + 20CD/3 = 12CD$

720 + 20CD = 36CD
36CD – 20CD = 720
16CD = 720
CD = 720/16
CD = 45

Therefore, the length of CD is 45 m.

20. The perimeter of a rhombus is 96 cm and obtuse angle of it is 120°. Find the lengths of its diagonals.
Solution:

As rhombus has all sides equal
Let's consider the diagram as below:



Hence, PQ = 96/4 = 24 cm
And, ∠PQR = 120°
We also know that,
In a rhombus, the diagonals bisect each other perpendicularly and the diagonal bisect the angle at vertex
Thus, ∠POR is a right-angle triangle
∠PQR = ½ (∠PQR)
= 60°
sin 60° = perpendicular/hypotenuse
√3/2 = PO/PQ
√3/2 = PO/24
PO = 24√3/2
= 12√3
= 20/784 cm
Hence,

PR = 2PO
= 2 x 20.784
= 41.568 cm
Also, we have
cos 60º = base/hypotenuse
½ = OQ/24
OQ = 24/2
= 12 cm
Hence, SQ = 2 x OQ
= 2 x 12
= 24 cm

Therefore, the length of the diagonals PR is 41.568 cm and of SQ is 24 cm