Exercise 7(A)

1. Evaluate:

(i) $3^3 \times (243)^{-2/3} \times 9^{-1/3}$

(ii) $5^{-4} \times (125)^{5/3} \div (25)^{-1/2}$

(iii) $(27/125)^{2/3} \times (9/25)^{-3/2}$

(iv) $7^0 \times (25)^{-3/2} - 5^{-3}$

(v) $(16/81)^{-3/4} \times (49/9)^{3/2} \div (343/216)^{2/3}$

Solution:

(i) $3^3 \times (243)^{-2/3} \times 9^{-1/3} = 3^3 \times (3 \times 3 \times 3 \times 3 \times 3)^{-2/3} \times (3 \times 3)^{-1/3}$

$= 3^3 \times (3^5)^{-2/3} \times (3^2)^{-1/3}$

$= 3^3 \times (3)^{-10/3} \times 3^{-2/3}$ [As $(a^m)^n = a^{mn}$]

$= 3^{3 - 10/3 - 2/3}$ [$a^m \times a^n \times a^p = a^{m+n+p}$]

$= 3^{(9 - 10 - 2)/3}$

$= 3^{-3/3}$

$= 3^{-1}$

$= 1/3$

(ii) $5^{-4} \times (125)^{5/3} \div (25)^{-1/2} = 5^{-4} \times (5 \times 5 \times 5)^{5/3} \div (5 \times 5)^{-1/2}$

$= 5^{-4} \times (5^3)^{5/3} \div (5^2)^{-1/2}$

$= 5^{-4} \times (5^{3 \times 5/3}) \div (5^{2 \times -1/2})$ [As $(a^m)^n = a^{mn}$]

$= 5^{-4} \times 5^5 \div 5^{-1}$

$= 5^{-4} \times 5^5 \times 5^{-(-1)}$ [As $1/a^{-m} = a^{-(-m)} = a^m$]

$= 5^{(-4 + 5 + 1)}$ [$a^m \times a^n \times a^p = a^{m+n+p}$]

$= 5^2$

$= 25$

(iii) $(27/125)^{2/3} \times (9/25)^{-3/2} = (3 \times 3 \times 3/5 \times 5 \times 5)^{2/3} \times (3 \times 3/5 \times 5)^{-3/2}$

$= (3^3/5^3)^{2/3} \times (3^2/5^2)^{-3/2}$

$= (3/5)^{3 \times 2/3} \times (3/5)^{2 \times -3/2}$ [As $(a^m)^n = a^{mn}$]

$= (3/5)^2 \times (3/5)^{-3}$

$= (3/5)^{2-3}$ [$a^m \times a^n = a^{m+n}$]

$= (3/5)^{-1}$

$= 5/3$

(iv) $7^0 \times (25)^{-3/2} - 5^{-3} = 1 \times (5 \times 5)^{-3/2} - 5^{-3}$ [As $a^0 = 1$]

$= (5^2)^{-3/2} - 5^{-3}$

$= (5)^{2 \times -3/2} - 5^{-3}$ [As $(a^m)^n = a^{mn}$]

$= 5^{-3} - 5^{-3}$

$= 1/5^3 - 1/5^3$

$= 0$

(v) $(16/81)^{-3/4} \times (49/9)^{3/2} \div (343/216)^{2/3}$

$= (2 \times 2 \times 2 \times 2/3 \times 3 \times 3 \times 3)^{-3/4} \times (7 \times 7/3 \times 3)^{3/2} \div (7 \times 7 \times 7/6 \times 6 \times 6)^{2/3}$

$= (2^4/3^4)^{-3/4} \times (7^2/3^2)^{3/2} \div (7^3/6^3)^{2/3}$

$= (2/3)^{4 \times -3/4} \times (7/3)^{2 \times 3/2} \div (7/6)^{3 \times 2/3}$
$= (2/3)^{-3} \times (7/3)^{3} \div (7/6)^{2}$ [As $(a^m)^n = a^{mn}$]
$= [1/(2/3)^3 \times (7/3)^3]/ (7/6)^2$ [As $a^{-m} = 1/a^m$]
$= [(3/2)^3 \times (7/3)^3] \times (7/6)^{-2}$
$= (3/2)^3 \times (7/3)^3 \times (6/7)^2$ [As $(a/b)^{-m} = (b/a)^m$]
$= 3/2 \times 3/2 \times 3/2 \times 7/3 \times 7/3 \times 7/3 \times 6/7 \times 6/7$
$= (7 \times 3 \times 3)/2$
$= 63/2$
$= 31.5$

2. Simplify:
(i) $(8x^3 \div 125y^3)^{2/3}$
(ii) $(a + b)^{-1}. (a^{-1} + b^{-1})$
(iii) $\dfrac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2}$
(iv) $(3x^2)^{-3} \times (x^9)^{2/3}$
Solution:

(i) $(8x^3 \div 125y^3)^{2/3} = (8x^3/125y^3)^{2/3}$
$= (2x \times 2x \times 2x/5y \times 5y \times 5y)^{2/3}$
$= (2x^3/5y^3)^{2/3}$
$= (2x/5y)^{3 \times 2/3}$
$= (2x/5y)^2$
$= 4x^2/25y^2$

(ii) $(a + b)^{-1}. (a^{-1} + b^{-1}) = 1/(a + b) \times (1/a + 1/b)$
$= 1/(a + b) \times (b + a)/ab$
$= 1/ab$

(iii)
$$\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2} = \frac{5^{n+1} \times 5^2 - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2}$$
$$= \frac{5^{n+1} \times (5^2 - 6)}{5^n \times (9 - 4)}$$
$$= \frac{5^n \times 5^1 \times (25 - 6)}{5^n \times (9 - 4)}$$
$= (5^1 \times 19)/5$
$= 19$

(iv) $(3x^2)^{-3} \times (x^9)^{2/3} = 3^{-3} \times (x^2)^{-3} \times (x)^{9 \times 2/3}$
$= 3^{-3} \times (x)^{2 \times -3} \times (x)^{9 \times 2/3}$
$= 3^{-3} \times x^{-6} \times x^6$
$= 3^{-3} \times 1$

$= 1/27$

3. Evaluate:
(i) $\sqrt{1/4} + (0.01)^{-1/2} - (27)^{2/3}$
(ii) $(27/8)^{2/3} - (1/4)^{-2} + 5^0$

Solution:

(i) $\sqrt{1/4} + (0.01)^{-1/2} - (27)^{2/3} = \sqrt{(1/2 \times 1/2)} + (0.1 \times 0.1)^{-1/2} - (3 \times 3 \times 3)^{2/3}$
$= \sqrt{(1/2)^2} + (0.1^2)^{-1/2} - (3^3)^{2/3}$
$= 1/2 + (0.1)^{2 \times -1/2} - (3)^{3 \times 2/3}$
$= 1/2 + (0.1)^{-1} - (3)^2$
$= 1/2 + 1/0.1 - 3^2$
$= 1/2 + 1/(1/10) - 9$
$= 1/2 + 10 - 9$
$= 1/2 + 1$
$= 3/2$

(ii) $(27/8)^{2/3} - (1/4)^{-2} + 5^0 = (3 \times 3 \times 3/2 \times 2 \times 2)^{2/3} - (1/2 \times 1/2)^{-2} + 5^0$
$= (3^3/2^3)^{2/3} - ((1/2)^2)^{-2} + 1$
$= (3/2)^{3 \times 2/3} - (1/2)^{-4} + 1$
$= (3/2)^2 - (1/2)^{-4} + 1$
$= (3/2)^2 - 2^4 + 1$
$= (3 \times 3)/(2 \times 2) - (2 \times 2 \times 2 \times 2) + 1$
$= 9/4 - 16 + 1$
$= (9 - 64 + 4)/4$
$= -51/4$

4. Simplify each of the following and express with positive index:
(i) $(3^{-4}/2^{-8})^{1/4}$
(ii) $(27^{-3}/9^{-3})^{1/5}$
(iii) $(32)^{-2/5} \div (125)^{-2/3}$
(iv) $[1 - \{1 - (1 - n)^{-1}\}^{-1}]^{-1}$
Solution:

(i) $(3^{-4}/2^{-8})^{1/4} = (2^8/3^4)^{1/4}$
$= (2^8)^{1/4}/(3^4)^{1/4}$
$= (2)^{8/4}/(3)^{4/4}$
$= 2^2/3$
$= 4/3$

(ii) $(27^{-3}/9^{-3})^{1/5} = (9^3/27^3)^{1/5}$
$= [(3 \times 3)^3/(3 \times 3 \times 3)^3]^{1/5}$
$= [(3^2)^3/(3^3)^3]^{1/5}$
$= [(3)^{2 \times 3}/(3)^{3 \times 3}]^{1/5}$
$= [(3)^6/(3)^9]^{1/5}$

$= [(3)^{6-9}]^{1/5}$
$= (3)^{-3 \times 1/5}$
$= (3)^{-3/5}$
$= 1/3^{3/5}$

(iii) $(32)^{-2/5} \div (125)^{-2/3} = (32)^{-2/5}/(125)^{-2/3}$
$= (125)^{2/3}/(32)^{2/5}$
$= (5 \times 5 \times 5)^{2/3}/(2 \times 2 \times 2 \times 2 \times 2)^{2/5}$
$= (5^3)^{2/3}/(2^5)^{2/5}$
$= 5^{3 \times 2/3}/2^{5 \times 2/5}$
$= 5^2/2^2$
$= 25/4$

(iv) $[1 – \{1 – (1 - n)^{-1}\}^{-1}]^{-1} = [1 – \{1 – 1/(1 - n)\}^{-1}]^{-1}$
$= [1 – \{((1 - n) – 1)/(1 - n)\}^{-1}]^{-1}$
$= [1 – \{– n/(1 - n)\}^{-1}]^{-1}$
$= [1 – \{– (1 - n)/n\}]^{-1}$
$= [1 + (1 - n)/n]^{-1}$
$= [(n + 1 - n)/n]^{-1}$
$= [1/n]^{-1}$
$= n$

5. If $2160 = 2^a . 3^b . 5^c$, find a, b and c. Hence, calculate the value of $3^a \times 2^{-b} \times 5^{-c}$.
Solution:

We have,
$2160 = 2^a \times 3^b \times 5^c$
$(2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3) \times 5 = 2^a \times 3^b \times 5^c$
$2^4 \times 3^3 \times 5^1 = 2^a \times 3^b \times 5^c$
$\Rightarrow 2^a \times 3^b \times 5^c = 2^4 \times 3^3 \times 5^1$
Comparing the exponents of 2, 3 and 5 on both sides, we get
a = 4, b = 3 and c = 1
Hence, the value
$3^a \times 2^{-b} \times 5^{-c} = 3^4 \times 2^{-3} \times 5^{-1}$
$= (3 \times 3 \times 3 \times 3) \times (½ \times ½ \times ½) \times 1/5$
$= 81 \times 1/8 \times 1/5$
$= 81/40$

6. If $1960 = 2^a . 5^b . 7^c$, calculate the value of $2^{-a} . 7^b . 5^{-c}$.
Solution:

We have,
$1960 = 2^a \times 5^b \times 7^c$
$(2 \times 2 \times 2) \times 5 \times (7 \times 7) = 2^a \times 5^b \times 7^c$
$2^3 \times 5^1 \times 7^2 = 2^a \times 5^b \times 7^c$
$\Rightarrow 2^a \times 5^b \times 7^c = 2^2 \times 5^1 \times 7^2$

Comparing the exponents of 2, 5 and 7 on both sides, we get
a = 3, b = 1 and c = 2
Hence, the value
$2^{-a}.\ 7^{b}.\ 5^{-c} = 2^{-3} \times 7^{1} \times 5^{-2}$
$= (½ \times ½ \times ½) \times 7 \times (1/5 \times 1/5)$
$= 1/8 \times 7 \times 1/25$
$= 7/200$

7. Simplify:

(i) $\dfrac{8^{3a} \times 2^{5} \times 2^{2a}}{4 \times 2^{11a} \times 2^{-2a}}$

(ii) $\dfrac{3 \times 27^{n+1} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times 27^{n}}$

Solution:

(i)

$$\frac{8^{3a} \times 2^{5} \times 2^{2a}}{4 \times 2^{11a} \times 2^{-2a}} = \frac{\left(2^{3}\right)^{3a} \times 2^{5} \times 2^{2a}}{2^{2} \times 2^{11a} \times 2^{-2a}}$$
$$= \frac{2^{3\times 3a} \times 2^{5} \times 2^{2a}}{2^{2} \times 2^{11a} \times 2^{-2a}}$$
$$= \frac{2^{9a} \times 2^{5} \times 2^{2a}}{2^{2} \times 2^{11a} \times 2^{-2a}}$$
$$= 2^{9a+5+2a-2-11a+2a}$$
$$= 2^{2a+3}$$

(ii)

$$\frac{3\times 27^{n+1}+9\times 3^{3n-1}}{8\times 3^{3n}-5\times 27^{n}}=\frac{3\times(3\times 3\times 3)^{n+1}+3\times 3\times 3^{3n-1}}{2\times 2\times 2\times 3^{3n}-5\times(3\times 3\times 3)^{n}}$$

$$=\frac{3\times\left(3^{3}\right)^{n+1}+3^{2}\times 3^{3n-1}}{2^{3}\times 3^{3n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3\times 3^{3n+3}+3^{3n+1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n+3+1}+3^{3n+1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n+4}+3^{3n+1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n}\times 3^{4}+3^{3n}\times 3^{1}}{2^{3}\times\left(3^{3}\right)^{n}-5\times\left(3^{3}\right)^{n}}$$

$$=\frac{3^{3n}\left(3^{4}+3^{1}\right)}{\left(3^{3}\right)^{n}(8-5)}$$

$$=\frac{3^{3n}\left(3^{4}+3^{1}\right)}{3^{3n}\times 3}$$

$$=\frac{3\times 3\times 3\times 3+3}{3}$$

$$=\frac{81+3}{3}$$

$$=\frac{84}{3}$$

$$=28$$

8. Show that:

$(a^m/a^{-n})^{m-n} \times (a^n/a^{-l})^{n-l} \times (a^l/a^{-m})^{l-m} = 1$

Solution:

Taking the L.H.S., we have

$(a^m/a^{-n})^{m-n} \times (a^n/a^{-l})^{n-l} \times (a^l/a^{-m})^{l-m}$

$= (a^m \times a^n)^{m-n} \times (a^n \times a^l)^{n-l} \times (a^l \times a^m)^{l-m}$

$= (a^{m+n})^{m-n} \times (a^{n+l})^{n-l} \times (a^{l+m})^{l-m}$

$= a^{m^2-n^2} \times a^{n^2-\ell^2} \times a^{\ell^2-m^2}$

$= a^{m^2-n^2+n^2-\ell^2+\ell^2-m^2}$

$= a^0$
$= 1$

9. If $a = x^{m+n}. x^l$; $b = x^{n+l}. x^m$ and $c = x^{l+m}. x^n$,
Prove that: $a^{m-n}. b^{n-l}. c^{l-m} = 1$
Solution:

We have,
$a = x^{m+n}. x^l$
$b = x^{n+l}. x^m$
$c = x^{l+m}. x^n$
Now,
Considering the L.H.S.,
$a^{m-n}. b^{n-l}. c^{l-m}$
$= (x^{m+n}. x^l)^{m-n}. (x^{n+l}. x^m)^{n-l}. (x^{l+m}. x^n)^{l-m}$
$= [x^{(m+n)(m-n)}. x^{l(m-n)}]. [x^{(n+l)(n-l)}. x^{m(n-l)}]. [x^{(l+m)(l-m)}. x^{n(l-m)}]$
$= x^{m^2 - n^2 + ml - nl + n^2 - l^2 + mn - nl + l^2 - m^2 + nl - mn}$
$= x^0$
$= 1$
$=$ R.H.S
- Hence proved.

10. Simplify:

(i) $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$

(ii) $\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$

Solution:

(i)
$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$
$= \left(x^{a-b}\right)^{a^2+ab+b^2} \times \left(x^{b-c}\right)^{b^2+bc+c^2} \times \left(x^{c-a}\right)^{c^2+ca+a^2}$
$= x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3}$
$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$
$= x^0$
$= 1$

(ii)

$$\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$$

$$= \left(x^{a+b}\right)^{a^2-ab+b^2} \times \left(x^{b+c}\right)^{b^2-bc+c^2} \times \left(x^{c+a}\right)^{c^2-ca+a^2}$$

$$= x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3}$$

$$= x^{a^3+b^3+b^3+c^3+c^3+a^3}$$

$$= x^{\left(a^3+b^3+b^3+c^3+c^3+a^3\right)}$$

$$= x^{2\left(a^3+b^3+c^3\right)}$$

Exercise 7(B)

1. Solve for x:

(i) $2^{2x+1} = 8$

(ii) $2^{5x-1} = 4 \times 2^{3x+1}$

(iii) $3^{4x+1} = (27)^{x+1}$

(iv) $(49)^{x+4} = 7^2 \times (343)^{x+1}$

Solution:

(i) We have, $2^{2x+1} = 8$

$\Rightarrow 2^{2x+1} = 2^3$

Now, if the bases are equal, then the powers must be equal

On comparing the exponents, we get

$2x + 1 = 3$

$2x = 3 - 1$

$2x = 2$

$x = 2/2$

$x = 1$

(ii) We have, $2^{5x-1} = 4 \times 2^{3x+1}$

$\Rightarrow 2^{5x-1} = 2^2 \times 2^{3x+1}$

$2^{5x-1} = 2^{(3x+1)+2}$

$2^{5x-1} = 2^{3x+3}$

Now, if the bases are equal, then the powers must be equal

On comparing the exponents, we get

$5x - 1 = 3x + 3$

$5x - 3x = 3 + 1$

$2x = 4$

$x = 4/2$

$x = 2$

(iii) We have, $3^{4x+1} = (27)^{x+1}$

$3^{4x+1} = (3^3)^{x+1}$

$3^{4x+1} = (3)^{3x+3}$

Now, if the bases are equal, then the powers must be equal

On comparing the exponents, we get

$4x + 1 = 3x + 3$

$4x - 3x = 3 - 1$

$x = 2$

(iv) We have, $(49)^{x+4} = 7^2 \times (343)^{x+1}$

$(7 \text{ x } 7)^{x+4} = 7^2 \times (7 \text{ x } 7 \text{ x } 7)^{x+1}$

$(7^2)^{x+4} = 7^2 \times (7^3)^{x+1}$

$(7)^{2x+8} = (7)^{3x+3+2}$

$(7)^{2x+8} = (7)^{3x+5}$

Now, if the bases are equal, then the powers must be equal

On comparing the exponents, we get

$2x + 8 = 3x + 5$
$3x – 2x = 8 – 5$
$x = 3$

2. Find x, if:
(i) $4^{2x} = 1/32$
(ii) $\sqrt{(2^{x+3})} = 16$
(iii) $[\sqrt{(3/5)}]^{x+1} = 125/27$
(iv) $[\sqrt[3]{(2/3)}]^{x-1} = 27/8$
Solution:

(i) We have, $4^{2x} = 1/32$
$(2 \times 2)^{2x} = 1/(2 \times 2 \times 2 \times 2 \times 2)$
$(2^2)^{2x} = 1/2^5$
$2^{4x} = 2^{-5}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$4x = -5$
$x = -5/4$

(ii) We have, $\sqrt{(2^{x+3})} = 16$
$(2^{x+3})^{1/2} = (2 \times 2 \times 2 \times 2)$
$2^{(x+3)/2} = 2^4$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$(x + 3)/2 = 4$
$x + 3 = 8$
$x = 8 – 3$
$x = 5$

(iii) We have, $[\sqrt{(3/5)}]^{x+1} = 125/27$
$[(3/5)^{1/2}]^{x+1} = (5 \times 5 \times 5)/(3 \times 3 \times 3)$
$(3/5)^{(x+1)/2} = 5^3/3^3$
$(3/5)^{(x+1)/2} = (5/3)^3$
$(3/5)^{(x+1)/2} = (3/5)^{-3}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$(x + 1)/2 = -3$
$x + 1 = -6$
$x = -6 – 1$
$x = -7$

(iv) We have, $[\sqrt[3]{(2/3)}]^{x-1} = 27/8$
$[(2/3)^{1/3}]^{x-1} = (3 \times 3 \times 3)/(2 \times 2 \times 2)$
$(2/3)^{(x-1)/3} = (3/2)^3$
$(2/3)^{(x-1)/3} = (2/3)^{-3}$

Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$(x - 1)/3 = -3$
$x – 1 = -9$
$x = -9 + 1$
$x = -8$

3. Solve:
(i) $4^{x-2} - 2^{x+1} = 0$
(ii) $3^{x^2} : 3^x = 9 : 1$
Solution:

(i) We have,
$4^{x-2} - 2^{x+1} = 0$
$(2^2)^{x-2} – 2^{x+1} = 0$
$2^{2x-4} – 2^{x+1} = 0$
$2^{2x-4} = 2^{x+1}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$2x – 4 = x + 1$
$2x – x = 4 + 1$
$x = 5$

(ii) We have,
$3^{x^2} : 3^x = 9 : 1$
$\frac{3^{x^2}}{3^x} = \frac{9}{1}$
$\Rightarrow 3^{x^2} = 9 \times 3^x$
$\Rightarrow 3^{x^2} = 3^2 \times 3^x$
$\Rightarrow 3^{x^2} = 3^{x+2}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$x^2 = x+ 2$
$x^2 – x – 2 = 0$
On factorization, we get
$x^2 – 2x + x – 2 = 0$
$x(x – 2 + 1(x – 2) = 0$
$(x + 1)(x - 2) = 0$
So, either $(x + 1) = 0$ or $(x - 2) = 0$
Thus, $x = -1$ or 2

4. Solve:
(i) $8 \times 2^{2x} + 4 \times 2^{x+1} = 1 + 2^x$

(ii) $2^{2x} + 2^{x+2} – 4 \times 2^3 = 0$
(iii) $(\sqrt{3})^{x-3} = (\sqrt[4]{3})^{x+1}$
Solution:

(i) We have, $8 \times 2^{2x} + 4 \times 2^{x+1} = 1 + 2^x$
$8 \times (2^x)^2 + 4 \times (2^x) \times 2^1 = 1 + 2^x$
Let us substitute $2^x = t$
Then,
$8 \times t^2 + 4 \times t \times 2 = 1 + t$
$8t^2 + 8t = 1 + t$
$8t^2 + 8t – t – 1 = 0$
$8t^2 + 7t – 1 = 0$
$8t^2 + 8t – t – 1 = 0$
$8t(t + 1) – 1(t + 1) = 0$
$(8t - 1)(t + 1) = 0$
So, either $8t – 1 = 0$ or $t + 1 = 0$
Thus, $t = 1/8$ or -1
Now, we have
$2^x = t$
So,
$2^x = 1/8$ or $2^x = -1$
The equation, $2^x = -1$ is not possible
Hence, for $2^x = 1/8$
$2^x = 1/(2 \times 2 \times 2)$
$2^x = 1/2^3$
$2^x = 2^{-3}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$x = -3$

(ii) We have,
$2^{2x} + 2^{x+2} – 4 \times 2^3 = 0$
$2^{2x} + 2^{x+2} – 2^2 \times 2^3 = 0$
$(2^x)^2 + 2^x.2^2 – 2^{3+2} = 0$
$(2^x)^2 + 2^x.2^2 – 2^5 = 0$
Now, let's assume $2^x = t$
So, the above equation becomes
$(t)^2 + t.2^2 – 2^5 = 0$
$t^2 + 4t – 32 = 0$
On factorization, we have
$t^2 + 8t – 4t – 32 = 0$
$t(t + 8) – 4(t + 8) = 0$
$(t - 4)(t + 8) = 0$
So, either $(t - 4) = 0$ or $(t + 8) = 0$
Thus, $t = 4$ or -8
Now, we have $t = 2^x$

So,
$2^x = 4$ or $2^x = -8$
The equation, $2^x = -8$ is not possible
Hence, for
$2^x = 4$
$2^x = 2^2$
On comparing the exponents, we get
$x = 2$

(iii) $(\sqrt{3})^{x-3} = (\sqrt[4]{3})^{x+1}$
$(3^{1/2})^{x-3} = (3^{1/4})^{x+1}$
$3^{(x-3)/2} = 3^{(x+1)/4}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$(x - 3)/2 = (x + 1)/4$
$2(x - 3) = (x + 1)$
$2x – 6 = x + 1$
$2x – x = 6 + 1$
$x = 7$

5. Find the values of m and n if:
$4^{2m} = (\sqrt[3]{16})^{-6/n} = (\sqrt{8})^2$
Solution:

We have, $4^{2m} = (\sqrt[3]{16})^{-6/n} = (\sqrt{8})^2$
Now, considering
$4^{2m} = (\sqrt{8})^2$
$(2^2)^{2m} = (8^{1/2})^2$
$2^{4m} = 8^{1/2 \times 2}$
$2^{4m} = 8$
$\Rightarrow 2^{4m} = 2^3$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$4m = 3$
$m = 3/4$

Now, from the given considering
$(\sqrt[3]{16})^{-6/n} = (\sqrt{8})^2$
$(16^{1/3})^{-6/n} = (8^{1/2})^2$
$(16)^{1/3 \times -6/n} = 8^{1/2 \times 2}$
$(16)^{-2/n} = 8$
$(2 \times 2 \times 2 \times 2)^{-2/n} = (2 \times 2 \times 2)$
$(2)^{4 \times -2/n} = 2^3$
$(2)^{-8/n} = 2^3$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get

-8/n = 3
n = -8/3

Therefore, the value of m and n are ¾ and -8/3

6. Solve x and y if:
$(\sqrt{32})^x \div 2^{y+1} = 1$ and $8^y - 16^{4 - x/2} = 0$
Solution:

Consider the equation, $(\sqrt{32})^x \div 2^{y+1} = 1$
$(\sqrt{(2 \times 2 \times 2 \times 2 \times 2)})^x \div 2^{y+1} = 1$
$(\sqrt{2^5})^x \div 2^{y+1} = 1$
$(2^5)^{1/2 \times x} \div 2^{y+1} = 1$
$2^{5x/2} \div 2^{y+1} = 1$
$(2^{5x/2})/(2^{y+1}) = 1$
$2^{5x/2 - (y+1)} = 2^0$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$5x/2 - (y+1) = 0$
$5x/2 - y - 1 = 0$
$5x - 2y - 2 = 0$ … (i)

Next, let's consider $8^y - 16^{4 - x/2} = 0$
$(2 \times 2 \times 2)^y - (2 \times 2 \times 2 \times 2)^{4 - x/2} = 0$
$(2^3)^y - (2^4)^{4 - x/2} = 0$
$2^{3y} - 2^{16 - 2x} = 0$
$2^{3y} = 2^{16 - 2x}$
Now, if the bases are equal, then the powers must be equal
So, on comparing the exponents, we get
$3y = 16 - 2x$
$2x + 3y - 16 = 0$ … (ii)
On solving equations (i) and (ii),
By manipulating by (i) × 3 + (ii) × 2, we have
$15x - 6y - 6 = 0$
$4x + 6y - 32 = 0$

$19x - 38 = 0$
$x = 38/19$
$x = 2$
Now, substituting the value of x in (i)
$5(2) - 2y - 2 = 0$
$10 - 2y - 2 = 0$
$8 = 2y$
$y = 8/2$
$y = 4$

Therefore, the values of x and y are 2 and 4 respectively

7. Prove that:
(i) $(x^a/x^b)^{a+b-c}. (x^b/x^c)^{b+c-a}. (x^c/x^a)^{c+a-b} = 1$
(ii) $x^{a(b-c)}/x^{b(a-c)} \div (x^b/x^a)^c = 1$
Solution:

(i) Taking L.H.S, we have
$(x^a/x^b)^{a+b-c}. (x^b/x^c)^{b+c-a}. (x^c/x^a)^{c+a-b}$
$= (x^a/x^b)^{a+b-c}. (x^b/x^c)^{b+c-a}. (x^c/x^a)^{c+a-b}$
$= x^{(a-b)(a+b-c)}. x^{(b-c)(b+c-a)}. x^{(c-a)(c+a-b)}$
$= x^{a^2+ab-ac-ab-b^2+bc} \times x^{b^2+bc-ab-cb-c^2+ac} \times x^{c^2+ac-bc-ac-a^2+ab}$
$= x^{a^2-ac-b^2+bc+b^2-ab-c^2+ac+c^2-bc-a^2+ab}$
$= x^0$
$= 1$
= R.H.S

(ii) Taking L.H.S, we have
$x^{a(b-c)}/x^{b(a-c)} \div (x^b/x^a)^c$
$= x^{a(b-c) - b(a-c)} \div x^{c(b-a)}$
$= x^{a(b-c) - b(a-c)}/x^{c(b-a)}$
$= x^{a(b-c) - b(a-c) - c(b-a)}$
$= x^{ab-ac-ba+bc-cb+ac}$
$= x^0$
$= 1$
= R.H.S

8. If $a^x = b$, $b^y = c$ and $c^z = a$, prove that: $xyz = 1$.
Solution:

We have, $a^x = b$, $b^y = c$ and $c^z = a$
Now, considering
$a^x = b$
On raising to the power yz on both sides, we get
$(a^x)^{yz} = (b)^{yz}$
$(a)^{xyz} = (b^y)^z$
$(a)^{xyz} = (c)^z$ [As, $b^y = c$]
$a^{xyz} = a$
$a^{xyz} = a^1$ [As, $c^z = a$]
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
Hence, $xyz = 1$

9. If $a^x = b^y = c^z$ and $b^2 = ac$, prove that: $y = 2az/(x + z)$.
Solution:

Let's assume $a^x = b^y = c^z = k$
So,
$a = k^{1/x}$; $b = k^{1/y}$ and $c = k^{1/z}$
Now,
It's given that $b^2 = ac$
$\Rightarrow (k^{1/y})^2 = (k^{1/x}) \times (k^{1/z})$
$(k^{2/y}) = k^{1/x + 1/z}$
$k^{2/y} = k^{(z+x)/xz}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$2/y = (z + x)/xz$
$2xz = y(z + x)$
Hence,
$y = 2xz/(x + z)$

10. If $5^{-p} = 4^{-q} = 20^r$, show that: $1/p + 1/q + 1/r = 0$.
Solution:

Let's assume $5^{-p} = 4^{-q} = 20^r = k$
Then, as
$5^{-p} = k \Rightarrow 5 = k^{-1/p}$
$4^{-q} = k \Rightarrow 4 = k^{-1/q}$
$20^r = k \Rightarrow 20 = k^{1/r}$
Now, we know
$5 \times 4 = 20$
$(k^{-1/p}) \times (k^{-1/q}) = k^{1/r}$
$k^{-1/p - 1/q} = k^{1/r}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$-1/p - 1/q = 1/r$
Hence,
$1/p + 1/q + 1/r = 0$

11. If $m \neq n$ and $(m + n)^{-1} (m^{-1} + n^{-1}) = m^x n^y$, show that: $x + y + 2 = 0$
Solution:

Given equation,
$(m + n)^{-1} (m^{-1} + n^{-1}) = m^x n^y$
$1/(m + n) \times (1/m + 1/n) = m^x n^y$
$1/(m + n) \times (m + n)/mn = m^x n^y$
$1/mn = m^x n^y$
$m^{-1}n^{-1} = m^x n^y$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$x = -1$ and $y = -1$

Substituting the values of x and y in the equation x + y + 2 = 0, we have
(-1) + (-1) + 2 = 0
0 = 0
L.H.S = R.H.S
Therefore, x + y + 2 = 0

12. If $5^{x+1} = 25^{x-2}$, find the value of $3^{x-3} \times 2^{3-x}$
Solution:

We have, $5^{x+1} = 25^{x-2}$
$5^{x+1} = (5^2)^{x-2}$
$5^{x+1} = 5^{2x-4}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
x + 1 = 2x - 4
2x – x = 4 + 1
x = 5
Hence, the value of
$3^{x-3} \times 2^{3-x} = 3^{5-3} \times 2^{3-5}$
$= 3^2 \times 2^{-2}$
$= 9 \times 1/4$
= 9/4

13. If $4^{x+3} = 112 + 8 \times 4^x$, find the value of $(18x)^{3x}$.
Solution:

We have,
$4^{x+3} = 112 + 8 \times 4^x$
$4^x .4^3 = 112 + 8 \times 4^x$
Let's assume $4^x = t$
Then,
$t .4^3 = 112 + 8 \times t$
64t = 112 + 8t
64t – 8t = 112
56t = 112
t = 112/56
t = 2
But we have taken $4^x = t$
So, $4^x = 2$
$(2^2)^x = 2^1$
$2^{2x} = 2^1$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
2x = 1
x = ½

Now, the value of $(18x)^{3x}$ will be
$= (18 \times ½)^{3 \times 1/2}$
$= (9)^{3/2}$
$= (3^2)^{3/2}$
$= 3^3$
$= 27$

14. Solve for x:
(i) $4^{x-1} \times (0.5)^{3 - 2x} = (1/8)^{-x}$
Solution:

We have,
$4^{x-1} \times (0.5)^{3 - 2x} = (1/8)^{-x}$
$(2^2)^{x-1} \times (1/2)^{3 - 2x} = (1/2^3)^{-x}$
$(2)^{2x-1} \times (2)^{-(3 - 2x)} = (2^{-3})^{-x}$
$(2)^{2x-2} \times (2)^{2x-3} = (2)^{3x}$
$2^{(2x-2) + (2x-3)} = (2)^{3x}$
$2^{4x-5} = 2^{3x}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$4x – 5 = 3x$
$4x – 3x = 5$
$x = 5$

(ii) $(a^{3x + 5})^2 \times (a^x)^4 = a^{8x + 12}$
Solution:

We have,
$(a^{3x + 5})^2 \times (a^x)^4 = a^{8x + 12}$
$a^{6x + 10} \times a^{4x} = a^{8x + 12}$
$a^{6x + 10 + 4x} = a^{8x + 12}$
$a^{10x + 10} = a^{8x + 12}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$10x + 10 = 8x + 12$
$10x – 8x = 12 – 10$
$2x = 2$
$x = 1$

(iii) $(81)^{3/4} – (1/32)^{-2/5} + x(1/2)^{-1} \times 2^0 = 27$
Solution:

We have,
$(81)^{3/4} – (1/32)^{-2/5} + x(1/2)^{-1} \times 2^0 = 27$
$(3^4)^{3/4} – (1/2^5)^{-2/5} + x(1/2)^{-1} \times 2^0 = 27$
$(3^4)^{3/4} – (2^{-5})^{-2/5} + x(2^{-1})^{-1} \times 2^0 = 27$

$3^3 - 2^2 + 2x \times 1 = 27$
$27 - 4 + 2x = 27$
$2x + 23 = 27$
$2x = 27 - 23$
$2x = 4$
$x = 4/2$
$x = 2$

(iv) $2^{3x+3} = 2^{3x+1} + 48$
Solution:

We have,
$2^{3x+3} = 2^{3x+1} + 48$
$2^{3x+3} - 2^{3x+1} = 48$
$2^{3x}(2^3 - 2^1) = 48$
$2^{3x}(8 - 2) = 48$
$2^{3x} \times 6 = 48$
$2^{3x} = 48/6$
$2^{3x} = 8$
$2^{3x} = 2^3$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$3x = 3$
$x = 1$

(v) $3(2^x + 1) - 2^{x+2} + 5 = 0$
Solution:

We have,
$3(2^x + 1) - 2^{x+2} + 5 = 0$
$3 \times 2^x + 3 - 2^x.2^2 + 5 = 0$
$2^x(3 - 2^2) + 5 + 3 = 0$
$2^x(3 - 4) + 8 = 0$
$-2^x + 8 = 0$
$2^x = 8$
$2^x = 2^3$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$x = 3$

(vi) $9^{x+2} = 720 + 9^x$
Solution:

We have,
$9^{x+2} = 720 + 9^x$
$9^{x+2} - 9^x = 720$

$9^x (9^2 - 1) = 720$
$9^x (81 - 1) = 720$
$9^x (80) = 720$
$9^x = 9$
$9^x = 9^1$
Therefore, $x = 1$

Exercise 7(C)

1. Evaluate:

(i) $9^{5/2} – 3 \times 8^{0} – (1/81)^{-1/2}$

(ii) $(64)^{2/3} - \sqrt[3]{125} – 1/2^{-5} + (27)^{-2/3} \times (25/9)^{-1/2}$

(iii) $[(-2/3)^{-2}]^{3} \times (1/3)^{-4} \times 3^{-1} \times 1/6$

Solution:

(i) We have, $9^{5/2} – 3 \times 8^{0} – (1/81)^{-1/2}$

$= (3^{2})^{5/2} – 3 \times 1 – (1/9^{2})^{-1/2}$

$= 3^{2 \times 5/2} – 3 – (1/9)^{-2 \times ½}$

$= 3^{5} – 3 – (1/9)^{-1}$

$= (3 \times 3 \times 3 \times 3 \times 3) – 3 – (9^{-1})^{-1}$

$= 243 – 3 – 9$

$= 231$

(ii) We have, $(64)^{2/3} - \sqrt[3]{125} – 1/2^{-5} + (27)^{-2/3} \times (25/9)^{-1/2}$

$= (4 \times 4 \times 4)^{2/3} - (5 \times 5 \times 5)^{1/3} – 1/2^{-5} + (3 \times 3 \times 3)^{-2/3} \times (5 \times 5/3 \times 3)^{-1/2}$

$= (4^{3})^{2/3} – (5^{3})^{1/3} – (2^{-1})^{-5} + (3^{3})^{-2/3} \times (5^{2}/3^{2})^{-1/2}$

$= (4)^{3 \times 2/3} – (5)^{3 \times 1/3} – (2)^{-1 \times -5} + (3)^{3 \times -2/3} \times (5/3)^{2 \times -1/2}$

$= (4)^{2} – 5 – 2^{5} + 3^{-2} \times (5/3)^{-1}$

$= 16 – 5 – 32 + 1/9 \times 3/5$

$= -21 + 1/15$

$= (-315 + 1)/15$

$= -314/15$

(iii) We have, $[(-2/3)^{-2}]^{3} \times (1/3)^{-4} \times 3^{-1} \times 1/6$

$= (-2/3)^{-6} \times (3^{-1})^{-4} \times 3^{-1} \times ½ \times 1/3$

$= (-3/2)^{6} \times 3^{4} \times 3^{-1} \times 2^{-1} \times 3^{-1}$

$= (-1)^{6} \times (3^{6} \times 3^{4} \times 3^{-1} \times 3^{-1}) \times (2^{-6} \times 2^{-1})$

$= 1 \times 3^{6+4-1-1} \times 2^{-6-1}$

$= 3^{8} \times 2^{-7}$

$= 3^{8}/2^{7}$

2. Simplify: $\dfrac{3 \times 9^{n+1} - 9 \times 3^{2n}}{3 \times 3^{2n+3} - 9^{n+1}}$

Solution:

We have,

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$$\frac{3 \times 9^{n+1} - 9 \times 3^{2n}}{3 \times 3^{2n+3} - 9^{n+1}}$$

$$= \frac{3 \times \left(3^2\right)^{n+1} - 3^2 \times 3^{2n}}{3 \times 3^{2n+3} - \left(3^2\right)^{n+1}}$$

$$= \frac{3^{1+2n+2} - 3^{2+2n}}{3^{1+2n+3} - 3^{2n+2}}$$

$$= \frac{3^{3+2n} - 3^{2+2n}}{3^{4+2n} - 3^{2n+2}}$$

$$= \frac{3^{2n}\left(3^3 - 3^2\right)}{3^{2n}\left(3^4 - 3^2\right)}$$

= (27 – 9)/(81 - 9)
= 18/72
= 1/4

3. Solve: $3^{x-1} \times 5^{2y-3} = 225$.
Solution:

Given, $3^{x-1} \times 5^{2y-3} = 225$
$3^{x-1} \times 5^{2y-3} = 9 \times 25$
$3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
x – 1 = 2 and 2y – 3 = 2
x = 2 + 1 and 2y = 2 + 3
x = 3 and 2y = 5
x = 3 and y = 5/2 = 2.5

4. If $[(a^{-1}b^2)/(a^2b^{-4})]^7 \div [(a^3b^{-5})/(a^{-2}b^3)] = a^x.b^y$, find x + y.
Solution:

We have,

$$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right)^{-5} = a^x \cdot b^y$$

$$\Rightarrow \left(\frac{b^6}{a^3}\right)^7 \div \left(\frac{a^5}{b^8}\right)^{-5} = a^x \cdot b^y$$

$$\Rightarrow \left(\frac{b^6}{a^3}\right)^7 \div \left(\frac{b^8}{a^5}\right)^{5} = a^x \cdot b^y$$

$(b^{42}/a^{21}) \div (b^{40}/a^{25}) = a^x. b^y$
$(b^{42}/a^{21}) \times (a^{25}/b^{40}) = a^x. b^y$
$b^{42 - 40} \times a^{25 - 21} = a^x. b^y$
$a^4 \times b^2 = a^x. b^y$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$x = 4$ and $y = 2$
$x + y = 4 + 2 = 6$

5. If $3^{x+1} = 9^{x-3}$, find the value of 2^{1+x}.
Solution:

We have,
$3^{x+1} = 9^{x-3}$
$3^x \times 3^1 = (3^2)^{x - 3}$
$3^x \times 3^1 = (3)^{2x - 6}$
$3^x = (3)^{2x - 6}/3$
$3^x = (3)^{2x - 6} \times 3^{-1}$
$3^x = (3)^{2x - 6 - 1}$
$3^x = 3^{2x - 7}$
Now, if the bases are equal, then the powers must be equal
On comparing the exponents, we get
$x = 2x - 7$
$x = 7$
Now,
$2^{1+x} = 2^{1+7} = 2^8 = 256$

6. If $2^x = 4^y = 8^z$ and $1/2x + 1/4y + 1/8z = 4$, find the value of x.
Solution:

Given, $2^x = 4^y = 8^z$
$2^x = (2^2)^y = (2^3)^z$
$2^x = 2^{2y} = 2^{3z}$
On comparing the powers, we get
$x = 2y = 3z$
$y = x/2$ and $z = x/3$
Now,

1/2x + 1/4y + 1/8z = 4
1/2x + 1/4(x/2) + 1/8(x/3) = 4
1/2x + 2/4x + 3/8x = 4
1/2x + 1/2x + 3/8x = 4
(4 + 4 + 3)/8x = 4
11/8x = 4
4 × 8x = 11
32x = 11
x = 11/32

7. If $\frac{9^n \cdot 3^2 \cdot 3^n - (27)^n}{(3^m \cdot 2)^3} = 3^{-3}$.
Show that: m – n = 1
Solution:

We have,

$$\frac{9^n . 3^2 . 3^n - (27)^n}{(3^m . 2)^3} = 3^{-3}$$

$$\frac{3^{2n} . 3^2 . 3^n - (3)^{3n}}{3^{3m} . (2)^3} = \frac{1}{3^3}$$

$$\frac{3^{3n} . 3^2 - 3^{3n}}{3^{3m} . 2^3} = \frac{1}{3^3}$$

$$\frac{3^{3n}(3^2 - 1)}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\frac{3^{3n} \times 8}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\frac{1}{3^{3(m-n)}} = \frac{1}{3^{3\times1}}$$

On comparing the powers, we get
m – n = 1

8. Solve for x: $(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$.
Solution:

We have,
$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$

$= 256 – 81 – 6$
$= 169$
$= 13^2$
$13^{\sqrt{x}} = 13^2$
On comparing the powers, we get
$\sqrt{x} = 2$
Squaring on both sides,
$x = 2^2$
$x = 4$

9. If $3^{4x} = (81)^{-1}$ and $(10)^{1/y} = 0.0001$, find the value of $2^{-x} \times 16^y$.
Solution:

We have, $3^{4x} = (81)^{-1}$ and $(10)^{1/y} = 0.0001$
$3^{4x} = (3^4)^{-1}$ and $(10)^{1/y} = 1/10000$
$3^{4x} = 3^{-4}$ and $10^{1/y} = 1/10^{-4}$
$3^{4x} = 3^{-4}$ and $10^{1/y} = 1/10^4$
$3^{4x} = 3^{-4}$ and $10^{1/y} = 10^{-4}$
On comparing the powers, we get
$4x = -4$ and $1/y = -4$
$x = -1$ and $y = -1/4$
Now, value of
$2^{-x} \times 16^y = 2^{-(-1)} \times 16^{-1/4}$
$= 2^1 \times (2^4)^{-1/4}$
$= 2 \times 2^{-1}$
$= 2/2$
$= 1$

10. Solve: $3(2^x + 1) - 2^{x+2} + 5 = 0$.
Solution:

We have, $3(2^x + 1) - 2^{x+2} + 5 = 0$
$(3 \times 2^x + 3) – (2^x \times 2^2) + 5 = 0$
$2^x(3 - 2^2) + 3 + 5 = 0$
$2^x(3 - 4) + 8 = 0$
$2^x(-1) + 8 = 0$
$-2^x + 8 = 0$
$2^x = 8$
$2^x = 2^3$
On comparing the powers, we get
$x = 3$

11. If $(a^m)^n = a^m .a^n$, find the value of: $m(n - 1) - (n - 1)$
Solution:

We have, $(a^m)^n = a^m .a^n$

$a^{mn} = a^{m+n}$
On comparing the powers, we get
$mn = m + n$ … (i)
Now,
$m(n - 1) – (n - 1) = mn – m – n + 1$
$= (m + n) – m – n + 1$ … [Form (i)]
$= 1$

12. If $m = \sqrt[3]{15}$ and $n = \sqrt[3]{14}$, find the value of $m – n – 1/(m^2 + mn + n^2)$
Solution:

We have,
$m = \sqrt[3]{15}$ and $n = \sqrt[3]{14}$
$\Rightarrow m = 15^3$ and $n = 14^3$

$$\therefore m - n - \frac{1}{m^2 + mn + n^2} = \frac{(m^3 + m^2n + mn^2) - (m^2n + mn^2 + n^3) - 1}{m^2 + mn + n^2}$$
$$= \frac{m^3 + m^2n + mn^2 - m^2n - mn^2 - n^3 - 1}{m^2 + mn + n^2}$$
$$= \frac{m^3 - n^3 - 1}{m^2 + mn + n^2}$$
$$= \frac{15 - 14 - 1}{m^2 + mn + n^2}$$
$$= \frac{1 - 1}{m^2 + mn + n^2}$$
$$= 0$$

13. Evaluate: $\dfrac{2^n \times 6^{m+1} \times 10^{m-n} \times 15^{m+n-2}}{4^m \times 3^{2m+n} \times 25^{m-1}}$
Solution:

We have,

$$\frac{2^n \times 6^{m+1} \times 10^{m-n} \times 15^{m+n-2}}{4^m \times 3^{2m+n} \times 25^{m-1}}$$

$$= \frac{2^n \times 6^m \times 6 \times 10^m \times 10^{-n} \times 15^m \times 15^n \times 15^{-2}}{4^m \times (3^2)^m \times 3^n \times 25^m \times 25^{-1}}$$

$$= \frac{\left(2 \times \frac{1}{10} \times 15\right)^n \times (6 \times 10 \times 15)^m \times 6 \times \frac{1}{15^2}}{3^n \times (4 \times 3^2 \times 25)^m \times \frac{1}{25}}$$

$$= \frac{3^n \times 900^m \times \frac{6}{225}}{3^n \times 900^m \times \frac{1}{25}}$$

= 6/225 x 25/1

= 6/9

= 2/3

14. Evaluate:

$(x^q/x^r)^{1/qr} \times (x^r/x^p)^{1/rp} \times (x^p/x^q)^{1/pq}$

Solution:

We have,

$$\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}}$$

$$= \frac{x^{q \times \frac{1}{qr}}}{x^{r \times \frac{1}{qr}}} \times \frac{x^{r \times \frac{1}{rp}}}{x^{p \times \frac{1}{rp}}} \times \frac{x^{p \times \frac{1}{pq}}}{x^{q \times \frac{1}{pq}}}$$

$$= \frac{x^{\frac{1}{r}}}{x^{\frac{1}{q}}} \times \frac{x^{\frac{1}{p}}}{x^{\frac{1}{r}}} \times \frac{x^{\frac{1}{q}}}{x^{\frac{1}{p}}}$$

$$= 1$$

15. (i) Prove that: $a^{-1}/(a^{-1} + b^{-1}) + a^{-1}/(a^{-1} - b^{-1}) = 2b^2/(b^2 - a^2)$

Solution:

We have,

$$\frac{a^{-1}}{a^{-1}+b^{-1}}+\frac{a^{-1}}{a^{-1}-b^{-1}}=\frac{2}{b^2-a^2}$$

L.H.S. = $\frac{a^{-1}}{a^{-1}+b^{-1}}+\frac{a^{-1}}{a^{-1}-b^{-1}}$

= $\frac{\frac{1}{a}}{\frac{1}{a}+\frac{1}{b}}+\frac{\frac{1}{a}}{\frac{1}{a}-\frac{1}{b}}$

= $\frac{\frac{1}{a}}{\frac{b+a}{ab}}+\frac{\frac{1}{a}}{\frac{b-a}{ab}}$

= 1/a x ab/(b + a) + 1/a x ab/(b - a)
= b/(b + a) + b/(b - a)
= (b^2 – ab + b^2 + ab)/(b^2 – a^2)
= $2b^2$/(b^2 – a^2)
= R.H.S

(ii) Prove that: (a + b + c)/($a^{-1}b^{-1}$ + $b^{-1}c^{-1}$ + $c^{-1}a^{-1}$) = abc
Solution:

Taking L.H.S., we have

L.H.S. = $\frac{a+b+c}{a^{-1}b^{-1}+b^{-1}c^{-1}+c^{-1}a^{-1}}$

= $\frac{a+b+c}{\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}}$

= $\frac{a+b+c}{\frac{c+a+b}{abc}}$

= $\frac{(a+b+c)(abc)}{a+b+c}$

= abc
= R.H.S

16. Evaluate: 4/$(216)^{-2/3}$ + 1/$(256)^{-3/4}$ + 2/$(243)^{-1/5}$
Solution:

We have,

$$\frac{4}{(216)^{-\frac{2}{3}}}+\frac{1}{(256)^{-\frac{3}{4}}}+\frac{2}{(243)^{-\frac{1}{5}}}$$

$$=\frac{4}{\left(6^{3}\right)^{-\frac{2}{3}}}+\frac{1}{\left(4^{4}\right)^{-\frac{3}{4}}}+\frac{2}{\left(3^{5}\right)^{-\frac{1}{5}}}$$

$= 4/6^{-2} + 1/4^{-3} + 2/3^{-1}$

$= 4 \times 6^2 + 4^3 + 2 \times 3^1$

$= 4 \times 36 + 64 + 2 \times 3$

$= 144 + 64 + 6$

$= 214$