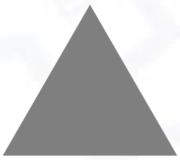
Mathematics (MA)

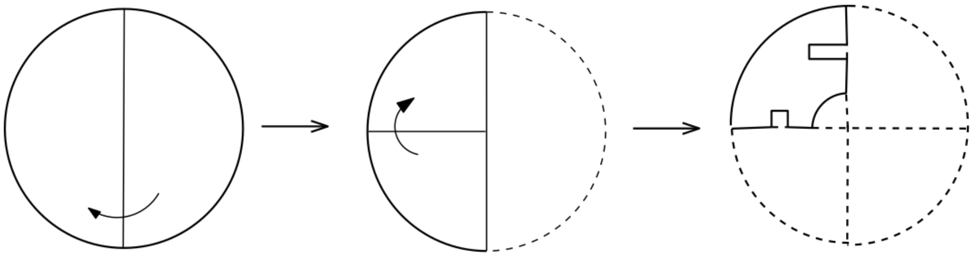
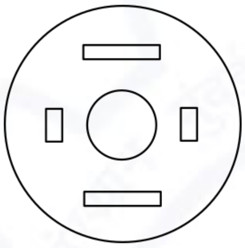
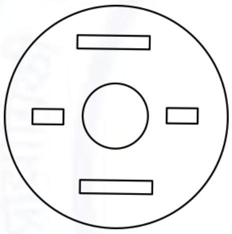
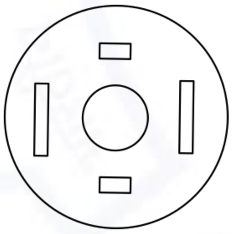
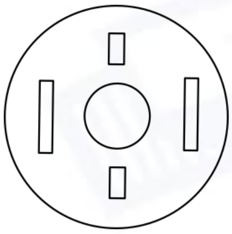
General Aptitude (GA)

Q.1 – Q.5 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: – 1/3).

Q.1	**The ratio of boys to girls in a class is 7 to 3.** **Among the options below, an acceptable value for the total number of students in the class is:**
(A)	21
(B)	37
(C)	50
(D)	73

Q.2	**A polygon is convex if, for every pair of points, P and Q belonging to the polygon, the line segment PQ lies completely inside or on the polygon.** **Which one of the following is <u>NOT</u> a convex polygon?**
(A)	
(B)	
(C)	
(D)	

Q.3	**Consider the following sentences:** **(i) Everybody in the class is prepared for the exam.** **(ii) Babu invited Danish to his home because he enjoys playing chess.** **Which of the following is the CORRECT observation about the above two sentences?**
(A)	(i) is grammatically correct and (ii) is unambiguous
(B)	(i) is grammatically incorrect and (ii) is unambiguous
(C)	(i) is grammatically correct and (ii) is ambiguous
(D)	(i) is grammatically incorrect and (ii) is ambiguous

Q.4	 **A circular sheet of paper is folded along the lines in the directions shown. The paper, after being punched in the final folded state as shown and unfolded in the reverse order of folding, will look like _______.**
(A)	
(B)	
(C)	
(D)	

Q.5	**_____ is to *surgery* as *writer* is to ________** **Which one of the following options maintains a similar logical relation in the above sentence?**
(A)	Plan, outline
(B)	Hospital, library
(C)	Doctor, book
(D)	Medicine, grammar

Q. 6 – Q.10 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: – 2/3).

Q.6	**We have 2 rectangular sheets of paper, M and N, of dimensions 6 cm x 1 cm each. Sheet M is rolled to form an open cylinder by bringing the short edges of the sheet together. Sheet N is cut into equal square patches and assembled to form the largest possible closed cube. Assuming the ends of the cylinder are closed, the ratio of the volume of the cylinder to that of the cube is ________**
(A)	$\frac{\pi}{2}$
(B)	$\frac{3}{\pi}$
(C)	$\frac{9}{\pi}$
(D)	3π

Q.7

Items	Cost (₹)	Profit %	Marked Price (₹)
P	5,400	---	5,860
Q	---	25	10,000

Details of prices of two items P and Q are presented in the above table. The ratio of cost of item P to cost of item Q is 3:4. Discount is calculated as the difference between the marked price and the selling price. The profit percentage is calculated as the ratio of the difference between selling price and cost, to the cost (Profit % $= \frac{\text{Selling price} - \text{Cost}}{\text{Cost}} \times 100$).

The discount on item Q, as a percentage of its marked price, is ______

(A)	25
(B)	12.5
(C)	10
(D)	5

Q.8 **There are five bags each containing identical sets of ten distinct chocolates. One chocolate is picked from each bag.**

The probability that at least two chocolates are identical is ___________

(A)	0.3024
(B)	0.4235
(C)	0.6976
(D)	0.8125

Q.9	**Given below are two statements 1 and 2, and two conclusions I and II.** **Statement 1: All bacteria are microorganisms.** **Statement 2: All pathogens are microorganisms.** **Conclusion I: Some pathogens are bacteria.** **Conclusion II: All pathogens are not bacteria.** **Based on the above statements and conclusions, which one of the following options is logically CORRECT?**
(A)	Only conclusion I is correct
(B)	Only conclusion II is correct
(C)	Either conclusion I or II is correct.
(D)	Neither conclusion I nor II is correct.

Q.10	**Some people suggest anti-obesity measures (AOM) such as displaying calorie information in restaurant menus. Such measures sidestep addressing the core problems that cause obesity: poverty and income inequality.** **Which one of the following statements summarizes the passage?**
(A)	The proposed AOM addresses the core problems that cause obesity.
(B)	If obesity reduces, poverty will naturally reduce, since obesity causes poverty.
(C)	AOM are addressing the core problems and are likely to succeed.
(D)	AOM are addressing the problem superficially.

Mathematics (MA)

Q.1 – Q.14 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: – 1/3).

Q.1	**Let A be a 3×4 matrix and B be a 4×3 matrix with real entries such that AB is non-singular. Consider the following statements:** **P: Nullity of A is 0.** **Q: BA is a non-singular matrix.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.2	**Let $f(z) = u(x, y) + i\,v(x, y)$ for $z = x + iy \in \mathbb{C}$, where x and y are real numbers, be a non-constant analytic function on the complex plane $\mathbb{C}$. Let u_x, v_x and u_y, v_y denote the first order partial derivatives of $u(x, y) = Re(f(z))$ and $v(x, y) = Im(f(z))$ with respect to real variables x and y, respectively. Consider the following two functions defined on $\mathbb{C}$:** $$g_1(z) = u_x(x, y) - i\,u_y(x, y) \text{ for } z = x + iy \in \mathbb{C},$$ $$g_2(z) = v_x(x, y) + i\,v_y(x, y) \text{ for } z = x + iy \in \mathbb{C}.$$ **Then**
(A)	both $g_1(z)$ and $g_2(z)$ are analytic in $\mathbb{C}$
(B)	$g_1(z)$ is analytic in $\mathbb{C}$ and $g_2(z)$ is NOT analytic in $\mathbb{C}$
(C)	$g_1(z)$ is NOT analytic in $\mathbb{C}$ and $g_2(z)$ is analytic in $\mathbb{C}$
(D)	neither $g_1(z)$ nor $g_2(z)$ is analytic in $\mathbb{C}$

Q.3	**Let $T(z) = \frac{az+b}{cz+d}$, $ad - bc \neq 0$, be the Möbius transformation which maps the points $z_1 = 0$, $z_2 = -i$, $z_3 = \infty$ in the z-plane onto the points $w_1 = 10$, $w_2 = 5 - 5i$, $w_3 = 5 + 5i$ in the w-plane, respectively. Then the image of the set $S = \{z \in \mathbb{C} : Re(z) < 0\}$ under the map $w = T(z)$ is**
(A)	$\{w \in \mathbb{C} : \|w\| < 5\}$
(B)	$\{w \in \mathbb{C} : \|w\| > 5\}$
(C)	$\{w \in \mathbb{C} : \|w - 5\| < 5\}$
(D)	$\{w \in \mathbb{C} : \|w - 5\| > 5\}$

Q.4	**Let R be the row reduced echelon form of a 4×4 real matrix A and let the third column of R be $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$. Consider the following statements:** **P: If $\begin{bmatrix} \alpha \\ \beta \\ \gamma \\ 0 \end{bmatrix}$ is a solution of $A\mathrm{x} = 0$, then $\gamma = 0$.** **Q: For all $\mathrm{b} \in \mathbb{R}^4$, $rank[A \mid \mathrm{b}] = rank[R \mid \mathrm{b}]$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.5	**The eigenvalues of the boundary value problem** $$\frac{d^2y}{dx^2} + \lambda y = 0, \qquad x \in (0,\pi), \;\; \lambda > 0,$$ $$y(0) = 0, \qquad y(\pi) - \frac{dy}{dx}(\pi) = 0,$$ **are given by**
(A)	$\lambda = (n\pi)^2, \qquad n = 1,2,3,\ldots$
(B)	$\lambda = n^2, \qquad n = 1,2,3,\ldots$
(C)	$\lambda = k_n^2$, where $k_n, n = 1,2,3,\ldots$ are the roots of $k - \tan(k\pi) = 0$
(D)	$\lambda = k_n^2$, where $k_n, n = 1,2,3,\ldots$ are the roots of $k + \tan(k\pi) = 0$

Q.6	**The family of surfaces given by $u = xy + f(x^2 - y^2)$, where $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function, satisfies**
(A)	$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = x^2 + y^2$
(B)	$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x^2 + y^2$
(C)	$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = x^2 - y^2$
(D)	$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x^2 - y^2$

Q.7	**The function $u(x,t)$ satisfies the initial value problem** $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},\ x \in \mathbb{R},\ t > 0,$ $u(x,0) = 0,\ \frac{\partial u}{\partial t}(x,0) = 4xe^{-x^2}$. **Then $u(5,5)$ is**
(A)	$1 - \frac{1}{e^{100}}$
(B)	$1 - e^{100}$
(C)	$1 - \frac{1}{e^{10}}$
(D)	$1 - e^{10}$

Q.8	**Consider the fixed-point iteration** $x_{n+1} = \varphi(x_n), \ n \geq 0,$ **with** $\varphi(x) = 3 + (x-3)^3, \ x \in (2.5, 3.5)$, **and the initial approximation** $x_0 = 3.25.$ **Then, the order of convergence of the fixed-point iteration method is**
(A)	1
(B)	2
(C)	3
(D)	4

Q.9	**Let $\{e_n : n = 1, 2, 3, \ldots\}$ be an orthonormal basis of a complex Hilbert space H. Consider the following statements:** **P: There exists a bounded linear functional $f: H \to \mathbb{C}$ such that $f(e_n) = \frac{1}{n}$ for $n = 1, 2, 3, \ldots$.** **Q: There exists a bounded linear functional $g: H \to \mathbb{C}$ such that $g(e_n) = \frac{1}{\sqrt{n}}$ for $n = 1, 2, 3, \ldots$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.10	**Let $f: \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ be given by $f(x) = \frac{\pi}{2} + x - \tan^{-1} x$. Consider the following statements:** **P: $\lvert f(x) - f(y)\rvert < \lvert x - y\rvert$ for all $x, y \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.** **Q: f has a fixed point.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.11	**Consider the following statements:** **P: $d_1(x,y) = \left\lvert \log\left(\frac{x}{y}\right) \right\rvert$ is a metric on $(0,1)$.** **Q: $d_2(x,y) = \begin{cases} \lvert x\rvert + \lvert y\rvert, & \text{if } x \neq y, \\ 0, & \text{if } x = y, \end{cases}$ is a metric on $(0,1)$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.12	**Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a twice continuously differentiable scalar field such that $div(\nabla f) = 6$. Let S be the surface $x^2 + y^2 + z^2 = 1$ and $\hat{n}$ be unit outward normal to S. Then the value of $\iint_S (\nabla f \cdot \hat{n})\ dS$ is**
(A)	2π
(B)	4π
(C)	6π
(D)	8π

Q.13	**Consider the following statements:** **P: Every compact metrizable topological space is separable.** **Q: Every Hausdorff topology on a finite set is metrizable.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.14	**Consider the following topologies on the set $\mathbb{R}$ of all real numbers:** $T_1 = \{U \subset \mathbb{R} : 0 \notin U \text{ or } U = \mathbb{R}\}$, $T_2 = \{U \subset \mathbb{R} : 0 \in U \text{ or } U = \emptyset\}$, $T_3 = T_1 \cap T_2$. **Then the closure of the set $\{1\}$ in $(\mathbb{R}, T_3)$ is**
(A)	$\{1\}$
(B)	$\{0,1\}$
(C)	$\mathbb{R}$
(D)	$\mathbb{R}\backslash\{0\}$

Q.15 – Q.25 Numerical Answer Type (NAT), carry ONE mark each (no negative marks).

Q.15	**Let $f: \mathbb{R}^2 \to \mathbb{R}$ be differentiable. Let $D_u f(0,0)$ and $D_v f(0,0)$ be the directional derivatives of f at $(0,0)$ in the directions of the unit vectors $u = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ and $v = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$, respectively. If $D_u f(0,0) = \sqrt{5}$ and $D_v f(0,0) = \sqrt{2}$, then $\frac{\partial f}{\partial x}(0,0) + \frac{\partial f}{\partial y}(0,0) =$ ______ .**

Q.16	**Let Γ denote the boundary of the square region R with vertices $(0,0)$, $(2,0)$, $(2,2)$ and $(0,2)$ oriented in the counter-clockwise direction. Then** $$\oint_\Gamma (1 - y^2)\, dx + x\, dy = \_\_\_\_\_\_\_\_\_.$$

Q.17	**The number of 5-Sylow subgroups in the symmetric group S_5 of degree 5 is ___ .**

Q.18	**Let I be the ideal generated by $x^2 + x + 1$ in the polynomial ring $R = \mathbb{Z}_3[x]$, where $\mathbb{Z}_3$ denotes the ring of integers modulo 3. Then the number of units in the quotient ring R/I is ________ .**

Q.19	**Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that** $$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\-1\\1\end{bmatrix},\ T^2\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\\1\end{bmatrix}, \text{ and } T^2\left(\begin{bmatrix}1\\1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\1\\1\end{bmatrix}.$$ **Then the rank of T is _____ .**

Q.20	Let $y(x)$ be the solution of the following initial value problem $$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0, \quad x > 0,$$ $$y(2) = 0, \quad \frac{dy}{dx}(2) = 4.$$ Then $y(4) =$ ___________ .

Q.21	Let $$f(x) = x^4 + 2x^3 - 11x^2 - 12x + 36 \quad \text{for } x \in \mathbb{R}.$$ The order of convergence of the Newton-Raphson method $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n \geq 0,$$ with $x_0 = 2.1$, for finding the root $\alpha = 2$ of the equation $f(x) = 0$ is _______ .

Q.22	If the polynomial $$p(x) = \alpha + \beta(x+2) + \gamma(x+2)(x+1) + \delta(x+2)(x+1)x$$ interpolates the data

x	-2	-1	0	1	2
$f(x)$	2	-1	8	5	-34

then $\alpha + \beta + \gamma + \delta =$ ___________ .

Q.23	**Consider the Linear Programming Problem P:** $$\text{Maximize } 2x_1 + 3x_2$$ **subject to** $$2x_1 + x_2 \le 6,$$ $$-x_1 + x_2 \le 1,$$ $$x_1 + x_2 \le 3,$$ $$x_1 \ge 0 \text{ and } x_2 \ge 0.$$ **Then the optimal value of the dual of P is equal to _________ .**

Q.24	**Consider the Linear Programming Problem P:** $$\text{Minimize } 2x_1 - 5x_2$$ **subject to** $$2x_1 + 3x_2 + s_1 = 12,$$ $$-x_1 + x_2 + s_2 = 1,$$ $$-x_1 + 2x_2 + s_3 = 3,$$ $$x_1 \ge 0,\ x_2 \ge 0,\ s_1 \ge 0,\ s_2 \ge 0, \text{ and } s_3 \ge 0.$$ **If $\begin{bmatrix} x_1 \\ 2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}$ is a basic feasible solution of P, then $x_1 + s_1 + s_2 + s_3 =$_________ .**

Q.25	**Let H be a complex Hilbert space. Let $u, v \in H$ be such that $\langle u, v\rangle = 2$. Then** $$\frac{1}{2\pi}\int_0^{2\pi} \left\lVert u + e^{it}v \right\rVert^2 e^{it}\, dt = \text{\_\_\_\_\_\_\_\_\_}.$$

Q.26 – Q.43 Multiple Choice Question (MCQ), carry TWO mark each (for each wrong answer: – 2/3).

Q.26	**Let $\mathbb{Z}$ denote the ring of integers. Consider the subring** $R = \{a + b\sqrt{-17} : \ a, b \in \mathbb{Z}\}$ **of the field $\mathbb{C}$ of complex numbers.** **Consider the following statements:** **P: $2 + \sqrt{-17}$ is an irreducible element.** **Q: $2 + \sqrt{-17}$ is a prime element.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.27	**Consider the second-order partial differential equation (PDE)** $$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + (x^2 + 4y^2)\frac{\partial^2 u}{\partial y^2} = \sin(x+y).$$ **Consider the following statements:** **P: The PDE is parabolic on the ellipse $\frac{x^2}{4} + y^2 = 1$.** **Q: The PDE is hyperbolic inside the ellipse $\frac{x^2}{4} + y^2 = 1$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.28	**If $u(x,y)$ is the solution of the Cauchy problem** $$x\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=1,\qquad u(x,0)=-x^2,\quad x>0,$$ **then $u(2,1)$ is equal to**
(A)	$1-2\,e^{-2}$
(B)	$1+4\,e^{-2}$
(C)	$1-4\,e^{-2}$
(D)	$1+2\,e^{-2}$

Q.29	**Let $y(t)$ be the solution of the initial value problem** $$\frac{d^2y}{dt^2}+a\frac{dy}{dt}+b\,y=f(t),\qquad a>0,\qquad b>0,\qquad a\neq b,\qquad a^2-4b=0,$$ $$y(0)=0,\quad \frac{dy}{dt}(0)=0,$$ **obtained by the method of Laplace transform. Then**
(A)	$y(t)=\int_0^t \tau\, e^{\frac{-a\tau}{2}}\, f(t-\tau)\, d\tau$
(B)	$y(t)=\int_0^t e^{\frac{-a\tau}{2}}\, f(t-\tau)\, d\tau$
(C)	$y(t)=\int_0^t \tau\, e^{\frac{-b\tau}{2}}\, f(t-\tau)\, d\tau$
(D)	$y(t)=\int_0^t e^{\frac{-b\tau}{2}}\, f(t-\tau)\, d\tau$

Q.30	**The critical point of the differential equation** $$\frac{d^2y}{dt^2} + 2\,\alpha\,\frac{dy}{dt} + \beta^2 y = 0,\ \alpha > \beta > 0,$$ **is a**
(A)	node and is asymptotically stable
(B)	spiral point and is asymptotically stable
(C)	node and is unstable
(D)	saddle point and is unstable

Q.31	**The initial value problem** $$\frac{dy}{dt} = f(t,y),\quad t > 0,\quad y(0) = 1,$$ **where** $f(t,y) = -10\,y$**, is solved by the following Euler method** $$y_{n+1} = y_n + h\,f(t_n, y_n),\ \ n \geq 0,$$ **with step-size** h**. Then** $y_n \to 0$ **as** $n \to \infty$**, provided**
(A)	$0 < h < 0.2$
(B)	$0.3 < h < 0.4$
(C)	$0.4 < h < 0.5$
(D)	$0.5 < h < 0.55$

Q.32	**Consider the Linear Programming Problem P:** $$\text{Maximize } c_1x_1 + c_2x_2$$ **subject to** $$a_{11}x_1 + a_{12}x_2 \le b_1,$$ $$a_{21}x_1 + a_{22}x_2 \le b_2,$$ $$a_{31}x_1 + a_{32}x_2 \le b_3,$$ $$x_1 \ge 0 \text{ and } x_2 \ge 0, \text{ where } a_{ij}, b_i \text{ and } c_j \text{ are real numbers } (i = 1,2,3; j = 1,2).$$ **Let $\begin{bmatrix} p \\ q \end{bmatrix}$ be a feasible solution of P such that $p\,c_1 + q\,c_2 = 6$ and let all feasible solutions $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ of P satisfy $-5 \le c_1x_1 + c_2x_2 \le 12$.** **Then, which one of the following statements is NOT true?**
(A)	P has an optimal solution
(B)	The feasible region of P is a bounded set
(C)	If $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ is a feasible solution of the dual of P, then $b_1y_1 + b_2y_2 + b_3y_3 \ge 6$
(D)	The dual of P has at least one feasible solution

Q.33	**Let $L^2[-1,1]$ be the Hilbert space of real valued square integrable functions on $[-1,1]$ equipped with the norm $\|f\| = \left(\int_{-1}^{1} \|f(x)\|^2\,dx\right)^{1/2}$.** **Consider the subspace $M = \{f \in L^2[-1,1] \,:\, \int_{-1}^{1} f(x)dx = 0\}$.** **For $f(x) = x^2$, define $d = \inf\{\|f - g\| : g \in M\}$. Then**
(A)	$d = \frac{\sqrt{2}}{3}$
(B)	$d = \frac{2}{3}$
(C)	$d = \frac{3}{\sqrt{2}}$
(D)	$d = \frac{3}{2}$

Q.34	**Let $C[0,1]$ be the Banach space of real valued continuous functions on $[0,1]$ equipped with the supremum norm. Define $T: C[0,1] \to C[0,1]$ by** $$(Tf)(x) = \int_0^x x\, f(t)\, dt.$$ **Let $R(T)$ denote the range space of T. Consider the following statements:** **P: T is a bounded linear operator.** **Q: $T^{-1}: R(T) \to C[0,1]$ exists and is bounded.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.35	**Let $\ell^1 = \{x = (x(1), x(2), \ldots, x(n), \ldots) \mid \sum_{n=1}^{\infty} \lvert x(n)\rvert < \infty\}$ be the sequence space equipped with the norm $\lVert x\rVert = \sum_{n=1}^{\infty} \lvert x(n)\rvert$. Consider the subspace** $$X = \left\{x \in \ell^1 : \sum_{n=1}^{\infty} n\, \lvert x(n)\rvert < \infty\right\},$$ **and the linear transformation $T: X \to \ell^1$ given by** **$(Tx)(n) = n\, x(n)$ for $n = 1, 2, 3, \ldots$. Then**
(A)	T is closed but NOT bounded
(B)	T is bounded
(C)	T is neither closed nor bounded
(D)	T^{-1} exists and is an open map

Q.36	**Let $f_n: [0, 10] \to \mathbb{R}$ be given by $f_n(x) = n\, x^3\, e^{-nx}$ for $n = 1, 2, 3, \dots$.** **Consider the following statements:** **P: (f_n) is equicontinuous on $[0, 10]$.** **Q: $\sum_{n=1}^{\infty} f_n$ does NOT converge uniformly on $[0, 10]$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.37	**Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by** $$f(x, y) = \begin{cases} \sqrt{x^2 + y^2}\ \sin(y^2/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$ **Consider the following statements:** **P: f is continuous at $(0, 0)$ but f is NOT differentiable at $(0, 0)$.** **Q: The directional derivative $D_u f(0, 0)$ of f at $(0, 0)$ exists in the direction of every unit vector $u \in \mathbb{R}^2$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.38	**Let V be the solid region in $\mathbb{R}^3$ bounded by the paraboloid $y = (x^2 + z^2)$ and the plane $y = 4$. Then the value of $\iiint_V 15\sqrt{x^2 + z^2}\, dV$ is**
(A)	$128\,\pi$
(B)	$64\,\pi$
(C)	$28\,\pi$
(D)	$256\,\pi$

Q.39	**Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x, y) = 4xy - 2x^2 - y^4$. Then f has**
(A)	a point of local maximum and a saddle point
(B)	a point of local minimum and a saddle point
(C)	a point of local maximum and a point of local minimum
(D)	two saddle points

Q.40	**The equation $xy - z\log y + e^{xz} = 1$ can be solved in a neighborhood of the point $(0, 1, 1)$ as $y = f(x, z)$ for some continuously differentiable function f. Then**
(A)	$\nabla f(0, 1) = (2, 0)$
(B)	$\nabla f(0, 1) = (0, 2)$
(C)	$\nabla f(0, 1) = (0, 1)$
(D)	$\nabla f(0, 1) = (1, 0)$

Q.41	**Consider the following topologies on the set $\mathbb{R}$ of all real numbers.** **T_1 is the upper limit topology having all sets $(a, b]$ as basis.** $\mathrm{T}_2 = \{\boldsymbol{U} \subset \mathbb{R} : \mathbb{R} \setminus \boldsymbol{U} \textbf{ is finite}\} \cup \{\emptyset\}$. **$\mathrm{T}_3$ is the standard topology having all sets (a, b) as basis.** **Then**
(A)	$\mathrm{T}_2 \subset \mathrm{T}_3 \subset \mathrm{T}_1$
(B)	$\mathrm{T}_1 \subset \mathrm{T}_2 \subset \mathrm{T}_3$
(C)	$\mathrm{T}_3 \subset \mathrm{T}_2 \subset \mathrm{T}_1$
(D)	$\mathrm{T}_2 \subset \mathrm{T}_1 \subset \mathrm{T}_3$

Q.42	**Let $\mathbb{R}$ denote the set of all real numbers. Consider the following topological spaces.** $\boldsymbol{X_1} = (\mathbb{R}, \mathrm{T}_1)$**, where T_1 is the upper limit topology having all sets $(a, b]$ as basis.** $\boldsymbol{X_2} = (\mathbb{R}, \mathrm{T}_2)$**, where** $\mathrm{T}_2 = \{\boldsymbol{U} \subset \mathbb{R} : \mathbb{R} \setminus \boldsymbol{U} \textbf{ is finite}\} \cup \{\emptyset\}$. **Then**
(A)	both X_1 and X_2 are connected
(B)	X_1 is connected and X_2 is NOT connected
(C)	X_1 is NOT connected and X_2 is connected
(D)	neither X_1 nor X_2 is connected

Q.43	**Let $\langle\cdot,\ \cdot\rangle: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be an inner product on the vector space $\mathbb{R}^n$ over $\mathbb{R}$. Consider the following statements:** **P: $|\langle u, v\rangle| \le \frac{1}{2}\left(\langle u, u\rangle + \langle v, v\rangle\right)$ for all $u, v \in \mathbb{R}^n$.** **Q: If $\langle u, v\rangle = \langle 2u, -v\rangle$ for all $v \in \mathbb{R}^n$, then $u = 0$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.44 -Q.55 Numerical Answer Type (NAT), carry TWO mark each (no negative marks).

Q.44	**Let G be a group of order 5^4 with center having 5^2 elements. Then the number of conjugacy classes in G is _________ .**

Q.45	**Let F be a finite field and $F^{\times}$ be the group of all nonzero elements of F under multiplication. If $F^{\times}$ has a subgroup of order 17, then the smallest possible order of the field F is ________ .**

Q.46	**Let $R = \{z = x + iy \in \mathbb{C} : 0 < x < 1 \text{ and } -11\pi < y < 11\pi\}$ and Γ be the positively oriented boundary of R. Then the value of the integral $$\frac{1}{2\pi i}\int_{\Gamma} \frac{e^z\, dz}{e^z - 2}$$ is ________ .**

Q.47	**Let $D = \{z \in \mathbb{C} : \lvert z\rvert < 2\pi\}$ and $f : D \to \mathbb{C}$ be the function defined by $$f(z) = \begin{cases} \dfrac{3z^2}{(1-\cos z)} & \text{if } z \neq 0, \\ 6 & \text{if } z = 0. \end{cases}$$ If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ for $z \in D$, then $6\, a_2 =$ __________.**

Q.48	**The number of zeroes (counting multiplicity) of $P(z) = 3z^5 + 2i\,z^2 + 7i\,z + 1$ in the annular region $\{z \in \mathbb{C} : 1 < \lvert z\rvert < 7\}$ is ________ .**

Q.49	**Let A be a square matrix such that $\det(xI - A) = x^4(x-1)^2(x-2)^3$, where $\det(M)$ denotes the determinant of a square matrix M. If $\operatorname{rank}(A^2) < \operatorname{rank}(A^3) = \operatorname{rank}(A^4)$, then the geometric multiplicity of the eigenvalue 0 of A is ________ .**

Q.50	**If $y = \sum_{k=0}^{\infty} a_k x^k$, $(a_0 \neq 0)$ is the power series solution of the differential equation $\frac{d^2y}{dx^2} - 24\,x^2 y = 0$, then $\frac{a_4}{a_0} =$ ________.**

Q.51	**If $u(x,t) = A\,e^{-t}\sin x$ solves the following initial boundary value problem** $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\quad 0 < x < \pi, \qquad t > 0,$$ $$u(0,t) = u(\pi,t) = 0, \qquad t > 0,$$ $$u(x,0) = \begin{cases} 60, & 0 < x \leq \frac{\pi}{2}, \\ 40, & \frac{\pi}{2} < x < \pi, \end{cases}$$ **then $\pi A =$ _______ .**

Q.52	**Let $V = \{p \,:\, p(x) = a_0 + a_1 x + a_2 x^2,\ a_0, a_1, a_2 \in \mathbb{R}\}$ be the vector space of all polynomials of degree at most 2 over the real field $\mathbb{R}$. Let $T: V \to V$ be the linear operator given by** $$T(p) = (p(0) - p(1)) + \big(p(0) + p(1)\big)\,x + p(0)\,x^2.$$ **Then the sum of the eigenvalues of T is _____ .**

Q.53	**The quadrature formula** $$\int_0^2 x\,f(x)\,dx \approx \alpha\,f(0) + \beta\,f(1) + \gamma\,f(2)$$ **is exact for all polynomials of degree ≤ 2. Then $2\beta - \gamma =$ _______ .**

Q.54	**For each $x \in (0,1]$, consider the decimal representation $x = \cdot\, d_1 d_2 d_3 \cdots d_n \cdots$. Define $f: [0,1] \to \mathbb{R}$ by $f(x) = 0$ if x is rational and $f(x) = 18\,n$ if x is irrational, where n is the number of zeroes immediately after the decimal point up to the first nonzero digit in the decimal representation of x. Then the Lebesgue integral $\int_0^1 f(x)\,dx =$ _______.**

Q.55	Let $\tilde{x} = \begin{bmatrix} 11/3 \\ 2/3 \\ 0 \end{bmatrix}$ be an optimal solution of the following Linear Programming Problem P: $$\text{Maximize } 4x_1 + x_2 - 3x_3$$ subject to $$2x_1 + 4x_2 + ax_3 \le 10,$$ $$x_1 - x_2 + bx_3 \le 3,$$ $$2x_1 + 3x_2 + 5x_3 \le 11,$$ $$x_1 \ge 0, x_2 \ge 0 \text{ and } x_3 \ge 0, \text{ where } a, b \text{ are real numbers.}$$ If $\tilde{y} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ is an optimal solution of the dual of P, then $p + q + r =$ ________ (round off to two decimal places).

END OF THE QUESTION PAPER