JEE Main 2021(July) Paper

(Memory Based)

Date of Exam: 20th July 2021
Time: 9:00 a.m.-12 a.m.
Subject: Mathematics

1. The mean of 6 numbers is 6.5 and its variance is 10.25. If 4 numbers are 2, 4, 5 and 7, then find the other two.

   Solution:
   Let a and b be the other two numbers.
   $\bar{x} = 6.5$
   $\Rightarrow \frac{a+b+2+4+5+7}{6} = 6.5$
   $\Rightarrow a + b = 21 \quad \cdots (i)$
   $\sigma^2 = 10.25$
   $\Rightarrow \frac{a^2+b^2+4+16+25+49}{6} - (6.5)^2 = 10.25$
   $\Rightarrow a^2 + b^2 = 221 \quad \cdots (ii)$
   On solving (i) and (ii),
   $a = 10,\ b = 11$ or $a = 11,\ b = 10$

2. Find the coefficient of x^{256} in $(1-x)^{101}(x^2+x+1)^{100}$

   Solution:
   $(1-x)^{101}(x^2+x+1)^{100}$
   $= (1-x)(1-x)^{100}(x^2+x+1)^{100}$
   $= (1-x)(1-x^3)^{100}$
   $= (1-x^3)^{100} - x(1-x^3)^{100}$
   First term will not have x^{256}.
   Coefficient of x^{256} is $-1 \times$ coefficient of x^{255} in $(1-x^3)^{100}$
   $= -1(-{}^{100}C_{85}) = {}^{100}C_{85}$

3. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors equally inclined to $\vec{a} + \vec{b} + \vec{c}$ at angle θ, then find the value of $36\cos^2 2\theta$

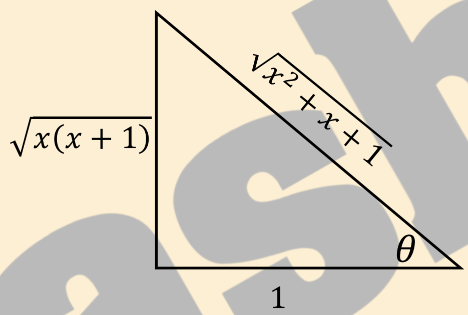
   Solution:
   $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$
   $(\vec{a}+\vec{b}+\vec{c}) \cdot (\vec{a}+\vec{b}+\vec{c}) = |\vec{a}+\vec{b}+\vec{c}|^2$
   $\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = |\vec{a}+\vec{b}+\vec{c}|^2$
   Now, $\vec{a} \cdot (\vec{a}+\vec{b}+\vec{c}) = |\vec{a}|\,|\vec{a}+\vec{b}+\vec{c}| \cos\theta$
   $\Rightarrow |\vec{a}|^2 = |\vec{a}||\vec{a}+\vec{b}+\vec{c}| \cos\theta$
   $\Rightarrow |\vec{a}| = |\vec{a}+\vec{b}+\vec{c}| \cos\theta$
   Similarly, $|\vec{b}| = |\vec{a}+\vec{b}+\vec{c}| \cos\theta$, $|\vec{c}| = |\vec{a}+\vec{b}+\vec{c}| \cos\theta$
   $\therefore 3|\vec{a}+\vec{b}+\vec{c}|^2 \cos^2\theta = |\vec{a}+\vec{b}+\vec{c}|^2$
   $\Rightarrow \cos^2\theta = \frac{1}{3} \Rightarrow 36\cos^2 2\theta = 36 \cdot \left(2 \times \frac{1}{3} - 1\right)^2 = 4$

4. If $f(x)=\begin{cases}\sin x-e^x, & x<0\\ a+[-x], & 0\le x<1\\ 2x-b, & x\ge 1\end{cases}$ is continuous in $(-\infty,1]$, find the value of $a+b$.

Solution:
$f(x)=\begin{cases}\sin x-e^x, & x<0\\ a+[-x], & 0\le x<1\\ 2x-b, & x\ge 1\end{cases}$
$f(1^-)=a+(-1)=a-1$
$f(1^+)=2-b$
$f(1^-)=f(1^+)$
$\Rightarrow a-1=2-b$
$\Rightarrow a+b=3$

5. The number of solutions of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is

Solution:
$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$
$\Rightarrow \tan^{-1}\sqrt{x(x+1)}=\frac{\pi}{2}-\sin^{-1}\sqrt{x^2+x+1}$
$\Rightarrow \tan^{-1}\sqrt{x(x+1)}=\cos^{-1}\sqrt{x^2+x+1}$
$\Rightarrow \cos^{-1}\frac{1}{\sqrt{x^2+x+1}}=\cos^{-1}\sqrt{x^2+x+1}$
$\Rightarrow \sqrt{x^2+x+1}=\frac{1}{\sqrt{x^2+x+1}}$
$\Rightarrow x^2+x+1=1$
$\Rightarrow x(x+1)=0\Rightarrow x=0,-1$
$\therefore$ Number of solutions is 2.



6. There are 6 batsmen, 7 bowlers and 2 wickets keepers. Find the number of ways of forming a team of 11 players having at least 4 batsmen, at least 5 bowlers and at least 1 wicket keeper.
a. 567
b. 525
c. 462
d. 777

Solution:
Team Size = 11 players

Case 1: 4 batsmen & 5 bowlers & 2 wicket keepers
$\Rightarrow {}^6C_4\times{}^7C_5\times{}^2C_2=\frac{6\times5}{2}\times\frac{7\times6}{2}\times1=315$
Case 2: 4 batsmen & 6 bowlers & 1 wickets keeper
$\Rightarrow {}^6C_4\times{}^7C_6\times{}^2C_1=\frac{6\times5}{2}\times7\times2=210$
Case 3: 5 batsmen & 5 bowlers & 1 wickets keepers
$\Rightarrow {}^6C_5\times{}^7C_5\times{}^2C_1=6\times\frac{7\times6}{2}\times2=252$
So total number of ways $=315+210+252=777$

7. All the letters of the word EXAMINATION are arranged. Find the probability that 'M' is at the 4^{th} position.
   a. $\frac{2}{11}$
   b. $\frac{1}{11}$
   c. $\frac{4}{11}$
   d. $\frac{8}{11}$

Solution:

EXAMINATION

Letters: AA II NN E T O X M

Total possible words

$n(S) = \frac{11!}{(2!)^3}$

Now when M is at 4^{th} position

$n(E) = \frac{10!}{(2!)^3}$

P (M is at 4^{th} position) $= \frac{n(E)}{n(S)} = \frac{1}{11}$

8. Find the number of integral terms in the expansion of $\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$
   a. 1
   b. 21
   c. 20
   d. 11

Solution:

$T_{r+1} = {}^{120}C_r\, 4^{\frac{120-r}{4}} \cdot 5^{\frac{r}{6}}$

$= {}^{120}C_r\, 2^{\frac{120-r}{2}} \cdot 5^{\frac{r}{6}}$

For integral terms, r should be even and multiple of 6

i.e., r should be a multiple of 6.

r can be $0, 6, 12, \ldots, 120$

$\therefore$ Total number of integral terms $= 21$

9. If $\lim\limits_{x\to 0}\{2 - \cos x\sqrt{\cos 2x}\}^{\frac{x+2}{x^2}} = e^{\alpha}$, then $\alpha =$?

Solution:

$\lim\limits_{x\to 0}\{2 - \cos x\sqrt{\cos 2x}\}^{\frac{x+2}{x^2}} = e^{\alpha}$ (1^{∞} form)

$\Rightarrow e^{\lim\limits_{x\to 0}\{1-\cos x\sqrt{\cos 2x}\}\times\left(\frac{x+2}{x^2}\right)} = e^{\alpha}$

$\Rightarrow e^{\lim\limits_{x\to 0}\left\{\frac{1-\cos^2 x\,(\cos 2x)}{1+\cos x\sqrt{\cos 2x}}\right\}\times\left(\frac{x+2}{x^2}\right)} = e^{\alpha}$

$\Rightarrow e^{\lim\limits_{x\to 0}\left\{\frac{1-\cos^2 x\,(1-2\sin^2 x)}{1+\cos x\sqrt{\cos 2x}}\right\}\times\left(\frac{x+2}{x^2}\right)} = e^{\alpha}$

$\Rightarrow e^{\lim\limits_{x\to 0}\left\{\frac{1-\cos^2 x+2\sin^2 x\cos^2 x}{1+\cos x\sqrt{\cos 2x}}\right\}\times\left(\frac{x+2}{x^2}\right)} = e^{\alpha}$

$\Rightarrow e^{\lim\limits_{x\to 0}\left\{\frac{\sin^2 x\,(2\cos^2 x+1)}{1+\cos x\sqrt{\cos 2x}}\right\}\times\left(\frac{x+2}{x^2}\right)} = e^{\alpha}$

$\Rightarrow e^{\lim\limits_{x\to 0}\left\{\frac{\sin^2 x}{x^2}\times\frac{(2\cos^2 x+1)}{1+\cos x\sqrt{\cos 2x}}\right\}\times(x+2)} = e^{\alpha}$

$\Rightarrow e^{\frac{2+1}{1+1}\times 2} = e^{\alpha} \Rightarrow e^3 = e^{\alpha} \Rightarrow \alpha = 3$

10. Evaluate $\int_{-1}^{1} \log(\sqrt{1-x}+\sqrt{1+x})dx$

Solution:

$f(-x) = f(x)$

$\Rightarrow f$ is even.

$\therefore I = 2\int_0^1 \log(\sqrt{1-x}+\sqrt{1+x})dx \quad \cdots (1)$

We will solve by integration by parts.

Let $u = \ln(\sqrt{1-x}+\sqrt{1+x})$

$du = \frac{\frac{1}{2}\left(\frac{-1}{\sqrt{1-x}}+\frac{1}{\sqrt{1+x}}\right)}{\sqrt{1-x}+\sqrt{1-x}}dx = \frac{1}{2}\frac{(\sqrt{1-x}-\sqrt{1+x})}{\sqrt{1-x^2}(\sqrt{1-x}+\sqrt{1+x})}dx$

$= \frac{-1}{4x}\frac{(\sqrt{1-x}-\sqrt{1+x})^2}{\sqrt{1-x^2}}dx = \frac{1}{2x}\frac{(\sqrt{1-x^2}-1)}{\sqrt{1-x^2}}dx = \frac{1}{2x}\left(1-\frac{1}{\sqrt{1-x^2}}\right)dx$

$I = 2\int_0^1 \log(\sqrt{1-x}+\sqrt{1+x})dx$

$I = 2x\ln(\sqrt{1-x}+\sqrt{1+x})|_0^1 - \int_0^1\left(1-\frac{1}{\sqrt{1-x^2}}\right)dx$

$I = 2x\ln(\sqrt{1-x}+\sqrt{1+x})|_0^1 - (x-\sin^{-1}x)|_0^1$

$I = 2\ln(2) - \left(1-\frac{\pi}{2}\right)$

11. The logical equivalence of the Boolean expression $\sim(p \wedge \sim q) \vee (q \vee \sim p)$ is

Solution:

$\sim(p \wedge \sim q) \vee (q \vee \sim p)$

$\equiv (\sim p \vee q) \vee (q \vee \sim p)$

$\equiv \sim p \vee q$

$\equiv p \rightarrow q$

12. Consider the curve $y^2 = 2x$ and point $A(2,\ 2)$. If the normal at A intersects the curve again at point B and the tangent at A intersects the x-axis at C, then area of ΔABC is

Solution:

Equation of tangent at A

$yy_1 = x + x_1$

$\Rightarrow 2y = x + 2$

Tangent at A intersects the x-axis at C

$C(-2, 0)$

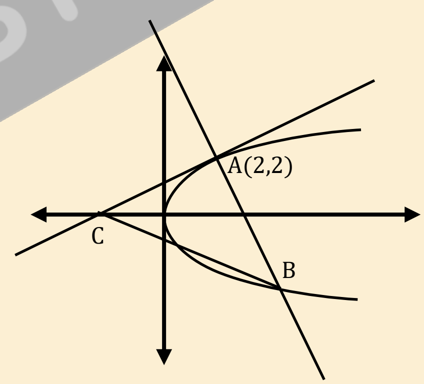
Equation of normal at A

$y + 2x = 6$

Solving normal equation with $y^2 = 2x$, we get $B\left(\frac{9}{2}, -3\right)$

So, area will be $\frac{1}{2}\times AC \times AB$

$= \frac{1}{2}\times 2\sqrt{5}\times\sqrt{\frac{25}{4}+25} = \frac{25}{2}$



13. If for vectors A and B, $A \cdot B = |A \times B|$, then $|A - B|$ is

a) $\sqrt{A^2 + B^2\sqrt{2(AB)}}$

b) $\sqrt{A^2 + B^2 - \sqrt{2(AB)}}$

c) $\sqrt{(A^2 + B^2 - \sqrt{2}\,AB)}$

d) $\sqrt{(A^2 + B^2 + \sqrt{2}\,AB)}$

Solution:

$A \cdot B = |A \times B| \Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \tan\theta = 1$

$\theta = \frac{\pi}{4}$

$|A - B|^2 = A^2 + B^2 - 2A \cdot B$

$= A^2 + B^2 - 2AB\cos\left(\frac{\pi}{4}\right)$

$= A^2 + B^2 - \sqrt{2}AB$

$\Rightarrow |A - B| = \sqrt{(A^2 + B^2 - \sqrt{2}\,AB)}$

14. If the roots of the quadratic equation $x^2 + 3^{1/4}x + 3^{1/2} = 0$ are $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, then the value of $\boldsymbol{\alpha}^{96}(\boldsymbol{\alpha}^{12} - 1) + \boldsymbol{\beta}^{96}(\boldsymbol{\beta}^{12} - 1)$

a) $50 \cdot 3^{24}$

b) $51 \cdot 3^{24}$

c) $52 \cdot 3^{24}$

d) $104 \cdot 3^{24}$

Solution:

Sol. $x^2 + 3^{1/2} = -3^{1/4}x$

$\Rightarrow x^4 + 3 = -3^{1/2}x^2$

$\Rightarrow x^8 + 9 + 6x^4 = 3x^4$

$\Rightarrow x^8 + 9 + 3x^4 = 0$

$\therefore \boldsymbol{\alpha}^8 = -9 - 3\boldsymbol{\alpha}^4$

and $\boldsymbol{\alpha}^{12} = -9\boldsymbol{\alpha}^4 - 3\boldsymbol{\alpha}^8 = -9\boldsymbol{\alpha}^4 - 3(-9 - 3\boldsymbol{\alpha}^4) = 27$

Similarly, $\boldsymbol{\beta}^{12} = 27$

$\boldsymbol{\alpha}^{96}(\boldsymbol{\alpha}^{12} - 1) + \boldsymbol{\beta}^{96}(\boldsymbol{\beta}^{12} - 1) = (27)^8.26 + (27)^8.26 = 52.3^{24}$

15. In ΔABC, if AB=5, $\angle B = cos^{-1}\left(\frac{3}{5}\right)$ and the radius of circumcircle of triangle is 5. Then the area of ΔABC is

a) $6 + 8\sqrt{3}$

b) $3 + 4\sqrt{3}$

c) $3 + 8\sqrt{3}$

d) $6 + 4\sqrt{3}$

Solution:

$\cos B = \frac{3}{5} \Rightarrow \sin B = \frac{4}{5}, R = 5$

$b = 2R\sin B = 8$

$AB = c = 5$

By cosine rule,

$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{3}{5}$

$\Rightarrow \frac{a^2+25-64}{2a(5)} = \frac{3}{5}$

$\Rightarrow a^2 - 39 = 6a$

$\Rightarrow a^2 - 6a - 39 = 0$

$\Rightarrow a = \frac{(6+8\sqrt{3})}{2} \Rightarrow a = 3 + 4\sqrt{3}$

$\Delta = \frac{abc}{4R} = 6 + 8\sqrt{3}$

16. If the shortest distance between the lines $r_1 = \alpha\hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + 2\hat{k}), \lambda \in \mathbb{R}, \alpha > 0$ and $r_2 = -4\hat{\imath} - \hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k}), \mu \in \mathbb{R}$ is 9, then the value of α is

a. 2
b. 4
c. 6
d. $\sqrt{6}$

Solution:

Shortest distance $= \left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_2} \times \overrightarrow{b_1})}{|\overrightarrow{b_2} \times \overrightarrow{b_1}|}\right|$

$9 = \left|\frac{\{((\alpha+4)\hat{\imath}+2\hat{\jmath}+3\hat{k})\ (8\hat{\imath}+8\hat{\jmath}+4\hat{k})\}}{\sqrt{64+64+16}}\right|$

$\Rightarrow |(\alpha + 4) \times 8 + 16 + 12| = 108$

$\Rightarrow (\alpha + 4) \times 8 = 80 \quad (\because \alpha > 0)$

$\Rightarrow \alpha = 6$

17. Let $y = mx + c, m > 0$ be the focal chord of $y^2 = -64x$ which is tangent to $(x+10)^2 + y^2 = 4$. Then the value of $4\sqrt{2}(m+c)$ is equal to

Solution:

Focus of parabola is $(-16, 0)$

So, $-16m + c = 0 \Rightarrow c = 16m \quad \dots(i)$

Now slope form of tangent to the circle is given by

$y = m(x+10) \pm 2\sqrt{1+m^2}$

$\therefore c = 10m \pm 2\sqrt{1+m^2} \quad \dots(ii)$

So, from (i) and (ii)

$16m = 10m \pm 2\sqrt{1+m^2}$

$\Rightarrow 9m^2 = 1 + m^2 \Rightarrow m = \frac{1}{2\sqrt{2}}$

$\Rightarrow c = 16m = \frac{8}{2\sqrt{2}}$

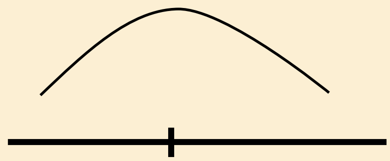
$\therefore 4\sqrt{2}\,(m+c) = 34$

18. A continuous differentiable function $f(x)$ is increasing in $\left(-\infty, \frac{3}{2}\right)$ and decreasing in $\left(\frac{3}{2}, \infty\right)$. Then $x = \frac{3}{2}$ is

a) point of local maxima
b) point of local minima
c) point of inflection
d) None of these

Solution:

Roughly graph of $f(x)$ can be drawn as



$\frac{3}{2}$

Thus $x = \frac{3}{2}$ is a point of local maxima.

19. If z and ω are complex numbers such that $|z\omega| = 1, \arg(z) - \arg(\omega) = \frac{3\pi}{2}$. then find $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$

a) $\frac{\pi}{4}$
b) $-\frac{\pi}{4}$
c) $\frac{3\pi}{4}$
d) $-\frac{3\pi}{4}$

Solution.

Let $z = re^{i\theta}$ and $\omega = \frac{1}{r}e^{\left(\theta - \frac{3\pi}{2}\right)}$

Then $\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega} = \frac{1-2re^{-i\theta}.\frac{1}{r}e^{i\left(\theta-\frac{3\pi}{2}\right)}}{1+3re^{-i\theta}.\frac{1}{r}e^{i\left(\theta-\frac{3\pi}{2}\right)}}$

$= \frac{1-2e^{i\left(-\frac{3\pi}{2}\right)}}{1+3e^{i\left(-\frac{3\pi}{2}\right)}} = \frac{1-2i}{1+3i}$

$= -\frac{1}{2} - \frac{1}{2}i$

$\arg\left(-\frac{1}{2} - \frac{1}{2}i\right) = -\frac{3\pi}{4}$

20. If an invertible function $f(x)$ is defined as $f(x) = 3x - 2, g(x)$ is also an invertible function such that $f^{-1}\left(g^{-1}(x)\right) = x - 2$, then $g(x)$ is

a) $\frac{x-8}{3}$
b) $\frac{x+8}{3}$
c) $\frac{x-3}{8}$
d) $\frac{x+3}{8}$

Solution:

$f^{-1}\left(g^{-1}(x)\right) = x - 2$

$\Rightarrow f(x-2) = g^{-1}(x)$

$\Rightarrow 3(x-2) - 2 = g^{-1}(x)$

$\Rightarrow 3x - 8 = g^{-1}(x)$

$\Rightarrow g^{-1}(x) = 3x - 8$

or $x = 3g(x) - 8$

$\therefore g(x) = \frac{x+8}{3}$

21. The probability of selecting integers $a \in [-5, 30]$, such that $x^2 + 2(a+4)x - 5a + 64 > 0$ for all $x \in \mathbf{R}$ is:

Solution:
$x^2 + 2(a+4)x - (5a - 64) > 0$
$D < 0$
$\therefore 4(a+4)^2 + 4(5a - 64) < 0$
$\Rightarrow a^2 + 13a - 48 < 0$
$\Rightarrow (a+16)(a-3) < 0$
So, $a \in (-16, 3)$
Total integers are 18
$\therefore$Probability $= \frac{18}{36} = \frac{1}{2}$

22. If $\int_0^a e^{x-[x]} dx = 10e - 9$, then the value of $'a'$ is (where $[.]$ is greatest integer function)

a) $9 + \ln 2$
b) $10 + \ln 2$
c) 10
d) 9

Solution:
Let $a = K + \lambda, 0 \le \lambda < 1$
$\int_0^a e^{\{x\}} dx = \int_0^K e^{\{x\}} dx + \int_K^{K+\lambda} e^{\{x\}} dx$
$= K \int_0^1 e^x \, dx + \int_0^\lambda e^x dx$
$= K(e-1) + (e^\lambda - 1)$
$= \left(Ke + e^\lambda\right) - (K+1)$
Given, $\int_0^a e^{x-[x]} dx = 10e - 9$
$\Rightarrow \left(Ke + e^\lambda\right) - (K+1) = 10e - 9$
$\Rightarrow K = 10$ and $e^\lambda = 11 - 9 = 2$
$\Rightarrow \lambda = \ln 2$
$\therefore a = 10 + \ln 2$

23. $a_{ij} = \begin{cases} 1, & i = j \\ -x, & |i-j| = 1 \\ 2x+1, & \text{otherwise} \end{cases}$ $A = \left[a_{ij}\right]_{3\times 3.}$ $f(x) = \operatorname{Det}(A)$. Then find the sum of local maximum and minimum values of $f(x)$.

a) $\frac{20}{27}$
b) $-\frac{20}{27}$
c) $\frac{88}{27}$
d) $-\frac{88}{27}$

Solution:

$$f(x) = \begin{vmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{vmatrix} = 4x^3 - 4x^2 - 4x$$

$f'(x) = 4(3x^2 - 2x - 1)$

$f'(x) = 4(x-1)\left(x + \frac{1}{3}\right)$

$\therefore f(1) = 4 - 4 - 4 = -4$

$f\left(-\frac{1}{3}\right) = \frac{-4}{27} - \frac{4}{9} + \frac{4}{3} = \frac{-4-12+36}{27} = \frac{20}{27}$

$\frac{20}{27} - 4 = \frac{20-108}{27} = -\frac{88}{27}$

24. Find the coefficient of $a^3b^4c^5$ in $(ab + bc + ca)^6$.

a) 60
b) 45
c) 40
d) 90

Solution:

$$(ab + bc + ca)^6 = \sum_{p+q+r=6} \frac{6!}{p!\,q!\,r!}(ab)^p(bc)^q(ca)^r$$

$$= \sum_{p+q+r=6} \frac{6!}{p!\,q!\,r!} a^{p+r} b^{p+q} c^{q+r}$$

For $a^3b^4c^5$, we need

$p + r = 3$

$p + q = 4$

$q + r = 5$

Solving we get, $p = 1, q = 3, r = 2$

$\therefore$ Co-efficient of $a^3b^4c^5$ in $(ab + bc + ca)^6$ is $\frac{6!}{1!3!2!} = 60$.

25. $x\frac{dy}{dx} \cdot \tan\frac{y}{x} = y\tan\frac{y}{x} + x,\ y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Area bounded by $x = 0, x = \frac{1}{\sqrt{2}}, y = y(x)$.

Solution:

$x\frac{dy}{dx} \cdot \tan\frac{y}{x} = y\tan\frac{y}{x} + x$

$\therefore \frac{dy}{dx}\tan\left(\frac{y}{x}\right) = \frac{y}{x}\tan\left(\frac{y}{x}\right) + 1$

Let $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\left(v + x\frac{dv}{dx}\right)\tan v = v\tan v + 1$

$\Rightarrow v + x\frac{dv}{dx} = v + \cot v$

$\Rightarrow \frac{dv}{\cot v} = \frac{dx}{x}$

$\Rightarrow \tan v \, dv = \frac{dx}{x}$

$-\log|\cos v| = \log|cx|$

$-\cos v = cx$

$\therefore y = x\cos^{-1} x$

$\int_0^{1/\sqrt{2}} x\cos^{-1} x = \frac{1}{8}(\pi - 1)$