JEE Main 2021(July) Paper

(Memory Based)

Date of Exam: 20^{th} July 2021
Time: 9:00 a.m.-12 a.m.
Subject: Physics

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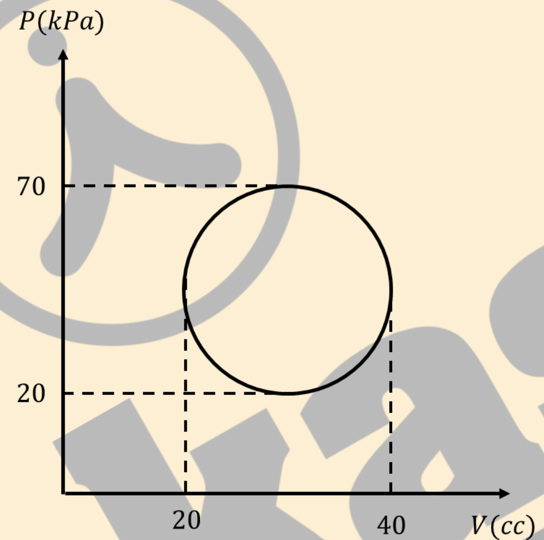
1. Consider the P-V diagram given below for a cyclic process. Find the net heat supplied to the system during the process

a. $0.625\pi\ J$
b. $25\pi\ J$
c. $0.25\pi\ J$
d. $0.2\pi\ J$



Solution: (c)

It is a cyclic process so the net change in internal energy of the system will zero. i.e.,

$$\Delta U = 0$$

From first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$\Delta Q = 0 + \Delta W$

$\therefore \Delta Q = \Delta W$

Where,

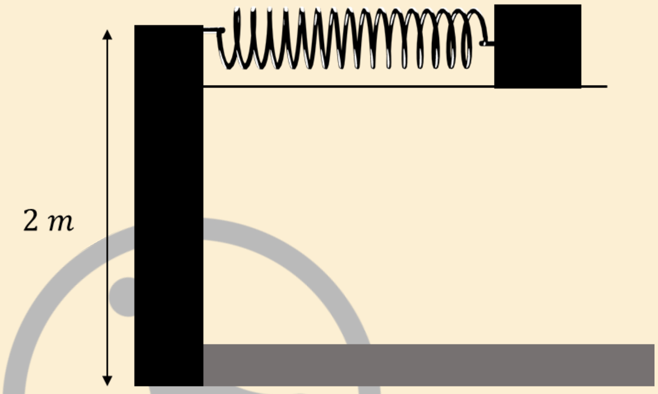
ΔW =Area of the shaded portion,

$=\pi[\frac{70-20}{2}] \times 10^3 \times [\frac{40-20}{2}] \times 10^{-6}$

$=\pi[25] \times 10^{-3} \times 10$

$=0.25\pi\ J$

2. A spring of force constant $k = 100\ N/m$ is compressed to $x = 0.5\ m$ by a block of mass $100\ g$ and released. Find the distance d where it falls



a. $5\ m$
b. $10\ m$
c. $15\ m$
d. $20\ m$

Solution: (d)

Here, we can find the horizontal velocity v with which the block will leave the surface. So, using principle of conservation of energy we have,

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2} \times 100 \times (5 \times 10^{-1})^2 = \frac{1}{2} \times 0.1 \times v^2$

$\Rightarrow\ v = 5\sqrt{10}\ m/s$

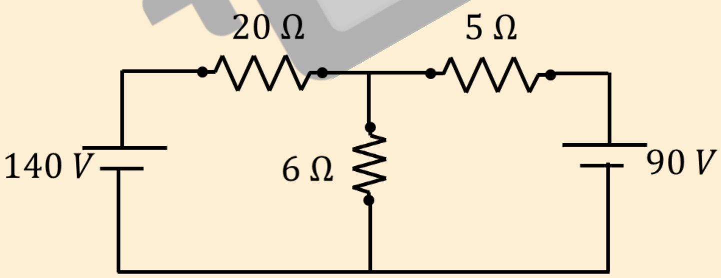
Now, the horizontal distance moved by the block is given by

$d = vt$, where t is the time taken by the block to fall the distance $2\ m$

Therefore, $d = 5\sqrt{10} \times \sqrt{(2h)/g}$

$\Rightarrow d = 5\sqrt{10} \times \sqrt{4/10} = 10\ m$

3. In the given circuit, find the current through $6\ \Omega$ resistor



a. $10\ A$
b. $7\ A$
c. $25\ A$
d. $30\ A$

Solution: (a)

Equivalent EMF of battery $EMF_{eq} = \frac{\frac{140}{20}+\frac{90}{5}}{\frac{1}{20}+\frac{1}{5}} = (7+18)4 = 100\ V$

Internal Equivalent resistant of battery $\frac{1}{r_{eq}} = \frac{1}{20} + \frac{1}{5} = \frac{1}{4} \Rightarrow r_{eq} = 4\ \Omega$

Equivalent resistance of the circuit $R_{eq} = 6 + 4 = 10\ \Omega$

$$\therefore i = \frac{100}{10} = 10\ A$$

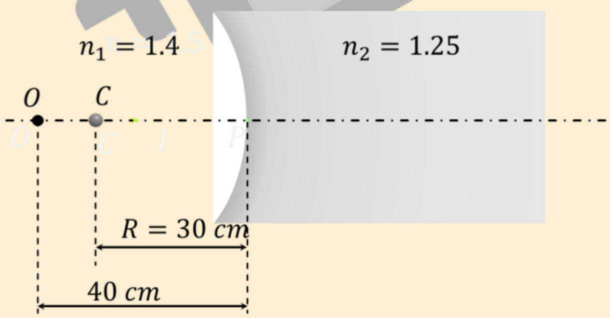
4. Four moles of a diatomic gas is heated from 0°C to 50°C. Find the heat supplied to the gas if work done by it is zero.
   a. 700 R
   b. 600 R
   c. 500 R
   d. 100 R

Solution: (c)

Given number of moles of gas, $n = 4$
Increase in temperature,$\Delta T = 50\ K$
For the diatomic gas ,
$C_V = \frac{5R}{2}$
The work done by gas is zero that means the given thermodynamic process is an isochoric process.

$$Q = W + \Delta U$$
$$Q = \Delta U$$
$$Q = nC_V\Delta T = 4 \times \frac{5R}{2} \times 50 = 500R$$

5. For the spherical interface shown in the figure, the two different media with refractive indices $n_1 = 1.4$ and $n_2 = 1.25$ are present as shown. The image will be formed at



a. $-\frac{125}{3}\ cm$
b. $-\frac{50}{6}\ cm$
c. $-\frac{25}{2}\ cm$
d. $-20\ cm$

Solution:(a)

Given:
Position of the object (u)= -40 cm
Refractive index of medium 1(n_1) = 1.4
Refractive index of medium 2 (n_2) = 1.25
Radius of the interface (R) = -30 cm
We know that for the spherical interface,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

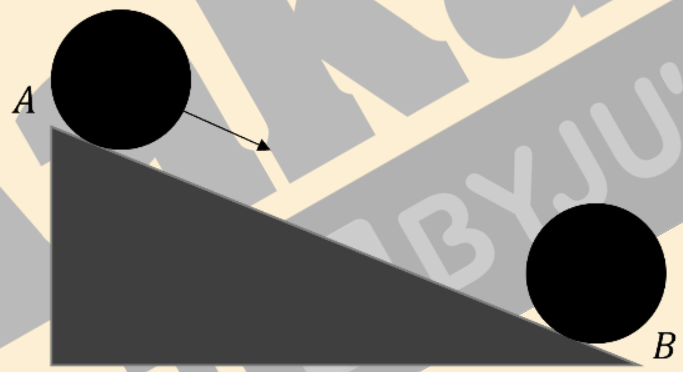
$$\frac{1.25}{v} - \frac{1.4}{-40} = \frac{1.25 - 1.4}{(-30)}$$

$\Rightarrow \frac{1.25}{v} = 0.005 - 0.035$

$\Rightarrow v = -\frac{1.25}{0.03}\ cm$

Or, $v = -\frac{125}{3}\ cm$

6. When a disc slides on smooth inclined surface from rest, the time taken to move from A to B is t_1. When disc performs pure rolling from rest then time taken to move from A to B is t_2. If $\frac{t_2}{t_1} = \sqrt{\frac{3}{x}}$ find x.



a. 2
b. 1
c. 5
d. 7

Solution:(a)

When disc slides $a_1 = g\sin\theta$

So, $S = ut_1 + \frac{1}{2}a_1t_1^2 = \frac{1}{2}g\sin\theta\, t_1^2$............(1)

When disc do pure rolling $a_2 = \frac{g\sin\theta}{1+k^2/R^2} = \frac{g\sin\theta}{1+1/2} = \frac{2}{3}g\sin\theta$

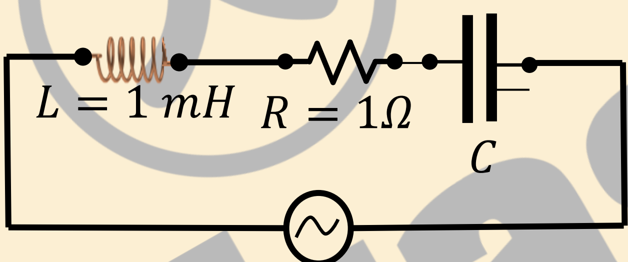
So, $S = ut_2 + \frac{1}{2}a_2 t_2^2 = \frac{1}{2}\times\frac{2}{3} g \sin\theta\, t_2^2$ ....(2)

From (1) & (2)

$$\frac{t_2}{t_1} = \sqrt{\frac{3}{2}}$$

So, $x = 2$

7. An AC circuit consists of a series combination of an inductance L 1 mH, a resistance R = 111 and a capacitance C. It is observed that the current leads the voltage by 45°. Find the value of capacitance 'C' if angular frequency of applied AC is 300 rad/s.
   a. $5.6\, mF$
   b. $3.92\, mF$
   c. $2.56\, mF$
   d. $5.2\, mF$



$\omega = 200\, rad/s$

Solution: (c)

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{\left(\frac{1}{\omega_C}\right) - \omega_L}{R}$$

$$1(1) = \frac{1}{300C} - 300(1\times10^{-3})$$

$$\frac{1}{300C} = 1 + 0.3 = 1.3$$

$$C = \frac{1}{300\times1.3} = 0.00256\, F = 2.56\, mF$$

8. An electron is projected into a magnetic field of $B = 5\times 10^{-3}$ T and rotates in a circle of radius of $R = 3\, mm$. Find the work done by the force due to magnetic field.
   a. $0\, J$
   b. $15\, mJ$
   c. $14\, mJ$
   d. $20\, mJ$

Solution: (a)

The work done by the force due to magnetic field is 0.

9. A charge Q is divided into q and $(Q - q)$. If $\frac{Q}{q} = x$, such that the repulsion between them is maximum, find x.

a. 1
b. 2
c. 3
d. 4

Solution: (b)

As we know, $F = \frac{k(Q-q)q}{d^2}$

For F to be maximum,

$$\frac{dF}{dq} = 0$$

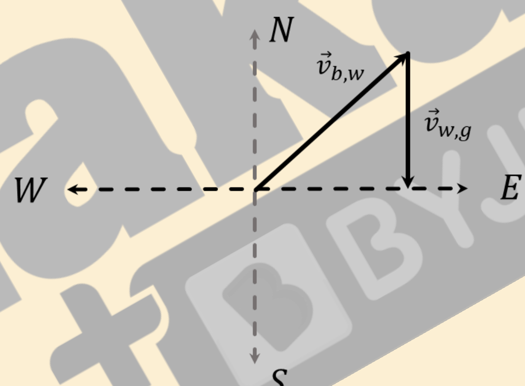
$$\Rightarrow Q - 2q = 0$$

$$\Rightarrow \frac{Q}{q} = 2$$

$$\Rightarrow x = 2$$

10. Bird is flying in north-east direction with or $v = 4\sqrt{2}\ m/s$ with respect to the wind and the wind blowing from north to south with speed $1\,m/s$. Find the magnitude of the displacement of bird in 3 sec.

a. $5\,m$
b. $15\,m$
c. $10\,m$
d. $20\,m$



Solution: (b)

Velocity of bird w.r.t. wind,

$$\vec{v}_{bw} = \left(4\sqrt{2}\cos 45°\,\hat{\imath} + 4\sqrt{2}\sin 45°\,\hat{\jmath}\right) m/s$$

Now velocity of wind w.r.t ground is,

$$\vec{v}_{wg} = -1\hat{\jmath}\ m/s$$

From relative motion relation, velocity of bird w.r.t. ground

$$\Rightarrow\ \vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg}$$

$$\vec{v}_{bg} = 4\hat{i} + 4\hat{j} - 1\hat{j} = 4\hat{i} + 3\hat{j}$$

$$|\vec{v}_{bg}| = \sqrt{(3)^2 + (4)^2} = 5\ m/s$$

Then displacement of bird in 3 sec,

$$\Rightarrow d = |\vec{v}_{bg}| \times t$$

$$\text{or, } d = 5 \times 3 = 15\ m/s$$

11. Deuteron and alpha particle having same $K.E.$ in magnetic field. If the ratio of radius of Deuteron and alpha particle is $x\sqrt{2}$. Then $x =$?

a. 5
b. 8
c. 3
d. 1

Solution: (d)

$$KE = \frac{B^2 q^2 r^2}{2m}$$

As B and KE are same

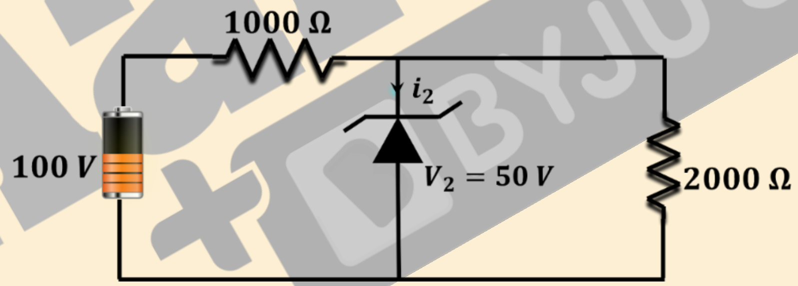
$$r^2 \propto \frac{m}{q^2} \Rightarrow r = \frac{\sqrt{m}}{q}$$

$$\frac{r_D}{r_{al}} = \sqrt{\frac{m_D}{q_{al}}} \times \frac{q_{al}}{q_D}$$

$$= \sqrt{\frac{2}{4}} \times \frac{2}{1} = \sqrt{2}$$

$$\Rightarrow \sqrt{2}x = \sqrt{2} \Rightarrow x = 1$$

12. In the circuit shown, find the current through the Zener diode.



a. $5\ mA$
b. $1\ mA$
c. $15\ mA$
d. $25\ mA$

Solution: (d)

Since 2000 Ω is parallel to Zener diode
So, the current passing through it as shown in the circuit,

$$i_3 = \frac{50}{2000} = 25\ mA$$

Potential difference across 1000 Ω,

$$V_1 = 100 - 50 = 50\ V$$

So, the electric current passing through it,

$$i_1 = \frac{50}{1000} = 50\ mA$$

So, current through Zener diode,

$$i_2 = 50 - 25 = 25\ mA$$

13. If $\vec{A}.\vec{B} = \vec{A} \times \vec{B}$, find $|\vec{A} - \vec{B}|$

a. $A - B$

b. $\sqrt{A^2 + B^2 - \sqrt{2AB}}$

c. $A + B$

d. $\sqrt{A^2 + B^2 - \sqrt{2}AB}$

Solution: (d)

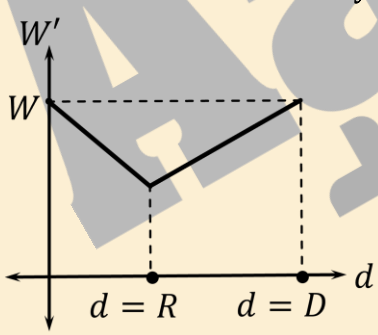
Since, $\vec{A}.\vec{B} = \vec{A} \times \vec{B}$

$\Rightarrow |\vec{A}||\vec{B}|\ cos\theta = |\vec{A}||\vec{B}|\ sin\theta\ \Rightarrow$ Angle between the vectors, $\theta = 45°$

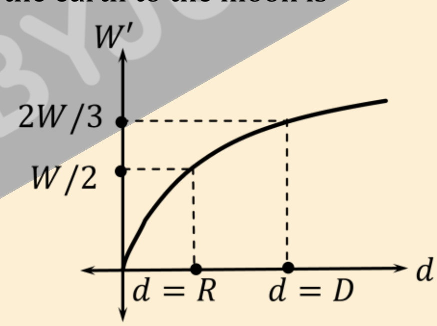
Hence,

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2ABcos\theta} = \sqrt{A^2 + B^2 - \sqrt{2}AB}$$

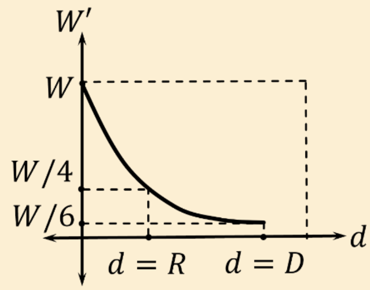
14. An object moves from earth's surface to the surface of the moon. The acceleration due to gravity on the earth's surface is $10\ m/s^2$. Considering the acceleration due to gravity on the moon to be $1/6th$ times of that of earth. If R be the earth's radius and its weight be W and the distance between the earth and the moon is D. The correct variation of the weight W' versus distance d for a body when it moves from the earth to the moon is

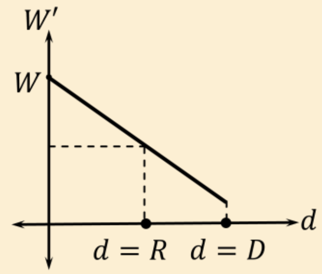


a.



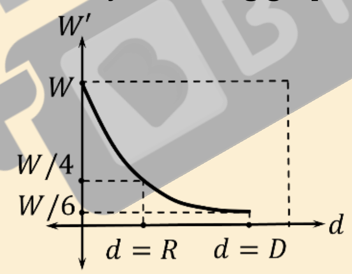
b.

c. 

d. 

Solution: (c)

At the earth's surface the weight of body is, $W = mg$
At the moon the distance of body from the earth's is $d = D$
At moon's surface the value of acceleration due to gravity is $g' = \frac{g}{6}$
$\Rightarrow W' = \frac{W}{6}$
From the relation of acceleration due to gravity at height d above earth's surface,
$g' = \frac{gR^2}{(R+d)^2} \ldots\ldots (i)$
For $d = 0$, $\Rightarrow g' = g$ or $W' = W$
At $d = R$,
$\Rightarrow g' = \frac{gR^2}{4R^2} = \frac{g}{4}$
Or, $W' = \frac{mg}{4} = \frac{W}{4}$
At distance $d = D$, when the body reaches surface of moon
$W' = \frac{mg}{6} = \frac{W}{6}$
Since Eq.(i) suggests that variation of g with distance(d) is non-linear, hence the graph of W' vs d will be non-linear as well. Also $d \uparrow$, $W' \downarrow$
Thus the correct variation is represented by following graph



15. For an element decaying through simultaneous reaction, the half-life for respective decaying path is 1400 s and 700 s. Find the time taken when the number of atoms become $N_0/3$ in the element sample. (N_0 is initial number of atoms in sample)

a. $\frac{1400}{5}\ln 3$
b. $\frac{1400}{3}\ln 3$
c. $\frac{1400}{3}\ln 2$
d. $\frac{700}{3}\ln 2$

Solution: (b)

$$N = N_o e^{-\frac{t}{\tau}}$$
$$\frac{N_o}{3} = N_o e^{-\frac{t}{\tau}}$$

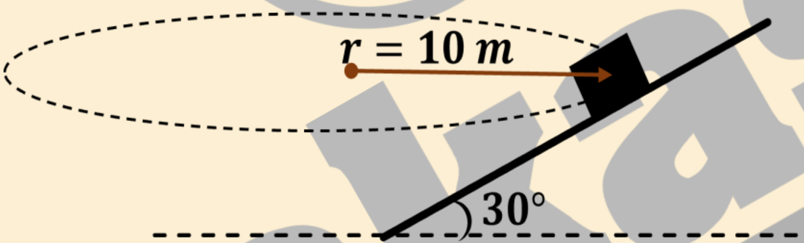
Taking natural log on both sides,

$$\ln\left(\frac{1}{3}\right) = \left(-\frac{t}{\tau}\right)\ln e$$

$$-\ln 3 = \left(-\frac{t}{\frac{1400}{3}}\right) \times 1$$

$$\therefore t = \frac{1400}{3}\ln 3$$

16. Consider a body of 800 kg moving with a maximum speed v on a road banked at $\theta = 30°$, given $cos30° = 0.87$. Find the normal reaction on the body. Coefficient of friction $\mu_s = 0.2$. [Take radius, $r = 10\ m$]



a. 10.4 kN
b. 12.6 kN
c. 11.6 kN
d. 8.3 kN

Solution: (a)

Since the circular motion is such that $v = v_{max}$ the tendency of body is to move up the inclined plane

$$[f_s]_{max} = \mu_s N = 0.2\ N$$

Resolving along X and Y axis, we have $N\cos 30° = mg + [f_s]_{max}\sin 30°$

$$\Rightarrow N[0.87] = 8000 + 0.2N\left[\frac{1}{2}\right]$$

$$N[0.87 - 0.1] = 8000$$

$$N = \frac{8000}{0.77} \approx 10{,}400\ newton$$

17. A spring with natural length l_0 has a tension T_1 when its length is l_1 and the tension is T_2 when its length is l_2.The natural length of spring will be:

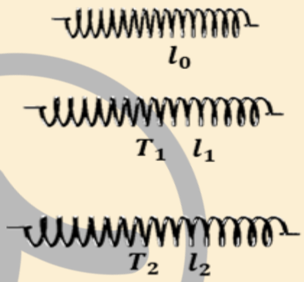
a. $\frac{T_1 l_2 - T_2 l_1}{l_1 - l_2}$

b. $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

c. $\frac{T_2 l_2 - T_1 l_1}{T_1 - T_2}$

d. $\frac{T_2 l_1 - T_1 l_2}{T_2 + T_1}$

Solution: (b)



Let the natural length be L_0

Using hook's law ,Y $= \frac{TL}{AdL}$, where dL =L – L_0

Case 1 : when tension is T_1 length of wire =L_1

$L_1 - L_0 = \frac{T_1 L_0}{AY}$ .....(1)

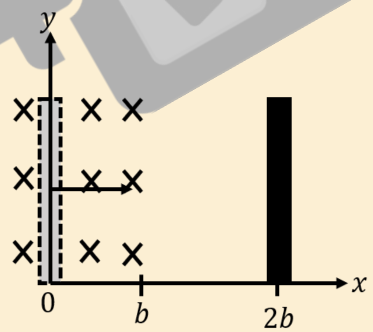
Case 2: Tension is T_2 and length of wire = L_2

L_2 - $l_0 = \frac{T_2}{A}\frac{l_0}{Y}$..........(2)

Dividing both equations :

$L_o = \frac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$

18. A conducting rod of length l is moving perpendicular to magnetic field. The rod moves from 0 to $2b$ while field exists only from 0 to b. Find the graph for emf and power dissipated w.r.t x.



Solution:

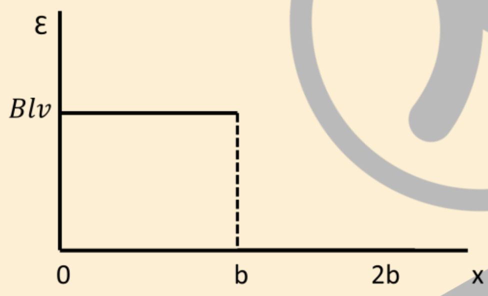
From the given figure it is clear that field exist from 0 to b only, therefore, the given conductor will experience field only from 0 to b. Here the given conductor is moving in a uniform magnetic field as long as field exist a constant emf(Ɛ) will be induced in the conductor.

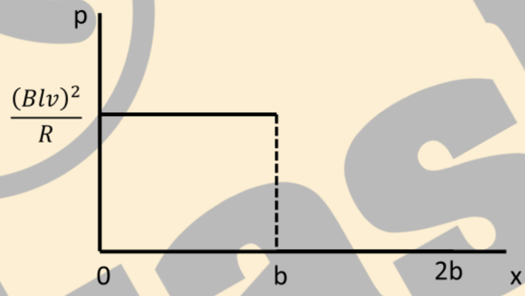
Induced emf in the conductor (Ɛ) $= Blv$

due to this emf current developed in the conductor as, $i = \frac{e.m.f}{R} = \frac{Blv}{R}$

Power dissipation exists as long as emf exists in the conductor, $p = i^2R = \frac{(Blv)^2}{R}$

Hence, the graph of Ɛ vs x and p vs x as follows,





19. A travelling wave is found to have the displacement by $y = \frac{1}{1+x^2}$ at $t = 0$, after 3 sec the wave pulse is represented by equation $y = \frac{1}{1+(1+x)^2}$. The velocity of wave is:

a. $1\ m/s$

b. $\frac{1}{3}\ m/s$

c. $\frac{2}{3}\ m/s$

d. $\frac{1}{4}\ m/s$

Solution: (b)

Displacement of wave, $\Delta x = 1\ m$

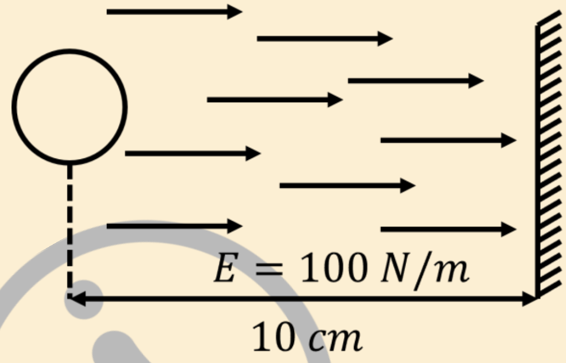
$$\Rightarrow v \times t = 1$$

$$t = \frac{1}{v}$$

$$3 = \frac{1}{v}$$

$$v = \frac{1}{3}\ m/s$$

20. A ball of charge to mass ratio $8\ \mu C/g$ is placed at a distance of $10\ cm$ from the ball. An electric field $100\ N/m$ is switched on in the direction of wall. Find the time period of its oscillations. Assume all collisions elastic.



a. 1 sec
b. 2 sec
c. 3 sec
d. 4 sec

Solution: (a)

$$a = \frac{qE}{m} = \frac{8 \times 10^{-6}}{10^{-3}} \times 100 = 0.8\ m/s^2$$

As electric field is switched ON, ball first strikes to wall and returns back.
One oscillation
Thus

$$s = ut + \frac{1}{2}at_1^2$$

$$0.1 = \frac{1}{2} \times 0.8t_1^2$$

$$t_1 = \frac{1}{2}\ sec$$

Thus, time period,

$$T = 2 \times \frac{1}{2} = 1\ sec$$

21. The wave number of the spectral line in the emission spectrum of hydrogen will be equal to $\frac{8}{9}$ times of the Rydberg's constant. Then the electron jumps from

a. $5 \to 2$
b. $5 \to 3$
c. $3 \to 1$
d. $4 \to 2$

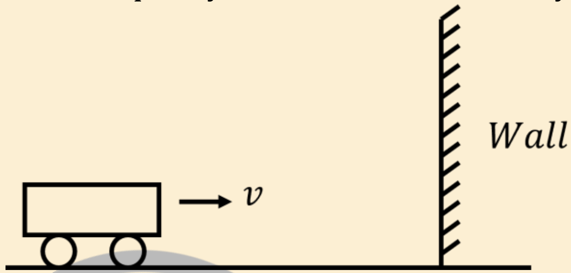
Solution: (c)

$$\bar{\nu} = Rz^2\left(\frac{1}{n_L^2} - \frac{1}{n_H^2}\right)$$

If $n_L = 1, n_H = 3$

$$\bar{\nu} = R.1^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8}{9}R$$

22. A vehicle moving with velocity v and releasing sound of frequency 400 Hz. Listening the reflected sound from a wall of frequency 500 Hz. Find the velocity of vehicle v.



a. 36.67 m/s
b. 30.12 m/s
c. 22.37 m/s
d. 20.25

Solution :-(a)

Frequency received by wall $f' = \left(\frac{v_s}{v_s - v}\right) f_o$

Reflected frequency received by man is $f'' = \left(\frac{v_s + v}{v_s}\right) f'$

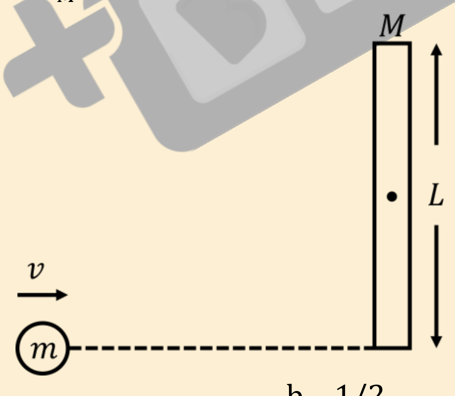
$$\Rightarrow f'' = \left(\frac{v_s + v}{v_s}\right)\left(\frac{v_s}{v_s - v}\right) f_o$$

$$\Rightarrow f'' = \left(\frac{v_s + v}{v_s - v}\right) f_o$$

$$\Rightarrow 500 = \left(\frac{330 + v}{330 - v}\right) 400$$

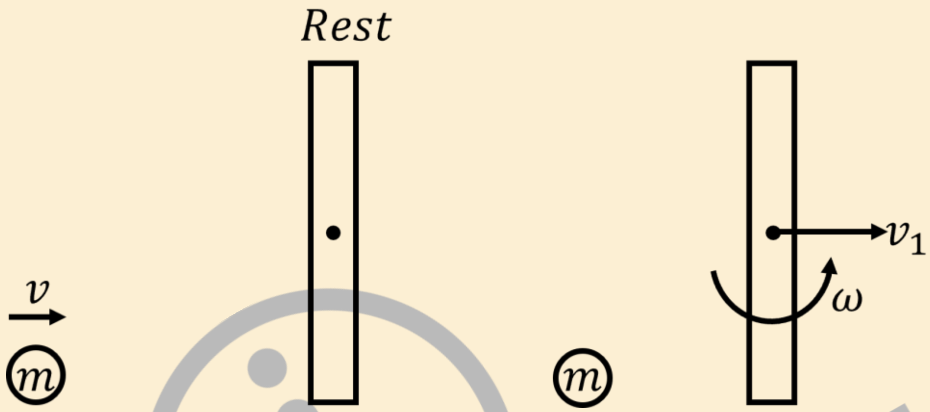
$$\Rightarrow v = \frac{330}{9} = 36.67\ m/s$$

23. A particle of mass moving with a speed v collide elastically with the end of a uniform rod of mass M and length L perpendicularly as shown in the figure. If the particle comes to rest after collision, find the value of $\frac{m}{M}$.



a. 1/3
b. 1/2
c. 1/4
d. 1/5

Solution: (c)



Applying conservation of angular momentum about the Centre of mass of rod,

$mv\left(\frac{L}{2}\right) = m\left(\frac{L^2}{12}\right)\omega \ldots\ldots..(i)$

Applying linear momentum conservation

$mv = Mv_1 \ldots\ldots.(ii)$

$1 = \frac{v_1+\omega\left(\frac{L}{2}\right)}{v} \ldots\ldots.(iii)$

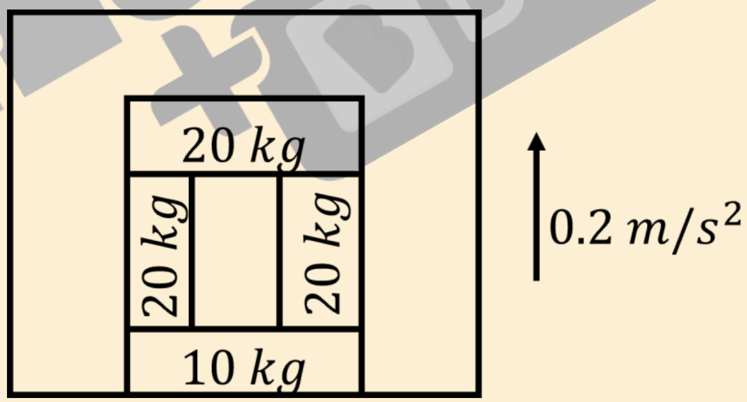
Putting v_1 from (ii) and ωL from (i) in (iii)

$v = \frac{m}{M}v + \frac{6mv}{2M}$

$\Rightarrow 1 = \frac{4m}{M}$

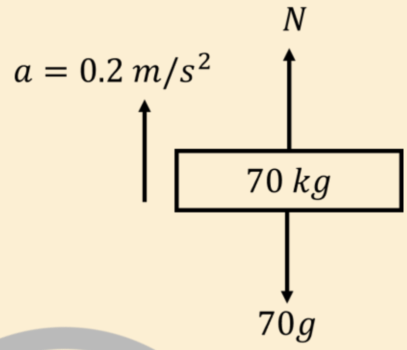
$\Rightarrow \frac{m}{M} = \frac{1}{4}$

24. Four planks are arranged in a lift going upwards with an acceleration of $0.2\, m/s^2$ as shown in figure. Find the normal reaction applied by the lift on $10\, kg$ block:
$(g = 9.8\, m/s^2)$



a. 500
b. 700
c. 672
d. 800

Solution: (b)



$N - 70g = 70 \times 0.2$

$N = 70(g + 0.2)$

$N = 700$

25. A body of mass m emits a photon of frequency v, then loss in its internal energy?

a. hv

b. $hv(1 - \frac{hv}{2mc^2})$

c. $hv\left(1 + \frac{hv}{2mc^2}\right)$

d. Zero

Solution: (c)

$v = \frac{c}{\lambda}$

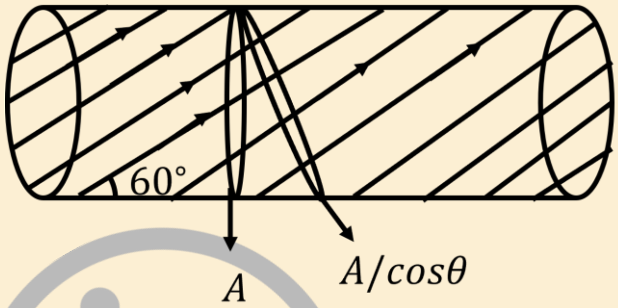
$mv = \frac{h}{\lambda} = \left(\frac{hv}{c}\right)$

Loss of energy $= \frac{1}{2}mv^2 + hv$

$= \frac{1}{2m}\left(\frac{hv}{c}\right)^2 + hv$

$= hv\left(1 + \frac{hv}{2mc^2}\right)$

26. In a magnesium rod of area $3m^2$, current $I = 5A$ is flowing angle of 60^o from axis of rod. Resistivity of material is $44 \times 10^{-2}\ ohm \times m$. Find electric field inside the rod



a. 0.567
b. 0.367
c. 0.667
d. 0.767

Solution: (b)

$$J = \sigma E$$

$$\frac{I}{A_{effective}} = \frac{E}{\rho}$$

$$E = \frac{\rho I}{A}\cos 60° = \frac{44\times10^{-2}\times5}{3\times2}ss$$

$$E = 0.367$$