**Notations :**

1.Options shown in green color and with ✔ icon are correct.

2.Options shown in red color and with ✖ icon are incorrect.

**Question Paper Name:** MA: MATHEMATICS 1st Feb shift2
**Number of Questions:** 65
**Total Marks:** 100.0

Wrong answer for MCQ will result in negative marks, (-1/3) for 1 mark Questions and (-2/3) for 2 marks Questions.

General Aptitude

| | |
|---|---|
| Number of Questions: | 10 |
| Section Marks: | 15.0 |

Q.1 to Q.5 carry 1 mark each & Q.6 to Q.10 carry 2 marks each.

**Question Number : 1 Question Type : MCQ**

Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence:

Apparent lifelessness ____________________ dormant life.

(A) harbours (B) leads to (C) supports (D) affects

**Options :**

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

**Question Number : 2 Question Type : MCQ**

Fill in the blank with the correct idiom/phrase.

That boy from the town was a ____________ in the sleepy village.

(A) dog out of herd (B) sheep from the heap
(C) fish out of water (D) bird from the flock

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 3 Question Type : MCQ**

Choose the statement where underlined word is used correctly.

(A) When the teacher eludes to different authors, he is being elusive.
(B) When the thief keeps eluding the police, he is being elusive.
(C) Matters that are difficult to understand, identify or remember are allusive.
(D) Mirages can be allusive, but a better way to express them is illusory.

**Options :**

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

**Question Number : 4 Question Type : MCQ**

Tanya is older than Eric.
Cliff is older than Tanya.
Eric is older than Cliff.

If the first two statements are true, then the third statement is:

(A) True
(B) False
(C) Uncertain
(D) Data insufficient

**Options :**

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

**Question Number : 5 Question Type : MCQ**

Five teams have to compete in a league, with every team playing every other team exactly once, before going to the next round. How many matches will have to be held to complete the league round of matches?

(A) 20 (B) 10 (C) 8 (D) 5

**Options :**

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

**Question Number : 6 Question Type : MCQ**

Select the appropriate option in place of underlined part of the sentence.

Increased productivity necessary reflects greater efforts made by the employees.

(A) Increase in productivity necessary
(B) Increase productivity is necessary
(C) Increase in productivity necessarily
(D) No improvement required

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 7 Question Type : MCQ**

Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.

Statements:

I. No manager is a leader.
II. All leaders are executives.

Conclusions:

I. No manager is an executive.
II. No executive is a manager.

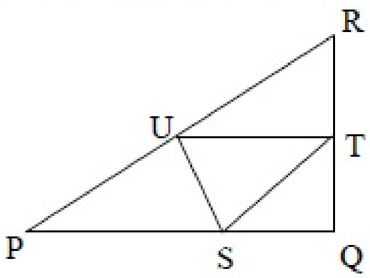
(A) Only conclusion I follows.
(B) Only conclusion II follows.
(C) Neither conclusion I nor II follows.
(D) Both conclusions I and II follow.

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 8 Question Type : NAT**

In the given figure angle Q is a right angle, PS:QS = 3:1, RT:QT = 5:2 and PU:UR = 1:1. If area of triangle QTS is 20 $cm^2$, then the area of triangle PQR in $cm^2$ is _______.




**Correct Answer :**

280

**Question Number : 9 Question Type : MCQ**

Right triangle PQR is to be constructed in the xy - plane so that the right angle is at P and line PR is parallel to the x-axis. The x and y coordinates of P, Q, and R are to be integers that satisfy the inequalities: $-4 \le x \le 5$ and $6 \le y \le 16$. How many different triangles could be constructed with these properties?

(A) 110 (B) 1,100 (C) 9,900 (D) 10,000

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 10 Question Type : MCQ**

A coin is tossed thrice. Let $X$ be the event that head occurs in each of the first two tosses. Let $Y$ be the event that a tail occurs on the third toss. Let $Z$ be the event that two tails occur in three tosses. Based on the above information, which one of the following statements is TRUE?

(A) $X$ and $Y$ are not independent
(B) $Y$ and $Z$ are dependent
(C) $Y$ and $Z$ are independent
(D) $X$ and $Z$ are independent

**Options :**

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

| | Mathematics |
|---|---|
| Number of Questions: | 55 |
| Section Marks: | 85.0 |

Q.11 to Q.35 carry 1 mark each & Q.36 to Q.65 carry 2 marks each.

**Question Number : 11 Question Type : NAT**

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear map defined by

$$T(x, y, z, w) = (x + z, 2x + y + 3z, 2y + 2z, w).$$

Then the rank of $T$ is equal to __________

**Correct Answer :**

3

**Question Number : 12 Question Type : NAT**

Let $M$ be a $3 \times 3$ matrix and suppose that $1, 2$ and $3$ are the eigenvalues of $M$. If

$$M^{-1} = \frac{M^2}{\alpha} - M + \frac{11}{\alpha} I_3$$

for some scalar $\alpha \neq 0$, then $\alpha$ is equal to ___________

**Correct Answer :**

6

**Question Number : 13 Question Type : NAT**

Let $M$ be a $3 \times 3$ singular matrix and suppose that 2 and 3 are eigenvalues of $M$. Then the number of linearly independent eigenvectors of $M^3 + 2M + I_3$ is equal to __________

**Correct Answer :**

3

**Question Number : 14 Question Type : NAT**

Let $M$ be a $3 \times 3$ matrix such that $M \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$ and suppose that $M^3 \begin{pmatrix} 1 \\ -1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ for some $\alpha, \beta, \gamma \in \mathbb{R}$. Then $|\alpha|$ is equal to _______

**Correct Answer:**

27

**Question Number : 15 Question Type : MCQ**

Let $f: [0, \infty) \to \mathbb{R}$ be defined by

$$f(x) = \int_0^x \sin^2(t^2)\, dt.$$

Then the function $f$ is

(A) uniformly continuous on $[0, 1)$ but NOT on $(0, \infty)$
(B) uniformly continuous on $(0, \infty)$ but NOT on $[0, 1)$
(C) uniformly continuous on both $[0, 1)$ and $(0, \infty)$
(D) neither uniformly continuous on $[0, 1)$ nor uniformly continuous on $(0, \infty)$

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 16 Question Type : NAT**

Consider the power series $\sum_{n=0}^{\infty} a_n z^n$, where $a_n = \begin{cases} \frac{1}{3^n} & \text{if } n \text{ is even} \\ \frac{1}{5^n} & \text{if } n \text{ is odd.} \end{cases}$

The radius of convergence of the series is equal to ___________

**Correct Answer :**

3

**Question Number : 17 Question Type : NAT**

Let $C = \{ z \in \mathbb{C} : |z - i| = 2\}$. Then $\frac{1}{2\pi} \oint_C \frac{z^2 - 4}{z^2 + 4} \, dz$ is equal to _____________

**Correct Answer :**

-2

**Question Number : 18 Question Type : NAT**

Let $X \sim B(5, \frac{1}{2})$ and $Y \sim U(0,1)$. Then $\frac{P(X+Y \leq 2)}{P(X+Y \geq 5)}$ is equal to ____________

**Correct Answer:**

6

**Question Number : 19 Question Type : NAT**

Let the random variable $X$ have the distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x}{2} & \text{if } 0 \leq x < 1 \\ \frac{3}{5} & \text{if } 1 \leq x < 2 \\ \frac{1}{2} + \frac{x}{8} & \text{if } 2 \leq x < 3 \\ 1 & \text{if } x \geq 3. \end{cases}$$

Then $P(2 \leq X < 4)$ is equal to ____________

**Correct Answer:**

4

**Question Number : 20 Question Type : NAT**

Let $X$ be a random variable having the distribution function

$$F(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ \frac{1}{4} & \text{if } 0 \leq x < 1 \\ \frac{1}{3} & \text{if } 1 \leq x < 2 \\ \frac{1}{2} & \text{if } 2 \leq x < \frac{11}{3} \\ 1 & \text{if} \quad x \geq \frac{11}{3}. \end{cases}$$

Then $E(X)$ is equal to ________

**Correct Answer :**

2.25

**Question Number : 21 Question Type : MCQ**

In an experiment, a fair die is rolled until two sixes are obtained in succession. The probability that the experiment will end in the fifth trial is equal to

(A) $\frac{125}{6^5}$ (B) $\frac{150}{6^5}$ (C) $\frac{175}{6^5}$ (D) $\frac{200}{6^5}$

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 22 Question Type : MCQ**

Let $x_1 = 2.2,\ x_2 = 4.3,\ x_3 = 3.1,\ x_4 = 4.5,\ x_5 = 1.1$ and $x_6 = 5.7$ be the observed values of a random sample of size 6 from a $U(\theta - 1,\ \theta + 4)$ distribution, where $\theta \in (0, \infty)$ is unknown. Then a maximum likelihood estimate of $\theta$ is equal to

(A) 1.8 (B) 2.3 (C) 3.1 (D) 3.6

**Options :**

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

**Question Number : 23 Question Type : MCQ**

Let $\Omega = \{(x, y) \in \mathbb{R}^2 | \, x^2 + y^2 < 1\}$ be the open unit disc in $\mathbb{R}^2$ with boundary $\partial\Omega$. If $u(x, y)$ is the solution of the Dirichlet problem

$$u_{xx} + u_{yy} = 0 \quad \text{in } \Omega$$
$$u(x, y) = 1 - 2y^2 \quad \text{on } \partial\Omega,$$

then $u\left(\frac{1}{2}, 0\right)$ is equal to

(A) $-1$ (B) $\frac{-1}{4}$ (C) $\frac{1}{4}$ (D) 1

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 24 Question Type : NAT**

Let $c \in \mathbb{Z}_3$ be such that $\frac{\mathbb{Z}_3[X]}{\langle X^3 + c\,X + 1 \rangle}$ is a field. Then $c$ is equal to ___________

**Correct Answer :**

2

**Question Number : 25 Question Type : MCQ**

Let $V = C^1[0,1]$, $X = (C[0,1], \| \quad \|_\infty)$ and $Y = (C[0,1], \| \quad \|_2)$. Then $V$ is

(A) dense in $X$ but NOT in $Y$
(B) dense in $Y$ but NOT in $X$
(C) dense in both $X$ and $Y$
(D) neither dense in $X$ nor dense in $Y$

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 26 Question Type : NAT**

Let $T : (C[0,1], \| \quad \|_\infty) \to \mathbb{R}$ be defined by $T(f) = \int_0^1 2x f(x)\, dx$ for all $f \in C[0,1]$. Then $\|T\|$ is equal to ___________

**Correct Answer :**

1

**Question Number : 27 Question Type : MCQ**

Let $\tau_1$ be the usual topology on $\mathbb{R}$. Let $\tau_2$ be the topology on $\mathbb{R}$ generated by $\mathcal{B} = \{[a, b) \subset \mathbb{R} : -\infty < a < b < \infty\}$. Then the set $\{x \in \mathbb{R} : 4 \sin^2 x \le 1\} \cup \left\{\frac{\pi}{2}\right\}$ is

(A) closed in $(\mathbb{R}, \tau_1)$ but NOT in $(\mathbb{R}, \tau_2)$
(B) closed in $(\mathbb{R}, \tau_2)$ but NOT in $(\mathbb{R}, \tau_1)$
(C) closed in both $(\mathbb{R}, \tau_1)$ and $(\mathbb{R}, \tau_2)$
(D) neither closed in $(\mathbb{R}, \tau_1)$ nor closed in $(\mathbb{R}, \tau_2)$

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 28 Question Type : MCQ**

Let $X$ be a connected topological space such that there exists a non-constant continuous function $f : X \to \mathbb{R}$, where $\mathbb{R}$ is equipped with the usual topology. Let $f(X) = \{f(x) : x \in X\}$. Then

(A) $X$ is countable but $f(X)$ is uncountable
(B) $f(X)$ is countable but $X$ is uncountable
(C) both $f(X)$ and $X$ are countable
(D) both $f(X)$ and $X$ are uncountable

**Options :**

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

**Question Number : 29 Question Type : MCQ**

Let $d_1$ and $d_2$ denote the usual metric and the discrete metric on $\mathbb{R}$, respectively. Let $f : (\mathbb{R}, d_1) \to (\mathbb{R}, d_2)$ be defined by $f(x) = x, \; x \in \mathbb{R}$. Then

(A) $f$ is continuous but $f^{-1}$ is NOT continuous
(B) $f^{-1}$ is continuous but $f$ is NOT continuous
(C) both $f$ and $f^{-1}$ are continuous
(D) neither $f$ nor $f^{-1}$ is continuous

**Options :**

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

**Question Number : 30 Question Type : NAT**

If the trapezoidal rule with single interval $[0, 1]$ is exact for approximating the integral $\int_0^1 (x^3 - c\,x^2)\,dx$, then the value of $c$ is equal to ________

Correct Answer :

1.5

**Question Number : 31 Question Type : NAT**

Suppose that the Newton-Raphson method is applied to the equation $2x^2 + 1 - e^{x^2} = 0$ with an initial approximation $x_0$ sufficiently close to zero. Then, for the root $x = 0$, the order of convergence of the method is equal to ________

**Correct Answer :**

1

**Question Number : 32 Question Type : NAT**

The minimum possible order of a homogeneous linear ordinary differential equation with real constant coefficients having $x^2 \sin(x)$ as a solution is equal to __________

**Correct Answer:**

6

**Question Number : 33 Question Type : MCQ**

The Lagrangian of a system in terms of polar coordinates $(r, \theta)$ is given by

$$L = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m (\dot{r}^2 + r^2\dot{\theta}^2) - m g r (1 - \cos(\theta)),$$

where $m$ is the mass, $g$ is the acceleration due to gravity and $\dot{s}$ denotes the derivative of $s$ with respect to time. Then the equations of motion are

(A) $2\ddot{r} = r\dot{\theta}^2 - g(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = -g r \sin(\theta)$

(B) $2\ddot{r} = r\dot{\theta}^2 + g(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = -g r \sin(\theta)$

(C) $2\ddot{r} = r\dot{\theta}^2 - g(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = g r \sin(\theta)$

(D) $2\ddot{r} = r\dot{\theta}^2 + g(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = g r \sin(\theta)$

**Options :**

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

**Question Number : 34 Question Type : NAT**

If $y(x)$ satisfies the initial value problem

$$(x^2 + y)dx = x\,dy, \qquad y(1) = 2,$$

then $y(2)$ is equal to __________

.

**Correct Answer:**

6

**Question Number : 35  Question Type : NAT**

It is known that Bessel functions $J_n(x)$, for $n \geq 0$, satisfy the identity

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = J_0(x) + \sum_{n=1}^{\infty} J_n(x)\left(t^n + \frac{(-1)^n}{t^n}\right)$$

for all $t > 0$ and $x \in \mathbb{R}$. The value of $J_0\left(\frac{\pi}{3}\right) + 2\sum_{n=1}^{\infty} J_{2n}\left(\frac{\pi}{3}\right)$ is equal to ________

**Correct Answer :**

1

**Question Number : 36  Question Type : MCQ**

Let $X$ and $Y$ be two random variables having the joint probability density function

$$f(x,y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Then the conditional probability $P\left(X \leq \frac{2}{3} \mid Y = \frac{3}{4}\right)$ is equal to

(A) $\frac{5}{9}$ (B) $\frac{2}{3}$ (C) $\frac{7}{9}$ (D) $\frac{8}{9}$

**Options :**

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

**Question Number : 37  Question Type : NAT**

Let $\Omega = (0,1]$ be the sample space and let $P(\cdot)$ be a probability function defined by

$$P((0,x]) = \begin{cases} \frac{x}{2} & \text{if } 0 \leq x < \frac{1}{2} \\ x & \text{if } \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then $P\left(\left\{\frac{1}{2}\right\}\right)$ is equal to ________

**Correct Answer :**

0.25

**Question Number : 38 Question Type : NAT**

Let $X_1, X_2$ and $X_3$ be independent and identically distributed random variables with $E(X_1) = 0$ and $E(X_1^2) = \frac{15}{4}$. If $\psi : (0,\infty) \to (0,\infty)$ is defined through the conditional expectation

$$\psi(t) = E(X_1^2 \mid X_1^2 + X_2^2 + X_3^2 = t), \quad t > 0,$$

then $E(\psi((X_1 + X_2)^2))$ is equal to ___________

**Correct Answer :**

2.5

**Question Number : 39 Question Type : NAT**

Let $X \sim \text{Poisson}(\lambda)$, where $\lambda > 0$ is unknown. If $\delta(X)$ is the unbiased estimator of $g(\lambda) = e^{-\lambda}(3\lambda^2 + 2\lambda + 1)$, then $\sum_{k=0}^{\infty} \delta(k)$ is equal to ____________

**Correct Answer :**

9

**Question Number : 40 Question Type : NAT**

Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, 1)$ distribution, where $\mu \in \{0, \frac{1}{2}\}$. For testing the null hypothesis $H_0 : \mu = 0$ against the alternative hypothesis $H_1 : \mu = \frac{1}{2}$, consider the critical region

$$R = \left\{(x_1, x_2, \ldots, x_n) : \sum_{i=1}^{n} x_i > c\right\},$$

where $c$ is some real constant. If the critical region $R$ has size $0.025$ and power $0.7054$, then the value of the sample size $n$ is equal to ____________

**Correct Answer :**

25

**Question Number : 41 Question Type : MCQ**

Let $X$ and $Y$ be independently distributed central chi-squared random variables with degrees of freedom $m\ (\geq 3)$ and $n\ (\geq 3)$, respectively. If $E\left(\frac{X}{Y}\right) = 3$ and $m + n = 14$, then $E\left(\frac{Y}{X}\right)$ is equal to

(A) $\frac{2}{7}$ (B) $\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{5}{7}$

**Options :**

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

**Question Number : 42 Question Type : NAT**

Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with $P(X_1 = 1) = \frac{1}{4}$ and $P(X_1 = 2) = \frac{3}{4}$. If $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, for $n = 1, 2, \ldots$, then $\lim_{n\to\infty} P(\overline{X}_n \le 1.8)$ is equal to ___________

**Correct Answer:**

1

**Question Number : 43 Question Type : MCQ**

Let $u(x, y) = 2f(y)\cos(x - 2y)$, $(x, y) \in \mathbb{R}^2$, be a solution of the initial value problem

$$2u_x + u_y = u$$
$$u(x, 0) = \cos(x).$$

Then $f(1)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{e}{2}$ (C) $e$ (D) $\frac{3e}{2}$

**Options :**

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

**Question Number : 44 Question Type : NAT**

Let $u(x, t)$, $x \in \mathbb{R}$, $t \ge 0$, be the solution of the initial value problem

$$u_{tt} = u_{xx}$$
$$u(x, 0) = x$$
$$u_t(x, 0) = 1.$$

Then $u(2,2)$ is equal to _________

**Correct Answer:**

4

**Question Number : 45 Question Type : NAT**

Let $W = \text{Span}\left\{\frac{1}{\sqrt{2}}(0,0,1,1), \frac{1}{\sqrt{2}}(1,-1,0,0)\right\}$ be a subspace of the Euclidean space $\mathbb{R}^4$. Then the square of the distance from the point $(1,1,1,1)$ to the subspace $W$ is equal to _________

**Correct Answer :**

2

**Question Number : 46  Question Type : NAT**

Let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be a linear map such that the null space of $T$ is

$$\{(x, y, z, w) \in \mathbb{R}^4 : x + y + z + w = 0\}$$

and the rank of $(T - 4 I_4)$ is 3. If the minimal polynomial of $T$ is $x(x-4)^{\alpha}$, then $\alpha$ is equal to ________

**Correct Answer :**

1

**Question Number : 47  Question Type : MCQ**

Let $M$ be an invertible Hermitian matrix and let $x, y \in \mathbb{R}$ be such that $x^2 < 4y$. Then

(A) both $M^2 + xM + yI$ and $M^2 - xM + yI$ are singular
(B) $M^2 + xM + yI$ is singular but $M^2 - xM + yI$ is non-singular
(C) $M^2 + xM + yI$ is non-singular but $M^2 - xM + yI$ is singular
(D) both $M^2 + xM + yI$ and $M^2 - xM + yI$ are non-singular

**Options :**

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

**Question Number : 48  Question Type : MCQ**

Let $G = \{e, x, x^2, x^3, y, xy, x^2y, x^3y\}$ with $o(x) = 4, o(y) = 2$ and $xy = yx^3$. Then the number of elements in the center of the group $G$ is equal to

(A) 1 (B) 2 (C) 4 (D) 8

**Options :**

1. ✖ A
2. ✔ B
3. ✖ C
4. ✖ D

**Question Number : 49  Question Type : NAT**

The number of ring homomorphisms from $\mathbb{Z}_2 \times \mathbb{Z}_2$ to $\mathbb{Z}_4$ is equal to __________

**Correct Answer:**

1

**Question Number : 50  Question Type : MCQ**

Let $p(x) = 9x^5 + 10x^3 + 5x + 15$ and $q(x) = x^3 - x^2 - x - 2$ be two polynomials in $\mathbb{Q}[x]$. Then, over $\mathbb{Q}$,

(A) $p(x)$ and $q(x)$ are both irreducible
(B) $p(x)$ is reducible but $q(x)$ is irreducible
(C) $p(x)$ is irreducible but $q(x)$ is reducible
(D) $p(x)$ and $q(x)$ are both reducible

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 51 Question Type : NAT**

Consider the linear programming problem

$$\text{Maximize } 3x + 9y,$$
$$\text{subject to} \quad 2y - x \leq 2$$
$$3y - x \geq 0$$
$$2x + 3y \leq 10$$
$$x, y \geq 0.$$

Then the maximum value of the objective function is equal to _______

**Correct Answer:**

24

**Question Number : 52 Question Type : MCQ**

Let $S = \{(x, \sin\frac{1}{x}) : 0 < x \leq 1\}$ and $T = S \cup \{(0,0)\}$. Under the usual metric on $\mathbb{R}^2$,

(A) $S$ is closed but $T$ is NOT closed
(B) $T$ is closed but $S$ is NOT closed
(C) both $S$ and $T$ are closed
(D) neither $S$ nor $T$ is closed

**Options :**

1. ✖ A
2. ✖ B
3. ✖ C
4. ✔ D

**Question Number : 53 Question Type : MCQ**

Let $H = \left\{ (x_n) \in \ell_2 : \sum_{n=1}^{\infty} \frac{x_n}{n} = 1 \right\}$. Then $H$

(A) is bounded
(B) is closed
(C) is a subspace
(D) has an interior point

**Options :**

1. ✖ A
2. ✔ B

3. ✖ C

4. ✖ D

**Question Number : 54 Question Type : MCQ**

Let $V$ be a closed subspace of $L^2[0,1]$ and let $f,g \in L^2[0,1]$ be given by $f(x) = x$ and $g(x) = x^2$. If $V^\perp = \text{Span}\{f\}$ and $Pg$ is the orthogonal projection of $g$ on $V$, then $(g - Pg)(x)$, $x \in [0,1]$, is

(A) $\frac{3}{4}x$ (B) $\frac{1}{4}x$ (C) $\frac{3}{4}x^2$ (D) $\frac{1}{4}x^2$

**Options :**

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

**Question Number : 55 Question Type : NAT**

Let $p(x)$ be the polynomial of degree at most 3 that passes through the points $(-2,12), (-1,1)$, $(0,2)$ and $(2,-8)$. Then the coefficient of $x^3$ in $p(x)$ is equal to __________

**Correct Answer :**

-2

**Question Number : 56 Question Type : NAT**

If, for some $\alpha, \beta \in \mathbb{R}$, the integration formula

$$\int_0^2 p(x)dx = p(\alpha) + p(\beta)$$

holds for all polynomials $p(x)$ of degree at most 3, then the value of $3(\alpha - \beta)^2$ is equal to ______

**Correct Answer :**

4

**Question Number : 57 Question Type : NAT**

Let $y(t)$ be a continuous function on $[0,\infty)$ whose Laplace transform exists. If $y(t)$ satisfies

$$\int_0^t (1 - \cos(t-\tau))\, y(\tau)\, d\tau = t^4,$$

then $y(1)$ is equal to ________

**Correct Answer :**

28

**Question Number : 58 Question Type : NAT**

Consider the initial value problem

$$x^2y'' - 6y = 0, \qquad y(1) = \alpha,\ y'(1) = 6.$$

If $y(x) \to 0$ as $x \to 0^+$, then $\alpha$ is equal to ___________

**Correct Answer :**

2

**Question Number : 59 Question Type : MCQ**

Define $f_1, f_2: [0,1] \to \mathbb{R}$ by

$$f_1(x) = \sum_{n=1}^{\infty} \frac{x \sin(n^2 x)}{n^2} \quad \text{and} \quad f_2(x) = \sum_{n=1}^{\infty} x^2(1-x^2)^{n-1}.$$

Then

(A) $f_1$ is continuous but $f_2$ is NOT continuous
(B) $f_2$ is continuous but $f_1$ is NOT continuous
(C) both $f_1$ and $f_2$ are continuous
(D) neither $f_1$ nor $f_2$ is continuous

**Options :**

1. ✔ A
2. ✖ B
3. ✖ C
4. ✖ D

**Question Number : 60 Question Type : NAT**

Consider the unit sphere $S = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ and the unit normal vector $\hat{n} = (x,y,z)$ at each point $(x,y,z)$ on $S$. The value of the surface integral

$$\iint_S \left\{ \left(\frac{2x}{\pi} + \sin(y^2)\right) x + \left(e^z - \frac{y}{\pi}\right) y + \left(\frac{2z}{\pi} + \sin^2 y\right) z \right\} d\sigma$$

is equal to ________

**Correct Answer :**

4

**Question Number : 61 Question Type : NAT**

Let $D = \{(x,y) \in \mathbb{R}^2 : 1 \le x \le 1000,\ 1 \le y \le 1000\}$. Define

$$f(x,y) = \frac{xy}{2} + \frac{500}{x} + \frac{500}{y}.$$

Then the minimum value of $f$ on $D$ is equal to _________

**Correct Answer:**
150

**Question Number : 62 Question Type : MCQ**

Let $\mathbb{D} = \{ z \in \mathbb{C} : |z| < 1 \}$. Then there exists a non-constant analytic function $f$ on $\mathbb{D}$ such that for all $n = 2, 3, 4, \ldots$

(A) $f\left(\frac{\sqrt{-1}}{n}\right) = 0$

(B) $f\left(\frac{1}{n}\right) = 0$

(C) $f\left(1 - \frac{1}{n}\right) = 0$

(D) $f\left(\frac{1}{2} - \frac{1}{n}\right) = 0$

**Options :**

1. ✖ A
2. ✖ B
3. ✔ C
4. ✖ D

**Question Number : 63 Question Type : NAT**

Let $\sum_{n=-\infty}^{\infty} a_n z^n$ be the Laurent series expansion of $f(z) = \frac{1}{2z^2 - 13z + 15}$ in the annulus $\frac{3}{2} < |z| < 5$. Then $\frac{a_1}{a_2}$ is equal to __________

**Correct Answer:**
5

**Question Number : 64 Question Type : NAT**

The value of $\frac{i}{4-\pi} \int_{|z|=4} \frac{dz}{z \cos(z)}$ is equal to __________

**Correct Answer :**
2

**Question Number : 65 Question Type : MCQ**

Suppose that among all continuously differentiable functions $y(x)$, $x \in \mathbb{R}$, with $y(0) = 0$ and $y(1) = \frac{1}{2}$, the function $y_0(x)$ minimizes the functional

$$\int_0^1 \left(e^{-(y'-x)} + (1+y)y'\right)dx.$$

Then $y_0\left(\frac{1}{2}\right)$ is equal to

(A) 0

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

**Options :**

1. ✖ A

2. ✔ B

3. ✖ C

4. ✖ D