MA : MATHEMATICS

Duration: Three Hours *Maximum Marks*: 100

Read the following instructions carefully.

1. Write your name and registration number in the space provided at the bottom of this page.
2. Take out the **O**ptical **R**esponse **S**heet **(ORS)** from this Question Booklet **without breaking the seal.**
3. Do not open the seal of the Question Booklet until you are asked to do so by the invigilator.
4. Write your registration number, your name and name of the examination centre at the specified locations on the right half of the **ORS**. Also, using HB pencil, darken the appropriate bubble under each digit of your registration number and the letters corresponding to your test paper code (MA).
5. This Question Booklet contains **20** pages including blank pages for rough work. After opening the seal at the specified time, please check all pages and report discrepancy, if any.
6. There are a total of 65 questions carrying 100 marks. All these questions are of objective type. Questions must be answered on the left hand side of the **ORS** by darkening the appropriate bubble (marked A, B, C, D) using HB pencil against the question number. **For each question darken the bubble of the correct answer**. In case you wish to change an answer, erase the old answer completely. More than one answer bubbled against a question will be treated as an incorrect response.
7. Questions Q.1 – Q.25 carry 1-mark each, and questions Q.26 – Q.55 carry 2-marks each.
8. Questions Q.48 – Q.51 (2 pairs) are common data questions and question pairs (Q.52, Q.53) and (Q.54, Q.55) are linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
9. Questions Q.56 – Q.65 belong to General Aptitude (GA). Questions Q.56 – Q.60 carry 1-mark each, and questions Q.61 – Q.65 carry 2-marks each. The GA questions begin on a fresh page starting from page **14**.
10. Unattempted questions will result in zero mark and wrong answers will result in **NEGATIVE** marks. For Q.1 – Q.25 and Q.56 – Q.60, ⅓ mark will be deducted for each wrong answer. For Q.26 – Q.51 and Q.61 – Q.65, ⅔ mark will be deducted for each wrong answer. The question pairs (Q.52, Q.53), and (Q.54, Q.55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair, i.e. for Q.52 and Q.54, ⅔ mark will be deducted for each wrong answer. There is no negative marking for Q.53 and Q.55.
11. Calculator is allowed whereas charts, graph sheets or tables are **NOT** allowed in the examination hall.
12. Rough work can be done on the question paper itself. Additionally, blank pages are provided at the end of the question paper for rough work.

Name								
Registration Number	**MA**							

Notations and Symbols used

$\mathbb{R}$	:	The set of all real numbers
$\mathbb{Z}$	:	The set of all integers
$\mathbb{C}$	:	The set of all complex numbers
$\mathbb{R}^n$	:	$\{(x_1,\ldots,x_n): x_i \in \mathbb{R} \text{ for } 1 \le i \le n\}$
l^p	:	The vector space of all scalar sequences $\{x_n\}$ such that $\sum_{i=1}^{\infty} \lvert x_i \rvert^p < \infty,\ 1 \le p < \infty$
c_{00}	:	Set of all sequences $x = \{x_n\}$ with finitely many non-zero terms
x^T	:	The transpose of the vector x
$N(\mu, \sigma^2)$	:	The normal distribution with mean μ and variance σ^2
χ_n^2	:	Chi-square distribution with n degrees of freedom
$P(E)$	:	Probability of an event E
$P(E \vert F)$	:	Conditional probability of E given F
$E(X)$	:	Expectation of a random variable X
$E(X \mid Y = y)$	:	Conditional expectation of X given $Y = y$
$\exp(x)$	:	Exponential of x (that is e^x)
$\langle x, y \rangle$	:	Inner product of x and y
y'	:	$\dfrac{dy}{dx}$

Q. 1 – Q. 25 carry one mark each.

Q.1 The distinct eigenvalues of the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

are

(A) 0 and 1 (B) 1 and -1 (C) 1 and 2 (D) 0 and 2

Q.2 The minimal polynomial of the matrix

$$\begin{bmatrix} 3 & 3 & 0 \\ 3 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

is

(A) $x(x-1)(x-6)$ (B) $x(x-3)$ (C) $(x-3)(x-6)$ (D) $x(x-6)$

Q.3 Which of the following is the imaginary part of a possible value of $\ln(\sqrt{i})$?

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

Q.4 Let $f:\mathbb{C}\to\mathbb{C}$ be analytic except for a simple pole at $z=0$ and let $g:\mathbb{C}\to\mathbb{C}$ be analytic.

Then, the value of $\dfrac{\operatorname{Res}_{z=0}\{f(z)\,g(z)\}}{\operatorname{Res}_{z=0} f(z)}$

is

(A) $g(0)$ (B) $g'(0)$ (C) $\lim_{z\to 0} z\,f(z)$ (D) $\lim_{z\to 0} z\,f(z)\,g(z)$

Q.5 Let $I=\oint_C (2x^2+y^2)\,dx+e^y\,dy$, where C is the boundary (oriented anticlockwise) of the region in the first quadrant bounded by $y=0,\ x^2+y^2=1$ and $x=0$. The value of I is

(A) -1 (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$ (D) 1

Q.6 The series $\sum_{m=1}^{\infty} x^{\ln m}$, $x>0$, is convergent on the interval

(A) $(0,\ 1/e)$ (B) $(1/e,\ e)$ (C) $(0,\ e)$ (D) $(1,\ e)$

Q.7 While solving the equation $x^2-3x+1=0$ using the Newton-Raphson method with the initial guess of a root as 1, the value of the root after one iteration is

(A) 1.5 (B) 1 (C) 0.5 (D) 0

Q.8 Consider the system of equations

$$\begin{bmatrix} 5 & 2 & 1 \\ -2 & 5 & 2 \\ -1 & 2 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 13 \\ -22 \\ 14 \end{bmatrix}.$$

With the initial guess of the solution $[x_1^{(0)}, x_2^{(0)}, x_3^{(0)}]^T = [1, 1, 1]^T$, the approximate value of the solution $[x_1^{(1)}, x_2^{(1)}, x_3^{(1)}]^T$ after one iteration by the Gauss-Seidel method is

(A) $[2, \; -4.4, \; 1.625]^T$
(B) $[2, \; -4, \; -3]^T$
(C) $[2, \; 4.4, \; 1.625]^T$
(D) $[2, \; -4, \; 3]^T$

Q.9 Let y be the solution of the initial value problem

$$\frac{dy}{dx} = (y^2 + x); \; y(0) = 1.$$

Using Taylor series method of order 2 with the step size $h = 0.1$, the approximate value of $y(0.1)$ is

(A) 1.315
(B) 1.415
(C) 1.115
(D) 1.215

Q.10 The partial differential equation

$$x^2 \frac{\partial^2 z}{\partial x^2} - (y^2 - 1)x \frac{\partial^2 z}{\partial x \partial y} + y(y-1)^2 \frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$$

is hyperbolic in a region in the XY - plane if

(A) $x \neq 0$ and $y = 1$
(B) $x = 0$ and $y \neq 1$
(C) $x \neq 0$ and $y \neq 1$
(D) $x = 0$ and $y = 1$

Q.11 Which of the following functions is a probability density function of a random variable X ?

(A) $f(x) = \begin{cases} x(2-x), & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$

(B) $f(x) = \begin{cases} x(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$

(C) $f(x) = \begin{cases} 2xe^{-x^2}, & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$

(D) $f(x) = \begin{cases} 2xe^{-x^2}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$

Q.12 Let X_1, X_2, X_3 and X_4 be independent standard normal random variables. The distribution of

$$W = \frac{1}{2}\{(X_1 - X_2)^2 + (X_3 - X_4)^2\}$$

is

(A) $N(0,1)$
(B) $N(0,2)$
(C) χ_2^2
(D) χ_4^2

Q.13 For $n \geq 1$, let $\{X_n\}$ be a sequence of independent random variables with

$$P(X_n = n) = P(X_n = -n) = \frac{1}{2n^2}, \qquad P(X_n = 0) = 1 - \frac{1}{n^2}.$$

Then, which of the following statements is **TRUE** for the sequence $\{X_n\}$?

(A) Weak Law of Large Numbers holds but Strong Law of Large Numbers does not hold
(B) Weak Law of Large Numbers does not hold but Strong Law of Large Numbers holds
(C) Both Weak Law of Large Numbers and Strong Law of Large Numbers hold
(D) Both Weak Law of Large Numbers and Strong Law of Large Numbers do not hold

Q.14 The Linear Programming Problem:

Maximize $z = x_1 + x_2$

subject to

$$x_1 + 2x_2 \le 20$$
$$x_1 + x_2 \le 15$$
$$x_2 \le 6$$
$$x_1, x_2 \ge 0$$

(A) has exactly one optimum solution
(B) has more than one optimum solutions
(C) has unbounded solution
(D) has no solution

Q.15 Consider the Primal Linear Programming Problem:

$$P: \begin{cases} \text{Maximize} \quad z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ \text{subject to} \\ \qquad a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1 \\ \qquad a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2 \\ \qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ \qquad a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m \\ x_j \ge 0,\ j = 1, \ldots, n. \end{cases}$$

The Dual of P is

$$D: \begin{cases} \text{Minimize} \quad z' = b_1w_1 + b_2w_2 + \cdots + b_mw_m \\ \text{subject to} \\ \qquad a_{11}w_1 + a_{21}w_2 + \cdots + a_{m1}w_m \ge c_1 \\ \qquad a_{12}w_1 + a_{22}w_2 + \cdots + a_{m2}w_m \ge c_2 \\ \qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ \qquad a_{1n}w_1 + a_{2n}w_2 + \cdots + a_{mn}w_m \ge c_n \\ w_i \ge 0,\ i = 1, \ldots, m. \end{cases}$$

Which of the following statements is **FALSE**?

(A) If P has an optimal solution, then D also has an optimal solution
(B) The dual of the dual problem is a primal problem
(C) If P has an unbounded solution, then D has no feasible solution
(D) If P has no feasible solution, then D has a feasible solution

Q.16 The number of irreducible quadratic polynomials over the field of two elements $\mathbf{F}_2$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.17 The number of elements in the conjugacy class of the 3-cycle (2 3 4) in the symmetric group S_6 is

(A) 20 (B) 40 (C) 120 (D) 216

Q.18 The initial value problem

$$x\frac{dy}{dx} = y + x^2,\ x > 0;\ \ y(0) = 0,$$

has

(A) infinitely many solutions
(B) exactly two solutions
(C) a unique solution
(D) no solution

Q.19 The subspace $P=\{(x,y,z)\in \mathbb{R}^3 : z = x^2 + y^2 + 1\}$ is

(A) compact and connected
(B) compact but not connected
(C) not compact but connected
(D) neither compact nor connected

Q.20 Let $P=(0,1)$; $Q=[0,1)$; $U=(0,1]$; $S=[0,1]$, $T=\mathbb{R}$ and $A=\{P,Q,U,S,T\}$. The equivalence relation 'homeomorphism' induces which one of the following as the partition of A?

(A) $\{P,Q,U,S\}, \{T\}$
(B) $\{P,T\}, \{Q\}, \{U\}, \{S\}$
(C) $\{P,T\}, \{Q,U,S\}$
(D) $\{P,T\}, \{Q,U\}, \{S\}$

Q.21 Let $x=(x_1,x_2,\ldots)\in l^4$, $x\neq 0$. For which one of the following values of p, the series $\sum_{i=1}^{\infty} x_i y_i$ converges for every $y=(y_1,y_2,\ldots)\in l^p$?

(A) 1
(B) 2
(C) 3
(D) 4

Q.22 Let H be a complex Hilbert space and H^* be its dual. The mapping $\phi : H \to H^*$ defined by $\phi(y)=f_y$ where $f_y(x)=\langle x, y\rangle$ is

(A) not linear but onto
(B) both linear and onto
(C) linear but not onto
(D) neither linear nor onto

Q.23 A horizontal lever is in static equilibrium under the application of vertical forces F_1 at a distance l_1 from the fulcrum and F_2 at a distance l_2 from the fulcrum. The equilibrium for the above quantities can be obtained if

(A) $F_1 l_1 = 2F_2 l_2$
(B) $2F_1 l_1 = F_2 l_2$
(C) $F_1 l_1 = F_2 l_2$
(D) $F_1 l_1 < F_2 l_2$

Q.24 Assume F to be a twice continuously differentiable function. Let $J(y)$ be a functional of the form

$$\int_0^1 F(x, y')\,dx, \quad 0\le x\le 1$$

defined on the set of all continuously differentiable functions y on $[0, 1]$ satisfying $y(0)=a$, $y(1)=b$. For some arbitrary constant c, a necessary condition for y to be an extremum of J is

(A) $\frac{\partial F}{\partial x} = c$
(B) $\frac{\partial F}{\partial y'} = c$
(C) $\frac{\partial F}{\partial y} = c$
(D) $\frac{\partial F}{\partial x} = 0$

Q.25 The eigenvalue λ of the following Fredholm integral equation

$$y(x) = \lambda \int_0^1 x^2 t\, y(t)\,dt,$$

is

(A) -2
(B) 2
(C) 4
(D) -4

Q. 26 to Q. 55 carry two marks each.

Q.26 The application of Gram-Schmidt process of orthonormalization to

$$u_1 = (1,1,0),\ u_2 = (1,0,0),\ u_3 = (1,1,1) \quad \text{yields}$$

(A) $\frac{1}{\sqrt{2}}(1,1,0), (1,0,0), (0,0,1)$

(B) $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(1,-1,0), \frac{1}{\sqrt{2}}(1,1,1)$

(C) $(0,1,0), (1,0,0), (0,0,1)$

(D) $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(1,-1,0), (0,0,1)$

Q.27 Let $T:\mathbb{C}^3 \to \mathbb{C}^3$ be defined by $T\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} z_1 - iz_2 \\ iz_1 + z_2 \\ z_1 + z_2 + iz_3 \end{pmatrix}$. Then, the adjoint T^* of T is given by $T^*\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} =$

(A) $\begin{pmatrix} z_1 + iz_2 \\ -iz_1 + z_2 \\ z_1 + z_2 - iz_3 \end{pmatrix}$

(B) $\begin{pmatrix} z_1 - iz_2 + z_3 \\ -iz_1 + z_2 + z_3 \\ iz_3 \end{pmatrix}$

(C) $\begin{pmatrix} z_1 - iz_2 + z_3 \\ iz_1 + z_2 + z_3 \\ -iz_3 \end{pmatrix}$

(D) $\begin{pmatrix} iz_1 + z_2 \\ z_1 - iz_2 \\ z_1 - z_2 - iz_3 \end{pmatrix}$

Q.28 Let $f(z)$ be an entire function such that $|f(z)| \le K|z|$, $\forall z \in \mathbb{C}$, for some $K > 0$. If $f(1) = i$, the value of $f(i)$ is

(A) 1

(B) -1

(C) i

(D) $-i$

Q.29 Let y be the solution of the initial value problem

$$\frac{d^2y}{dx^2} + y = 6\cos 2x, \quad y(0) = 3,\ y'(0) = 1.$$

Let the Laplace transform of y be $F(s)$. Then, the value of $F(1)$ is

(A) $\frac{17}{5}$

(B) $\frac{13}{5}$

(C) $\frac{11}{5}$

(D) $\frac{9}{5}$

Q.30 For $0 \le x \le 1$, let

$$f_n(x) = \begin{cases} \frac{n}{1+n}, & \text{if } x \text{ is irrational} \\ 0, & \text{if } x \text{ is rational} \end{cases}$$

and $f(x) = \lim_{n\to\infty} f_n(x)$. Then, on the interval [0, 1]

(A) f is measurable and Riemann integrable

(B) f is measurable and Lebesgue integrable

(C) f is not measurable

(D) f is not Lebesgue integrable

Q.31 If x, y and z are positive real numbers, then the minimum value of

$$x^2+8y^2+27z^2 \quad \text{where} \quad \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$$

is

(A) 108 (B) 216 (C) 405 (D) 1048

Q.32 Let $T:\mathbb{R}^4 \to \mathbb{R}^4$ be defined by

$$T(x,y,z,w)=(x+y+5w, x+2y+w, -z+2w, 5x+y+2z).$$

The dimension of the eigenspace of T is

(A) 1 (B) 2 (C) 3 (D) 4

Q.33 Let y be a polynomial solution of the differential equation

$$(1-x^2)y''-2xy'+6y=0.$$

If $y(1)=2$, then the value of the integral $\int_{-1}^{1} y^2\,dx$ is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{4}{5}$ (D) $\frac{8}{5}$

Q.34 The value of the integral

$$I=\int_{-1}^{1}\exp(x^2)\,dx$$

using a rectangular rule is approximated as 2. Then, the approximation error $|I-2|$ lies in the interval

(A) $(2e, 3e]$ (B) $(2/3,\ 2e]$ (C) $(e/8,\ 2/3]$ (D) $(0,\ e/8]$

Q.35 The integral surface for the Cauchy problem

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=1,$$

which passes through the circle $z=0,\ x^2+y^2=1$ is

(A) $x^2+y^2+2z^2+2zx-2yz-1=0$
(B) $x^2+y^2+2z^2+2zx+2yz-1=0$
(C) $x^2+y^2+2z^2-2zx-2yz-1=0$
(D) $x^2+y^2+2z^2+2zx+2yz+1=0$

Q.36 The vertical displacement $u(x,t)$ of an infinitely long elastic string is governed by the initial value problem

$$\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}, \quad -\infty<x<\infty,\ t>0,$$

$$u(x,0)=-x \quad \text{and} \quad \frac{\partial u}{\partial t}(x,0)=0.$$

The value of $u(x,t)$ at $x=2$ and $t=2$ is equal to

(A) 2 (B) 4 (C) −2 (D) −4

Q.37 We have to assign four jobs I, II, III, IV to four workers A, B, C and D. The time taken by different workers (in hours) in completing different jobs is given below:

		I	II	III	IV
	A	5	3	2	8
Workers	B	7	9	2	6
	C	6	4	5	7
	D	5	7	7	8

The optimal assignment is as follows:

Job III to worker A; Job IV to worker B; Job II to worker C and Job I to worker D and hence the time taken by different workers in completing different jobs is now changed as:

		I	II	III	IV
	A	5	3	2	5
Workers	B	7	9	2	3
	C	4	2	3	2
	D	5	7	7	5

Then the minimum time (in hours) taken by the workers to complete all the jobs is

(A) 10 (B) 12 (C) 15 (D) 17

Q.38 The following table shows the information on the availability of supply to each warehouse, the requirement of each market and unit transportation cost (in rupees) from each warehouse to each market.

		Market				
		M_1	M_2	M_3	M_4	Supply
	W_1	6	3	5	4	22
Warehouse	W_2	5	9	2	7	15
	W_3	5	7	8	6	8
Requirement		7	12	17	9	

The present transportation schedule is as follows:

W_1 to M_2: 12 units; W_1 to M_3: 1 unit; W_1 to M_4: 9 units; W_2 to M_3: 15 units; W_3 to M_1: 7 units and W_3 to M_3: 1 unit. Then the minimum total transportation cost (in rupees) is

(A) 150 (B) 149 (C) 148 (D) 147

Q.39 If $\mathbb{Z}[i]$ is the ring of Gaussian integers, the quotient $\mathbb{Z}[i]/(3-i)$ is isomorphic to

(A) $\mathbb{Z}$ (B) $\mathbb{Z}/3\mathbb{Z}$ (C) $\mathbb{Z}/4\mathbb{Z}$ (D) $\mathbb{Z}/10\mathbb{Z}$

Q.40 For the rings $L = \dfrac{\mathbb{R}[x]}{\langle x^2 - x + 1\rangle}$; $M = \dfrac{\mathbb{R}[x]}{\langle x^2 + x + 1\rangle}$; $N = \dfrac{\mathbb{R}[x]}{\langle x^2 + 2x + 1\rangle}$;

which one of the following is **TRUE**?

(A) L is isomorphic to M; L is not isomorphic to N; M is not isomorphic to N
(B) M is isomorphic to N; M is not isomorphic to L; N is not isomorphic to L
(C) L is isomorphic to M; M is isomorphic to N
(D) L is not isomorphic to M; L is not isomorphic to N; M is not isomorphic to N

Q.41 The time to failure (in hours) of a component is a continuous random variable T with the probability density function

$$f(t)=\begin{cases}\frac{1}{10}e^{-\frac{t}{10}}, & t>0,\\ 0, & t\le 0.\end{cases}$$

Ten of these components are installed in a system and they work independently. Then, the probability that **NONE** of these fail before ten hours, is

(A) e^{-10} (B) $1-e^{-10}$ (C) $10e^{-10}$ (D) $1-10e^{-10}$

Q.42 Let X be the real normed linear space of all real sequences with finitely many non-zero terms, with supremum norm and $T: X \to X$ be a one to one and onto linear operator defined by

$$T(x_1, x_2, x_3 \ldots)=\left(x_1, \frac{x_2}{2^2}, \frac{x_3}{3^2}, \ldots\right).$$

Then, which of the following is **TRUE**?

(A) T is bounded but T^{-1} is not bounded
(B) T is not bounded but T^{-1} is bounded
(C) Both T and T^{-1} are bounded
(D) Neither T nor T^{-1} is bounded

Q.43 Let $e_i=(0,\ldots,0,1,0,\ldots)$ (i.e., e_i is the vector with 1 at the i^{th} place and 0 elsewhere) for $i=1,2,\ldots$.

Consider the statements:

P: $\{f(e_i)\}$ converges for every continuous linear functional on l^2.

Q: $\{e_i\}$ converges in l^2.

Then, which of the following holds?

(A) Both P and Q are TRUE
(B) P is TRUE but Q is not TRUE
(C) P is not TRUE but Q is TRUE
(D) Neither P nor Q is TRUE

Q.44 For which subspace $X \subseteq \mathbb{R}$ with the usual topology and with $\{0,1\} \subseteq X$, will a continuous function $f: X \to \{0,1\}$ satisfying $f(0)=0$ and $f(1)=1$ exist?

(A) $X=[0,1]$ (B) $X=[-1,1]$ (C) $X=\mathbb{R}$ (D) $[0,1] \not\subset X$

Q.45 Suppose X is a finite set with more than five elements. Which of the following is **TRUE**?

(A) There is a topology on X which is T_3
(B) There is a topology on X which is T_2 but not T_3
(C) There is a topology on X which is T_1 but not T_2
(D) There is no topology on X which is T_1

Q.46 A massless wire is bent in the form of a parabola $z = r^2$ and a bead slides on it smoothly. The wire is rotated about z-axis with a constant angular acceleration α. Assume that m is the mass of the bead, ω is the initial angular velocity and g is the acceleration due to gravity. Then, the Lagrangian at any time t is

(A) $\frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2(1+4r^2)+r^2(\omega+\alpha t)^2+2gr^2\right]$

(B) $\frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2(1+4r^2)-r^2(\omega+\alpha t)^2+2gr^2\right]$

(C) $\frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2(1+4r^2)-r^2(\omega+\alpha t)^2-2gr^2\right]$

(D) $\frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2(1+4r^2)+r^2(\omega+\alpha t)^2-2gr^2\right]$

Q.47 On the interval [0, 1], let y be a twice continuously differentiable function which is an extremal of the functional

$$J(y) = \int_0^1 \frac{\sqrt{1+2y'^2}}{x}\,dx$$

with $y(0) = 1,\ y(1) = 2$. Then, for some arbitrary constant c, y satisfies

(A) $y'^2(2-c^2x^2) = c^2x^2$

(B) $y'^2(2+c^2x^2) = c^2x^2$

(C) $y'^2(1-c^2x^2) = c^2x^2$

(D) $y'^2(1+c^2x^2) = c^2x^2$

Common Data Questions

Common Data for Questions 48 and 49:

Let X and Y be two continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} 2, & 0<x+y<1, \quad x>0, \quad y>0, \\ 0, & \text{elsewhere.} \end{cases}$$

Q.48 $P\left(X+Y<\frac{1}{2}\right)$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

Q.49 $E\left(X \mid Y=\frac{1}{2}\right)$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

Common Data for Questions 50 and 51:

Let $f(z)=\frac{z}{8-z^3}$, $z=x+iy$.

Q.50 $\operatorname*{Res}_{z=2} f(z)$ is

(A) $-\frac{1}{8}$ (B) $\frac{1}{8}$ (C) $-\frac{1}{6}$ (D) $\frac{1}{6}$

Q.51 The Cauchy principal value of $\int_{-\infty}^{\infty} f(x)\,dx$ is

(A) $-\frac{\pi}{6}\sqrt{3}$ (B) $-\frac{\pi}{8}\sqrt{3}$ (C) $\pi\sqrt{3}$ (D) $-\pi\sqrt{3}$

Linked Answer Questions

Statement for Linked Answer Questions 52 and 53:

Let $f_n(x) = \dfrac{x}{\{(n-1)x+1\}\{nx+1\}}$ and $s_n(x) = \sum_{j=1}^{n} f_j(x)$ for $x \in [0,1]$.

Q.52 The sequence $\{s_n\}$

(A) converges uniformly on $[0,1]$

(B) converges pointwise on $[0,1]$ but not uniformly

(C) converges pointwise for $x = 0$ but not for $x \in (0, 1]$

(D) does not converge for $x \in [0, 1]$

Q.53 $\lim_{n\to\infty} \int_0^1 s_n(x)\,dx = 1$

(A) by dominated convergence theorem

(B) by Fatou's lemma

(C) by the fact that $\{s_n\}$ converges uniformly on $[0, 1]$

(D) by the fact that $\{s_n\}$ converges pointwise on $[0, 1]$

Statement for Linked Answer Questions 54 and 55:

The matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ can be decomposed into the product of a lower triangular matrix L and an upper triangular matrix U as $A = LU$ where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$$

Let $x, z \in \mathbb{R}^3$ and $b = [1, \ 1, \ 1]^T$.

Q.54 The solution $z = [z_1, \ z_2, \ z_3]^T$ of the system $L\,z = b$ is

(A) $[-1, \ -1, \ -2]^T$ (B) $[1, \ -1, \ 2]^T$ (C) $[1, \ -1, \ -2]^T$ (D) $[-1, \ 1, \ 2]^T$

Q.55 The solution $x = [x_1, \ x_2, \ x_3]^T$ of the system $U\,x = z$ is

(A) $[2, \ 1, \ -2]^T$ (B) $[2, \ 1, \ 2]^T$ (C) $[-2, \ -1, \ -2]^T$ (D) $[-2, \ 1, \ -2]^T$

General Aptitude (GA) Questions

Q. 56 – Q. 60 carry one mark each.

Q.56 Choose the most appropriate word from the options given below to complete the following sentence:
It was her view that the country's problems had been —————— by foreign technocrats, so that to invite them to come back would be counter-productive.

(A) identified
(B) ascertained
(C) exacerbated
(D) analysed

Q.57 There are two candidates P and Q in an election. During the campaign, 40% of the voters promised to vote for P, and rest for Q. However, on the day of election 15% of the voters went back on their promise to vote for P and instead voted for Q. 25% of the voters went back on their promise to vote for Q and instead voted for P. Suppose, P lost by 2 votes, then what was the total number of voters?

(A) 100 (B) 110 (C) 90 (D) 95

Q.58 The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair:
Gladiator : Arena

(A) dancer : stage
(B) commuter : train
(C) teacher : classroom
(D) lawyer : courtroom

Q.59 Choose the most appropriate word from the options given below to complete the following sentence:
Under ethical guidelines recently adopted by the Indian Medical Association, human genes are to be manipulated only to correct diseases for which —————— treatments are unsatisfactory.

(A) similar
(B) most
(C) uncommon
(D) available

Q.60 Choose the word from the options given below that is most nearly opposite in meaning to the given word:
Frequency

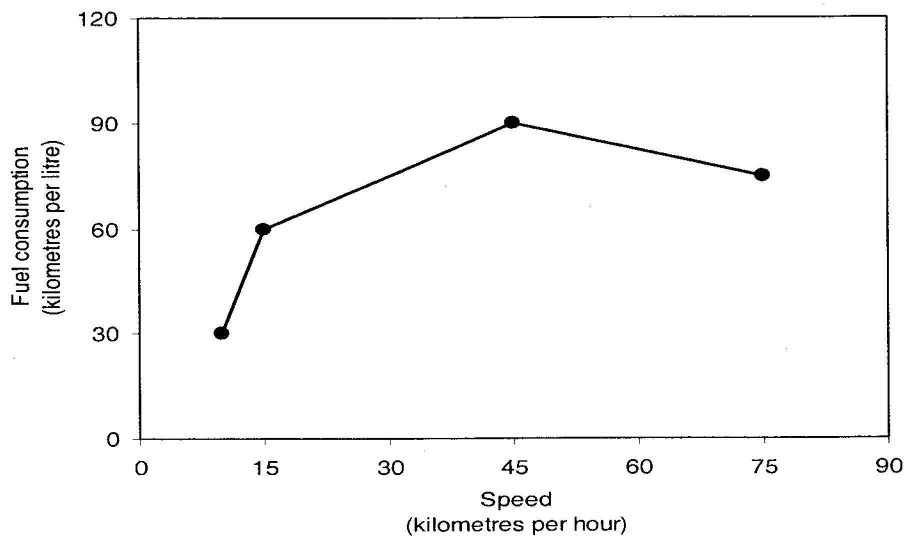
(A) periodicity
(B) rarity
(C) gradualness
(D) persistency

Q. 61 to Q. 65 carry two marks each.

Q.61 Three friends, R, S and T shared toffee from a bowl. R took $1/3^{rd}$ of the toffees, but returned four to the bowl. S took $1/4^{th}$ of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?

(A) 38 (B) 31 (C) 48 (D) 41

Q.62 The fuel consumed by a motorcycle during a journey while traveling at various speeds is indicated in the graph below.



The distances covered during four laps of the journey are listed in the table below

Lap	Distance (kilometres)	Average speed (kilometres per hour)
P	15	15
Q	75	45
R	40	75
S	10	10

From the given data, we can conclude that the fuel consumed per kilometre was least during the lap

(A) P (B) Q (C) R (D) S

Q.63 **The horse has played a little known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way.**

It can be inferred from the passage, that horses were

(A) given immunity to diseases
(B) generally quite immune to diseases
(C) given medicines to fight toxins
(D) given diphtheria and tetanus serums

Q.64 The sum of n terms of the series 4+44+444+.... is

(A) $(4/81)\,[10^{n+1} - 9n - 1]$
(B) $(4/81)\,[10^{n-1} - 9n - 1]$
(C) $(4/81)\,[10^{n+1} - 9n - 10]$
(D) $(4/81)\,[10^{n} - 9n - 10]$

Q.65 Given that $f(y) = |\,y\,| / y$, and q is any non-zero real number, the value of $|\,f(q) - f(-q)\,|$ is

(A) 0 (B) –1 (C) 1 (D) 2

END OF THE QUESTION PAPER