# Marking Scheme

## Mathematics (Term-I)

## Class-XII (Code-041)

| Q.N. | Correct Option | Hints / Solutions |
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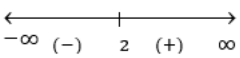
| 1 | d | $sin\left(\frac{\pi}{3}-\left(\frac{-\pi}{6}\right)\right)=\sin\left(\frac{\pi}{2}\right)=1$ |
| 2 | b | $\lim_{x\to o}\left(\frac{1-\cos kx}{x\sin x}\right)=\frac{1}{2}$<br>$\Rightarrow \lim_{x\to o}\left(\frac{2sin^2\frac{kx}{2}}{x\sin x}\right)=\frac{1}{2}$<br>$\Rightarrow \lim_{x\to o} 2\left(\frac{k}{2}\right)^2\left(\frac{sin\frac{kx}{2}}{\frac{kx}{2}}\right)^2\left(\frac{x}{sinx}\right)=\frac{1}{2}$<br>$\Rightarrow k^2$=1$\Rightarrow$ k $=\pm1$ $but\ k<0$ $\Rightarrow$ k= -1 |
| 3 | d | $A^2=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ |
| 4 | c | As $A$ is singular matrix<br>$\Rightarrow \lvert A\rvert=0$<br>$\Rightarrow 2k^2-32=0 \Rightarrow$ k $=\pm4$ |
| 5 | b | $f(x)=x^2-4x+6$<br>$f'(x)=2x-4$<br>$let\ f'(x)=0\Rightarrow$ x $=2$<br><br>as $f'(x)>0\ \forall\ x\in(2,\infty)$<br>$\Rightarrow f(x)$ is Strictly increasing in $(2,\infty)$ |
| 6 | d | as $\lvert adj\ A\rvert=\lvert A\rvert^{n-1}$, where $n$ is order of matrix $A$<br>$=(-4)^2=16$ |
| 7 | b | $(1,2)$ |
| 8 | a | $\left.\begin{matrix}2a+b=4\\a-2b=-3\\5c-d=11\\4c+3d=24\end{matrix}\right\}\Rightarrow \begin{matrix}a=1\\b=2\\c=3\\d=4\end{matrix}$<br>$\therefore a+b-c+2d=8$ |
| 9 | a | $f(x)=x+\frac{1}{x},x>0\Rightarrow f'(x)=1-\frac{1}{x^2}=\frac{x^2-1}{x^2},x>0$<br>As normal to $f(x)$ is ⊥ to given line<br>$\Rightarrow\left(\frac{x^2}{1-x^2}\right)\times\frac{3}{4}=-1\ \ (m_1.m_2=\ -1)$<br>$\Rightarrow\ x^2=4\Rightarrow$ x $=\pm2$<br>But $x>0,\therefore x=2$<br>Therefore point=$\left(2,\ \frac{5}{2}\right)$ |
| 10 | d | $\sin(\tan^{-1}x)=\sin\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)=\frac{x}{\sqrt{1+x^2}}$ |
| 11 | a | $\{1,5,9\}$ |
| 12 | c | $e^x+e^y=e^{x+y}$<br>$\Rightarrow e^{-y}+e^{-x}=1$<br>Differentiating *w.r.t.* $x$: |

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| | | $\Rightarrow\ -e^{-y}\frac{dy}{dx} - e^{-x} = 0 \Rightarrow \frac{dy}{dx} = -e^{y-x}$ |
| **13** | **b** | $3 \times 5$ |
| **14** | **a** | $y = 5\cos x - 3\sin x \Rightarrow \frac{dy}{dx} = -5\sin x - 3\cos x$<br>$\Rightarrow \frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -y$ |
| **15** | **c** | adj $A = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix} \Rightarrow (adjA)' = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ |
| **16** | **c** | $\frac{x^2}{9} + \frac{y^2}{16} = 1 \Rightarrow \frac{2x}{9} + \frac{2y}{16}\frac{dy}{dx} = 0$<br>$\Rightarrow slope\ of\ normal = \frac{-dx}{dy} = \frac{9y}{16x}$<br>As curve's tangent is parallel to *y*-axes<br>$\Rightarrow \frac{9y}{16x} = 0 \Rightarrow y = 0$ and $x = \pm 3$<br>$\therefore\ points = (\pm 3, 0)$ |
| **17** | **b** | $\|A\| = -7$<br>$\therefore \sum_{i=1}^{3} a_{i2}A_{i2} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} = \|A\| = -7$ |
| **18** | **d** | $y = \log(cos\ e^x)$<br>Differentiating wrt $x$:<br>$\frac{dy}{dx} = \frac{1}{\cos(e^x)}.(-\sin e^x).\ e^x$ (chain rule)<br>$\Rightarrow \frac{dy}{dx} = -e^x \tan e^x$ |
| **19** | **d** | Z is maximum 180 at points *C (15,15)* and *D(0, 20).*<br>⇒ Z is maximum at every point on the line segment *CD* |
| **20** | **c** | $f(x) = 2cosx + x\ ,\ x \in [0, \frac{\pi}{2}]$<br>$f'(x) = -2sinx + 1$<br>Let$f'(x) = 0 \Rightarrow x = \frac{\pi}{6} \in [0, \frac{\pi}{2}]$<br>$f(0) = 2$<br>$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \sqrt{3}$<br>$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$ ⇒ least value of $f(x)$ is $\frac{\pi}{2}$ at $x = \frac{\pi}{2}$ |
| | | **Section-B** |
| **21** | **d** | $let\ f(x_1) = f(x_2)\ \forall x_1 x_2 \in R$<br>$\Rightarrow x_1^3 = x_2^3$<br>$\Rightarrow x_1 = x_2$<br>$\Rightarrow$ $f$ is one - one<br>$let\ f(x) = x^3 = y\ \ \forall\ y \in R$<br>$\Rightarrow x = y^{\frac{1}{3}}$<br>every image $y \in R$ has a unique pre image in $R$<br>$\Rightarrow f$ is onto<br>$\therefore f$ is one-one and onto |
| **22** | **a** | $x = a\ sec\theta \Rightarrow \frac{dx}{d\theta} = a\tan\theta\sec\theta$<br>$y = b\tan\theta \Rightarrow \frac{dy}{d\theta} = b\ sec^2\theta$<br>$\therefore \frac{dy}{dx} = \frac{b}{a}cosec\theta$<br>$\Rightarrow \frac{d^2y}{dx^2} = \frac{-b}{a}\ cosec\theta.cot\theta.\frac{d\theta}{dx} = \frac{-b}{a^2}cot^3\theta$<br>$\therefore \left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{6}} = \frac{-3\sqrt{3}b}{a^2}$ |
| **23** | **c** | $Z$ is minimum -24 at (0, 8) |
| **24** | **a** | $let\ u = sin^{-1}(2x\sqrt{1-x^2})$ |

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| | | and $v = sin^{-1}x,\ \frac{1}{\sqrt{2}} < x < 1\ \Rightarrow \sin v = x$……(1)<br>Using (1), we get :<br>$=sin^{-1}(2\sin v\ \cos v)$<br>$\Rightarrow \mathrm{u} = 2\mathrm{v}$<br>Differentiating with respect to v, we get: $\frac{du}{dv} = 2$ |
| **25** | **d** | $AB = 6I\ \Rightarrow \mathrm{B}^{-1} = \frac{1}{6}A$ |
| **26** | **b** | $f'(x) = 6(x^2 - x - 6)\ = 6(x-3)(x+2)$<br>$-\infty$ $(+)$ $-2$ $(-)$ $3$ $(+)$ $\infty$<br>As $f'(x) < 0\ \forall\ x \in (-2,3)$<br>$\Rightarrow$ $f(x)$ is strictly decreasing in $(-2,3)$ |
| **27** | **a** | $tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-cosx}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)$<br>$= tan^{-1}\left(\frac{-\sqrt{2}\cos\frac{x}{2}+\sqrt{2}\sin\frac{x}{2}}{-\sqrt{2}\cos -\sqrt{2}\sin\frac{x}{2}}\right)$ , $\pi < x < \frac{3\pi}{2}$<br>$= tan^{-1}\left(\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)$<br>$= \frac{\pi}{4} - \frac{x}{2}$ |
| **28** | **c** | $A^2 = 2A$<br>$\Rightarrow \|A^2\| = \|2A\|$<br>$\Rightarrow \|A\|^2 = 2^3\|A\|$ as $\|kA\| = k^n\|A\|\ for\ a\ matrix\ of\ order\ n$<br>$\Rightarrow$ either $\|A\| = 0\ or\ \|A\| = 8$<br>But A is non-singular matrix<br>$\therefore \|A\| = 8^2 = 64$ |
| **29** | **b** | $f'(x) = 1 - sin\ x \Rightarrow f'(x) > 0\ \ \forall x \in R$<br>$\Rightarrow$ no value of b exists |
| **30** | **c** | $a = b - 2\ and\ b > 6$<br>$\Rightarrow (6,8) \in R$ |
| **31** | **a** | $f(x) = \begin{cases} \frac{x}{-x} = -1\ , & x < 0 \\ -1\ , & x \geq 0 \end{cases}$<br>$\Rightarrow f(x) = -1\ \forall\ x \in R$<br>$\Rightarrow f(x) is\ continous\ \forall\ x \in R\ as\ it\ is\ a\ constant\ function$ |
| **32** | **b** | $kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$<br>$\Rightarrow k =\ -6, a =\ -4\ and\ b =\ -9$ |
| **33** | **d** | Corner points of feasible region $Z = 30x + 50y$<br>(5,0) 150<br>(9,0) 270<br>(0,3) 150<br>(0,6) 300<br>Minimum value of $Z$occurs at two points |
| **34** | **c** | $f'(x) = \frac{-2x^2-10x+100}{\sqrt{100-x^2}}$<br>$f'(x) = 0\ \Rightarrow \mathrm{x} = -10$ or 5 , But $x > 0 \Rightarrow \mathrm{x} = 5$<br>$f''(x) = \frac{2x^3-300x-1000}{(100-x)^{\frac{3}{2}}} \Rightarrow f''(5) = \frac{-30}{\sqrt{75}} < 0$<br>$\Rightarrow$ Maximum area of trapezium is $75\sqrt{3}\ cm^2$ when x = 5 |
| **35** | **d** | $(I-A)^3 - 7A =\ I + A + 3A + 3A - 7A\ = I$ |
| **36** | **c** | $\frac{-\pi}{2} < y < \frac{\pi}{2}$ |

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| 37 | b | As every per-image $x \in$ A has a unique image y ∈ B<br>$\Rightarrow f$ is injective function |
| 38 | b | $\lvert A\rvert = 7,\ adjA = \begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}$<br>$\therefore 14A^{-1} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$ |
| 39 | b | $y = x^3 - 11x + 5 \ \Rightarrow \frac{dy}{dx} = 3x^2 - 11$<br>Slope of line $y = x - 11\ is\ 1 \ \Rightarrow 3x^2 - 11 = 1 \Rightarrow x = \pm 2$<br>∴ point is (2, -9) as (-2, 19) does not satisfy given line |
| 40 | c | $A^2 = 3I$<br>$\Rightarrow \begin{bmatrix} \alpha^2 + \beta r & 0 \\ 0 & \beta r + \alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad \Rightarrow 3 - \alpha^2 - \beta r = 0$ |
| | | **Section C** |
| 41 | a | As $Z$ is maximum at (30, 30) and (0, 40)<br>$\Rightarrow 30a + 30b = 40b \Rightarrow b - 3a = 0$ |
| 42 | b | $y = mx + 1 \ldots..(1)$ and $y^2 = 4x \ldots\ldots(2)$<br>Substituting (1) in (2) : $(mx+1)^2 = 4x$<br>$\Rightarrow m^2x^2 + (2m-4)x + 1 = 0 \ldots\ldots..(3)$<br>As line is tangent to the curve<br>⇒ line touches the curve at only one point<br>$\Rightarrow (2m-4)^2 - 4m^2 = 0 \Rightarrow m = 1$ |
| 43 | c | Let $f(x) = [x(x-1)+1]^{\frac{1}{3}}, \quad 0 \ \leq x \leq 1$<br>$f'(x) = \frac{2x-1}{3(x^2-x+1)^{\frac{2}{3}}}$ let $f'(x) = 0 \quad \Rightarrow \ x = \frac{1}{2} \in [0,1]$<br>$f(o) = 1, f\left(\frac{1}{2}\right) = \left(\frac{3}{4}\right)^{\frac{1}{3}}$ and $f(1) = 1$<br>∴ Maximum value of $f(x)$ is 1 |
| 44 | b | Feasible region is bounded in the first quadrant |
| 45 | d | $\lvert A\rvert = 2 + 2sin^2\theta$<br>As $-1 \leq \sin\theta \leq 1, \forall\ 0 \leq \theta \leq 2\pi$<br>$\Rightarrow 2 \leq 2 + 2sin^2\theta \leq 4 \Rightarrow \lvert A\rvert \in [2,4]$ |
| 46 | d | Fuel cost= $k(speed)^2$<br>$\Rightarrow 48 = k.16^2 \Rightarrow k = \frac{3}{16}$ |
| 47 | b | Total cost of running train (let C) =$\frac{3}{16}v^2t + 1200t$<br>Distance covered = 500km $\Rightarrow$ time $= \frac{500}{v} hrs$<br>Total cost of running train 500 km= $\frac{3}{16}v^2\left(\frac{500}{v}\right) + 1200\left(\frac{500}{v}\right)$<br>$\Rightarrow \ C = \frac{375}{4}v + \frac{600000}{v}$ |
| 48 | c | $\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2}$<br>Let $\frac{dC}{dv} = 0 \quad \Rightarrow v = 80\ km/h$ |
| 49 | c | Fuel cost for running 500 km $\frac{375}{4}v = \frac{375}{4} \times 80 = Rs.7500/-$ |
| 50 | d | Total cost for running 500 km = $\frac{375}{4}v + \frac{600000}{v}$<br>$= \frac{375\times 80}{4} + \frac{600000}{80} = Rs.15000/-$ |