MATHEMATICS

Textbook For Class X

Jammu and Kashmir
Board of School Education

11
CONSTRUCTIONS

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11.1 Introduction

In Class IX, you have done certain constructions using a straight edge (ruler) and a compass, e.g., bisecting an angle, drawing the perpendicular bisector of a line segment, some constructions of triangles etc. and also gave their justifications. In this chapter, we shall study some more constructions by using the knowledge of the earlier constructions. You would also be expected to give the mathematical reasoning behind why such constructions work.

11.2 Division of a Line Segment

Suppose a line segment is given and you have to divide it in a given ratio, say 3 : 2. You may do it by measuring the length and then marking a point on it that divides it in the given ratio. But suppose you do not have any way of measuring it precisely, how would you find the point? We give below two ways for finding such a point.

Construction 11.1 To divide a line segment in a given ratio.

Given a line segment AB, we want to divide it in the ratio m : n, where both m and n are positive integers. To help you to understand it, we shall take m = 3 and n = 2.

Steps of Construction :

1. Draw any ray AX, making an acute angle with AB.
2. Locate 5 (= m + n) points A_1, A_2, A_3, A_4 and A_5 on AX so that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.
3. Join BA_5.
4. Through the point A_3 (m = 3), draw a line parallel to A_5B (by making an angle equal to $\angle AA_5B$) at A_3 intersecting AB at the point C (see Fig. 11.1). Then, AC : CB = 3 : 2.

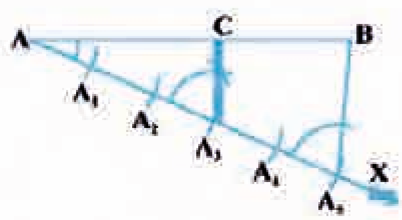


Fig. 11.1

Let us see how this method gives us the required division.

Since A_3C is parallel to A_5B, therefore,

$$\frac{AA_3}{A_3A_5} = \frac{AC}{CB} \qquad \text{(By the Basic Proportionality Theorem)}$$

By construction, $\frac{AA_3}{A_3A_5} = \frac{3}{2}\cdot$ Therefore, $\frac{AC}{CB} = \frac{3}{2}\cdot$

This shows that C divides AB in the ratio 3 : 2.

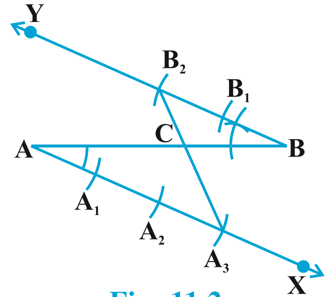


Fig. 11.2

Alternative Method

Steps of Construction :

1. Draw any ray AX making an acute angle with AB.
2. Draw a ray BY parallel to AX by making $\angle$ ABY equal to $\angle$ BAX.
3. Locate the points A_1, A_2, A_3 ($m = 3$) on AX and B_1, B_2 ($n = 2$) on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2$.
4. Join A_3B_2. Let it intersect AB at a point C (see Fig. 11.2).

Then AC : CB = 3 : 2.

Why does this method work? Let us see.

Here $\Delta\ AA_3C$ is similar to $\Delta\ BB_2C$. (Why ?)

Then $$\frac{AA_3}{BB_2} = \frac{AC}{BC}.$$

Since by construction, $\frac{AA_3}{BB_2} = \frac{3}{2}$, therefore, $\frac{AC}{BC} = \frac{3}{2}\cdot$

In fact, the methods given above work for dividing the line segment in any ratio.

We now use the idea of the construction above for constructing a triangle similar to a given triangle whose sides are in a given ratio with the corresponding sides of the given triangle.

Construction 11.2 : *To construct a triangle similar to a given triangle as per given scale factor.*

This construction involves two different situations. In one, the triangle to be constructed is smaller and in the other it is larger than the given triangle. Here, the **scale factor** means the ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle (see also Chapter 6). Let us take the following examples for understanding the constructions involved. **The same methods would apply for the general case also.**

Example 1 : Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{3}{4}$ of the corresponding sides of the triangle ABC (i.e., of scale factor $\frac{3}{4}$).

Solution : Given a triangle ABC, we are required to construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

Steps of Construction :

1. Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
2. Locate 4 (the greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
3. Join B_4C and draw a line through B_3 (the 3rd point, 3 being smaller of 3 and 4 in $\frac{3}{4}$) parallel to B_4C to intersect BC at C′.
4. Draw a line through C′ parallel to the line CA to intersect BA at A′ (see Fig. 11.3).

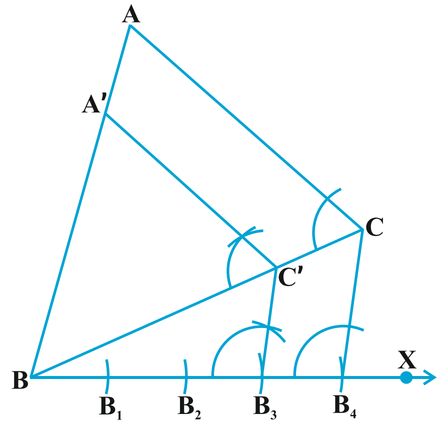


Fig. 11.3

Then, Δ A′BC′ is the required triangle.

Let us now see how this construction gives the required triangle.

By Construction 11.1, $\frac{BC'}{C'C} = \frac{3}{1}$.

Therefore, $\frac{BC}{BC'} = \frac{BC' + C'C}{BC'} = 1 + \frac{C'C}{BC'} = 1 + \frac{1}{3} = \frac{4}{3}$, i.e., $\frac{BC'}{BC} = \frac{3}{4}$.

Also C′A′ is parallel to CA. Therefore, Δ A′BC′ ~ Δ ABC. (Why ?)

So, $\frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{4}$.

Example 2 : Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{5}{3}$ of the corresponding sides of the triangle ABC (i.e., of scale factor $\frac{5}{3}$).

Solution : Given a triangle ABC, we are required to construct a triangle whose sides are $\frac{5}{3}$ of the corresponding sides of Δ ABC.

Steps of Construction :

1. Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
2. Locate 5 points (the greater of 5 and 3 in $\frac{5}{3}$) B_1, B_2, B_3, B_4 and B_5 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
3. Join B_3(the 3rd point, 3 being smaller of 3 and 5 in $\frac{5}{3}$) to C and draw a line through B_5 parallel to B_3C, intersecting the extended line segment BC at C′.
4. Draw a line through C′ parallel to CA intersecting the extended line segment BA at A′ (see Fig. 11.4).

   Then A′BC′ is the required triangle.

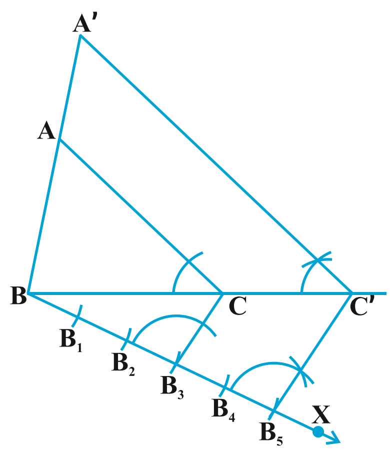


Fig. 11.4

For justification of the construction, note that Δ ABC ~ Δ A′BC′. (Why ?)

Therefore, $\frac{AB}{A'B} = \frac{AC}{A'C'} = \frac{BC}{BC'}$.

But, $\frac{BC}{BC'} = \frac{BB_3}{BB_5} = \frac{3}{5}$,

So, $\frac{BC'}{BC} = \frac{5}{3}$, and, therefore, $\frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{5}{3}$.

Remark : In Examples 1 and 2, you could take a ray making an acute angle with AB or AC and proceed similarly.

EXERCISE 11.1

In each of the following, give the justification of the construction also:

1. Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts.

2. Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

3. Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

4. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

5. Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle$ ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

6. Draw a triangle ABC with side BC = 7 cm, $\angle$ B = 45°, $\angle$ A = 105°. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of Δ ABC.

7. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

11.3 Construction of Tangents to a Circle

You have already studied in the previous chapter that if a point lies inside a circle, there cannot be a tangent to the circle through this point. However, if a point lies on the circle, then there is only one tangent to the circle at this point and it is perpendicular to the radius through this point. Therefore, if you want to draw a tangent at a point of a circle, simply draw the radius through this point and draw a line perpendicular to this radius through this point and this will be the required tangent at the point.

You have also seen that if the point lies outside the circle, there will be two tangents to the circle from this point.

We shall now see how to draw these tangents.

Construction 11.3 : *To construct the tangents to a circle from a point outside it.*

We are given a circle with centre O and a point P outside it. We have to construct the two tangents from P to the circle.

Steps of Construction:

1. Join PO and bisect it. Let M be the mid-point of PO.
2. Taking M as centre and MO as radius, draw a circle. Let it intersect the given circle at the points Q and R.
3. Join PQ and PR.

   Then PQ and PR are the required two tangents (see Fig. 11.5).

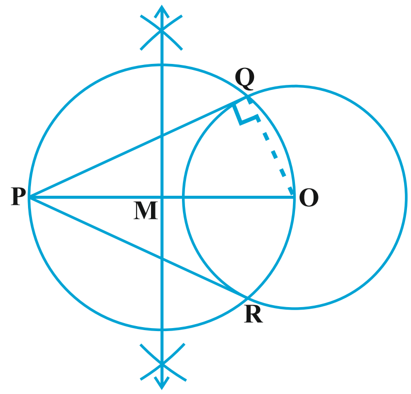


Fig. 11.5

Now let us see how this construction works. Join OQ. Then ∠ PQO is an angle in the semicircle and, therefore,

$$\angle PQO = 90^\circ$$

Can we say that PQ ⊥ OQ?

Since, OQ is a radius of the given circle, PQ has to be a tangent to the circle. Similarly, PR is also a tangent to the circle.

Note : If centre of the circle is not given, you may locate its centre first by taking any two non-parallel chords and then finding the point of intersection of their perpendicular bisectors. Then you could proceed as above.

EXERCISE 11.2

In each of the following, give also the justification of the construction:

1. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.
2. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.
3. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.
4. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.
5. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

6. Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and ∠ B = 90°. BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

7. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

11.4 Summary

In this chapter, you have learnt how to do the following constructions:

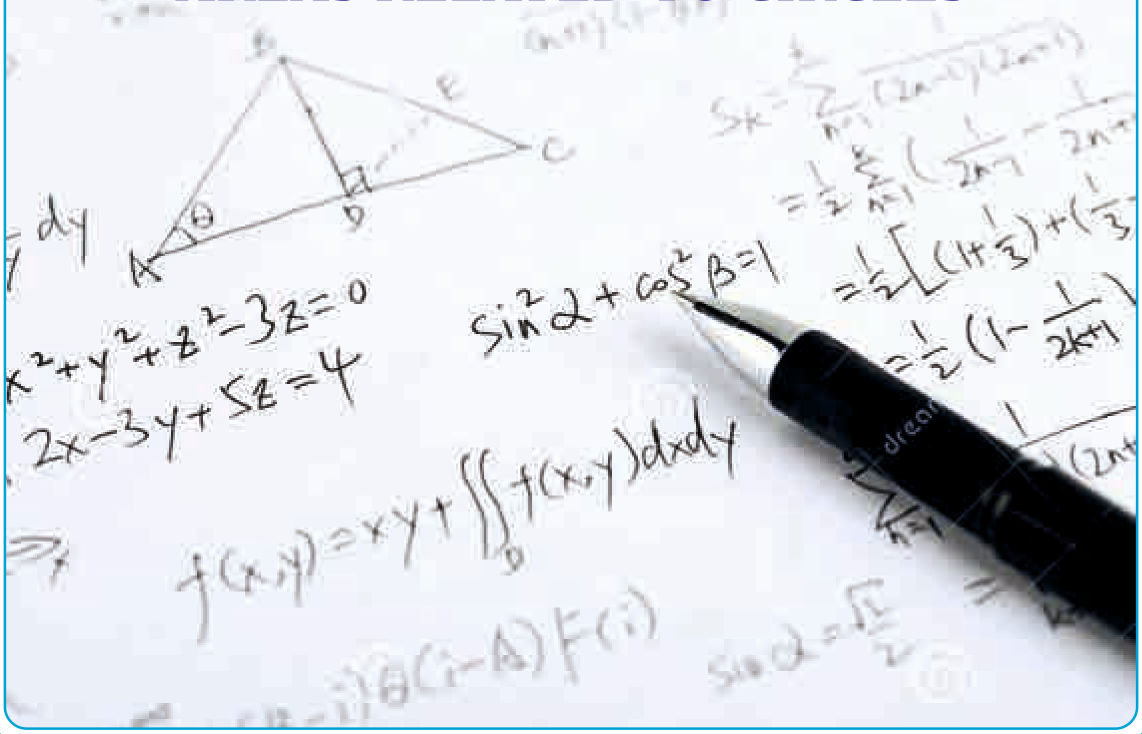
1. To divide a line segment in a given ratio.
2. To construct a triangle similar to a given triangle as per a given scale factor which may be less than 1 or greater than 1.
3. To construct the pair of tangents from an external point to a circle.

A Note to the Reader

Construction of a quadrilateral (or a polygon) similar to a given quadrilateral (or a polygon) with a given scale factor can also be done following the similar steps as used in Examples 1 and 2 of Construction 11.2.

12

AREAS RELATED TO CIRCLES



12 AREAS RELATED TO CIRCLES

12.1 Introduction

You are already familiar with some methods of finding perimeters and areas of simple plane figures such as rectangles, squares, parallelograms, triangles and circles from your earlier classes. Many objects that we come across in our daily life are related to the circular shape in some form or the other. Cycle wheels, wheel barrow (*thela*), dartboard, round cake, *papad*, drain cover, various designs, bangles, brooches, circular paths, washers, flower beds, etc. are some examples of such objects (see Fig. 12.1). So, the problem of finding perimeters and areas related to circular figures is of great practical importance. In this chapter, we shall begin our discussion with a review of the concepts of perimeter (circumference) and area of a circle and apply this knowledge in finding the areas of two special 'parts' of a circular region (or briefly of a circle) known as *sector* and *segment*. We shall also see how to find the areas of some combinations of plane figures involving circles or their parts.

Fig. 12.1

12.2 Perimeter and Area of a Circle — A Review

Recall that the distance covered by travelling once around a circle is its *perimeter,* usually called its *circumference*. You also know from your earlier classes, that circumference of a circle bears a constant ratio with its diameter. This constant ratio is denoted by the Greek letter π (read as 'pi'). In other words,

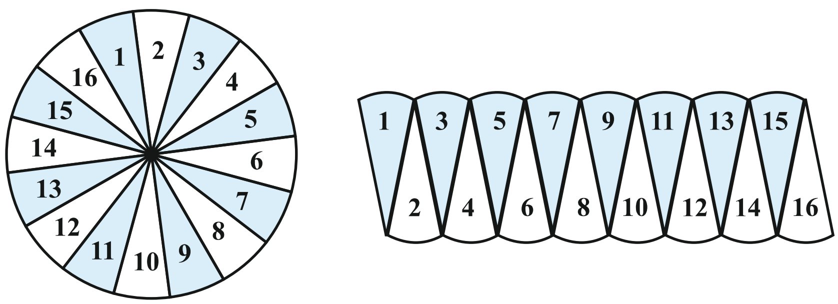
$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

or,

$$\begin{aligned}\text{circumference} &= \pi \times \text{diameter}\\ &= \pi \times 2r \quad (\text{where } r \text{ is the radius of the circle})\\ &= 2\pi r\end{aligned}$$

The great Indian mathematician Aryabhata (C.E. 476 – 550) gave an approximate value of π. He stated that $\pi = \frac{62832}{20000}$, which is nearly equal to 3.1416. It is also interesting to note that using an identity of the great mathematical genius Srinivas Ramanujan (1887–1920) of India, mathematicians have been able to calculate the value of π correct to million places of decimals. As you know from Chapter 1 of Class IX, π is an irrational number and its decimal expansion is non-terminating and non-recurring (non-repeating). However, for practical purposes, we generally take the value of π as $\frac{22}{7}$ or 3.14, approximately.

You may also recall that area of a circle is πr^2, where r is the radius of the circle. Recall that you have verified it in Class VII, by cutting a circle into a number of sectors and rearranging them as shown in Fig. 12.2.



(i) (ii)

Fig 12.2

You can see that the shape in Fig. 12.2 (ii) is nearly a rectangle of length $\frac{1}{2} \times 2\pi r$ and breadth r. This suggests that the area of the circle $= \frac{1}{2} \times 2\pi r \times r = \pi r^2$. Let us recall the concepts learnt in earlier classes, through an example.

Example 1 : The cost of fencing a circular field at the rate of ₹ 24 per metre is ₹ 5280. The field is to be ploughed at the rate of ₹ 0.50 per m². Find the cost of ploughing the field (Take $\pi = \frac{22}{7}$).

Solution : Length of the fence (in metres) $= \frac{\text{Total cost}}{\text{Rate}} = \frac{5280}{24} = 220$

So, circumference of the field = 220 m

Therefore, if r metres is the radius of the field, then

$$2\pi r = 220$$

or,
$$2 \times \frac{22}{7} \times r = 220$$

or,
$$r = \frac{220 \times 7}{2 \times 22} = 35$$

i.e., radius of the field is 35 m.

Therefore, area of the field $= \pi r^2 = \frac{22}{7} \times 35 \times 35 \text{ m}^2 = 22 \times 5 \times 35 \text{ m}^2$

Now, cost of ploughing 1 m² of the field = ₹ 0.50

So, total cost of ploughing the field = ₹ $22 \times 5 \times 35 \times 0.50$ = ₹ 1925

EXERCISE 12.1

Unless stated otherwise, use $\pi = \frac{22}{7}$.

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.
2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
3. Fig. 12.3 depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Fig. 12.3

4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

5. Tick the correct answer in the following and justify your choice : If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 2 units (B) π units (C) 4 units (D) 7 units

12.3 Areas of Sector and Segment of a Circle

You have already come across the terms *sector* and *segment* of a circle in your earlier classes. Recall that the portion (or part) of the circular region enclosed by two radii and the corresponding arc is called a *sector* of the circle and the portion (or part) of the circular region enclosed between a chord and the corresponding arc is called a *segment* of the circle. Thus, in Fig. 12.4, shaded region OAPB is a *sector* of the circle with centre O. $\angle$ AOB is called the *angle* of the sector. Note that in this figure, unshaded region OAQB is also a sector of the circle. For obvious reasons, OAPB is called the *minor sector* and OAQB is called the *major sector*. You can also see that angle of the major sector is $360° - \angle$ AOB.

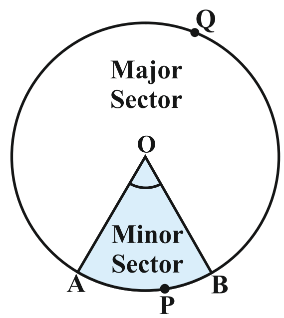


Fig. 12.4

Now, look at Fig. 12.5 in which AB is a chord of the circle with centre O. So, shaded region APB is a segment of the circle. You can also note that unshaded region AQB is another segment of the circle formed by the chord AB. For obvious reasons, APB is called the *minor segment* and AQB is called the *major segment*.

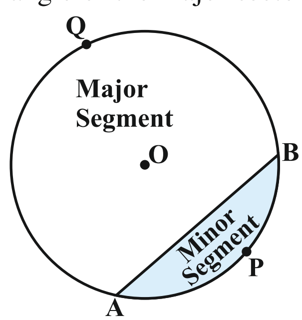


Fig. 12.5

Remark : When we write 'segment' and 'sector' we will mean the 'minor segment' and the 'minor sector' respectively, unless stated otherwise.

Now with this knowledge, let us try to find some relations (or formulae) to calculate their areas.

Let OAPB be a sector of a circle with centre O and radius r (see Fig. 12.6). Let the degree measure of $\angle$ AOB be θ.

You know that area of a circle (in fact of a circular region or disc) is πr^2.

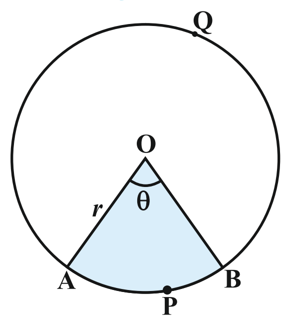


Fig. 12.6

In a way, we can consider this circular region to be a sector forming an angle of 360° (i.e., of degree measure 360) at the centre O. Now by applying the Unitary Method, we can arrive at the area of the sector OAPB as follows:

When degree measure of the angle at the centre is 360, area of the sector = πr^2

So, when the degree measure of the angle at the centre is 1, area of the sector = $\frac{\pi r^2}{360}$.

Therefore, when the degree measure of the angle at the centre is θ, area of the sector = $\frac{\pi r^2}{360} \times \theta = \frac{\theta}{360} \times \pi r^2$.

Thus, we obtain the following relation (or formula) for area of a sector of a circle:

Area of the sector of angle θ = $\boldsymbol{\frac{\theta}{360} \times \pi r^2}$,

where r is the radius of the circle and θ the angle of the sector in degrees.

Now, a natural question arises : Can we find the length of the arc APB corresponding to this sector? Yes. Again, by applying the Unitary Method and taking the whole length of the circle (of angle 360°) as $2\pi r$, we can obtain the required length of the arc APB as $\frac{\theta}{360} \times 2\pi r$.

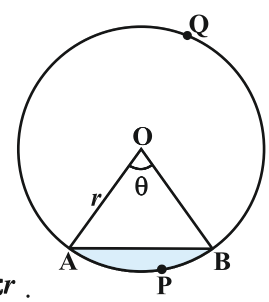


Fig. 12.7

So, **length of an arc of a sector of angle θ =** $\boldsymbol{\frac{\theta}{360} \times 2\pi r}$.

Now let us take the case of the area of the segment APB of a circle with centre O and radius r (see Fig. 12.7). You can see that :

Area of the segment APB = Area of the sector OAPB – Area of Δ OAB

$$= \frac{\theta}{360} \times \pi r^2 - \text{area of } \Delta OAB$$

Note : From Fig. 12.6 and Fig. 12.7 respectively, you can observe that :

Area of the major sector OAQB = πr^2 – Area of the minor sector OAPB

and Area of major segment AQB = πr^2 – Area of the minor segment APB

Let us now take some examples to understand these concepts (or results).

Example 2 : Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use $\pi = 3.14$).

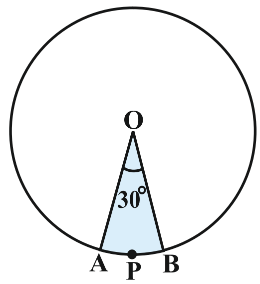


Fig. 12.8

Solution : Given sector is OAPB (see Fig. 12.8).

$$\text{Area of the sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{30}{360} \times 3.14 \times 4 \times 4 \text{ cm}^2$$

$$= \frac{12.56}{3} \text{ cm}^2 = 4.19 \text{cm}^2 \text{ (approx.)}$$

Area of the corresponding major sector

$$= \pi r^2 - \text{area of sector OAPB}$$

$$= (3.14 \times 16 - 4.19) \text{ cm}^2$$

$$= 46.05 \text{ cm}^2 = 46.1 \text{ cm}^2 \text{(approx.)}$$

Alternatively, area of the major sector $= \frac{(360 - \theta)}{360} \times \pi r^2$

$$= \left(\frac{360 - 30}{360}\right) \times 3.14 \times 16 \text{ cm}^2$$

$$= \frac{330}{360} \times 3.14 \times 16 \text{cm}^2 = 46.05 \text{ cm}^2$$

$$= 46.1 \text{ cm}^2 \text{ (approx.)}$$

Example 3 : Find the area of the segment AYB shown in Fig. 12.9, if radius of the circle is 21 cm and $\angle$ AOB = 120°. (Use $\pi = \frac{22}{7}$)

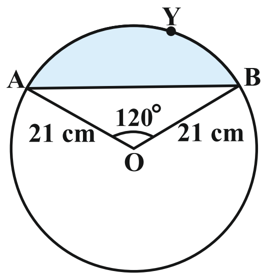


Fig. 12.9

Solution : Area of the segment AYB

$$= \text{Area of sector OAYB} - \text{Area of } \Delta \text{ OAB} \quad (1)$$

Now, area of the sector OAYB $= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 462 \text{ cm}^2$ (2)

For finding the area of Δ OAB, draw OM ⊥ AB as shown in Fig. 12.10.

Note that OA = OB. Therefore, by RHS congruence, Δ AMO ≅ Δ BMO.

So, M is the mid-point of AB and $\angle \text{AOM} = \angle \text{BOM} = \frac{1}{2} \times 120° = 60°$.

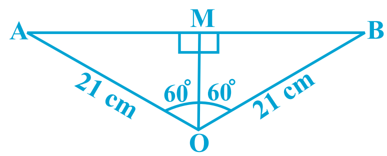


Fig. 12.10

Let $\text{OM} = x \text{ cm}$

So, from Δ OMA, $\frac{\text{OM}}{\text{OA}} = \cos 60°$

or, $\frac{x}{21} = \frac{1}{2} \quad \left(\cos 60° = \frac{1}{2}\right)$

or, $x = \frac{21}{2}$

So, $\text{OM} = \frac{21}{2} \text{ cm}$

Also, $\frac{\text{AM}}{\text{OA}} = \sin 60° = \frac{\sqrt{3}}{2}$

So, $\text{AM} = \frac{21\sqrt{3}}{2} \text{ cm}$

Therefore, $\text{AB} = 2 \text{ AM} = \frac{2 \times 21\sqrt{3}}{2} \text{ cm} = 21\sqrt{3} \text{ cm}$

So, area of $\Delta \text{ OAB} = \frac{1}{2} \text{AB} \times \text{OM} = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2} \text{ cm}^2$

$$= \frac{441}{4}\sqrt{3} \text{ cm}^2 \quad (3)$$

Therefore, area of the segment AYB $= \left(462 - \frac{441}{4}\sqrt{3}\right) \text{ cm}^2$ [From (1), (2) and (3)]

$$= \frac{21}{4}(88 - 21\sqrt{3}) \text{ cm}^2$$

EXERCISE 12.2

Unless stated otherwise, use $\pi = \frac{22}{7}$.

1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.
2. Find the area of a quadrant of a circle whose circumference is 22 cm.
3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.
4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (Use $\pi = 3.14$)
5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
   (i) the length of the arc (ii) area of the sector formed by the arc
   (iii) area of the segment formed by the corresponding chord
6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.
   (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)
7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.
   (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)
8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 12.11). Find
   (i) the area of that part of the field in which the horse can graze.
   (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)

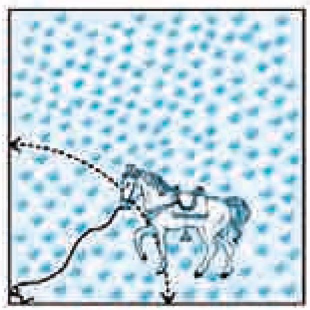


Fig. 12.11

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 12.12. Find :
   (i) the total length of the silver wire required.
   (ii) the area of each sector of the brooch.

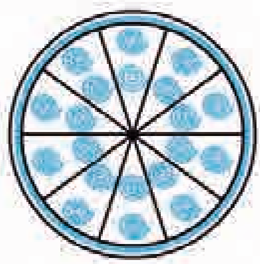


Fig. 12.12

10. An umbrella has 8 ribs which are equally spaced (see Fig. 12.13). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Fig. 12.13

12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

13. A round table cover has six equal designs as shown in Fig. 12.14. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm². (Use $\sqrt{3} = 1.7$)

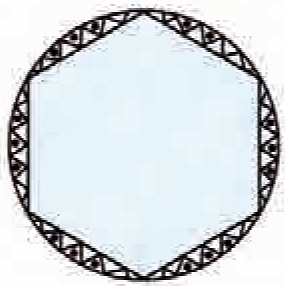


Fig. 12.14

14. Tick the correct answer in the following :

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$ (B) $\frac{p}{180} \times \pi R^2$ (C) $\frac{p}{360} \times 2\pi R$ (D) $\frac{p}{720} \times 2\pi R^2$

12.4 Areas of Combinations of Plane Figures

So far, we have calculated the areas of different figures separately. Let us now try to calculate the areas of some combinations of plane figures. We come across these types of figures in our daily life and also in the form of various interesting designs. Flower beds, drain covers, window designs, designs on table covers, are some of such examples. We illustrate the process of calculating areas of these figures through some examples.

Example 4 : In Fig. 12.15, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.

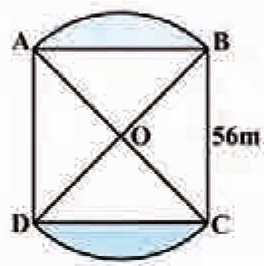


Fig. 12.15

Solution : Area of the square lawn ABCD = 56 × 56 m² (1)

$$\text{Let} \qquad OA = OB = x \text{ metres}$$

$$\text{So,} \qquad x^2 + x^2 = 56^2$$

$$\text{or,} \qquad 2x^2 = 56 \times 56$$

$$\text{or,} \qquad x^2 = 28 \times 56 \qquad (2)$$

Now, area of sector OAB $= \dfrac{90}{360} \times \pi x^2 = \dfrac{1}{4} \times \pi x^2$

$$= \frac{1}{4} \times \frac{22}{7} \times 28 \times 56 \text{ m}^2 \qquad \text{[From (2)]} \qquad (3)$$

Also, area of Δ OAB $= \dfrac{1}{4} \times 56 \times 56 \text{ m}^2$ (∠ AOB = 90°) (4)

So, area of flower bed AB $= \left(\dfrac{1}{4} \times \dfrac{22}{7} \times 28 \times 56 - \dfrac{1}{4} \times 56 \times 56\right) \text{m}^2$

[From (3) and (4)]

$$= \frac{1}{4} \times 28 \times 56 \left(\frac{22}{7} - 2\right) \text{m}^2$$

$$= \frac{1}{4} \times 28 \times 56 \times \frac{8}{7} \text{ m}^2 \qquad (5)$$

Similarly, area of the other flower bed

$$= \frac{1}{4} \times 28 \times 56 \times \frac{8}{7} \text{ m}^2 \qquad (6)$$

Therefore, total area $= \left(56 \times 56 + \dfrac{1}{4} \times 28 \times 56 \times \dfrac{8}{7}\right.$

$$\left. + \frac{1}{4} \times 28 \times 56 \times \frac{8}{7}\right) \text{m}^2 \qquad \text{[From (1), (5) and (6)]}$$

$$= 28 \times 56 \left(2 + \frac{2}{7} + \frac{2}{7}\right) \text{m}^2$$

$$= 28 \times 56 \times \frac{18}{7} \text{ m}^2 = 4032 \text{ m}^2$$

Alternative Solution :

Total area = Area of sector OAB + Area of sector ODC + Area of Δ OAD + Area of Δ OBC

$$= \left(\frac{90}{360}\times\frac{22}{7}\times 28\times 56 + \frac{90}{360}\times\frac{22}{7}\times 28\times 56\right.$$

$$\left. + \frac{1}{4}\times 56\times 56 + \frac{1}{4}\times 56\times 56\right)\text{m}^2$$

$$= \frac{1}{4}\times 28\times 56\left(\frac{22}{7}+\frac{22}{7}+2+2\right)\text{m}^2$$

$$= \frac{7\times 56}{7}(22+22+14+14)\,\text{m}^2$$

$$= 56\times 72\ \text{m}^2 = 4032\ \text{m}^2$$

Example 5 : Find the area of the shaded region in Fig. 12.16, where ABCD is a square of side 14 cm.

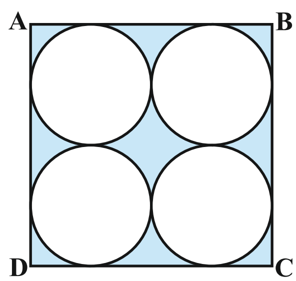


Fig. 12.16

Solution : Area of square ABCD

$$= 14 \times 14\ \text{cm}^2 = 196\ \text{cm}^2$$

$$\text{Diameter of each circle} = \frac{14}{2}\text{cm} = 7\,\text{cm}$$

So, radius of each circle $= \frac{7}{2}\text{cm}$

So, area of one circle $= \pi r^2 = \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\ \text{cm}^2$

$$= \frac{154}{4}\text{cm} = \frac{77}{2}\ \text{cm}^2$$

Therefore, area of the four circles $= 4\times\frac{77}{2}\text{cm}^2 = 154\ \text{cm}^2$

Hence, area of the shaded region $= (196 - 154)\ \text{cm}^2 = 42\ \text{cm}^2$.

Example 6 : Find the area of the shaded design in Fig. 12.17, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter. (Use $\pi = 3.14$)

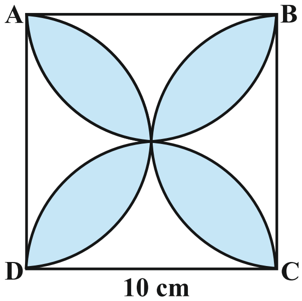


Fig. 12.17

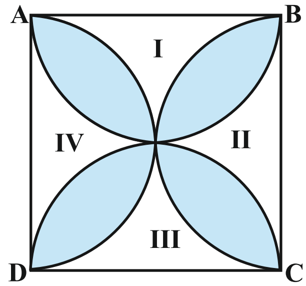


Fig. 12.18

Solution : Let us mark the four unshaded regions as I, II, III and IV (see Fig. 12.18).

Area of I + Area of III

= Area of ABCD – Areas of two semicircles of each of radius 5 cm

$$= \left(10 \times 10 - 2 \times \frac{1}{2} \times \pi \times 5^2\right) \text{cm}^2 = (100 - 3.14 \times 25) \text{ cm}^2$$

$$= (100 - 78.5) \text{ cm}^2 = 21.5 \text{ cm}^2$$

Similarly, Area of II + Area of IV = 21.5 cm^2

So, area of the shaded design = Area of ABCD – Area of (I + II + III + IV)

$$= (100 - 2 \times 21.5) \text{ cm}^2 = (100 - 43) \text{ cm}^2 = 57 \text{ cm}^2$$

EXERCISE 12.3

Unless stated otherwise, use $\pi = \frac{22}{7}$.

1. Find the area of the shaded region in Fig. 12.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

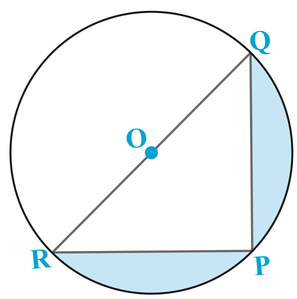


Fig. 12.19

2. Find the area of the shaded region in Fig. 12.20, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40°$.

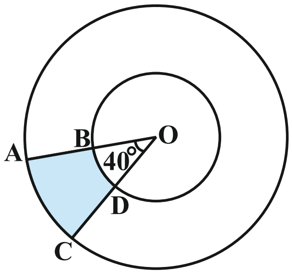


Fig. 12.20

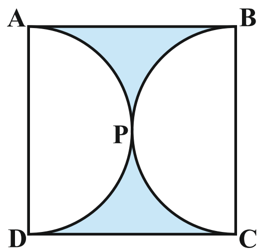


Fig. 12.21

3. Find the area of the shaded region in Fig. 12.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

4. Find the area of the shaded region in Fig. 12.22, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

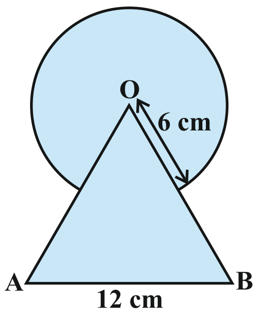


Fig. 12.22

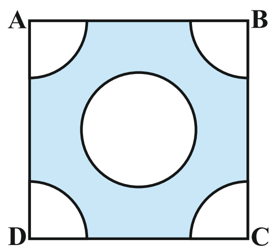


Fig. 12.23

5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 12.23. Find the area of the remaining portion of the square.

6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 12.24. Find the area of the design.

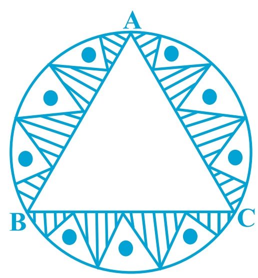


Fig. 12.24

7. In Fig. 12.25, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

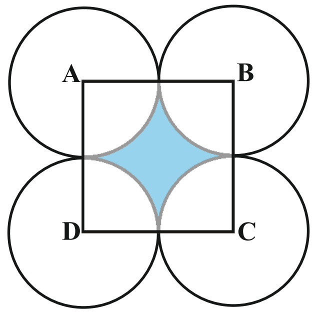


Fig. 12.25

8. Fig. 12.26 depicts a racing track whose left and right ends are semicircular.

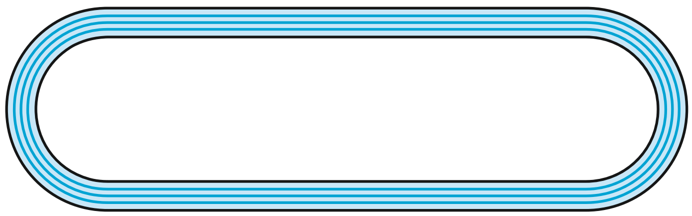


Fig. 12.26

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find :

(i) the distance around the track along its inner edge

(ii) the area of the track.

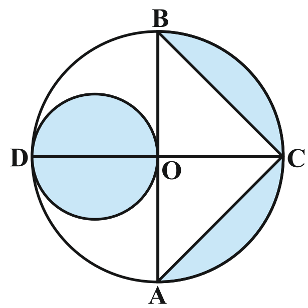


Fig. 12.27

9. In Fig. 12.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

10. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region. (Use π = 3.14 and $\sqrt{3}$ = 1.73205)

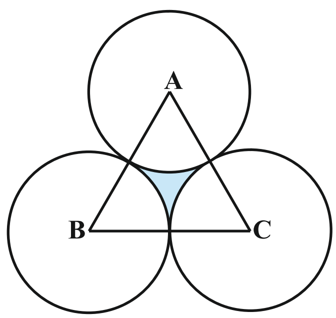


Fig. 12.28

11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig. 12.29). Find the area of the remaining portion of the handkerchief.

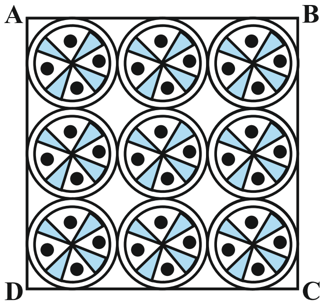


Fig. 12.29

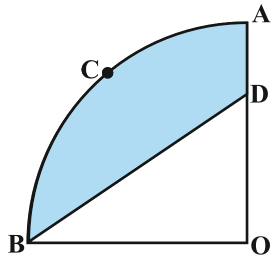


Fig. 12.30

12. In Fig. 12.30, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

(i) quadrant OACB, (ii) shaded region.

13. In Fig. 12.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)

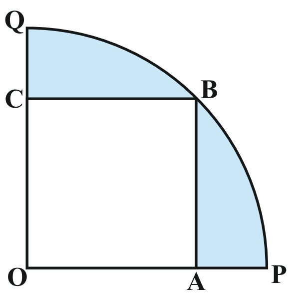


Fig. 12.31

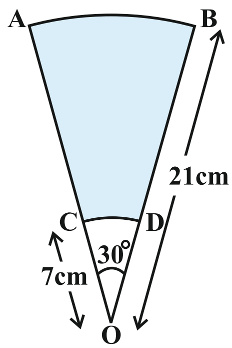


Fig. 12.32

14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 12.32). If $\angle AOB = 30°$, find the area of the shaded region.

15. In Fig. 12.33, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

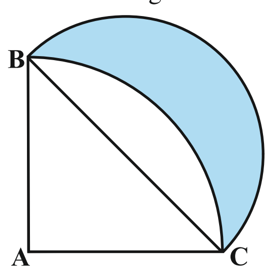


Fig. 12.33

16. Calculate the area of the designed region in Fig. 12.34 common between the two quadrants of circles of radius 8 cm each.

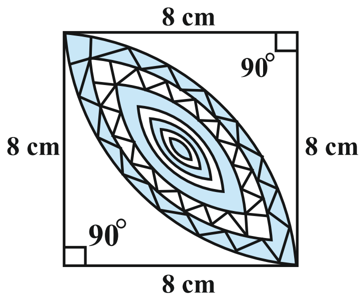


Fig. 12.34

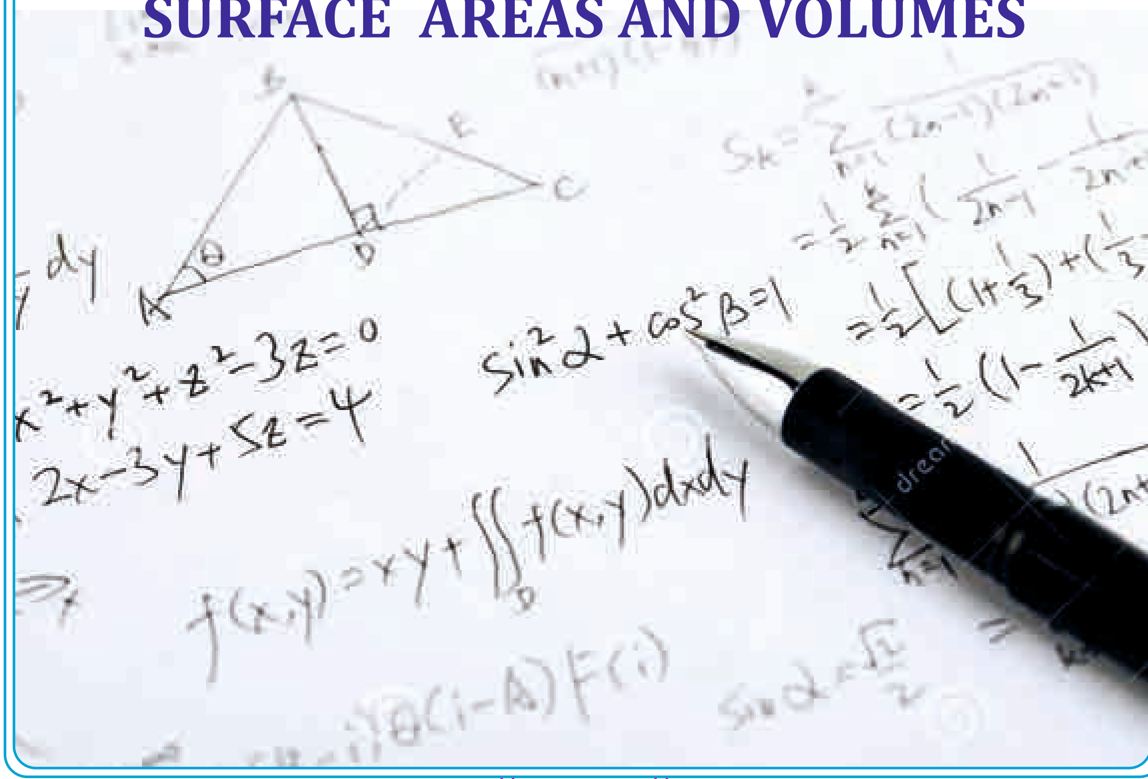
12.5 Summary

In this chapter, you have studied the following points :

1. Circumference of a circle = $2\pi r$.
2. Area of a circle = πr^2.
3. Length of an arc of a sector of a circle with radius r and angle with degree measure θ is $\frac{\theta}{360} \times 2\pi r$.
4. Area of a sector of a circle with radius r and angle with degree measure θ is $\frac{\theta}{360} \times \pi r^2$.
5. Area of segment of a circle
   = Area of the corresponding sector – Area of the corresponding triangle.

13

SURFACE AREAS AND VOLUMES



SURFACE AREAS AND VOLUMES 13

13.1 Introduction

From Class IX, you are familiar with some of the solids like cuboid, cone, cylinder, and sphere (see Fig. 13.1). You have also learnt how to find their surface areas and volumes.

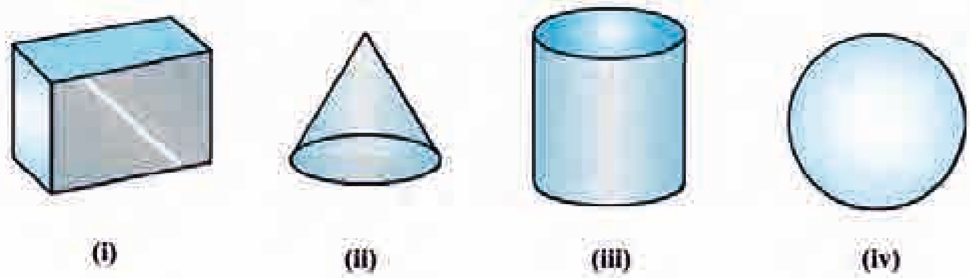


Fig. 13.1

In our day-to-day life, we come across a number of solids made up of combinations of two or more of the basic solids as shown above.

You must have seen a truck with a container fitted on its back (see Fig. 13.2), carrying oil or water from one place to another. Is it in the shape of any of the four basic solids mentioned above? You may guess that it is made of a cylinder with two hemispheres as its ends.

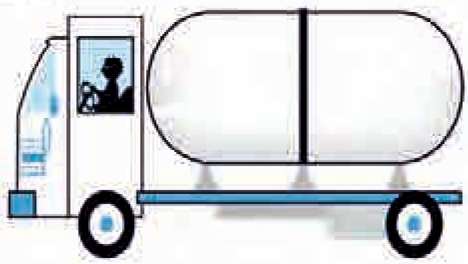


Fig. 13.2

Again, you may have seen an object like the one in Fig. 13.3. Can you name it? A test tube, right! You would have used one in your science laboratory. This tube is also a combination of a cylinder and a hemisphere. Similarly, while travelling, you may have seen some big and beautiful buildings or monuments made up of a combination of solids mentioned above.

If for some reason you wanted to find the surface areas, or volumes, or capacities of such objects, how would you do it? We cannot classify these under any of the solids you have already studied.

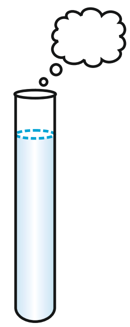


Fig. 13.3

In this chapter, you will see how to find surface areas and volumes of such objects.

13.2 Surface Area of a Combination of Solids

Let us consider the container seen in Fig. 13.2. How do we find the surface area of such a solid? Now, whenever we come across a new problem, we first try to see, if we can break it down into smaller problems, we have earlier solved. We can see that this solid is made up of a cylinder with two hemispheres stuck at either end. It would look like what we have in Fig. 13.4, after we put the pieces all together.

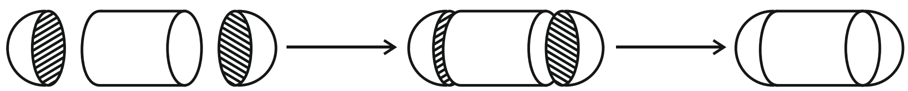


Fig. 13.4

If we consider the surface of the newly formed object, we would be able to see only the curved surfaces of the two hemispheres and the curved surface of the cylinder.

So, the *total* surface area of the new solid is the sum of the *curved* surface areas of each of the individual parts. This gives,

TSA of new solid = CSA of one hemisphere + CSA of cylinder
+ CSA of other hemisphere

where TSA, CSA stand for 'Total Surface Area' and 'Curved Surface Area' respectively.

Let us now consider another situation. Suppose we are making a toy by putting together a hemisphere and a cone. Let us see the steps that we would be going through.

First, we would take a cone and a hemisphere and bring their flat faces together. Here, of course, we would take the base radius of the cone equal to the radius of the hemisphere, for the toy is to have a smooth surface. So, the steps would be as shown in Fig. 13.5.

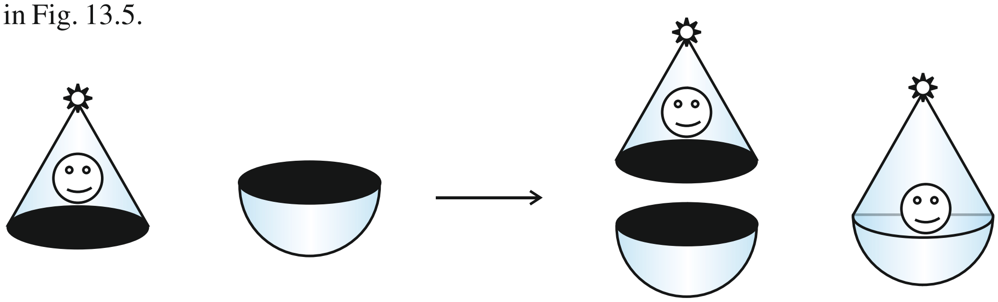


Fig. 13.5

At the end of our trial, we have got ourselves a nice round-bottomed toy. Now if we want to find how much paint we would require to colour the surface of this toy, what would we need to know? We would need to know the surface area of the toy, which consists of the CSA of the hemisphere and the CSA of the cone.

So, we can say:

Total surface area of the toy = CSA of hemisphere + CSA of cone

Now, let us consider some examples.

Example 1 : Rasheed got a playing top (*lattu*) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere (see Fig 13.6). The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (Take $\pi = \frac{22}{7}$)

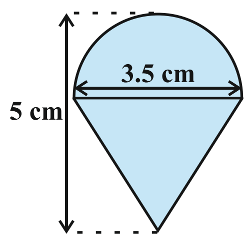


Fig. 13.6

Solution : This top is exactly like the object we have discussed in Fig. 13.5. So, we can conveniently use the result we have arrived at there. That is :

TSA of the toy = CSA of hemisphere + CSA of cone

Now, the curved surface area of the hemisphere $= \frac{1}{2}(4\pi r^2) = 2\pi r^2$

$$= \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) \text{cm}^2$$

Also, the height of the cone = height of the top – height (radius) of the hemispherical part

$$= \left(5 - \frac{3.5}{2}\right) \text{cm} = 3.25 \text{ cm}$$

So, the slant height of the cone (l) = $\sqrt{r^2 + h^2} = \sqrt{\left(\frac{3.5}{2}\right)^2 + (3.25)^2}$ cm = 3.7 cm (approx.)

Therefore, CSA of cone = $\pi r l = \left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right) \text{cm}^2$

This gives the surface area of the top as

$$= \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) \text{cm}^2 + \left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right) \text{cm}^2$$

$$= \frac{22}{7} \times \frac{3.5}{2}\left(3.5 + 3.7\right) \text{cm}^2 = \frac{11}{2} \times (3.5 + 3.7) \text{ cm}^2 = 39.6 \text{ cm}^2 \text{ (approx.)}$$

You may note that 'total surface area of the top' is *not* the sum of the total surface areas of the cone and hemisphere.

Example 2 : The decorative block shown in Fig. 13.7 is made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block. (Take $\pi = \frac{22}{7}$)

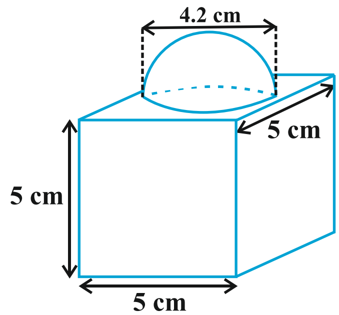


Fig. 13.7

Solution : The total surface area of the cube = 6 × (edge)^2 = 6 × 5 × 5 cm^2 = 150 cm^2.

Note that the part of the cube where the hemisphere is attached is not included in the surface area.

So, the surface area of the block = TSA of cube – base area of hemisphere + CSA of hemisphere

$$= 150 - \pi r^2 + 2\pi r^2 = (150 + \pi r^2) \text{ cm}^2$$

$$= 150 \text{ cm}^2 + \left(\frac{22}{7} \times \frac{4.2}{2} \times \frac{4.2}{2}\right) \text{cm}^2$$

$$= (150 + 13.86) \text{ cm}^2 = 163.86 \text{ cm}^2$$

Example 3 : A wooden toy rocket is in the shape of a cone mounted on a cylinder, as shown in Fig. 13.8. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take $\pi = 3.14$)

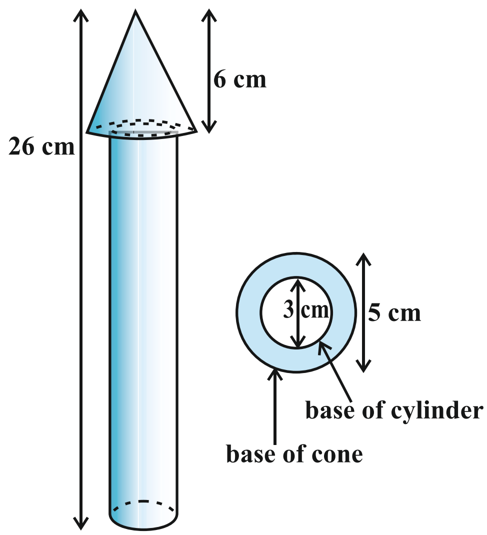


Fig. 13.8

Solution : Denote radius of cone by r, slant height of cone by l, height of cone by h, radius of cylinder by r' and height of cylinder by h'. Then $r = 2.5$ cm, $h = 6$ cm, $r' = 1.5$ cm, $h' = 26 - 6 = 20$ cm and

$$l = \sqrt{r^2 + h^2} = \sqrt{2.5^2 + 6^2} \text{ cm} = 6.5 \text{ cm}$$

Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder. So, a part of the base of the cone (a ring) is to be painted.

So, the area to be painted orange = CSA of the cone + base area of the cone – base area of the cylinder

$$= \pi rl + \pi r^2 - \pi(r')^2$$

$$= \pi[(2.5 \times 6.5) + (2.5)^2 - (1.5)^2] \text{ cm}^2$$

$$= \pi[20.25] \text{ cm}^2 = 3.14 \times 20.25 \text{ cm}^2$$

$$= 63.585 \text{ cm}^2$$

Now, the area to be painted yellow = CSA of the cylinder + area of one base of the cylinder

$$= 2\pi r'h' + \pi(r')^2$$

$$= \pi r'(2h' + r')$$

$$= (3.14 \times 1.5)(2 \times 20 + 1.5) \text{ cm}^2$$

$$= 4.71 \times 41.5 \text{ cm}^2$$

$$= 195.465 \text{ cm}^2$$

Example 4 : Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end (see Fig. 13.9). The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)

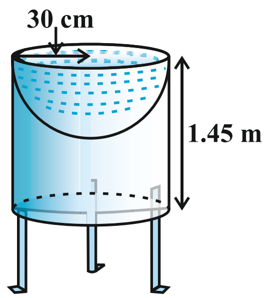


Fig. 13.9

Solution : Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere. Then,

the total surface area of the bird-bath = CSA of cylinder + CSA of hemisphere

$$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 30(145 + 30) \text{ cm}^2$$

$$= 33000 \text{ cm}^2 = 3.3 \text{ m}^2$$

EXERCISE 13.1

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.
2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.
3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.
4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.
5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.
6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig. 13.10). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

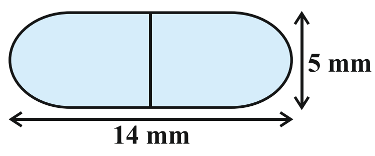


Fig. 13.10

7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹500 per m^2. (Note that the base of the tent will not be covered with canvas.)

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 13.11. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

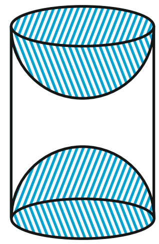


Fig. 13.11

13.3 Volume of a Combination of Solids

In the previous section, we have discussed how to find the surface area of solids made up of a combination of two basic solids. Here, we shall see how to calculate their volumes. It may be noted that in calculating the surface area, we have not added the surface areas of the two constituents, because some part of the surface area disappeared in the process of joining them. However, this will not be the case when we calculate the volume. The volume of the solid formed by joining two basic solids will actually be the sum of the volumes of the constituents, as we see in the examples below.

Example 5 : Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder (see Fig. 13.12). If the base of the shed is of dimension 7 m × 15 m, and the height of the cuboidal portion is 8 m, find the volume of air that the shed can hold. Further, suppose the machinery in the shed occupies a total space of 300 m^3, and there are 20 workers, each of whom occupy about 0.08 m^3 space on an average. Then, how much air is in the shed? (Take $\pi = \frac{22}{7}$)

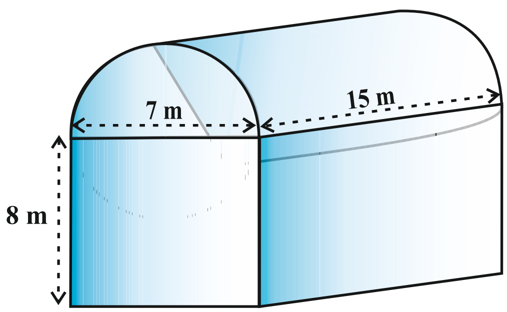


Fig. 13.12

Solution : The volume of air inside the shed (when there are no people or machinery) is given by the volume of air inside the cuboid and inside the half cylinder, taken together.

Now, the length, breadth and height of the cuboid are 15 m, 7 m and 8 m, respectively.

Also, the diameter of the half cylinder is 7 m and its height is 15 m.

So, the required volume = volume of the cuboid + $\frac{1}{2}$ volume of the cylinder

$$= \left[15 \times 7 \times 8 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 15\right] \text{m}^3 = 1128.75 \text{ m}^3$$

Next, the total space occupied by the machinery = 300 m^3

And the total space occupied by the workers = 20 × 0.08 m^3 = 1.6 m^3

Therefore, the volume of the air, when there are machinery and workers

$$= 1128.75 - (300.00 + 1.60) = 827.15 \text{ m}^3$$

Example 6 : A juice seller was serving his customers using glasses as shown in Fig. 13.13. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14.)

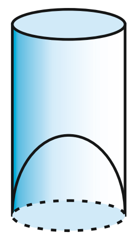


Fig. 13.13

Solution : Since the inner diameter of the glass = 5 cm and height = 10 cm,

the apparent capacity of the glass = $\pi r^2 h$

$$= 3.14 \times 2.5 \times 2.5 \times 10 \text{ cm}^3 = 196.25 \text{ cm}^3$$

But the actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.

i.e., it is less by $\frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5 \text{ cm}^3 = 32.71 \text{ cm}^3$

So, the actual capacity of the glass = apparent capacity of glass – volume of the hemisphere

$= (196.25 - 32.71)$ cm^3

$= 163.54$ cm^3

Example 7 : A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$)

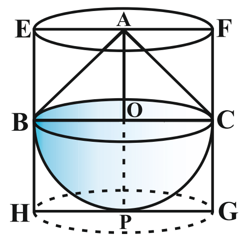


Fig. 13.14

Solution : Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere (see Fig. 13.14). The radius BO of the hemisphere (as well as of the cone) $= \frac{1}{2} \times 4$ cm = 2 cm.

So, volume of the toy $= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$$= \left[\frac{2}{3} \times 3.14 \times (2)^3 + \frac{1}{3} \times 3.14 \times (2)^2 \times 2\right] \text{cm}^3 = 25.12 \text{ cm}^3$$

Now, let the right circular cylinder EFGH circumscribe the given solid. The radius of the base of the right circular cylinder = HP = BO = 2 cm, and its height is

$$\text{EH} = \text{AO} + \text{OP} = (2 + 2) \text{ cm} = 4 \text{ cm}$$

So, the volume required = volume of the right circular cylinder – volume of the toy

$$= (3.14 \times 2^2 \times 4 - 25.12) \text{ cm}^3$$

$$= 25.12 \text{ cm}^3$$

Hence, the required difference of the two volumes = 25.12 cm^3.

EXERCISE 13.2

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

3. A *gulab jamun,* contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns,* each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Fig. 13.15).

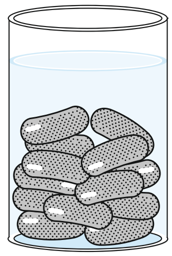


Fig. 13.15

4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig. 13.16).

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

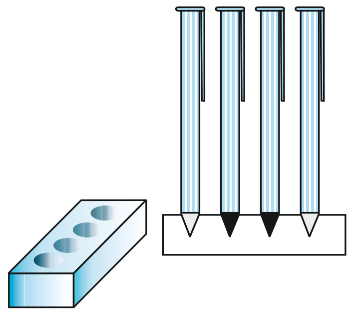


Fig. 13.16

6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass. (Use $\pi = 3.14$)

7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

13.4 Conversion of Solid from One Shape to Another

We are sure you would have seen candles. Generally, they are in the shape of a cylinder. You may have also seen some candles shaped like an animal (see Fig. 13.17).

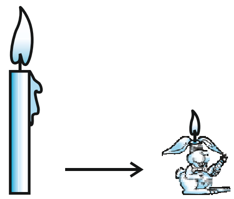


Fig. 13.17

How are they made? If you want a candle of any special shape, you will have to heat the wax in a metal container till it becomes completely liquid. Then you will have to pour it into another container which has the special shape that you want. For example, take a candle in the shape of a solid cylinder, melt it and pour whole of the molten wax into another container shaped like a rabbit. On cooling, you will obtain a candle in the shape of the rabbit. The volume of the new candle will be the same as the volume of the earlier candle. This is what we have to remember when we come across objects which are converted from one shape to another, or when a liquid which originally filled one container of a particular shape is poured into another container of a different shape or size, as you see in Fig 13.18.

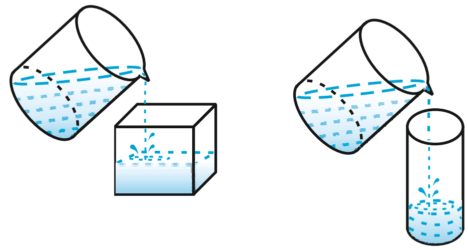


Fig. 13.18

To understand what has been discussed, let us consider some examples.

Example 8: A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

Solution : Volume of cone = $\frac{1}{3} \times \pi \times 6 \times 6 \times 24 \text{ cm}^3$

If r is the radius of the sphere, then its volume is $\frac{4}{3}\pi r^3$.

Since, the volume of clay in the form of the cone and the sphere remains the same, we have

$$\frac{4}{3} \times \pi \times r^3 = \frac{1}{3} \times \pi \times 6 \times 6 \times 24$$

i.e., $r^3 = 3 \times 3 \times 24 = 3^3 \times 2^3$

So, $r = 3 \times 2 = 6$

Therefore, the radius of the sphere is 6 cm.

Example 9 : Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (an underground tank) which is in the shape of a cuboid. The sump has dimensions 1.57 m × 1.44 m × 95cm. The overhead tank has its radius 60 cm and height 95 cm. Find the height of the water left in the sump after the overhead tank has been completely filled with water from the sump which had been full. Compare the capacity of the tank with that of the sump. (Use $\pi = 3.14$)

Solution : The volume of water in the overhead tank equals the volume of the water removed from the sump.

Now, the volume of water in the overhead tank (cylinder) = $\pi r^2 h$

$$= 3.14 \times 0.6 \times 0.6 \times 0.95 \text{ m}^3$$

The volume of water in the sump when full = $l \times b \times h = 1.57 \times 1.44 \times 0.95 \text{ m}^3$

The volume of water left in the sump after filling the tank

$= [(1.57 \times 1.44 \times 0.95) - (3.14 \times 0.6 \times 0.6 \times 0.95)] \text{ m}^3 = (1.57 \times 0.6 \times 0.6 \times 0.95 \times 2) \text{ m}^3$

So, the height of the water left in the sump = $\dfrac{\text{volume of water left in the sump}}{l \times b}$

$$= \frac{1.57 \times 0.6 \times 0.6 \times 0.95 \times 2}{1.57 \times 1.44} \text{ m}$$

$$= 0.475 \text{ m} = 47.5 \text{ cm}$$

Also, $\dfrac{\text{Capacity of tank}}{\text{Capacity of sump}} = \dfrac{3.14 \times 0.6 \times 0.6 \times 0.95}{1.57 \times 1.44 \times 0.95} = \dfrac{1}{2}$

Therefore, the capacity of the tank is half the capacity of the sump.

Example 10 : A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

Solution : The volume of the rod = $\pi \times \left(\frac{1}{2}\right)^2 \times 8 \text{ cm}^3 = 2\pi \text{ cm}^3$.

The length of the new wire of the same volume = 18 m = 1800 cm

If r is the radius (in cm) of cross-section of the wire, its volume = $\pi \times r^2 \times 1800 \text{ cm}^3$

Therefore, $\pi \times r^2 \times 1800 = 2\pi$

i.e., $r^2 = \dfrac{1}{900}$

i.e., $r = \dfrac{1}{30}$

So, the diameter of the cross section, i.e., the thickness of the wire is $\dfrac{1}{15}$ cm, i.e., 0.67mm (approx.).

Example 11 : A hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to empty half the tank, if it is 3m in diameter? (Take $\pi = \frac{22}{7}$)

Solution : Radius of the hemispherical tank = $\frac{3}{2}$ m

$$\text{Volume of the tank} = \frac{2}{3} \times \frac{22}{7} \times \left(\frac{3}{2}\right)^3 \text{ m}^3 = \frac{99}{14} \text{m}^3$$

So, the volume of the water to be emptied $= \frac{1}{2} \times \frac{99}{14} \text{ m}^3 = \frac{99}{28} \times 1000$ litres

$$= \frac{99000}{28} \text{ litres}$$

Since, $\frac{25}{7}$ litres of water is emptied in 1 second, $\frac{99000}{28}$ litres of water will be emptied in $\frac{99000}{28} \times \frac{7}{25}$ seconds, i.e., in 16.5 minutes.

EXERCISE 13.3

Take $\pi = \frac{22}{7}$, unless stated otherwise.

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.
2. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.
3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.
4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.
5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.
6. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.
8. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?
9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

13.5 Frustum of a Cone

In Section 13.2, we observed objects that are formed when two basic solids were joined together. Let us now do something different. We will take a right circular cone and *remove* a portion of it. There are so many ways in which we can do this. But one particular case that we are interested in is the removal of a smaller right circular cone by cutting the given cone by a plane parallel to its base. You must have observed that the glasses (tumblers), in general, used for drinking water, are of this shape. (See Fig. 13.19)

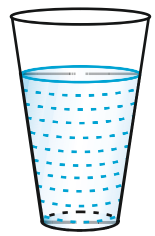


Fig. 13.19

Activity 1 : Take some clay, or any other such material (like plasticine, etc.) and form a cone. Cut it with a knife parallel to its base. Remove the smaller cone. What are you left with?You are left with a solid called a *frustum* of the cone. You can see that this has two circular ends with different radii.

So, given a cone, when we slice (or cut) through it with a plane parallel to its base (see Fig. 13.20) and remove the cone that is formed on one side of that plane, the part that is now left over on the other side of the plane is called a **frustum* of the cone**.

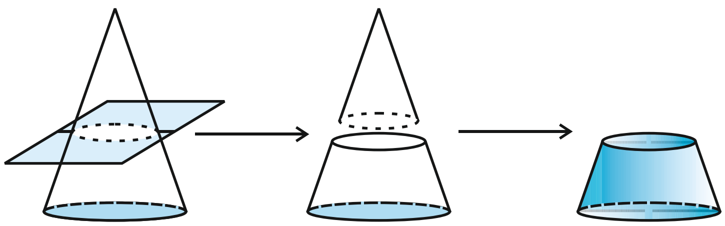


Fig. 13.20

*'Frustum' is a latin word meaning 'piece cut off', and its plural is 'frusta'.

How can we find the surface area and volume of a frustum of a cone? Let us explain it through an example.

Example 12 : The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm (see Fig. 13.21). Find its volume, the curved surface area and the total suface area (Take $\pi = \frac{22}{7}$).

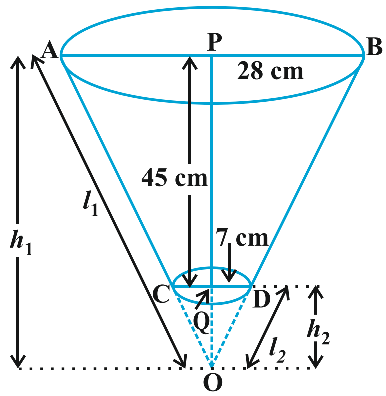


Fig. 13.21

Solution : The frustum can be viewed as a difference of two right circular cones OAB and OCD (see Fig. 13.21). Let the height (in cm) of the cone OAB be h_1 and its slant height l_1, i.e., OP = h_1 and OA = OB = l_1. Let h_2 be the height of cone OCD and l_2 its slant height.

We have : $r_1 = 28$ cm, $r_2 = 7$ cm

and the height of frustum (h) = 45 cm. Also,

$$h_1 = 45 + h_2 \tag{1}$$

We first need to determine the respective heights h_1 and h_2 of the cone OAB and OCD.

Since the triangles OPB and OQD are similar (Why?), we have

$$\frac{h_1}{h_2} = \frac{28}{7} = \frac{4}{1} \tag{2}$$

From (1) and (2), we get $h_2 = 15$ and $h_1 = 60$.

Now, the volume of the frustum

= volume of the cone OAB – volume of the cone OCD

$$= \left[\frac{1}{3}\cdot\frac{22}{7}\cdot(28)^2\cdot(60) - \frac{1}{3}\cdot\frac{22}{7}\cdot(7)^2\cdot(15)\right]\text{cm}^3 = 48510 \text{ cm}^3$$

The respective slant height l_2 and l_1 of the cones OCD and OAB are given by

$$l_2 = \sqrt{(7)^2 + (15)^2} = 16.55 \text{ cm (approx.)}$$

$$l_1 = \sqrt{(28)^2 + (60)^2} = 4\sqrt{(7)^2 + (15)^2} = 4 \times 16.55 = 66.20 \text{ cm}$$

Thus, the curved surface area of the frustum = $\pi r_1 l_1 - \pi r_2 l_2$

$$= \frac{22}{7}(28)(66.20) - \frac{22}{7}(7)(16.55) = 5461.5 \text{ cm}^2$$

Now, the total surface area of the frustum

$$= \text{the curved surface area} + \pi r_1^2 + \pi r_2^2$$

$$= 5461.5 \text{ cm}^2 + \frac{22}{7}(28)^2 \text{ cm}^2 + \frac{22}{7}(7)^2 \text{ cm}^2$$

$$= 5461.5 \text{ cm}^2 + 2464 \text{ cm}^2 + 154 \text{ cm}^2 = 8079.5 \text{ cm}^2.$$

Let h be the height, l the slant height and r_1 and r_2 the radii of the ends ($r_1 > r_2$) of the frustum of a cone. Then we can directly find the volume, the curved surace area and the total surface area of frustum by using the formulae given below :

(i) *Volume of the frustum of the cone* = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$.

(ii) *the curved surface area of the frustum of the cone* = $\pi(r_1 + r_2)l$

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$.

(iii) *Total surface area of the frustum of the cone* = $\pi l\,(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$,

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$.

These formulae can be derived using the idea of similarity of triangles but we shall not be doing derivations here.

Let us solve Example 12, using these formulae :

(i) Volume of the frustum $= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$$= \frac{1}{3} \cdot \frac{22}{7} \cdot 45 \cdot \left[(28)^2 + (7)^2 + (28)(7)\right] \text{ cm}^3$$

$$= 48510 \text{ cm}^3$$

(ii) We have $l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{(45)^2 + (28-7)^2}$ cm

$$= 3\sqrt{(15)^2 + (7)^2} = 49.65 \text{ cm}$$

So, the curved surface area of the frustum

$$= \pi(r_1 + r_2)\, l = \frac{22}{7}(28 + 7)(49.65) = 5461.5 \text{ cm}^2$$

(iii) Total surface area of the frustum

$$= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$$

$$= \left[5461.5 + \frac{22}{7}(28)^2 + \frac{22}{7}(7)^2\right] \text{cm}^2 = 8079.5 \text{ cm}^2$$

Let us apply these formulae in some examples.

Example 13 : Hanumappa and his wife Gangamma are busy making jaggery out of sugarcane juice. They have processed the sugarcane juice to make the molasses, which is poured into moulds in the shape of a frustum of a cone having the diameters of its two circular faces as 30 cm and 35 cm and the vertical height of the mould is 14 cm (see Fig. 13.22). If each cm^3 of molasses has mass about 1.2 g, find the mass of the molasses that can be poured into each mould. $\left(\text{Take } \pi = \frac{22}{7}\right)$

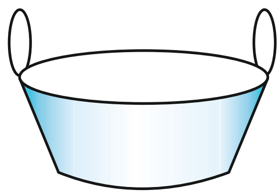


Fig. 13.22

Solution : Since the mould is in the shape of a frustum of a cone, the quantity (volume) of molasses that can be poured into it $= \frac{\pi}{3}h\left(r_1^2 + r_2^2 + r_1 r_2\right)$,

where r_1 is the radius of the larger base and r_2 is the radius of the smaller base.

$$= \frac{1}{3} \times \frac{22}{7} \times 14\left[\left(\frac{35}{2}\right)^2 + \left(\frac{30}{2}\right)^2 + \left(\frac{35}{2} \times \frac{30}{2}\right)\right]\text{cm}^3 = 11641.7 \text{ cm}^3.$$

It is given that 1 cm^3 of molasses has mass 1.2g. So, the mass of the molasses that can be poured into each mould = (11641.7 × 1.2) g

= 13970.04 g = 13.97 kg = 14 kg (approx.)

Example 14 : An open metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet (see Fig. 13.23). The diameters of the two circular ends of the bucket are 45 cm and 25 cm, the total vertical height of the bucket is 40 cm and that of the cylindrical base is 6 cm. Find the area of the metallic sheet used to make the bucket, where we do not take into account the handle of the bucket. Also, find the volume of water the bucket can hold. Take $\pi = \frac{22}{7}$.

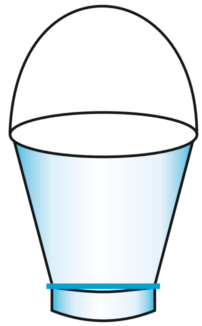


Fig. 13.23

Solution : The total height of the bucket = 40 cm, which includes the height of the base. So, the height of the frustum of the cone = (40 – 6) cm = 34 cm.

Therefore, the slant height of the frustum, $l = \sqrt{h^2 + (r_1 - r_2)^2}$,

where r_1 = 22.5 cm, r_2 = 12.5 cm and h = 34 cm.

So,
$$l = \sqrt{34^2 + (22.5 - 12.5)^2} \text{ cm}$$
$$= \sqrt{34^2 + 10^2} = 35.44 \text{ cm}$$

The area of metallic sheet used = curved surface area of frustum of cone

+ area of circular base

+ curved surface area of cylinder

$$= [\pi \times 35.44\,(22.5 + 12.5) + \pi \times (12.5)^2 + 2\pi \times 12.5 \times 6] \text{ cm}^2$$

$$= \frac{22}{7}(1240.4 + 156.25 + 150) \text{ cm}^2$$

$$= 4860.9 \text{ cm}^2$$

Now, the volume of water that the bucket can hold (also, known as the capacity of the bucket)

$$= \frac{\pi \times h}{3} \times (r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{22}{7} \times \frac{34}{3} \times [(22.5)^2 + (12.5)^2 + 22.5 \times 12.5] \text{ cm}^3$$

$$= \frac{22}{7} \times \frac{34}{3} \times 943.75 = 33615.48 \text{ cm}^3$$

$$= 33.62 \text{ litres (approx.)}$$

EXERCISE 13.4

Use $\pi = \frac{22}{7}$ unless stated otherwise.

1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.
2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.
3. A *fez*, the cap used by the Turks, is shaped like the frustum of a cone (see Fig. 13.24). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Fig. 13.24

4. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of ₹ 20 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 8 per 100 cm². (Take $\pi = 3.14$)
5. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

EXERCISE 13.5 (Optional)*

1. A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.
2. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate.)
3. A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?
4. In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 7280 km^2, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.
5. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see Fig. 13.25).

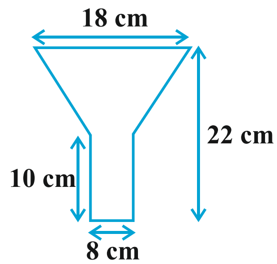


Fig. 13.25

6. Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.
7. Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

13.6 Summary

In this chapter, you have studied the following points:

1. To determine the surface area of an object formed by combining any two of the basic solids, namely, cuboid, cone, cylinder, sphere and hemisphere.
2. To find the volume of objects formed by combining any two of a cuboid, cone, cylinder, sphere and hemisphere.

* These exercises are not from the examination point of view.

3. Given a right circular cone, which is sliced through by a plane parallel to its base, when the smaller conical portion is removed, the resulting solid is called a *Frustum of a Right Circular Cone*.

4. The formulae involving the frustum of a cone are:

   (i) Volume of a frustum of a cone = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$.

   (ii) Curved surface area of a frustum of a cone = $\pi l(r_1 + r_2)$ where $l = \sqrt{h^2 + (r_1 - r_2)^2}$.

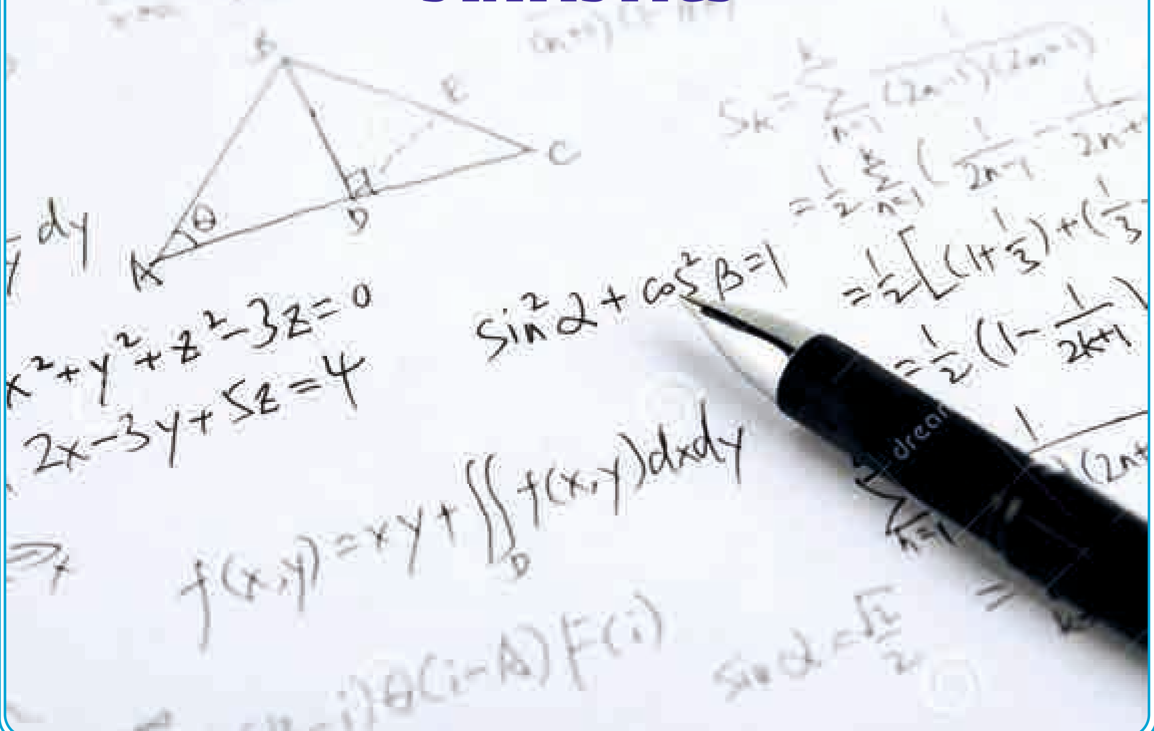
   (iii) Total surface area of frustum of a cone = $\pi l(r_1 + r_2) + \pi(r_1^2 + r_2^2)$ where

   h = vertical height of the frustum, l = slant height of the frustum

   r_1 and r_2 are radii of the two bases (ends) of the frustum.

14

STATISTICS



STATISTICS 14

14.1 Introduction

In Class IX, you have studied the classification of given data into ungrouped as well as grouped frequency distributions. You have also learnt to represent the data pictorially in the form of various graphs such as bar graphs, histograms (including those of varying widths) and frequency polygons. In fact, you went a step further by studying certain numerical representatives of the ungrouped data, also called measures of central tendency, namely, mean, median and mode. In this chapter, we shall extend the study of these three measures, i.e., mean, median and mode from ungrouped data to that of grouped data. We shall also discuss the concept of cumulative frequency, the cumulative frequency distribution and how to draw cumulative frequency curves, called ogives.

14.2 Mean of Grouped Data

The mean (or average) of observations, as we know, is the sum of the values of all the observations divided by the total number of observations. From Class IX, recall that if $x_1, x_2, \ldots, x_n$ are observations with respective frequencies $f_1, f_2, \ldots, f_n$, then this means observation x_1 occurs f_1 times, x_2 occurs f_2 times, and so on.

Now, the sum of the values of all the observations $= f_1x_1 + f_2x_2 + \ldots + f_nx_n$, and the number of observations $= f_1 + f_2 + \ldots + f_n$.

So, the mean $\overline{x}$ of the data is given by

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + \ldots + f_n}$$

Recall that we can write this in short form by using the Greek letter Σ (capital sigma) which means summation. That is,

$$\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

which, more briefly, is written as $\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$, if it is understood that i varies from 1 to n.

Let us apply this formula to find the mean in the following example.

Example 1 : The marks obtained by 30 students of Class X of a certain school in a Mathematics paper consisting of 100 marks are presented in table below. Find the mean of the marks obtained by the students.

Marks obtained (x_i)	10	20	36	40	50	56	60	70	72	80	88	92	95
Number of students (f_i)	1	1	3	4	3	2	4	4	1	1	2	3	1

Solution: Recall that to find the mean marks, we require the product of each x_i with the corresponding frequency f_i. So, let us put them in a column as shown in Table 14.1.

Table 14.1

Marks obtained (x_i)	**Number of students (f_i)**	$f_i x_i$
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
Total	$\Sigma f_i = 30$	$\Sigma f_i x_i = 1779$

Now, $$\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1779}{30} = 59.3$$

Therefore, the mean marks obtained is 59.3.

In most of our real life situations, data is usually so large that to make a meaningful study it needs to be condensed as grouped data. So, we need to convert given ungrouped data into grouped data and devise some method to find its mean.

Let us convert the ungrouped data of Example 1 into grouped data by forming class-intervals of width, say 15. Remember that, while allocating frequencies to each class-interval, students falling in any upper class-limit would be considered in the next class, e.g., 4 students who have obtained 40 marks would be considered in the class-interval 40-55 and not in 25-40. With this convention in our mind, let us form a grouped frequency distribution table (see Table 14.2).

Table 14.2

Class interval	10 - 25	25 - 40	40 - 55	55 - 70	70 - 85	85 - 100
Number of students	2	3	7	6	6	6

Now, for each class-interval, we require a point which would serve as the representative of the whole class. *It is assumed that the frequency of each class-interval is centred around its mid-point*. So the *mid-point* (or *class mark*) of each class can be chosen to represent the observations falling in the class. Recall that we find the mid-point of a class (or its class mark) by finding the average of its upper and lower limits. That is,

$$\textbf{Class mark} = \frac{\textbf{Upper class limit + Lower class limit}}{\textbf{2}}$$

With reference to Table 14.2, for the class 10-25, the class mark is $\frac{10+25}{2}$, i.e., 17.5. Similarly, we can find the class marks of the remaining class intervals. We put them in Table 14.3. These class marks serve as our x_i's. Now, in general, for the ith class interval, we have the frequency f_i corresponding to the class mark x_i. We can now proceed to compute the mean in the same manner as in Example 1.

Table 14.3

Class interval	Number of students (f_i)	Class mark (x_i)	$f_i x_i$
10 - 25	2	17.5	35.0
25 - 40	3	32.5	97.5
40 - 55	7	47.5	332.5
55 - 70	6	62.5	375.0
70 - 85	6	77.5	465.0
85 - 100	6	92.5	555.0
Total	$\Sigma f_i = 30$		$\Sigma f_i x_i = 1860.0$

The sum of the values in the last column gives us $\Sigma f_i x_i$. So, the mean $\overline{x}$ of the given data is given by

$$\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1860.0}{30} = 62$$

This new method of finding the mean is known as the **Direct Method**.

We observe that Tables 14.1 and 14.3 are using the same data and employing the same formula for the calculation of the mean but the results obtained are different. Can you think why this is so, and which one is more accurate? The difference in the two values is because of the mid-point assumption in Table 14.3, 59.3 being the exact mean, while 62 an approximate mean.

Sometimes when the numerical values of x_i and f_i are large, finding the product of x_i and f_i becomes tedious and time consuming. So, for such situations, let us think of a method of reducing these calculations.

We can do nothing with the f_i's, but we can change each x_i to a smaller number so that our calculations become easy. How do we do this? What about subtracting a fixed number from each of these x_i's? Let us try this method.

The first step is to choose one among the x_i's as the *assumed mean*, and denote it by 'a'. Also, to further reduce our calculation work, we may take 'a' to be that x_i which lies in the centre of $x_1, x_2, \ldots, x_n$. So, we can choose $a = 47.5$ or $a = 62.5$. Let us choose $a = 47.5$.

The next step is to find the difference d_i between a and each of the x_i's, that is, the **deviation** of 'a' from each of the x_i's.

i.e., $$d_i = x_i - a = x_i - 47.5$$

The third step is to find the product of d_i with the corresponding f_i, and take the sum of all the $f_i d_i$'s. The calculations are shown in Table 14.4.

Table 14.4

Class interval	Number of students (f_i)	Class mark (x_i)	$d_i = x_i - 47.5$	$f_i d_i$
10 - 25	2	17.5	–30	–60
25 - 40	3	32.5	–15	–45
40 - 55	7	47.5	0	0
55 - 70	6	62.5	15	90
70 - 85	6	77.5	30	180
85 - 100	6	92.5	45	270
Total	$\Sigma f_i = 30$			$\Sigma f_i d_i = 435$

So, from Table 14.4, the mean of the deviations, $\bar{d} = \dfrac{\Sigma f_i d_i}{\Sigma f_i}$.

Now, let us find the relation between $\bar{d}$ and $\bar{x}$.

Since in obtaining d_i, we subtracted 'a' from each x_i, so, in order to get the mean $\bar{x}$, we need to add 'a' to $\bar{d}$. This can be explained mathematically as:

Mean of deviations, $\bar{d} = \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

So,
$$\bar{d} = \frac{\Sigma f_i (x_i - a)}{\Sigma f_i}$$
$$= \frac{\Sigma f_i x_i}{\Sigma f_i} - \frac{\Sigma f_i a}{\Sigma f_i}$$
$$= \bar{x} - a\frac{\Sigma f_i}{\Sigma f_i}$$
$$= \bar{x} - a$$

So, $\bar{x} = a + \bar{d}$

i.e., $\bar{x} = a + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

Substituting the values of a, $\Sigma f_i d_i$ and Σf_i from Table 14.4, we get

$$\overline{x} = 47.5 + \frac{435}{30} = 47.5 + 14.5 = 62.$$

Therefore, the mean of the marks obtained by the students is 62.

The method discussed above is called the **Assumed Mean Method**.

Activity 1 : From the Table 14.3 find the mean by taking each of x_i (i.e., 17.5, 32.5, and so on) as 'a'. What do you observe? You will find that the mean determined in each case is the same, i.e., 62. (Why ?)

So, we can say that the value of the mean obtained does not depend on the choice of 'a'.

Observe that in Table 14.4, the values in Column 4 are all multiples of 15. So, if we divide the values in the entire Column 4 by 15, we would get smaller numbers to multiply with f_i. (Here, 15 is the class size of each class interval.)

So, let $u_i = \dfrac{x_i - a}{h}$, where a is the assumed mean and h is the class size.

Now, we calculate u_i in this way and continue as before (i.e., find $f_i u_i$ and then $\Sigma f_i u_i$). Taking $h = 15$, let us form Table 14.5.

Table 14.5

Class interval	f_i	x_i	$d_i = x_i - a$	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$
10 - 25	2	17.5	–30	–2	–4
25 - 40	3	32.5	–15	–1	–3
40 - 55	7	47.5	0	0	0
55 - 70	6	62.5	15	1	6
70 - 85	6	77.5	30	2	12
85 - 100	6	92.5	45	3	18
Total	$\Sigma f_i = 30$				$\Sigma f_i u_i = 29$

Let $$\overline{u} = \frac{\Sigma f_i u_i}{\Sigma f_i}$$

Here, again let us find the relation between $\overline{u}$ and $\overline{x}$.

We have, $$u_i = \frac{x_i - a}{h}$$

Therefore, $$\bar{u} = \frac{\Sigma f_i \frac{(x_i - a)}{h}}{\Sigma f_i} = \frac{1}{h}\left[\frac{\Sigma f_i x_i - a\Sigma f_i}{\Sigma f_i}\right]$$

$$= \frac{1}{h}\left[\frac{\Sigma f_i x_i}{\Sigma f_i} - a\frac{\Sigma f_i}{\Sigma f_i}\right]$$

$$= \frac{1}{h}[\bar{x} - a]$$

So, $$h\bar{u} = \bar{x} - a$$

i.e., $$\bar{x} = a + h\bar{u}$$

So, $$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

Now, substituting the values of a, h, $\Sigma f_i u_i$ and Σf_i from Table 14.5, we get

$$\bar{x} = 47.5 + 15 \times \left(\frac{29}{30}\right)$$

$$= 47.5 + 14.5 = 62$$

So, the mean marks obtained by a student is 62.

The method discussed above is called the **Step-deviation** method.

We note that :

- the step-deviation method will be convenient to apply if all the d_i's have a common factor.
- The mean obtained by all the three methods is the same.
- The assumed mean method and step-deviation method are just simplified forms of the direct method.
- The formula $\bar{x} = a + h\bar{u}$ still holds if a and h are not as given above, but are any non-zero numbers such that $u_i = \dfrac{x_i - a}{h}$.

Let us apply these methods in another example.

Example 2 : The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean percentage of female teachers by all the three methods discussed in this section.

Percentage of female teachers	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of States/U.T.	6	11	7	4	4	2	1

Source : *Seventh All India School Education Survey conducted by NCERT*

Solution : Let us find the class marks, x_i, of each class, and put them in a column (see Table 14.6):

Table 14.6

Percentage of female teachers	**Number of States /U.T. (f_i)**	x_i
15 - 25	6	20
25 - 35	11	30
35 - 45	7	40
45 - 55	4	50
55 - 65	4	60
65 - 75	2	70
75 - 85	1	80

Here we take $a = 50$, $h = 10$, then $d_i = x_i - 50$ and $u_i = \frac{x_i - 50}{10}$.

We now find d_i and u_i and put them in Table 14.7.

Table 14.7

Percentage of female teachers	Number of states/U.T. (f_i)	x_i	$d_i = x_i - 50$	$u_i = \frac{x_i - 50}{10}$	$f_i x_i$	$f_i d_i$	$f_i u_i$
15 - 25	6	20	–30	–3	120	–180	–18
25 - 35	11	30	–20	–2	330	–220	–22
35 - 45	7	40	–10	–1	280	–70	–7
45 - 55	4	50	0	0	200	0	0
55 - 65	4	60	10	1	240	40	4
65 - 75	2	70	20	2	140	40	4
75 - 85	1	80	30	3	80	30	3
Total	**35**				**1390**	**–360**	**–36**

From the table above, we obtain $\Sigma f_i = 35$, $\Sigma f_i x_i = 1390$,

$$\Sigma f_i d_i = -360, \quad \Sigma f_i u_i = -36.$$

Using the direct method, $\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1390}{35} = 39.71$

Using the assumed mean method,

$$\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 50 + \frac{(-360)}{35} = 39.71$$

Using the step-deviation method,

$$\overline{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h = 50 + \left(\frac{-36}{35}\right) \times 10 = 39.71$$

Therefore, the mean percentage of female teachers in the primary schools of rural areas is 39.71.

Remark : The result obtained by all the three methods is the same. So the choice of method to be used depends on the numerical values of x_i and f_i. If x_i and f_i are sufficiently small, then the direct method is an appropriate choice. If x_i and f_i are numerically large numbers, then we can go for the assumed mean method or step-deviation method. If the class sizes are unequal, and x_i are large numerically, we can still apply the step-deviation method by taking h to be a suitable divisor of all the d_i's.

Example 3 : The distribution below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean number of wickets by choosing a suitable method. What does the mean signify?

Number of wickets	20 - 60	60 - 100	100 - 150	150 - 250	250 - 350	350 - 450
Number of bowlers	7	5	16	12	2	3

Solution : Here, the class size varies, and the x_is are large. Let us still apply the step-deviation method with $a = 200$ and $h = 20$. Then, we obtain the data as in Table 14.8.

Table 14.8

Number of wickets taken	**Number of bowlers (f_i)**	x_i	$d_i = x_i - 200$	$u_i = \frac{d_i}{20}$	$u_i f_i$
20 - 60	7	40	–160	–8	–56
60 - 100	5	80	–120	–6	–30
100 - 150	16	125	–75	–3.75	–60
150 - 250	12	200	0	0	0
250 - 350	2	300	100	5	10
350 - 450	3	400	200	10	30
Total	**45**				**–106**

So, $\bar{u} = \frac{-106}{45}$. Therefore, $\bar{x} = 200 + 20\left(\frac{-106}{45}\right) = 200 - 47.11 = 152.89$.

This tells us that, on an average, the number of wickets taken by these 45 bowlers in one-day cricket is 152.89.

Now, let us see how well you can apply the concepts discussed in this section!

Activity 2 :

Divide the students of your class into three groups and ask each group to do one of the following activities.

1. Collect the marks obtained by all the students of your class in Mathematics in the latest examination conducted by your school. Form a grouped frequency distribution of the data obtained.
2. Collect the daily maximum temperatures recorded for a period of 30 days in your city. Present this data as a grouped frequency table.
3. Measure the heights of all the students of your class (in cm) and form a grouped frequency distribution table of this data.

After all the groups have collected the data and formed grouped frequency distribution tables, the groups should find the mean in each case by the method which they find appropriate.

EXERCISE 14.1

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14
Number of houses	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

2. Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages (in ₹)	500 - 520	520 -540	540 - 560	560 - 580	580 -600
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs 18. Find the missing frequency f.

Daily pocket allowance (in ₹)	11 - 13	13 - 15	15 - 17	17 - 19	19 - 21	21 - 23	23 - 25
Number of children	7	6	9	13	f	5	4

4. Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method.

Number of heartbeats per minute	65 - 68	68 - 71	71 - 74	74 - 77	77 - 80	80 - 83	83 - 86
Number of women	2	4	3	8	7	4	2

5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50 - 52	53 - 55	56 - 58	59 - 61	62 - 64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

6. The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in ₹)	100 - 150	150 - 200	200 - 250	250 - 300	300 - 350
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO_2 (in ppm)	**Frequency**
0.00 - 0.04	4
0.04 - 0.08	9
0.08 - 0.12	9
0.12 - 0.16	2
0.16 - 0.20	4
0.20 - 0.24	2

Find the mean concentration of SO_2 in the air.

8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0 - 6	6 - 10	10 - 14	14 - 20	20 - 28	28 - 38	38 - 40
Number of students	11	10	7	4	4	3	1

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 - 55	55 - 65	65 - 75	75 - 85	85 - 95
Number of cities	3	10	11	8	3

14.3 Mode of Grouped Data

Recall from Class IX, a mode is that value among the observations which occurs most often, that is, the value of the observation having the maximum frequency. Further, we discussed finding the mode of ungrouped data. Here, we shall discuss ways of obtaining a mode of grouped data. It is possible that more than one value may have the same maximum frequency. In such situations, the data is said to be multimodal. Though grouped data can also be multimodal, we shall restrict ourselves to problems having a single mode only.

Let us first recall how we found the mode for ungrouped data through the following example.

Example 4 : The wickets taken by a bowler in 10 cricket matches are as follows:

2 6 4 5 0 2 1 3 2 3

Find the mode of the data.

Solution : Let us form the frequency distribution table of the given data as follows:

Number of wickets	0	1	2	3	4	5	6
Number of matches	1	1	3	2	1	1	1

Clearly, 2 is the number of wickets taken by the bowler in the maximum number (i.e., 3) of matches. So, the mode of this data is 2.

In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. Here, we can only locate a class with the maximum frequency, called the **modal class**. The mode is a value inside the modal class, and is given by the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where l = lower limit of the modal class,

h = size of the class interval (assuming all class sizes to be equal),

f_1 = frequency of the modal class,

f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class.

Let us consider the following examples to illustrate the use of this formula.

Example 5 : A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household:

Family size	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11
Number of families	7	8	2	2	1

Find the mode of this data.

Solution : Here the maximum class frequency is 8, and the class corresponding to this frequency is 3 – 5. So, the modal class is 3 – 5.

Now

modal class = 3 – 5, lower limit (l) of modal class = 3, class size (h) = 2

frequency (f_1) of the modal class = 8,

frequency (f_0) of class preceding the modal class = 7,

frequency (f_2) of class succeeding the modal class = 2.

Now, let us substitute these values in the formula :

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 3 + \left(\frac{8 - 7}{2 \times 8 - 7 - 2} \right) \times 2 = 3 + \frac{2}{7} = 3.286$$

Therefore, the mode of the data above is 3.286.

Example 6 : The marks distribution of 30 students in a mathematics examination are given in Table 14.3 of Example 1. Find the mode of this data. Also compare and interpret the mode and the mean.

Solution : Refer to Table 14.3 of Example 1. Since the maximum number of students (i.e., 7) have got marks in the interval 40 - 55, the modal class is 40 - 55. Therefore,

the lower limit (l) of the modal class = 40,

the class size (h) = 15,

the frequency (f_1) of modal class = 7,

the frequency (f_0) of the class preceding the modal class = 3,

the frequency (f_2) of the class succeeding the modal class = 6.

Now, using the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h,$$

we get

$$\text{Mode} = 40 + \left(\frac{7 - 3}{14 - 6 - 3} \right) \times 15 = 52$$

So, the mode marks is 52.

Now, from Example 1, you know that the mean marks is 62.

So, the maximum number of students obtained 52 marks, while on an average a student obtained 62 marks.

Remarks :

1. In Example 6, the mode is less than the mean. But for some other problems it may be equal or more than the mean also.

2. It depends upon the demand of the situation whether we are interested in finding the average marks obtained by the students or the average of the marks obtained by most

of the students. In the first situation, the mean is required and in the second situation, the mode is required.

Activity 3 : Continuing with the same groups as formed in Activity 2 and the situations assigned to the groups. Ask each group to find the mode of the data. They should also compare this with the mean, and interpret the meaning of both.

Remark : The mode can also be calculated for grouped data with unequal class sizes. However, we shall not be discussing it.

EXERCISE 14.2

1. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components :

Lifetimes (in hours)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure :

Expenditure (in ₹)	Number of families
1000 - 1500	24
1500 - 2000	40
2000 - 2500	33
2500 - 3000	28
3000 - 3500	30
3500 - 4000	22
4000 - 4500	16
4500 - 5000	7

4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students per teacher	Number of states / U.T.
15 - 20	3
20 - 25	8
25 - 30	9
30 - 35	10
35 - 40	3
40 - 45	0
45 - 50	0
50 - 55	2

5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	Number of batsmen
3000 - 4000	4
4000 - 5000	18
5000 - 6000	9
6000 - 7000	7
7000 - 8000	6
8000 - 9000	3
9000 - 10000	1
10000 - 11000	1

Find the mode of the data.

6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data :

Number of cars	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	7	14	13	12	20	11	15	8

14.4 Median of Grouped Data

As you have studied in Class IX, the median is a measure of central tendency which gives the value of the middle-most observation in the data. Recall that for finding the median of ungrouped data, we first arrange the data values of the observations in ascending order. Then, if n is odd, the median is the $\left(\frac{n+1}{2}\right)$th observation. And, if n is even, then the median will be the average of the $\frac{n}{2}$th and the $\left(\frac{n}{2}+1\right)$th observations.

Suppose, we have to find the median of the following data, which gives the marks, out of 50, obtained by 100 students in a test :

Marks obtained	20	29	28	33	42	38	43	25
Number of students	6	28	24	15	2	4	1	20

First, we arrange the marks in ascending order and prepare a frequency table as follows :

Table 14.9

Marks obtained	**Number of students (Frequency)**
20	6
25	20
28	24
29	28
33	15
38	4
42	2
43	1
Total	**100**

Here $n = 100$, which is even. The median will be the average of the $\frac{n}{2}$ th and the $\left(\frac{n}{2}+1\right)$th observations, i.e., the 50th and 51st observations. To find these observations, we proceed as follows:

Table 14.10

Marks obtained	Number of students
20	6
upto 25	6 + 20 = 26
upto 28	26 + 24 = 50
upto 29	50 + 28 = 78
upto 33	78 + 15 = 93
upto 38	93 + 4 = 97
upto 42	97 + 2 = 99
upto 43	99 + 1 = 100

Now we add another column depicting this information to the frequency table above and name it as *cumulative frequency column*.

Table 14.11

Marks obtained	Number of students	Cumulative frequency
20	6	6
25	20	26
28	24	50
29	28	78
33	15	93
38	4	97
42	2	99
43	1	100

From the table above, we see that:

50th observaton is 28 (Why?)

51st observation is 29

So, $\text{Median} = \dfrac{28 + 29}{2} = 28.5$

Remark : The part of Table 14.11 consisting Column 1 and Column 3 is known as *Cumulative Frequency Table*. The median marks 28.5 conveys the information that about 50% students obtained marks less than 28.5 and another 50% students obtained marks more than 28.5.

Now, let us see how to obtain the median of grouped data, through the following situation.

Consider a grouped frequency distribution of marks obtained, out of 100, by 53 students, in a certain examination, as follows:

Table 14.12

Marks	Number of students
0 - 10	5
10 - 20	3
20 - 30	4
30 - 40	3
40 - 50	3
50 - 60	4
60 - 70	7
70 - 80	9
80 - 90	7
90 - 100	8

From the table above, try to answer the following questions:

How many students have scored marks less than 10? The answer is clearly 5.

How many students have scored less than 20 marks? Observe that the number of students who have scored less than 20 include the number of students who have scored marks from 0 - 10 as well as the number of students who have scored marks from 10 - 20. So, the total number of students with marks less than 20 is 5 + 3, i.e., 8. We say that the cumulative frequency of the class 10-20 is 8.

Similarly, we can compute the cumulative frequencies of the other classes, i.e., the number of students with marks less than 30, less than 40, . . ., less than 100. We give them in Table 14.13 given below:

Table 14.13

Marks obtained	Number of students (Cumulative frequency)
Less than 10	5
Less than 20	5 + 3 = 8
Less than 30	8 + 4 = 12
Less than 40	12 + 3 = 15
Less than 50	15 + 3 = 18
Less than 60	18 + 4 = 22
Less than 70	22 + 7 = 29
Less than 80	29 + 9 = 38
Less than 90	38 + 7 = 45
Less than 100	45 + 8 = 53

The distribution given above is called the *cumulative frequency distribution of the less than type*. Here 10, 20, 30, . . . 100, are the upper limits of the respective class intervals.

We can similarly make the table for the number of students with scores, more than or equal to 0, more than or equal to 10, more than or equal to 20, and so on. From Table 14.12, we observe that all 53 students have scored marks more than or equal to 0. Since there are 5 students scoring marks in the interval 0 - 10, this means that there are 53 – 5 = 48 students getting more than or equal to 10 marks. Continuing in the same manner, we get the number of students scoring 20 or above as 48 – 3 = 45, 30 or above as 45 – 4 = 41, and so on, as shown in Table 14.14.

Table 14.14

Marks obtained	Number of students (Cumulative frequency)
More than or equal to 0	53
More than or equal to 10	53 – 5 = 48
More than or equal to 20	48 – 3 = 45
More than or equal to 30	45 – 4 = 41
More than or equal to 40	41 – 3 = 38
More than or equal to 50	38 – 3 = 35
More than or equal to 60	35 – 4 = 31
More than or equal to 70	31 – 7 = 24
More than or equal to 80	24 – 9 = 15
More than or equal to 90	15 – 7 = 8

The table above is called a *cumulative frequency distribution of the more than type*. Here 0, 10, 20, . . ., 90 give the lower limits of the respective class intervals.

Now, to find the median of grouped data, we can make use of any of these cumulative frequency distributions.

Let us combine Tables 14.12 and 14.13 to get Table 14.15 given below:

Table 14.15

Marks	Number of students (f)	Cumulative frequency (cf)
0 - 10	5	5
10 - 20	3	8
20 - 30	4	12
30 - 40	3	15
40 - 50	3	18
50 - 60	4	22
60 - 70	7	29
70 - 80	9	38
80 - 90	7	45
90 - 100	8	53

Now in a grouped data, we may not be able to find the middle observation by looking at the cumulative frequencies as the middle observation will be some value in a class interval. It is, therefore, necessary to find the value inside a class that divides the whole distribution into two halves. But which class should this be?

To find this class, we find the cumulative frequencies of all the classes and $\frac{n}{2}$. We now locate the class whose cumulative frequency is greater than (and nearest to) $\frac{n}{2}$. This is called the *median class*. In the distribution above, $n = 53$. So, $\frac{n}{2} = 26.5$. Now 60 – 70 is the class whose cumulative frequency 29 is greater than (and nearest to) $\frac{n}{2}$, i.e., 26.5.

Therefore, 60 – 70 is the **median class**.

After finding the median class, we use the following formula for calculating the median.

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h,$$

where
- l = lower limit of median class,
- n = number of observations,
- cf = cumulative frequency of class preceding the median class,
- f = frequency of median class,
- h = class size (assuming class size to be equal).

Substituting the values $\frac{n}{2} = 26.5$, $l = 60$, cf $= 22$, $f = 7$, $h = 10$ in the formula above, we get

$$\text{Median} = 60 + \left(\frac{26.5 - 22}{7}\right) \times 10$$

$$= 60 + \frac{45}{7}$$

$$= 66.4$$

So, about half the students have scored marks less than 66.4, and the other half have scored marks more than 66.4.

Example 7 : A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained:

Height (in cm)	Number of girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Find the median height.

Solution : To calculate the median height, we need to find the class intervals and their corresponding frequencies.

The given distribution being of the *less than type*, 140, 145, 150, . . ., 165 give the upper limits of the corresponding class intervals. So, the classes should be below 140, 140 - 145, 145 - 150, . . ., 160 - 165. Observe that from the given distribution, we find that there are 4 girls with height less than 140, i.e., the frequency of class interval below 140 is 4. Now, there are 11 girls with heights less than 145 and 4 girls with height less than 140. Therefore, the number of girls with height in the interval 140 - 145 is 11 – 4 = 7. Similarly, the frequency of 145 - 150 is 29 – 11 = 18, for 150 - 155, it is 40 – 29 = 11, and so on. So, our frequency distribution table with the given cumulative frequencies becomes:

Table 14.16

Class intervals	Frequency	Cumulative frequency
Below 140	4	4
140 - 145	7	11
145 - 150	18	29
150 - 155	11	40
155 - 160	6	46
160 - 165	5	51

Now $n = 51$. So, $\frac{n}{2} = \frac{51}{2} = 25.5$. This observation lies in the class 145 - 150. Then,

l (the lower limit) = 145,

cf (the cumulative frequency of the class preceding 145 - 150) = 11,

f (the frequency of the median class 145 - 150) = 18,

h (the class size) = 5.

Using the formula, Median $= l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h$, we have

$$\text{Median} = 145 + \left(\frac{25.5 - 11}{18}\right) \times 5$$

$$= 145 + \frac{72.5}{18} = 149.03.$$

So, the median height of the girls is 149.03 cm.

This means that the height of about 50% of the girls is less than this height, and 50% are taller than this height.

Example 8 : The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class interval	Frequency
0 - 100	2
100 - 200	5
200 - 300	x
300 - 400	12
400 - 500	17
500 - 600	20
600 - 700	y
700 - 800	9
800 - 900	7
900 - 1000	4

Solution :

Class intervals	Frequency	Cumulative frequency
0 - 100	2	2
100 - 200	5	7
200 - 300	x	$7 + x$
300 - 400	12	$19 + x$
400 - 500	17	$36 + x$
500 - 600	20	$56 + x$
600 - 700	y	$56 + x + y$
700 - 800	9	$65 + x + y$
800 - 900	7	$72 + x + y$
900 - 1000	4	$76 + x + y$

It is given that $n = 100$

So, $76 + x + y = 100$, i.e., $x + y = 24$ (1)

The median is 525, which lies in the class 500 – 600

So, $l = 500$, $f = 20$, $\text{cf} = 36 + x$, $h = 100$

Using the formula : $$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right)h, \text{ we get}$$

$$525 = 500 + \left(\frac{50 - 36 - x}{20}\right) \times 100$$

i.e., $525 - 500 = (14 - x) \times 5$

i.e., $25 = 70 - 5x$

i.e., $5x = 70 - 25 = 45$

So, $x = 9$

Therefore, from (1), we get $9 + y = 24$

i.e., $y = 15$

Now, that you have studied about all the three measures of central tendency, let us discuss **which measure would be best suited for a particular requirement**.

The mean is the most frequently used measure of central tendency because it takes into account all the observations, and lies between the extremes, i.e., the largest and the smallest observations of the entire data. It also enables us to compare two or more distributions. For example, by comparing the average (mean) results of students of different schools of a particular examination, we can conclude which school has a better performance.

However, extreme values in the data affect the mean. For example, the mean of classes having frequencies more or less the same is a good representative of the data. But, if one class has frequency, say 2, and the five others have frequency 20, 25, 20, 21, 18, then the mean will certainly not reflect the way the data behaves. So, in such cases, the mean is not a good representative of the data.

In problems where individual observations are not important, and we wish to find out a 'typical' observation, the median is more appropriate, e.g., finding the typical productivity rate of workers, average wage in a country, etc. These are situations where extreme values may be there. So, rather than the mean, we take the median as a better measure of central tendency.

In situations which require establishing the most frequent value or most popular item, the mode is the best choice, e.g., to find the most popular T.V. programme being watched, the consumer item in greatest demand, the colour of the vehicle used by most of the people, etc.

Remarks :

1. There is a empirical relationship between the three measures of central tendency :

$$\textbf{3 Median = Mode + 2 Mean}$$

2. The median of grouped data with unequal class sizes can also be calculated. However, we shall not discuss it here.

EXERCISE 14.3

1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 - 85	4
85 - 105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	8
185 - 205	4

2. If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	Frequency
0 - 10	5
10 - 20	x
20 - 30	20
30 - 40	15
40 - 50	y
50 - 60	5
Total	60

3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

4. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table :

Length (in mm)	Number of leaves
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

Find the median length of the leaves.

(**Hint :** The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 - 126.5, 126.5 - 135.5, . . ., 171.5 - 180.5.)

5. The following table gives the distribution of the life time of 400 neon lamps :

Life time (in hours)	Number of lamps
1500 - 2000	14
2000 - 2500	56
2500 - 3000	60
3000 - 3500	86
3500 - 4000	74
4000 - 4500	62
4500 - 5000	48

Find the median life time of a lamp.

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1 - 4	4 - 7	7 - 10	10 - 13	13 - 16	16 - 19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75
Number of students	2	3	8	6	6	3	2

14.5 Graphical Representation of Cumulative Frequency Distribution

As we all know, pictures speak better than words. A graphical representation helps us in understanding given data at a glance. In Class IX, we have represented the data through bar graphs, histograms and frequency polygons. Let us now represent a cumulative frequency distribution graphically.

For example, let us consider the cumulative frequency distribution given in Table 14.13.

Recall that the values 10, 20, 30, . . ., 100 are the upper limits of the respective class intervals. To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis), choosing a convenient scale. The scale may not be the same on both the axis. Let us now plot the points corresponding to the ordered pairs given by (upper limit, corresponding cumulative frequency), i.e., (10, 5), (20, 8), (30, 12), (40, 15), (50, 18), (60, 22), (70, 29), (80, 38), (90, 45), (100, 53) on a graph paper and join them by a free hand smooth curve. The curve we get is called a **cumulative frequency curve**, or an **ogive** (of the less than type). (See Fig. 14.1)

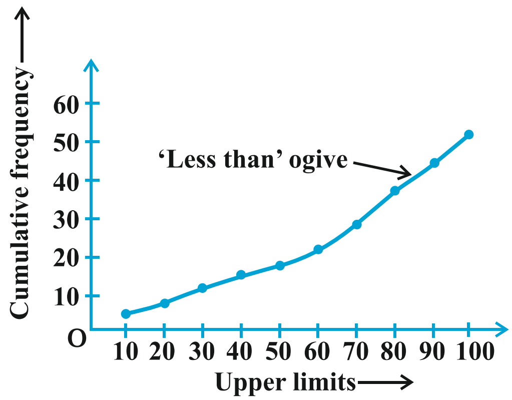


Fig. 14.1

The term 'ogive' is pronounced as 'ojeev' and is derived from the word **ogee**. An ogee is a shape consisting of a concave arc flowing into a convex arc, so forming an S-shaped curve with vertical ends. In architecture, the ogee shape is one of the characteristics of the 14th and 15th century Gothic styles.

Next, again we consider the cumulative frequency distribution given in Table 14.14 and draw its ogive (of the more than type).

Recall that, here 0, 10, 20, . . ., 90 are the lower limits of the respective class intervals 0 - 10, 10 - 20, . . ., 90 - 100. To represent 'the more than type' graphically, we plot the lower limits on the x-axis and the corresponding cumulative frequencies on the y-axis. Then we plot the points (lower limit, corresponding cumulative frequency), i.e., (0, 53), (10, 48), (20, 45), (30, 41), (40, 38), (50, 35), (60, 31), (70, 24), (80, 15), (90, 8), on a graph paper, and join them by a free hand smooth curve. The curve we get is a *cumulative frequency curve,* or an *ogive* (*of the more than type*). (See Fig. 14.2)

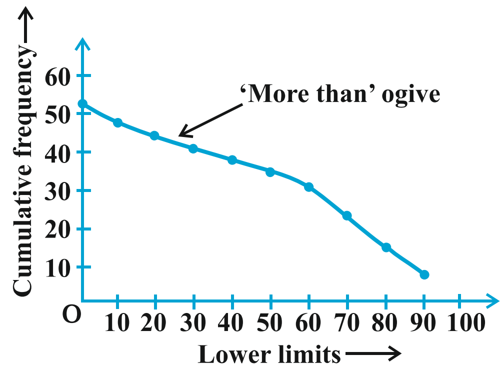


Fig. 14.2

Remark : Note that both the ogives (in Fig. 14.1 and Fig. 14.2) correspond to the same data, which is given in Table 14.12.

Now, are the ogives related to the median in any way? Is it possible to obtain the median from these two cumulative frequency curves corresponding to the data in Table 14.12? Let us see.

One obvious way is to locate $\frac{n}{2} = \frac{53}{2} = 26.5$ on the y-axis (see Fig. 14.3). From this point, draw a line parallel to the x-axis cutting the curve at a point. From this point, draw a perpendicular to the x-axis. The point of intersection of this perpendicular with the x-axis determines the median of the data (see Fig. 14.3).

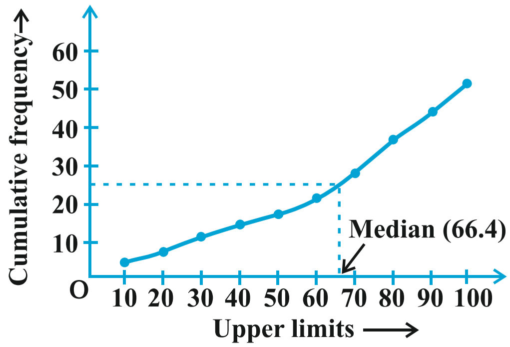


Fig. 14.3

Another way of obtaining the median is the following :

Draw both ogives (i.e., of the less than type and of the more than type) on the same axis. The two ogives will intersect each other at a point. From this point, if we draw a perpendicular on the x-axis, the point at which it cuts the x-axis gives us the median (see Fig. 14.4).

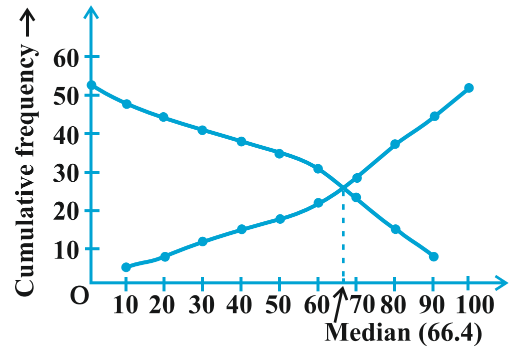


Fig. 14.4

Example 9 : The annual profits earned by 30 shops of a shopping complex in a locality give rise to the following distribution :

Profit (Rs in lakhs)	Number of shops (frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Draw both ogives for the data above. Hence obtain the median profit.

Solution : We first draw the coordinate axes, with lower limits of the profit along the horizontal axis, and the cumulative frequency along the vertical axes. Then, we plot the points (5, 30), (10, 28), (15, 16), (20, 14), (25, 10), (30, 7) and (35, 3). We join these points with a smooth curve to get the 'more than' ogive, as shown in Fig. 14.5.

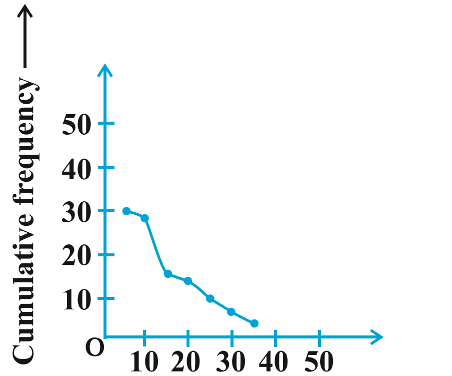


Fig. 14.5

Now, let us obtain the classes, their frequencies and the cumulative frequency from the table above.

Table 14.17

Classes	5 - 10	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35	35 - 40
No. of shops	2	12	2	4	3	4	3
Cumulative frequency	2	14	16	20	23	27	30

Using these values, we plot the points (10, 2), (15, 14), (20, 16), (25, 20), (30, 23), (35, 27), (40, 30) on the same axes as in Fig. 14.5 to get the 'less than' ogive, as shown in Fig. 14.6.

The abcissa of their point of intersection is nearly 17.5, which is the median. This can also be verified by using the formula. Hence, the median profit (in lakhs) is ₹ 17.5.

Remark : In the above examples, it may be noted that the class intervals were continuous. For drawing ogives, it should be ensured that the class intervals are continuous. (Also see constructions of histograms in Class IX)

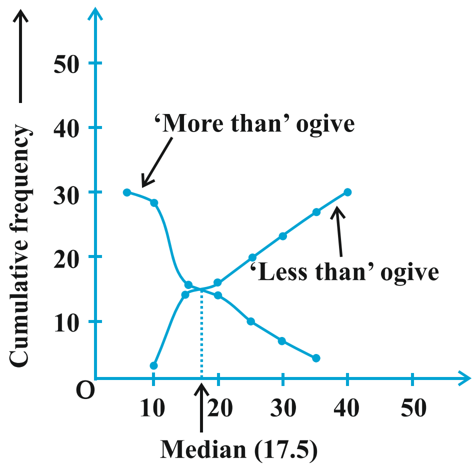


Fig. 14.6

EXERCISE 14.4

1. The following distribution gives the daily income of 50 workers of a factory.

Daily income (in ₹)	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

2. During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

3. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

14.6 Summary

In this chapter, you have studied the following points:

1. The mean for grouped data can be found by :

   (i) the direct method : $\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

(ii) the assumed mean method : $\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

(iii) the step deviation method : $\overline{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$,

with the assumption that the frequency of a class is centred at its mid-point, called its class mark.

2. The mode for grouped data can be found by using the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where symbols have their usual meanings.

3. The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class.

4. The median for grouped data is formed by using the formula:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h,$$

where symbols have their usual meanings.

5. Representing a cumulative frequency distribution graphically as a cumulative frequency curve, or an ogive of the less than type and of the more than type.

6. The median of grouped data can be obtained graphically as the x-coordinate of the point of intersection of the two ogives for this data.

A Note to the Reader

For calculating mode and median for grouped data, it should be ensured that the class intervals are continuous before applying the formulae. Same condition also apply for construction of an ogive. Further, in case of ogives, the scale may not be the same on both the axes.