

### Exercise 1.3

**1. Prove that  $\sqrt{5}$  is irrational.**

**Solutions:** Let us assume, that  $\sqrt{5}$  is rational number.

i.e.  $\sqrt{5} = x/y$  (where, x and y are co-primes)

$$y\sqrt{5} = x$$

Squaring both the sides, we get,

$$(y\sqrt{5})^2 = x^2$$

$$\Rightarrow 5y^2 = x^2 \dots\dots\dots (1)$$

Thus,  $x^2$  is divisible by 5, so x is also divisible by 5.

Let us say,  $x = 5k$ , for some value of k and substituting the value of x in equation (1), we get,

$$5y^2 = (5k)^2$$

$$\Rightarrow y^2 = 5k^2$$

$y^2$  is divisible by 5 it means y is divisible by 5.

Clearly, x and y are not co-primes. Thus, our assumption about  $\sqrt{5}$  is rational is incorrect.

Hence,  $\sqrt{5}$  is irrational number.

**2. Prove that  $3 + 2\sqrt{5}$  is irrational.**

**Solutions:** Let us assume  $3 + 2\sqrt{5}$  is rational.

Then we can find co-prime x and y ( $y \neq 0$ ) such that  $3 + 2\sqrt{5} = x/y$

Rearranging, we get,

$$2\sqrt{5} = \frac{x}{y} - 3$$

$$\sqrt{5} = \frac{1}{2} \left( \frac{x}{y} - 3 \right)$$

Since, x and y are integers, thus,

$$\frac{1}{2} \left( \frac{x}{y} - 3 \right) \text{ is a rational number.}$$

Therefore,  $\sqrt{5}$  is also a rational number. But this contradicts the fact that  $\sqrt{5}$  is irrational.

So, we conclude that  $3 + 2\sqrt{5}$  is irrational.

**3. Prove that the following are irrationals:**

(i)  $1/\sqrt{2}$

(ii)  $7\sqrt{5}$

(iii)  $6 + \sqrt{2}$

**Solutions:****(i)  $1/\sqrt{2}$** 

Let us assume  $1/\sqrt{2}$  is rational.

Then we can find co-prime  $x$  and  $y$  ( $y \neq 0$ ) such that  $1/\sqrt{2} = x/y$

Rearranging, we get,

$$\sqrt{2} = y/x$$

Since,  $x$  and  $y$  are integers, thus,  $\sqrt{2}$  is a rational number, which contradicts the fact that  $\sqrt{2}$  is irrational.

Hence, we can conclude that  $1/\sqrt{2}$  is irrational.

**(ii)  $7\sqrt{5}$** 

Let us assume  $7\sqrt{5}$  is a rational number.

Then we can find co-prime  $a$  and  $b$  ( $b \neq 0$ ) such that  $7\sqrt{5} = x/y$

Rearranging, we get,

$$\sqrt{5} = x/7y$$

Since,  $x$  and  $y$  are integers, thus,  $\sqrt{5}$  is a rational number, which contradicts the fact that  $\sqrt{5}$  is irrational.

Hence, we can conclude that  $7\sqrt{5}$  is irrational.

**(iii)  $6 + \sqrt{2}$** 

Let us assume  $6 + \sqrt{2}$  is a rational number.

Then we can find co-primes  $x$  and  $y$  ( $y \neq 0$ ) such that  $6 + \sqrt{2} = x/y$ .

Rearranging, we get,

$$\sqrt{2} = (x/y) - 6$$

Since,  $x$  and  $y$  are integers, thus  $(x/y) - 6$  is a rational number and therefore,  $\sqrt{2}$  is rational. This contradicts the fact that  $\sqrt{2}$  is an irrational number.

Hence, we can conclude that  $6 + \sqrt{2}$  is irrational.