Exercise: 12.2   **(Page No: 230)**

1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Solution:
It is given that the angle of the sector is 60°
We know that the area of sector = $(θ/360°) × πr^2$
∴ Area of the sector with angle 60° = $(60°/360°) × πr^2$ cm^2
= $(36/6)π$ cm^2
= $6 × 22/7$ cm^2 = $132/7$ cm^2

2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Solution:
Circumference of the circle, C = 22 cm (given)
It should be noted that a quadrant of a circle is a sector which is making an angle of 90°.
Let the radius of the circle = r
As C = 2πr = 22,
R = 22/2π cm = 7/2 cm
∴ Area of the quadrant = $(θ/360°) × πr^2$
Here, θ = 90°
So, A = $(90°/360°) × π r^2$ cm^2
= $(49/16) π$ cm^2
= 77/8 cm^2 = 9.6 cm^2

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Solution:
Length of minute hand = radius of the clock (circle)
∴ Radius (r) of the circle = 14 cm (given)
Angle swept by minute hand in 60 minutes = 360°
So, the angle swept by the minute hand in 5 minutes = 360° × 5/60 = 30°
We know,
Area of a sector = $(θ/360°) × πr^2$
Now, area of the sector making an angle of 30° = $(30°/360°) × πr^2$ cm^2
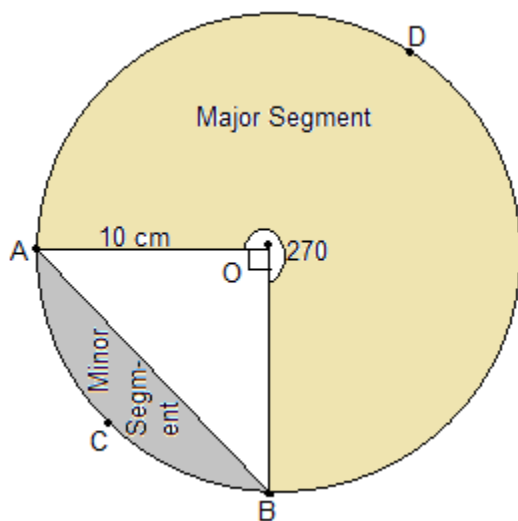= $(1/12) × π14^2$

= (49/3)×(22/7) cm²
= 154/3 cm²

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:
(i) minor segment
(ii) major sector. (Use π = 3.14)

Solution:

Here AB be the chord which is subtending an angle 90° at the center O.
It is given that the radius (r) of the circle = 10 cm

(i) Area of minor sector = (90/360°)×πr²
= (¼)×(22/7)×10²
Or, Area of minor sector = 78.5 cm²
Also, area of ΔAOB = ½×OB×OA
Here, OB and OA are the radii of the circle i.e. = 10 cm
So, area of ΔAOB = ½×10×10
= 50 cm²
Now, area of minor segment = area of minor sector - area of ΔAOB
= 78.5 - 50
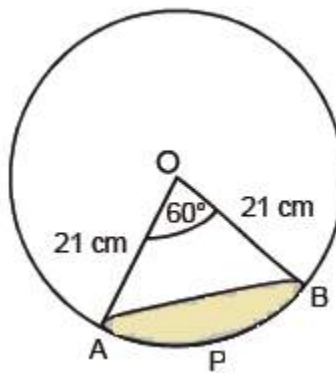= 28.5 cm²

(ii) Area of major sector = Area of circle - Area of minor sector

= (3.14×10²)-78.5

= 235.5 cm²

5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
(i) the length of the arc
(ii) area of the sector formed by the arc
(iii) area of the segment formed by the corresponding chord

Solution:

Given,
Radius = 21 cm
θ = 60°
(i) Length of an arc = θ/360°×Circumference(2πr)
∴ Length of an arc AB = (60°/360°)×2×(22/7)×21
= (1/6)×2×(22/7)×21
Or Arc AB Length = 22cm

(ii) It is given that the angle subtend by the arc = 60°
So, area of the sector making an angle of 60° = (60°/360°)×π r² cm²
= 441/6×22/7 cm²
Or, the area of the sector formed by the arc APB is 231 cm²

(iii) Area of segment APB = Area of sector OAPB - Area of ΔOAB
Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60°, ΔOAB is an equilateral triangle. So, its area will be √3/4×a² sq. Units.

Area of segment APB = 231-(√3/4)×(OA)²

= 231-(√3/4)×21²

Or, Area of segment APB = [231-(441×√3)/4] cm²

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)

Solution:

Given,

Radius = 15 cm

θ = 60°

So,
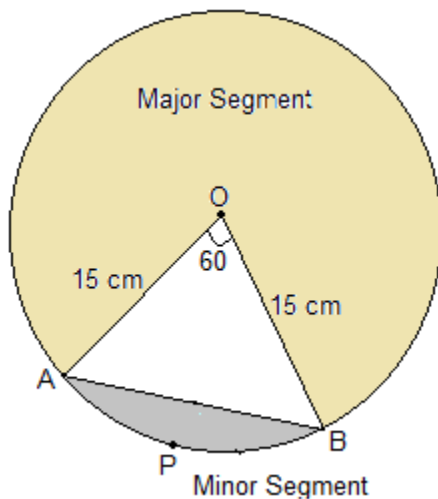
Area of sector OAPB = (60°/360°)×πr² cm²

= 225/6 πcm²

Now, ΔAOB is equilateral as two sides are the radii of the circle and hence equal and one angle is 60°

So, Area of ΔAOB = (√3/4) ×a²

Or, (√3/4) ×15²

∴ Area of ΔAOB = 97.31 cm²

Now, area of minor segment APB = Area of OAPB - Area of ΔAOB

Or, area of minor segment APB = ((225/6)π - 97.31) cm² = 20.43 cm²

And,

Area of major segment = Area of circle - Area of segment APB

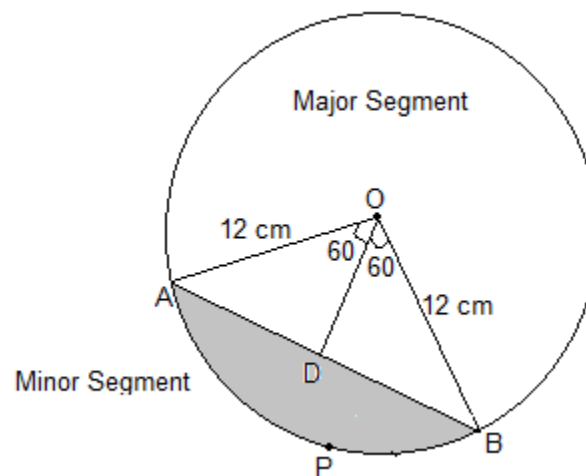Or, area of major segment = $(\pi\times15^2)$ - 20.4 = 686.06 cm²

7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and √3 = 1.73)

Solution:
Radius, r = 12 cm
Now, draw a perpendicular OD on chord AB and it will bisect chord AB.
So, AD = DB

Now, the area of the minor sector = $(\theta/360°)\times\pi r^2$
= $(120/360)\times(22/7)\times12^2$
= 150.72 cm²
Consider the ∆AOB,
∠ OAB = 180°-(90°+60°) = 30°
Now, cos 30° = AD/OA
⟹√3/2 = AD/12
Or, AD = 6√3 cm
We know OD bisects AB. So,
AB = 2×AD = 12√3 cm
Now, sin 30° = OD/OA
Or, ½ = OD/12
∴ OD = 6 cm
So, the area of ∆AOB = ½ × base × height
Here, base = AB = 12√3 and
Height = OD = 6

So, area of ΔAOB = ½×12√3×6 = 36√3 cm = 62.28 cm²

∴ Area of the corresponding Minor segment = Area of the Minor sector - Area of ΔAOB
= 150.72 cm²- 62.28 cm² = 88.44 cm²

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 12.11). Find
(i) the area of that part of the field in which the horse can graze.
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

Fig. 12.11

Solution:
As the horse is tied at one end of a square field, it will graze only a quarter (i.e. sector with θ = 90°) of the field with radius 5 m.
Here, the length of rope will be the radius of the circle i.e. r = 5 m
It is also known that the side of square field = 15 m

(i) Area of circle = πr² = 22/7 × 5² = 78.5 m²
Now, the area of the part of the field where the horse can graze = ¼ (the area of the circle) = 78.5/4 = 19.625 m²

(ii) If the rope is increased to 10 m,
Area of circle will be = πr² =22/7×10² = 314 m²
Now, the area of the part of the field where the horse can graze = ¼ (the area of the circle)
= 314/4 = 78.5 m²
∴ Increase in the grazing area = 78.5 m² - 19.625 m² = 58.875 m²

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 12.12. Find:

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

Fig. 12.12

Solution:

Diameter (D) = 35 mm

Total number of diameters to be considered= 5

Now, the total length of 5 diameters that would be required = 35×5 = 175

Circumference of the circle = 2πr

Or, C = πD = 22/7×35 = 110

Area of the circle = πr²

Or, A = (22/7)×(35/2)² = 1925/2 mm²

(i) Total length of silver wire required = Circumference of the circle + Length of 5 diameter

= 110+175 = 285 mm

(ii) Total Number of sectors in the brooch = 10

So, the area of each sector = total area of the circle/number of sectors

∴ Area of each sector = (1925/2)×1/10 = 385/4 mm²

10. An umbrella has 8 ribs which are equally spaced (see Fig. 12.13). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.
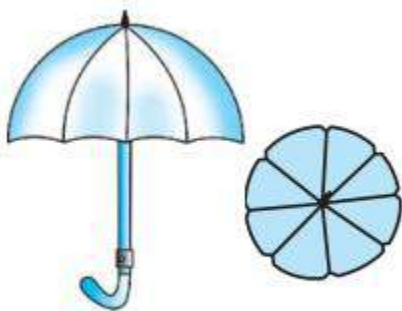

Fig. 12.13

Solution:

The radius (r) of the umbrella when flat = 45 cm

So, the area of the circle (A) = πr² = (22/7)×(45)² =6364.29 cm²

Total number of ribs (n) = 8

∴ The area between the two consecutive ribs of the umbrella = A/n

⟹ 6364.29/8 cm²

Or, The area between the two consecutive ribs of the umbrella = 795.5 cm²

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Solution:

Given,

Radius (r) = 25 cm

Sector angle (θ) = 115°

Since there are 2 blades,

The total area of the sector made by wiper = 2×(θ/360°)×π r²
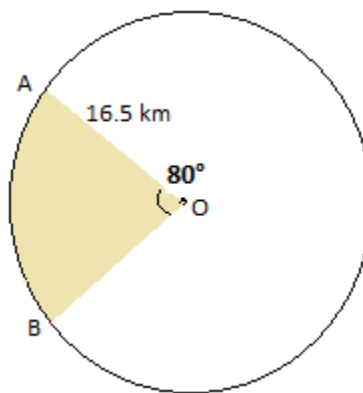
= 2×(115/360)×(22/7)×25²

= 2×158125/252 cm²

= 158125/126 = 1254.96 cm²

12. To warn ships for underwater rocks, a lighthouse spreads a red colored light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned.

(Use π = 3.14)

Solution:

Let O bet the position of Lighthouse.

Here the radius will be the distance over which light spreads.

Given, radius (r) = 16.5 km

Sector angle (θ) = 80°

Now, the total area of the sea over which the ships are warned = Area made by the sector

Or, Area of sector = $(\theta/360°) \times \pi r^2$

= $(80°/360°) \times \pi r^2$ km^2

= 189.97 km^2

13. A round table cover has six equal designs as shown in Fig. 12.14. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2 . (Use √3 = 1.7)
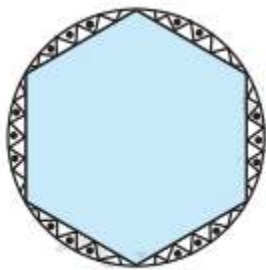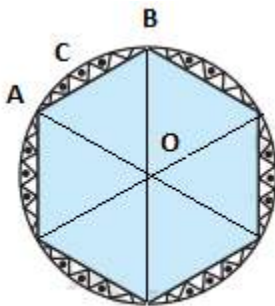

Fig. 12.14

Solution:

Fig. 12.14

Total number of equal designs = 6

∠AOB= 360°/6 = 60°

Radius of the cover = 28 cm

Cost of making design = ₹ 0.35 per cm^2

Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60°, ΔAOB is an equilateral triangle. So, its area will be (√3/4)×a^2 sq. units

Here, a = OA

∴ Area of equilateral ΔAOB = (√3/4)×28² = 333.2 cm²

Area of sector ACB = (60°/360°)×πr² cm²

= 410.66 cm²

So, area of a single design = area of sector ACB - area of ΔAOB

 = 410.66 cm² - 333.2 cm² = 77.46 cm²

∴ Area of 6 designs = 6×77.46 cm² = 464.76 cm²

So, total cost of making design = 464.76 cm² ×Rs.0.35 per cm²

= Rs. 162.66

14. Tick the correct solution in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) p/180 × 2πR

(B) p/180 × π R²

(C) p/360 × 2πR

(D) p/720 × 2πR²

Solution:

The area of a sector = (θ/360°)×πr²

Given, θ = p

So, area of sector = p/360×πR²

Multiplying and dividing by 2 simultaneously,

= (p/360)×2/2×πR²

  = (2p/720)×2πR²

So, option (D) is correct.