**Exercise: 12.3** **(Page No: 234)**

**1. Find the area of the shaded region in Fig. 12.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.**

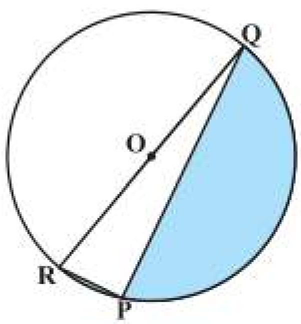



Fig. 12.19

**Solution:**

Here, ∠P is in the semi-circle and so,

∠P = 90°

So, it can be concluded that QR is hypotenuse of the circle and is equal to the diameter of the circle.

∴ QR = D

Using Pythagorean theorem,

$QR^2 = PR^2+PQ^2$

Or, $QR^2 = 7^2+24^2$

⇒ QR= 25 cm = Diameter

Hence, the radius of the circle = 25/2 cm

Now, the area of the semicircle = $(\pi R^2)/2$

= $(22/7)\times(25/2)\times(25/2)/2$ $cm^2$

= 13750/56 $cm^2$ = 245.54 $cm^2$

Also, area of the ΔPQR = ½×PR×PQ

=(½)×7×24 cm$^2$

= 84 cm$^2$

Hence, the area of the shaded region = 245.54 cm$^2$-84 cm$^2$

= 161.54 cm$^2$

**2. Find the area of the shaded region in Fig. 12.20, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠AOC = 40°.**

**Solution:**

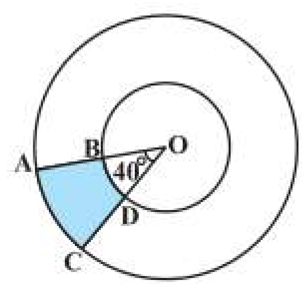



Fig. 12.20

Given,

Angle made by sector = 40°,

Radius the inner circle = r = 7 cm, and

Radius of the outer circle = R = 14 cm

We know,

Area of the sector = (θ/360°)×πr$^2$

So, Area of OAC = (40°/360°)×πr$^2$ cm$^2$

= 68.44 cm$^2$

Area of the sector OBD = (40°/360°)×πr$^2$ cm$^2$

$= (1/9)\times(22/7)\times7^2 = 17.11\ cm^2$

Now, area of the shaded region ABDC = Area of OAC - Area of the OBD

$= 68.44\ cm^2 - 17.11\ cm^2 = 51.33\ cm^2$

**3. Find the area of the shaded region in Fig. 12.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles.**

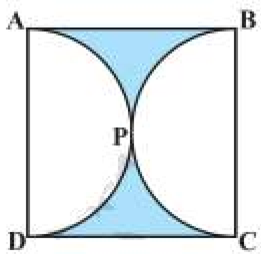



Fig. 12.21

**Solution:**

Side of the square ABCD (as given) = 14 cm

So, Area of ABCD = $a^2$

$= 14\times14\ cm^2 = 196\ cm^2$

We know that the side of the square = diameter of the circle = 14 cm

So, side of the square = diameter of the semicircle = 14 cm

∴ Radius of the semicircle = 7 cm

Now, area of the semicircle = $(\pi R^2)/2$

$= (22/7\times7\times7)/2\ cm^2$

$= 77\ cm^2$

∴ Area of two semicircles = $2\times77\ cm^2 = 154\ cm^2$

Hence, area of the shaded region = Area of the Square - Area of two semicircles

$= 196\ cm^2 - 154\ cm^2$

= 42 $cm^2$

**4. Find the area of the shaded region in Fig. 12.22, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.**

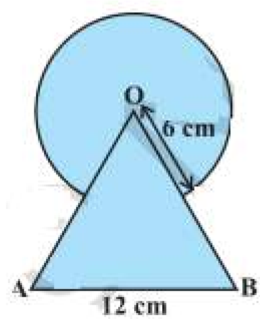


Fig. 12.22

**Solution:**

It is given that OAB is an equilateral triangle having each angle as 60°

Area of the sector is common in both.

Radius of the circle = 6 cm.

Side of the triangle = 12 cm.

Area of the equilateral triangle = $(\sqrt{3}/4)\ (OA)^2 = (\sqrt{3}/4)\times 12^2 = 36\sqrt{3}\ cm^2$

Area of the circle = $\pi R^2 = (22/7)\times 6^2 = 792/7\ cm^2$

Area of the sector making angle 60° = $(60°/360°)\times\pi r^2\ cm^2$

$= (1/6)\times(22/7)\times 6^2\ cm^2 = 132/7\ cm^2$

Area of the shaded region = Area of the equilateral triangle + Area of the circle - Area of the sector

$= 36\sqrt{3}\ cm^2 + 792/7\ cm^2 - 132/7\ cm^2$

$= (36\sqrt{3} + 660/7)\ cm^2$

**5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 12.23. Find the area of the remaining portion of the square.**

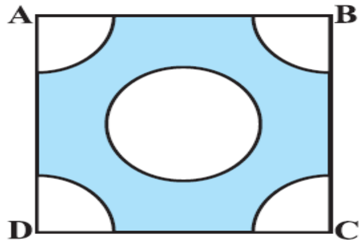



Fig. 12.23

**Solution:**

Side of the square = 4 cm

Radius of the circle = 1 cm

Four quadrant of a circle are cut from corner and one circle of radius are cut from middle.

Area of square = $(\text{side})^2 = 4^2 = 16\text{ cm}^2$

Area of the quadrant = $(\pi R^2)/4\text{ cm}^2 = (22/7)\times(1^2)/4 = 11/14\text{ cm}^2$

$\therefore$ Total area of the 4 quadrants = $4 \times(11/14)\text{ cm}^2 = 22/7\text{ cm}^2$

Area of the circle = $\pi R^2\text{ cm}^2 = (22/7\times1^2) = 22/7\text{ cm}^2$

Area of the shaded region = Area of square - (Area of the 4 quadrants + Area of the circle)

$= 16\text{ cm}^2-(22/7)\text{ cm}^2-(22/7)\text{ cm}^2$

$= 68/7\text{ cm}^2$

**6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 12.24. Find the area of the design.**

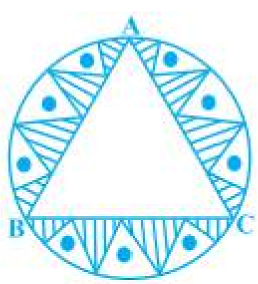



Fig. 12.24

**Solution:**

Radius of the circle = 32 cm

Draw a median AD of the triangle passing through the centre of the circle.

⇒ BD = AB/2

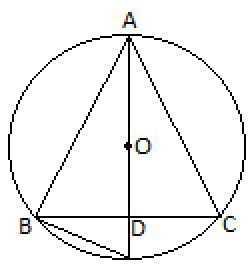
Since, AD is the median of the triangle

∴ AO = Radius of the circle = (2/3) AD

⇒ (2/3)AD = 32 cm

⇒ AD = 48 cm

In ΔADB,




By Pythagoras theorem,

$AB^2 = AD^2 + BD^2$

⇒ $AB^2 = 48^2 + (AB/2)^2$

⇒ $AB^2 = 2304 + AB^2/4$

⇒ $3/4\ (AB^2) = 2304$

⇒ $AB^2 = 3072$

⇒ AB= 32√3 cm

Area of ΔADB = $\sqrt{3}/4 \times (32\sqrt{3})^2\ cm^2 = 768\sqrt{3}\ cm^2$

Area of circle = $\pi R^2 = (22/7) \times 32 \times 32 = 22528/7\ cm^2$

Area of the design = Area of circle - Area of ΔADB

= $(22528/7 - 768\sqrt{3})\ cm^2$

**7. In Fig. 12.25, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.**

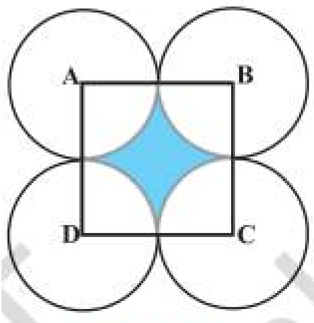



Fig. 12.25

**Solution:**

Side of square = 14 cm

Four quadrants are included in the four sides of the square.

∴ Radius of the circles = 14/2 cm = 7 cm

Area of the square ABCD = $14^2 = 196\ cm^2$

Area of the quadrant = $(\pi R^2)/4\ cm^2 = (22/7) \times 7^2/4\ cm^2$

$= 77/2\ cm^2$

Total area of the quadrant = $4 \times 77/2\ cm^2 = 154 cm^2$

Area of the shaded region = Area of the square ABCD - Area of the quadrant

$= 196\ cm^2 - 154\ cm^2$

$= 42\ cm^2$

**8. Fig. 12.26 depicts a racing track whose left and right ends are semicircular.**

**The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:**

**(i) the distance around the track along its inner edge**

**(ii) the area of the track.**

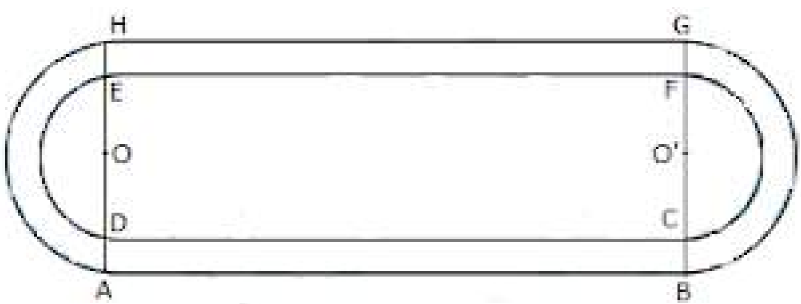

Fig. 12.26

**Solution:**

Width of the track = 10 m

Distance between two parallel lines = 60 m

Length of parallel tracks = 106 m




DE = CF = 60 m

Radius of inner semicircle, r = OD = O'C

= 60/2 m = 30 m

Radius of outer semicircle, R = OA = O'B

= 30+10 m = 40 m

Also, AB = CD = EF = GH = 106 m

Distance around the track along its inner edge = CD+EF+2×(Circumference of inner semicircle)

= 106+106+(2×πr) m = 212+(2×22/7×30) m

= 212+1320/7 m = 2804/7 m

Area of the track = Area of ABCD + Area EFGH + 2 × (area of outer semicircle) - 2 × (area of inner semicircle)

= (AB×CD)+(EF×GH)+2×($\pi r^2$/2) -2×($\pi R^2$/2) $m^2$

= (106×10)+(106×10)+2×π/2($r^2$-$R^2$) $m^2$

= 2120+22/7×70×10 $m^2$

= 4320 $m^2$

**9. In Fig. 12.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.**

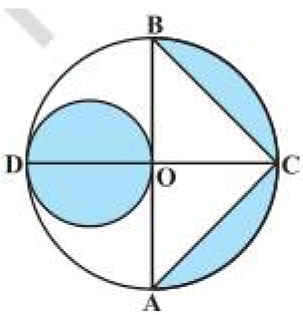



Fig. 12.27

**Solution:**

Radius of larger circle, R = 7 cm

Radius of smaller circle, r = 7/2 cm

Height of ΔBCA = OC = 7 cm

Base of ΔBCA = AB = 14 cm

Area of ΔBCA = 1/2 × AB × OC = (½)×7×14 = 49 $cm^2$

Area of larger circle = $\pi R^2$ = (22/7)×$7^2$ = 154 $cm^2$

Area of larger semicircle = 154/2 $cm^2$ = 77 $cm^2$

Area of smaller circle = $\pi r^2 = (22/7)\times(7/2)\times(7/2) = 77/2\ cm^2$

Area of the shaded region = Area of larger circle - Area of triangle - Area of larger semicircle + Area of smaller circle

Area of the shaded region = $(154-49-77+77/2)\ cm^2$

$= 133/2\ cm^2 = 66.5\ cm^2$

**10. The area of an equilateral triangle ABC is 17320.5 cm$^2$. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region. (Use π = 3.14 and √3 = 1.73205)**

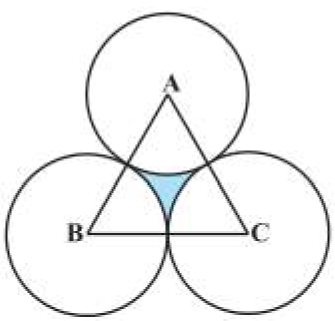


Fig. 12.28

**Solution:**

ABC is an equilateral triangle.

$\therefore \angle A = \angle B = \angle C = 60°$

There are three sectors each making 60°.

Area of ΔABC = 17320.5 cm$^2$

$\Rightarrow \sqrt{3}/4 \times(\text{side})^2 = 17320.5$

$\Rightarrow (\text{side})^2 = 17320.5\times4/1.73205$

$\Rightarrow (\text{side})^2 = 4\times10^4$

$\Rightarrow \text{side} = 200$ cm

Radius of the circles = 200/2 cm = 100 cm

Area of the sector = $(60°/360°)\times\pi r^2$ cm$^2$

= $1/6\times3.14\times(100)^2$ cm$^2$

= 15700/3cm$^2$

Area of 3 sectors = 3×15700/3 = 15700 cm$^2$

Thus, area of the shaded region = Area of equilateral triangle ABC - Area of 3 sectors

= 17320.5-15700 cm$^2$ = 1620.5 cm$^2$

**11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig. 12.29). Find the area of the remaining portion of the handkerchief.**

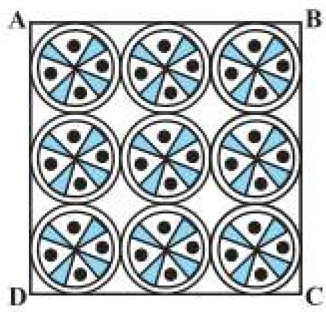


Fig. 12.29

**Solution:**

Number of circular designs = 9

Radius of the circular design = 7 cm

There are three circles in one side of square handkerchief.

∴ Side of the square = 3×diameter of circle = 3×14 = 42 cm

Area of the square = 42×42 cm$^2$ = 1764 cm$^2$

Area of the circle = $\pi r^2 = (22/7)\times7\times7$ = 154 cm$^2$

Total area of the design = 9×154 = 1386 cm$^2$

Area of the remaining portion of the handkerchief = Area of the square - Total area of the design = 1764 - 1386 = 378 $cm^2$

**12. In Fig. 12.30, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the**

**(i) quadrant OACB,**

**(ii) shaded region.**

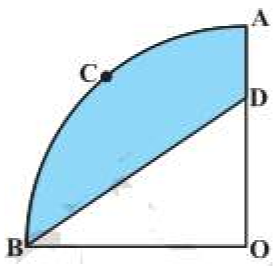



Fig. 12.30

**Solution:**

Radius of the quadrant = 3.5 cm = 7/2 cm

**(i)** Area of quadrant OACB = $(\pi R^2)/4$ $cm^2$

= $(22/7)\times(7/2)\times(7/2)/4$ $cm^2$

= 77/8 $cm^2$

**(ii)** Area of triangle BOD = $(½)\times(7/2)\times 2$ $cm^2$

= 7/2 $cm^2$

Area of shaded region = Area of quadrant - Area of triangle BOD

= (77/8)-(7/2) $cm^2$ = 49/8 $cm^2$

= 6.125 $cm^2$

**13. In Fig. 12.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)**

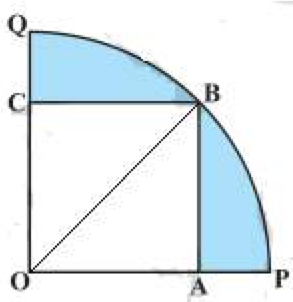


Fig. 12.31

**Solution:**

Side of square = OA = AB = 20 cm

Radius of the quadrant = OB

OAB is right angled triangle

By Pythagoras theorem in ΔOAB,

$OB^2 = AB^2+OA^2$

$\Rightarrow OB^2 = 20^2 +20^2$

$\Rightarrow OB^2 = 400+400$

$\Rightarrow OB^2 = 800$

$\Rightarrow OB= 20\sqrt{2}$ cm

Area of the quadrant = $(\pi R^2)/4$ $cm^2$ = $(3.14/4)\times(20\sqrt{2})^2$ $cm^2$ = $628 cm^2$

Area of the square = 20×20 = 400 $cm^2$

Area of the shaded region = Area of the quadrant - Area of the square

= 628-400 $cm^2$ = $228 cm^2$

**14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 12.32). If ∠AOB = 30°, find the area of the shaded region.**

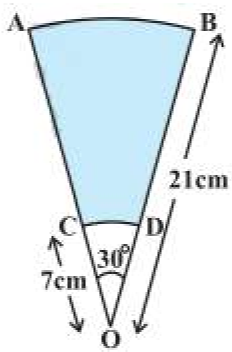



Fig. 12.32

**Solution:**

Radius of the larger circle, R = 21 cm

Radius of the smaller circle, r = 7 cm

Angle made by sectors of both concentric circles = 30°

Area of the larger sector = $(30°/360°)\times\pi R^2$ $cm^2$

= $(1/12)\times(22/7)\times21^2$ $cm^2$

= $231/2cm^2$

Area of the smaller circle = $(30°/360°)\times\pi r^2$ $cm^2$

= $1/12\times22/7\times7^2$ $cm^2$

=$77/6$ $cm^2$

Area of the shaded region = $(231/2) - (77/6)$ $cm^2$

= $616/6$ $cm^2$ = $308/3cm^2$

**15. In Fig. 12.33, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region**.

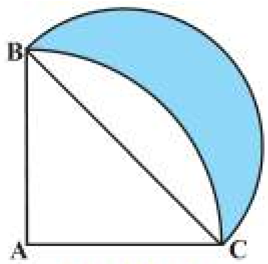



Fig. 12.33

**Solution:**

Radius of the quadrant ABC of circle = 14 cm

AB = AC = 14 cm

BC is diameter of semicircle.

ABC is right angled triangle.

By Pythagoras theorem in ΔABC,

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = 14^2 + 14^2$

$\Rightarrow BC = 14\sqrt{2}$ cm

Radius of semicircle = $14\sqrt{2}/2$ cm = $7\sqrt{2}$ cm

Area of ΔABC =( ½)×14×14 = 98 $cm^2$

Area of quadrant = (¼)×(22/7)×(14×14) = 154 $cm^2$

Area of the semicircle = (½)×(22/7)×$7\sqrt{2}$×$7\sqrt{2}$ = 154 $cm^2$

Area of the shaded region =Area of the semicircle + Area of ΔABC - Area of quadrant

= 154 +98-154 $cm^2$ = 98$cm^2$

**16. Calculate the area of the designed region in Fig. 12.34 common between the two quadrants of circles of radius 8 cm each.**

**Solution:**

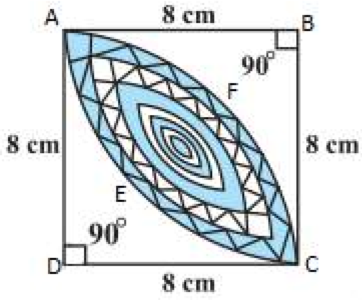



Fig. 12.34

AB = BC = CD = AD = 8 cm

Area of ΔABC = Area of ΔADC = $(½)\times8\times8 = 32\ cm^2$

Area of quadrant AECB = Area of quadrant AFCD = $(¼)\times22/7\times8^2$

$= 352/7\ cm^2$

Area of shaded region = (Area of quadrant AECB - Area of ΔABC) = (Area of quadrant AFCD - Area of ΔADC)

$= (352/7 - 32)+(352/7 - 32)\ cm^2$

$= 2\times(352/7-32)\ cm^2$

$= 256/7\ cm^2$