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1. In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

Solution:

We can write the given condition as;
Taxi fare for 1 km = 15
Taxi fare for first 2 kms = 15+8 = 23
Taxi fare for first 3 kms = 23+8 = 31
Taxi fare for first 4 kms = 31+8 = 39
And so on……

Thus, 15, 23, 31, 39 … forms an A.P. because every next term is 8 more than the preceding term.

(ii) The amount of air present in a cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.

Solution:

Let the volume of air in a cylinder, initially, be V litres.
In each stroke, the vacuum pump removes 1/4th of air remaining in the cylinder at a time. Or we can say, after every stroke, 1-1/4 = 3/4th part of air will remain.

Therefore, volumes will be V, $3V/4$, $(3V/4)^2$, $(3V/4)^3$...and so on
Clearly, we can see here, the adjacent terms of this series do not have the common difference between them. Therefore, this series is not an A.P.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

Solution:
We can write the given condition as;
Cost of digging a well for first metre = Rs.150
Cost of digging a well for first 2 metres = Rs.150+50 = Rs.200
Cost of digging a well for first 3 metres = Rs.200+50 = Rs.250
Cost of digging a well for first 4 metres =Rs.250+50 = Rs.300
And so on..

Clearly, 150, 200, 250, 300 … forms an A.P. with a common difference of 50 between each term.

(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Solution:

We know that if Rs. P is deposited at *r*% compound interest per annum for n years, the amount of money will be:
$P(1+r/100)^n$
Therefore, after each year, the amount of money will be;
$10000(1+8/100)$, $10000(1+8/100)^2$, $10000(1+8/100)^3$......

Clearly, the terms of this series do not have the common difference between them. Therefore, this is not an A.P.

2. Write first four terms of the A.P. when the first term a and the common difference are given as follows:
(i) $a = 10$, $d = 10$
(ii) $a = -2$, $d = 0$
(iii) $a = 4$, $d = -3$
(iv) $a = -1$ $d = 1/2$
(v) $a = -1.25$, $d = -0.25$

Solutions:

(i) $a = 10$, $d = 10$

Let us consider, the Arithmetic Progression series be a_1, a_2, a_3, a_4, a_5 ...
$a_1 = a = 10$
$a_2 = a_1+d = 10+10 = 20$
$a_3 = a_2+d = 20+10 = 30$
$a_4 = a_3+d = 30+10 = 40$
$a_5 = a_4+d = 40+10 = 50$
And so on…
Therefore, the A.P. series will be 10, 20, 30, 40, 50 …
And First four terms of this A.P. will be 10, 20, 30, and 40.

(ii) $a = -2$, $d = 0$

Let us consider, the Arithmetic Progression series be a_1, a_2, a_3, a_4, a_5 ...
$a_1 = a = -2$
$a_2 = a_1+d = -2+0 = -2$
$a_3 = a_2+d = -2+0 = -2$

$a_4 = a_3 + d = -2+0 = -2$
Therefore, the A.P. series will be - 2, - 2, - 2, - 2 …
And, First four terms of this A.P. will be - 2, - 2, - 2 and - 2.

(iii) $a = 4$, $d = -3$

Let us consider, the Arithmetic Progression series be a_1, a_2, a_3, a_4, a_5 …
$a_1 = a = 4$
$a_2 = a_1 + d = 4-3 = 1$
$a_3 = a_2 + d = 1-3 = -2$
$a_4 = a_3 + d = -2-3 = -5$

Therefore, the A.P. series will be 4, 1, - 2 - 5 …
And, first four terms of this A.P. will be 4, 1, - 2 and - 5.

(iv) $a = -1$, $d = 1/2$

Let us consider, the Arithmetic Progression series be a_1, a_2, a_3, a_4, a_5 …
$a_2 = a_1 + d = -1+1/2 = -1/2$
$a_3 = a_2 + d = -1/2+1/2 = 0$
$a_4 = a_3 + d = 0+1/2 = 1/2$

Thus, the A.P. series will be-1, -1/2, 0, 1/2
And First four terms of this A.P. will be -1, -1/2, 0 and 1/2.

(v) $a = -1.25$, $d = -0.25$

Let us consider, the Arithmetic Progression series be a_1, a_2, a_3, a_4, a_5 …
$a_1 = a = -1.25$
$a_2 = a_1 + d = -1.25-0.25 = -1.50$
$a_3 = a_2 + d = -1.50-0.25 = -1.75$
$a_4 = a_3 + d = -1.75-0.25 = -2.00$

Therefore, the A.P series will be 1.25, - 1.50, - 1.75, - 2.00 ……..
And first four terms of this A.P. will be - 1.25, - 1.50, - 1.75 and - 2.00.

3. For the following A.P.s, write the first term and the common difference.
(i) 3, 1, - 1, - 3 …
(ii) -5, - 1, 3, 7 …
(iii) 1/3, 5/3, 9/3, 13/3 ....
(iv) 0.6, 1.7, 2.8, 3.9 …
Solutions

(i) Given series,
3, 1, - 1, - 3 …

First term, $a = 3$
Common difference, d = Second term - First term
$\Rightarrow$ 1 - 3 = -2
$\Rightarrow$ d = -2

(ii) Given series, - 5, - 1, 3, 7 …

First term, $a = -5$
Common difference, d = Second term - First term
$\Rightarrow$ (- 1)-(- 5) = - 1+5 = 4

(iii) Given series, 1/3, 5/3, 9/3, 13/3 ....

First term, $a = 1/3$
Common difference, d = Second term - First term
$\Rightarrow$ 5/3 - 1/3 = 4/3

(iv) Given series, 0.6, 1.7, 2.8, 3.9 …

First term, $a = 0.6$
Common difference, d = Second term - First term
$\Rightarrow$ 1.7 - 0.6
$\Rightarrow$ 1.1

4. Which of the following are APs? If they form an A.P. find the common difference *d* and write three more terms.
(i) 2, 4, 8, 16 …
(ii) 2, 5/2, 3, 7/2 ....
(iii) -1.2, -3.2, -5.2, -7.2 …
(iv) -10, - 6, - 2, 2 …
(v) 3, 3 + √2, 3 + 2√2, 3 + 3√2
(vi) 0.2, 0.22, 0.222, 0.2222 ….
(vii) 0, - 4, - 8, - 12 …
(viii) -1/2, -1/2, -1/2, -1/2 ....
(ix) 1, 3, 9, 27 …
(x) a, $2a$, $3a$, $4a$ …
(xi) a, a^2, a^3, a^4 …
(xii) √2, √8, √18, √32 ...
(xiii) √3, √6, √9, √12 ...
(xiv) 1^2, 3^2, 5^2, 7^2 …
(xv) 1^2, 5^2, 7^2, 7^3 …

Solution

(i) Given to us,

2, 4, 8, 16 …

Here, the common difference is;
$a_2 - a_1 = 4 - 2 = 2$
$a_3 - a_2 = 8 - 4 = 4$
$a_4 - a_3 = 16 - 8 = 8$

Since, $a_{n+1} - a_n$ or the common difference is not the same every time.
Therefore, the given series are not forming an A.P.

(ii) Given, 2, 5/2, 3, 7/2 ....

Here,
$a_2 - a_1 = 5/2-2 = 1/2$
$a_3 - a_2 = 3-5/2 = 1/2$
$a_4 - a_3 = 7/2-3 = 1/2$

Since, $a_{n+1} - a_n$ or the common difference is same every time.
Therefore, $d = 1/2$ and the given series are in A.P.

The next three terms are;
$a_5 = 7/2+1/2 = 4$
$a_6 = 4 +1/2 = 9/2$
$a_7 = 9/2 +1/2 = 5$

(iii) Given, -1.2, - 3.2, -5.2, -7.2 …

Here,
$a_2 - a_1 = (-3.2)-(-1.2) = -2$
$a_3 - a_2 = (-5.2)-(-3.2) = -2$
$a_4 - a_3 = (-7.2)-(-5.2) = -2$

Since, $a_{n+1} - a_n$ or common difference is same every time.
Therefore, $d = -2$ and the given series are in A.P.

Hence, next three terms are;
$a_5 = - 7.2-2 = -9.2$
$a_6 = - 9.2-2 = - 11.2$
$a_7 = - 11.2-2 = - 13.2$

(iv) Given, -10, - 6, - 2, 2 …
Here, the terms and their difference are;

$a_2 - a_1 = (-6)-(-10) = 4$
$a_3 - a_2 = (-2)-(-6) = 4$

$a_4 - a_3 = (2 - (-2) = 4$

Since, $a_{n+1} - a_n$ or the common difference is same every time.
Therefore, $d = 4$ and the given numbers are in A.P.
Hence, next three terms are;
$a_5 = 2+4 = 6$
$a_6 = 6+4 = 10$
$a_7 = 10+4 = 14$

(v) Given, 3, 3+√2, 3+2√2, 3+3√2
Here,
$a_2 - a_1 = 3+\sqrt{2}-3 = \sqrt{2}$
$a_3 - a_2 = (3+2\sqrt{2})-(3+\sqrt{2}) = \sqrt{2}$
$a_4 - a_3 = (3+3\sqrt{2}) - (3+2\sqrt{2}) = \sqrt{2}$

Since, $a_{n+1} - a_n$ or the common difference is same every time.
Therefore, $d = \sqrt{2}$ and the given series forms a A.P.

Hence, next three terms are;
$a_5 = (3+\sqrt{2}) +\sqrt{2} = 3+4\sqrt{2}$
$a_6 = (3+4\sqrt{2})+\sqrt{2} = 3+5\sqrt{2}$
$a_7 = (3+5\sqrt{2})+\sqrt{2} = 3+6\sqrt{2}$

(vi) 0.2, 0.22, 0.222, 0.2222 ….
Here,
$a_2 - a_1 = 0.22-0.2 = 0.02$
$a_3 - a_2 = 0.222-0.22 = 0.002$
$a_4 - a_3 = 0.2222-0.222 = 0.0002$

Since, $a_{n+1} - a_n$ or the common difference is not same every time.
Therefore, and the given series doesn't forms a A.P.

(vii) 0, -4, -8, -12 …
Here,
$a_2 - a_1 = (-4)-0 = -4$
$a_3 - a_2 = (-8)-(-4) = -4$
$a_4 - a_3 = (-12)-(-8) = -4$

Since, $a_{n+1} - a_n$ or the common difference is same every time.
Therefore, $d = -4$ and the given series forms a A.P.

Hence, next three terms are;
$a_5 = -12-4 = -16$
$a_6 = -16-4 = -20$
$a_7 = -20-4 = -24$

(viii) -1/2, -1/2, -1/2, -1/2 ....
Here,
$a_2 - a_1 = (-1/2) - (-1/2) = 0$
$a_3 - a_2 = (-1/2) - (-1/2) = 0$
$a_4 - a_3 = (-1/2) - (-1/2) = 0$

Since, $a_{n+1} - a_n$ or the common difference is same every time.
Therefore, $d = 0$ and the given series forms a A.P.

Hence, next three terms are;
$a_5 = (-1/2)-0 = -1/2$
$a_6 = (-1/2)-0 = -1/2$
$a_7 = (-1/2)-0 = -1/2$

(ix) 1, 3, 9, 27 …
Here,
$a_2 - a_1 = 3-1 = 2$
$a_3 - a_2 = 9-3 = 6$
$a_4 - a_3 = 27-9 = 18$

Since, $a_{n+1} - a_n$ or the common difference is not same every time.
Therefore, and the given series doesn't form a A.P.

(x) *a*, 2*a*, 3*a*, 4*a* …
Here,
$a_2 - a_1 = 2a-a = a$
$a_3 - a_2 = 3a-2a = a$
$a_4 - a_3 = 4a-3a = a$

Since, $a_{n+1} - a_n$ or the common difference is same every time.
Therefore, $d = a$ and the given series forms a A.P.

Hence, next three terms are;
$a_5 = 4a+a = 5a$
$a_6 = 5a+a = 6a$
$a_7 = 6a+a = 7a$

(xi) a, a^2, a^3, a^4 …
Here,
$a_2 - a_1 = a^2-a = a(a-1)$
$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$
$a_4 - a_3 = a^4 - a^3 = a^3(a-1)$

Since, $a_{n+1} - a_n$ or the common difference is not same every time.
Therefore, the given series doesn't forms a A.P.

(xii) √2, √8, √18, √32 ...
Here,
$a_2 - a_1 = \sqrt{8}-\sqrt{2} = 2\sqrt{2}-\sqrt{2} = \sqrt{2}$
$a_3 - a_2 = \sqrt{18}-\sqrt{8} = 3\sqrt{2}-2\sqrt{2} = \sqrt{2}$
$a_4 - a_3 = 4\sqrt{2}-3\sqrt{2} = \sqrt{2}$

Since, $a_{n+1} - a_n$ or the common difference is same every time.
Therefore, $d = \sqrt{2}$ and the given series forms a A.P.
Hence, next three terms are;
$a_5 = \sqrt{32}+\sqrt{2} = 4\sqrt{2}+\sqrt{2} = 5\sqrt{2} = \sqrt{50}$
$a_6 = 5\sqrt{2}+\sqrt{2} = 6\sqrt{2} = \sqrt{72}$
$a_7 = 6\sqrt{2}+\sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) √3, √6, √9, √12 ...
Here,
$a_2 - a_1 = \sqrt{6}-\sqrt{3} = \sqrt{3}\times\sqrt{2}-\sqrt{3} = \sqrt{3}(\sqrt{2}-1)$
$a_3 - a_2 = \sqrt{9}-\sqrt{6} = 3-\sqrt{6} = \sqrt{3}(\sqrt{3}-\sqrt{2})$
$a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - \sqrt{3}\times\sqrt{3} = \sqrt{3}(2-\sqrt{3})$

Since, $a_{n+1} - a_n$ or the common difference is not same every time.
Therefore, the given series doesn't form a A.P.

(xiv) $1^2, 3^2, 5^2, 7^2$ ...
Or, 1, 9, 25, 49 …..
Here,
$a_2 - a_1 = 9-1 = 8$
$a_3 - a_2 = 25-9 = 16$
$a_4 - a_3 = 49-25 = 24$

Since, $a_{n+1} - a_n$ or the common difference is not same every time.
Therefore, the given series doesn't form a A.P.

(xv) $1^2, 5^2, 7^2$, 73 …
Or 1, 25, 49, 73 …
Here,
$a_2 - a_1 = 25-1 = 24$
$a_3 - a_2 = 49-25 = 24$
$a_4 - a_3 = 73-49 = 24$

Since, $a_{n+1} - a_n$ or the common difference is same every time.
Therefore, $d = 24$ and the given series forms a A.P.
Hence, next three terms are;
$a_5 = 73+24 = 97$
$a_6 = 97+24 = 121$
$a_7 = 121+24 = 145$