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1. Evaluate the following:

(i) sin 60° cos 30° + sin 30° cos 60°

(ii) 2 tan^2 45° + cos^2 30° – sin^2 60

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Solution:

(i) sin 60° cos 30° + sin 30° cos 60°
First, find the values of the given trigonometric ratios
sin 30° = 1/2
cos 30° = √3/2
sin 60° = 3/2
cos 60°= 1/2
Now, substitute the values in the given problem
sin 60° cos 30° + sin 30° cos 60° = √3/2 ×√3/2 + (1/2) ×(1/2) = 3/4+1/4 = 4/4 = 1

(ii) 2 tan^2 45° + cos^2 30° – sin^2 60

We know that, the values of the trigonometric ratios are:
sin 60° = √3/2
cos 30° = √3/2
tan 45° = 1
Substitute the values in the given problem
2 tan^2 45° + cos^2 30° – sin^2 60 = 2(1)^2 + (√3/2)^2-(√3/2)^2
2 tan^2 45° + cos^2 30° – sin^2 60 = 2 + 0
2 tan^2 45° + cos^2 30° – sin^2 60 = 2

(iii) cos 45°/(sec 30°+cosec 30°)
We know that,
cos 45° = 1/√2
sec 30° = 2/√3
cosec 30° = 2

Substitute the values, we get

$$\frac{\cos 45°}{\sec 30°+\operatorname{cosec} 30°} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2(1+\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)}$$

Now, rationalize the terms we get,

$$= \frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3-\sqrt{3}}{2\sqrt{2}\,(3-1)} = \frac{3-\sqrt{3}}{2\sqrt{2}\,(2)}$$

Now, multiply both the numerator and denominator by √2 , we get

$$= \frac{3-\sqrt{3}}{2\sqrt{2}\,(2)} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}-\sqrt{3}\sqrt{2}}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

Therefore, cos 45°/(sec 30°+cosec 30°) = (3√2 – √6)/8

(iv) $\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

We know that,
sin 30° = 1/2
tan 45° = 1
cosec 60° = 2/√3
sec 30° = 2/√3
cos 60° = 1/2
cot 45° = 1

Substitute the values in the given problem, we get

$$\frac{\sin 30°+\tan 45°-\operatorname{cosec} 60°}{\sec 30°+\cos 60°+\cot 45°} = \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$$

Now, cancel the term 2√3, in numerator and denominator, we get

$$= \frac{\sqrt{3}+2\sqrt{3}-4}{4+\sqrt{3}+2\sqrt{3}} = \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

Now, rationalize the terms

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$= \frac{27-12\sqrt{3}-12\sqrt{3}+16}{27-12\sqrt{3}+12\sqrt{3}+16} = \frac{27-24\sqrt{3}+16}{11} = \frac{43-24\sqrt{3}}{11}$$

Therefore,

$$\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°} = \frac{43-24\sqrt{3}}{11}$$

(v) $\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

We know that,
cos 60° = 1/2
sec 30° = 2/√3

tan 45° = 1
sin 30° = 1/2
cos 30° = √3/2
Now, substitute the values in the given problem, we get

$= (5\cos^2 60° + 4\sec^2 30° - \tan^2 45°)/(\sin^2 30° + \cos^2 30°)$

$= 5(1/2)^2+4(2/\sqrt{3})^2-1^2/(1/2)^2+(\sqrt{3}/2)^2$

$= (5/4+16/3-1)/(1/4+3/4)$

$= (15+64-12)/12/(4/4)$

$= 67/12$

2. Choose the correct option and justify your choice :

(i) $2\tan 30°/1+\tan^2 30°$ =

(A) sin 60° (B) cos 60° (C) tan 60° (D) sin 30°

(ii) $1-\tan^2 45°/1+\tan^2 45°$ =

(A) tan 90° (B) 1 (C) sin 45° (D) 0

(iii) sin 2A = 2 sin A is true when A =

(A) 0° (B) 30° (C) 45° (D) 60°

(iv) $2\tan 30°/1-\tan^2 30°$ =

(A) cos 60° (B) sin 60° (C) tan 60° (D) sin 30°

Solution:
(i) (A) is correct.
Substitute the of tan 30° in the given equation
tan 30° = 1/√3

$2\tan 30°/1+\tan^2 30° = 2(1/\sqrt{3})/1+(1/\sqrt{3})^2$

$= (2/\sqrt{3})/(1+1/3) = (2/\sqrt{3})/(4/3)$

$= 6/4\sqrt{3} = \sqrt{3}/2 = \sin 60°$

The obtained solution is equivalent to the trigonometric ratio sin 60°

(ii) (D) is correct.
Substitute the of tan 45° in the given equation

$\tan 45° = 1$

$1-\tan^2 45°/1+\tan^2 45° = (1-1^2)/(1+1^2)$

$= 0/2 = 0$

The solution of the above equation is 0.

(iii) (A) is correct.
To find the value of A, substitute the degree given in the options one by one
$\sin 2A = 2 \sin A$ is true when $A = 0°$
As $\sin 2A = \sin 0° = 0$
$2 \sin A = 2 \sin 0° = 2 \times 0 = 0$
or,
Apply the sin 2A formula, to find the degree value
$\sin 2A = 2\sin A \cos A$
$\Rightarrow 2\sin A \cos A = 2 \sin A$
$\Rightarrow 2\cos A = 2 \Rightarrow \cos A = 1$
Now, we have to check, to get the solution as 1, which degree value has to be applied.
When 0 degree is applied to cos value, i.e., $\cos 0 = 1$
Therefore, $\Rightarrow A = 0°$

(iv) (C) is correct.
Substitute the of tan 30° in the given equation
$\tan 30° = 1/\sqrt{3}$

$2\tan 30°/1-\tan^2 30° = 2(1/\sqrt{3})/1-(1/\sqrt{3})^2$

$= (2/\sqrt{3})/(1-1/3) = (2/\sqrt{3})/(2/3) = \sqrt{3} = \tan 60°$

The value of the given equation is equivalent to tan 60°.

3. If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = 1/\sqrt{3}$, $0° < A + B \leq 90°$; $A > B$, find A and B.

Solution:

$\tan (A + B) = \sqrt{3}$
Since $\sqrt{3} = \tan 60°$
Now substitute the degree value
$\Rightarrow \tan (A + B) = \tan 60°$
$(A + B) = 60°$ ... (i)
The above equation is assumed as equation (i)
$\tan (A - B) = 1/\sqrt{3}$
Since $1/\sqrt{3} = \tan 30°$
Now substitute the degree value
$\Rightarrow \tan (A - B) = \tan 30°$
$(A - B) = 30°$ ... equation (ii)
Now add the equation (i) and (ii), we get
$A + B + A - B = 60° + 30°$
Cancel the terms B

$2A = 90°$
$A = 45°$
Now, substitute the value of A in equation (i) to find the value of B
$45° + B = 60°$
$B = 60° - 45°$
$B = 15°$
Therefore $A = 45°$ and $B = 15°$

4. State whether the following are true or false. Justify your answer.
(i) sin (A + B) = sin A + sin B.
(ii) The value of sin θ increases as θ increases.
(iii) The value of cos θ increases as θ increases.
(iv) sin θ = cos θ for all values of θ.
(v) cot A is not defined for A = 0°.

Solution:
(i) False.
Justification:
Let us take $A = 30°$ and $B = 60°$, then
Substitute the values in the sin (A + B) formula, we get
$\sin(A + B) = \sin(30° + 60°) = \sin 90° = 1$ and,
$\sin A + \sin B = \sin 30° + \sin 60°$
$= 1/2 + \sqrt{3}/2 = 1+\sqrt{3}/2$
Since the values obtained are not equal, the solution is false.
(ii) True.
Justification:
According to the values obtained as per the unit circle, the values of sin are:
$\sin 0° = 0$
$\sin 30° = 1/2$
$\sin 45° = 1/\sqrt{2}$
$\sin 60° = \sqrt{3}/2$
$\sin 90° = 1$
Thus the value of sin θ increases as θ increases. Hence, the statement is true

(iii) False.
According to the values obtained as per the unit circle, the values of cos are:
$\cos 0° = 1$
$\cos 30° = \sqrt{3}/2$
$\cos 45° = 1/\sqrt{2}$
$\cos 60° = 1/2$
$\cos 90° = 0$
Thus, the value of cos θ decreases as θ increases. So, the statement given above is false.

(iv) False

$\sin\theta = \cos\theta$, when a right triangle has 2 angles of $(\pi/4)$. Therefore, the above statement is false.

(v) True.

Since cot function is the reciprocal of the tan function, it is also written as:

$\cot A = \cos A / \sin A$

Now substitute $A = 0°$

$\cot 0° = \cos 0° / \sin 0° = 1/0$ = undefined.

Hence, it is true