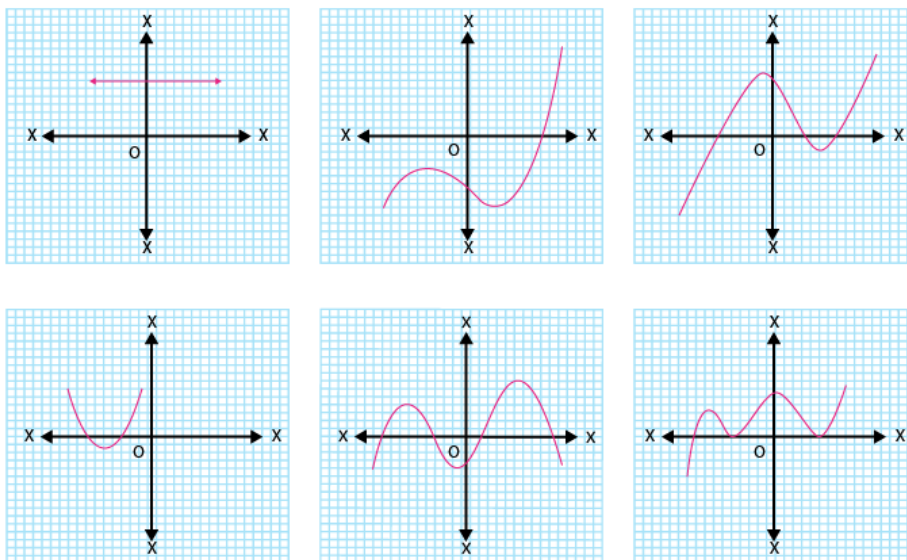Exercise 2.1   Page: 28

1. The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.

Solutions:

Graphical method to find zeroes:-

Total number of zeroes in any polynomial equation = total number of times the curve intersects x-axis.

(i)   In the given graph, the number of zeroes of p(x) is 0 because the graph is parallel to x-axis does not cut it at any point.

(ii)   In the given graph, the number of zeroes of p(x) is 1 because the graph intersects the x-axis at only one point.

(iii)   In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at any three points.

(iv)   In the given graph, the number of zeroes of p(x) is 2 because the graph intersects the x-axis at two points.

(v)   In the given graph, the number of zeroes of p(x) is 4 because the graph intersects the x-axis at four points.

(vi)   In the given graph, the number of zeroes of p(x) is 3 because the graph intersects the x-axis at three points.

Exercise 2.2   Page: 33

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solutions:

(i)$x^2–2x–8$
⇒$x^2– 4x+2x–8 = x(x–4)+2(x–4) = (x-4)(x+2)$

Therefore, zeroes of polynomial equation $x^2–2x–8$ are (4, -2)

Sum of zeroes = $4–2 = 2 = -(-2)/1 =$ -(Coefficient of x)/(Coefficient of x^2)

Product of zeroes = $4×(-2) = -8 =-(8)/1 =$ (Constant term)/(Coefficient of x^2)

(ii)$4s^2–4s+1$
⇒$4s^2–2s–2s+1 = 2s(2s–1)–1(2s-1) = (2s–1)(2s–1)$

Therefore, zeroes of polynomial equation $4s^2–4s+1$ are (1/2, 1/2)

Sum of zeroes = $(½)+(1/2) = 1 = -(-4)/4 =$ -(Coefficient of s)/(Coefficient of

s^2) Product of zeros = $(1/2)×(1/2) = 1/4 =$ (Constant term)/(Coefficient of

s^2)

(iii) $6x^2–3–7x$
⇒$6x^2–7x–3 = 6x^2 – 9x + 2x – 3 = 3x(2x - 3) +1(2x - 3) = (3x+1)(2x-3)$
Therefore, zeroes of polynomial equation $6x^2–3–7x$ are (-1/3, 3/2)

Sum of zeroes = $-(1/3)+(3/2) = (7/6) =$ -(Coefficient of x)/(Coefficient of x^2)

Product of zeroes = $-(1/3)×(3/2) = -(3/6) =$ (Constant term) /(Coefficient of x^2)

(iv)$4u^2+8u$

⇒ $4u(u+2)$

Therefore, zeroes of polynomial equation $4u^2 + 8u$ are (0, -2).

Sum of zeroes = $0+(-2) = -2 = -(8/4) = =$ -(Coefficient of u)/(Coefficient of u^2)

Product of zeroes = 0×-2 = 0 = 0/4 = (Constant term)/(Coefficient of u^2)

(v) t^2–15

⇒ t^2 = 15 or t = ±√15
Therefore, zeroes of polynomial equation t^2 –15 are (√15, -√15)

Sum of zeroes =√15+(-√15) = 0= -(0/1)= -(Coefficient of t) / (Coefficient of t^2)

Product of zeroes = √15×(-√15) = -15 = -15/1 = (Constant term) / (Coefficient of t^2)

(vi) $3x^2$–x–4

⇒ $3x^2$–4x+3x–4 = x(3x-4)+1(3x-4) = (3x - 4)(x + 1)

Therefore, zeroes of polynomial equation$3x^2$ – x – 4 are (4/3, -1)

Sum of zeroes = (4/3)+(-1) = (1/3)= -(-1/3) = -(Coefficient of x) / (Coefficient of x^2)

Product of zeroes=(4/3)×(-1) = (-4/3) = (Constant term) /(Coefficient of x^2)

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.
(i) 1/4 , -1

Solution:

From the formulas of sum and product of zeroes, we know,
Sum of zeroes = α+β
Product of zeroes = α β
Sum of zeroes = α+β = 1/4
Product of zeroes = α β = -1

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-
x^2–(α+β)x +αβ = 0

x^2–(1/4)x +(-1) = 0

$4x^2$–x-4 = 0

Thus,$4x^2$–x–4 is the quadratic polynomial.

(ii)√2, 1/3

Solution:

Sum of zeroes = α + β =√2
Product of zeroes = α β = 1/3

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

x^2–(α+β)x +αβ = 0

x^2 –(√2)x + (1/3) = 0

$3x^2$-3√2x+1 = 0

Thus, $3x^2$-3√2x+1 is the quadratic polynomial.

(iii) 0, √5

Solution:
Given,
Sum of zeroes = α+β = 0
Product of zeroes = α β = √5
∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

x^2–(α+β)x +αβ = 0

x^2–(0)x +√5= 0

Thus, x^2+√5 is the quadratic polynomial.

(iv) 1, 1

Solution:

Given,
Sum of zeroes = α+β = 1
Product of zeroes = α β = 1

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-
$x^2−(α+β)x +αβ = 0$

$x^2−x+1 = 0$

Thus , $x^2−x+1$ is the quadratic polynomial.

(v) -1/4, 1/4

Solution:

Given,
Sum of zeroes = $α+β$ = -1/4
Product of zeroes = $α β$ = 1/4

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2−(α+β)x +αβ = 0$

$x^2−(-1/4)x +(1/4) = 0$

$4x^2+x+1 = 0$

Thus,$4x^2+x+1$ is the quadratic polynomial.

(vi) 4, 1

Solution:

Given,
Sum of zeroes = $α+β$ = 4
Product of zeroes = $αβ$ = 1

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2−(α+β)x+αβ = 0$

$x^2−4x+1 = 0$

Thus,$x^2−4x+1$ is the quadratic polynomial.

Exercise 2.3   Page: 36

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) p(x) = x³-3x²+5x–3 , g(x) = x²–2

Solution:
Given,
Dividend = p(x) = x³-3x²+5x–3
Divisor = g(x) = x²– 2

$$
\begin{array}{r}
x \quad -3 \\[2pt]
\hline
x^2 - 2 \enclose{longdiv}{x^3 \quad -3x^2 \quad +5x \quad -3} \\[2pt]
\underline{x^3 \qquad +0x^2 \quad -2x} \\[2pt]
-3x^2 \quad +7x \quad -3 \\[2pt]
\underline{-3x^2 \quad +0x \quad +6} \\[2pt]
7x \quad -9
\end{array}
$$

Therefore, upon division we get,
Quotient = x–3
Remainder = 7x–9

(ii) p(x) = x⁴-3x²+4x+5 , g(x) = x²+1-x

Solution:
Given,
Dividend = p(x) = x⁴ - 3x² + 4x +5
Divisor = g(x) = x² +1-x

$$x^2 - x + 1 \overline{\smash{\big)}\ \begin{array}{c} x^2 \ +x \ -3 \\ \hline x^4 \ +0x^3 \ -3x^2 \ +4x \ +5 \end{array}}$$

$$\begin{array}{r} - \\ \underline{x^4 \ -x^3 \ +x^2} \\ x^3 \ -4x^2 \ +4x \ +5 \\ - \\ \underline{x^3 \ -x^2 \ +x} \\ -3x^2 \ +3x \ +5 \\ - \\ \underline{-3x^2 \ +3x \ -3} \\ 8 \end{array}$$

Therefore, upon division we get,
Quotient = $x^2 + x – 3$
Remainder = 8

(iii) p(x) =x⁴–5x+6, g(x) = 2–x²
Solution:
Given,

Dividend = p(x) =x⁴ - 5x + 6 = x⁴ +0x²–5x+6
Divisor = g(x) = 2–x² = –x²+2

$$-x^2 + 2 \overline{\smash{\big)}\ \begin{array}{c} -x^2 \ -2 \\ \hline x^4 \ +0x^3 \ +0x^2 \ -5x \ +6 \end{array}}$$

$$\begin{array}{r} - \\ \underline{x^4 \ +0x^3 \ -2x^2} \\ 2x^2 \ -5x \ +6 \\ - \\ \underline{2x^2 \ +0x \ -4} \\ -5x \ +10 \end{array}$$

Therefore, upon division we get,
Quotient = -x²-2

Remainder = -5x + 10

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) t^2-3, $2t^4$ +$3t^3$-$2t^2$-9t-12
Solutions:
Given,

First polynomial = t^2-3
Second polynomial = $2t^4$ +$3t^3$-$2t^2$ -9t-12

$$
\begin{array}{r}
2t^2 \quad +3t \quad +4 \\
t^2 - 3 \enclose{longdiv}{2t^4 \quad +3t^3 \quad -2t^2 \quad -9t \quad -12} \\
\underline{2t^4 \quad +0t^3 \quad -6t^2 \quad\quad\quad\quad\quad} \\
3t^3 \quad +4t^2 \quad -9t \quad -12 \\
\underline{3t^3 \quad +0t^2 \quad -9t \quad\quad\quad} \\
4t^2 \quad +0t \quad -12 \\
\underline{4t^2 \quad +0t \quad -12} \\
0
\end{array}
$$

As we can see, the remainder is left as 0. Therefore, we say that, t^2-3 is a factor of $2t^4$ +$3t^3$-$2t^2$ -9t-12 .

(ii)x^2+3x+1 , $3x^4$+$5x^3$-$7x^2$+2x+2

Solutions:
Given,

First polynomial = x^2+3x+1
Second polynomial = $3x^4$+$5x^3$-$7x^2$+2x+2

$$\begin{array}{r} 3x^2 \quad -4x \quad +2 \\ \hline x^2 + 3x + 1 \quad \overline{)\; 3x^4 \quad +5x^3 \quad -7x^2 \quad +2x \quad +2} \end{array}$$

$$\begin{array}{r} - \\ 3x^4 \quad +9x^3 \quad +3x^2 \\ \hline -4x^3 \quad -10x^2 \quad +2x \quad +2 \\ - \\ -4x^3 \quad -12x^2 \quad -4x \\ \hline 2x^2 \quad +6x \quad +2 \\ - \\ 2x^2 \quad +6x \quad +2 \\ \hline 0 \end{array}$$

As we can see, the remainder is left as 0. Therefore, we say that, $x^2 + 3x + 1$ is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) x^3-3x+1, $x^5-4x^3+x^2+3x+1$

Solutions:
Given,

First polynomial = x^3-3x+1

Second polynomial = $x^5-4x^3+x^2+3x+1$

$$\begin{array}{r} x^2 \quad -1 \\ \hline x^3 - 3x + 1 \quad \overline{)\; x^5 \quad +0x^4 \quad -4x^3 \quad +x^2 \quad +3x \quad +1} \end{array}$$

$$\begin{array}{r} - \\ x^5 \quad +0x^4 \quad -3x^3 \quad +x^2 \\ \hline -x^3 \quad +0x^2 \quad +3x \quad +1 \\ - \\ -x^3 \quad +0x^2 \quad +3x \quad -1 \\ \hline 2 \end{array}$$

As we can see, the remainder is not equal to 0. Therefore, we say that, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

3. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$.

Solutions:
Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

$\sqrt{(5/3)}$ **and** $-\sqrt{(5/3)}$ are zeroes of polynomial f(x).
$\therefore \left(x -\sqrt{(5/3)}\right) \left(x+\sqrt{(5/3)}\right) = x^2-(5/3) = 0$
$(3x^2-5)=0$, is a factor of given polynomial f(x).

Now, when we will divide f(x) by $(3x^2-5)$ the quotient obtained will also be a factor of f(x) and the remainder will be 0.

$$
\begin{array}{r}
x^2 + 2x + 1 \\
\hline
3x^2-5\ \big)\ 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
3x^4 \qquad\quad - 5x^2 \\
(-)\qquad\quad (+) \\
\hline
+\ 6x^3 + 3x^2 - 10x - 5 \\
-\ 6x^3 \qquad\quad -10x \\
(+)\qquad\quad (-) \\
\hline
3x^2 \qquad\quad -5 \\
3x^2 \qquad\quad -5 \\
(-)\qquad\quad (+) \\
\hline
0
\end{array}
$$

Therefore, $3x^4 +6x^3 -2x^2 -10x-5 = (3x^2 -5)(x^2+2x+1)$

Now, on further factorizing (x^2+2x+1) we get,

$x^2+2x+1 = x^2+x+x+1 = 0$

$x(x+1)+1(x+1) = 0$

$(x+1)(x+1) = 0$

So, its zeroes are given by: $x= -1$ and $x = -1$.

Therefore, all four zeroes of given polynomial equation are:

$\sqrt{(5/3)},- \sqrt{(5/3)}$, −1 and −1.

Hence, is the answer.

4. On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x–2$ and $–2x+4$, respectively. Find $g(x)$.

Solutions:
Given,
Dividend, $p(x) = x^3-3x^2+x+2$
Quotient $= x-2$
Remainder $= –2x+4$
We have to find the value of Divisor, $g(x) =$?

As we know,
Dividend = Divisor × Quotient + Remainder

∴ $x^3-3x^2+x+2 = g(x)\times(x-2) + (-2x+4)$
$x^3-3x^2+x+2-(-2x+4) = g(x)\times(x-2)$
Therefore, $g(x) \times (x-2) = x^3-3x^2+3x-2$
Now, for finding $g(x)$ we will divide x^3-3x^2+3x-2 with $(x-2)$

$$
\begin{array}{r}
x^2 - x + 1 \\
x-2 \enclose{longdiv}{x^3 - 3x^2 + 3x - 2} \\
\underline{x^3 - 2x^2} \\
(-) \quad (+) \\
-x^2 + 3x - 2 \\
\underline{-x^2 + 2x} \\
(+) \quad (-) \\
x - 2 \\
\underline{x - 2} \\
(-) \quad (+) \\
0
\end{array}
$$

Therefore, **$g(x) = (x^2–x+1)$**

5. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and
(i) deg $p(x)$ = deg $q(x)$
(ii) deg $q(x)$ = deg $r(x)$
(iii) deg $r(x)$ = 0

Solutions:
According to the division algorithm, dividend $p(x)$ and divisor $g(x)$ are two polynomials, where $g(x)\neq0$. Then we

can find the value of quotient q(x) and remainder r(x), with the help of below given formula;

Dividend = Divisor × Quotient + Remainder
∴ p(x) = g(x)×q(x)+r(x)
Where r(x) = 0 or degree of r(x)< degree of g(x).
Now let us proof the three given cases as per division algorithm by taking examples for each.

(i) deg p(x) = deg q(x)
Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.
Let us take an example, p(x) = $3x^2 + 3x + 3$ is a polynomial to be divided by g(x) = 3.
So, $(3x^2 + 3x + 3)/3 = x^2 + x + 1$ = q(x)
Thus, you can see, the degree of quotient q(x) = 2, which also equal to the degree of dividend p(x).
Hence, division algorithm is satisfied here.

(ii) deg q(x) = deg r(x)
Let us take an example, p(x) = $x^2 + 3$ is a polynomial to be divided by g(x) = x - 1.
So, $x^2 + 3 = (x - 1)×(x) + (x + 3)$
Hence, quotient q(x) = x
Also, remainder r(x) = x + 3
Thus, you can see, the degree of quotient q(x) = 1, which is also equal to the degree of remainder r(x).
Hence, division algorithm is satisfied here.

(iii) deg r(x) = 0
The degree of remainder is 0 only when the remainder left after division algorithm is constant.
Let us take an example, p(x) = $x^2 + 1$ is a polynomial to be divided by g(x) = x.
So, $x^2 + 1 = (x)×(x) + 1$
Hence, quotient q(x) = x
And, remainder r(x) = 1
Clearly, the degree of remainder here is 0.
Hence, division algorithm is satisfied here.

Exercise 2.4   Page: 36

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2$; -1/2, 1, -2

Solution:
Given, $p(x) = 2x^3+x^2-5x+2$

And zeroes for $p(x)$ are = 1/2, 1, -2

∴ $p(1/2) = 2(1/2)^3+(1/2)^2-5(1/2)+2 = (1/4)+(1/4)-(5/2)+2 = 0$

$p(1) = 2(1)^3+(1)^2-5(1)+2 = 0$

$p(-2) = 2(-2)^3+(-2)^2-5(-2)+2 = 0$

Hence, proved 1/2, 1, -2 are the zeroes of $2x^3+x^2-5x+2$.

Now, comparing the given polynomial with general expression, we get;

∴ $ax^3+bx^2+cx+d = 2x^3+x^2-5x+2$

a=2, b=1, c= -5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

α +β+γ = –b/a

αβ+βγ+γα = c/a

α βγ = – d/a.

Therefore, putting the values of zeroes of the polynomial,

α+β+γ = ½+1+(-2) = -1/2 = –b/a

αβ+βγ+γα = (1/2×1)+(1 ×-2)+(-2×1/2) = -5/2 = c/a

α β γ = ½×1×(-2) = -2/2 = -d/a

Hence, the relationship between the zeroes and the coefficients are satisfied.

(ii) x³-4x²+5x-2 ;2, 1, 1

Solution:
Given, p(x) = x³-4x²+5x-2

And zeroes for p(x) are 2,1,1.

∴ p(2)= 2³-4(2)²+5(2)-2 = 0

p(1) = 1³-(4×1²)+(5×1)-2 = 0
Hence proved, 2, 1, 1 are the zeroes of x³-4x²+5x-2

Now, comparing the given polynomial with general expression, we get;

∴ ax³+bx²+cx+d = x³-4x²+5x-2
a = 1, b = -4, c = 5 and d = -2

As we know, if α, β, γ are the zeroes of the cubic polynomial ax³+bx²+cx+d , then;

α + β + γ = –b/a
αβ + βγ + γα = c/a
α β γ = – d/a.

Therefore, putting the values of zeroes of the polynomial,

α +β+γ = 2+1+1 = 4 = -(-4)/1 = –b/a

αβ+βγ+γα = 2×1+1×1+1×2 = 5 = 5/1= c/a

αβγ = 2×1×1 = 2 = -(-2)/1 = -d/a

Hence, the relationship between the zeroes and the coefficients are satisfied.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Solution:
Let us consider the cubic polynomial is ax³+bx²+cx+d and the values of the zeroes of the polynomials be α, β, γ.

As per the given question,

α+β+γ = -b/a = 2/1

αβ +βγ+γα = c/a = -7/1

α βγ = -d/a = -14/1

Thus, from above three expressions we get the values of coefficient of polynomial.
a = 1, b = -2, c = -7, d = 14

Hence, the cubic polynomial is $x^3-2x^2-7x+14$

3. If the zeroes of the polynomial x^3-3x^2+x+1 are a – b, a, a + b, find a and b.

Solution:
We are given with the polynomial here,
$p(x) = x^3-3x^2+x+1$

And zeroes are given as a – b, a, a + b

Now, comparing the given polynomial with general expression, we get;

$\therefore px^3+qx^2+rx+s = x^3-3x^2+x+1$

p = 1, q = -3, r = 1 and s = 1

Sum of zeroes = a – b + a + a + b

-q/p = 3a

Putting the values q and p.

-(-3)/1 = 3a

a=1

Thus, the zeroes are 1-b, 1, 1+b.

Now, product of zeroes = 1(1-b)(1+b)

$-s/p = 1-b^2$

$-1/1 = 1-b^2$

$b^2 = 1+1 = 2$

$b = \pm\sqrt{2}$

Hence,1-√2, 1 ,1+√2 are the zeroes of x^3-3x^2+x+1.

4. If two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are 2 ±√3, find other zeroes.

Solution:
Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

Let f(x) = $x^4-6x^3-26x^2+138x-35$

Since 2 +√**3** and 2-√**3** are zeroes of given polynomial f(x).
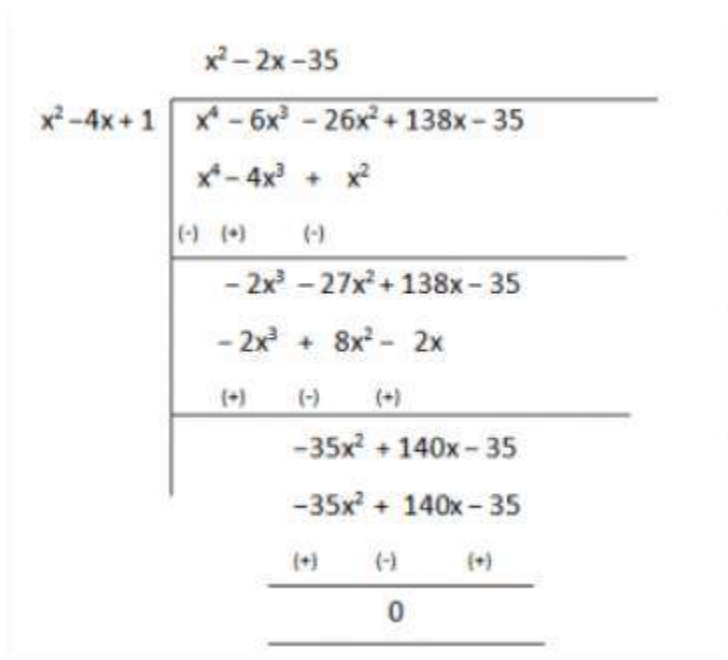
∴ [x−(2+√**3**)] [x−(2-√**3**)] = 0

(x−2−√**3**)(x−2+√**3**) = 0

On multiplying the above equation we get,

x^2-4x+1, this is a factor of a given polynomial f(x).

Now, if we will divide f(x) by g(x), the quotient will also be a factor of f(x) and the remainder will be 0.

$$
\begin{array}{r}
x^2 - 2x - 35 \\
x^2 - 4x + 1\overline{\smash{\big)}\ x^4 - 6x^3 - 26x^2 + 138x - 35} \\
\underline{x^4 - 4x^3 + x^2} \\
(-)\ (+)\ (-) \\
-2x^3 - 27x^2 + 138x - 35 \\
\underline{-2x^3 + 8x^2 - 2x} \\
(+)\ (-)\ (+) \\
-35x^2 + 140x - 35 \\
\underline{-35x^2 + 140x - 35} \\
(+)\ (-)\ (+) \\
0
\end{array}
$$

So, $x^4-6x^3-26x^2+138x-35$ = (x²-4x+1)(x² −2x−35)

Now, on further factorizing (x²–2x−35) we get,

x²–(7−5)x −35 = x²− 7x+5x+35 = 0

x(x −7)+5(x−7) = 0

(x+5)(x−7) = 0

So, its zeroes are given by:

x= −5 and x = 7.

Therefore, all four zeroes of given polynomial equation are: 2+√**3** , 2-√**3**, −5 and 7.