

M A T H E M A T I C S



Introduction to Trigonometry





Topics

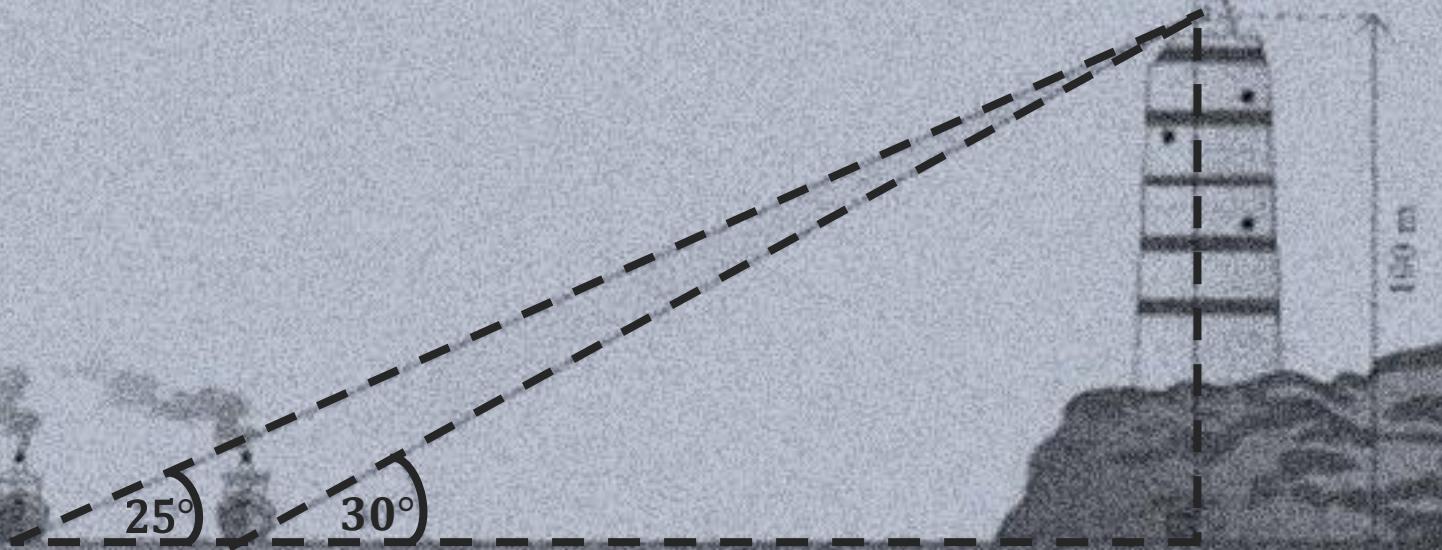


1. Trigonometric Ratios

2. Trigonometric Ratios of standard angles

3. Trigonometric Ratios of Complementary Angles

4. Trigonometric Identities





Trigonometric Ratios

SOH

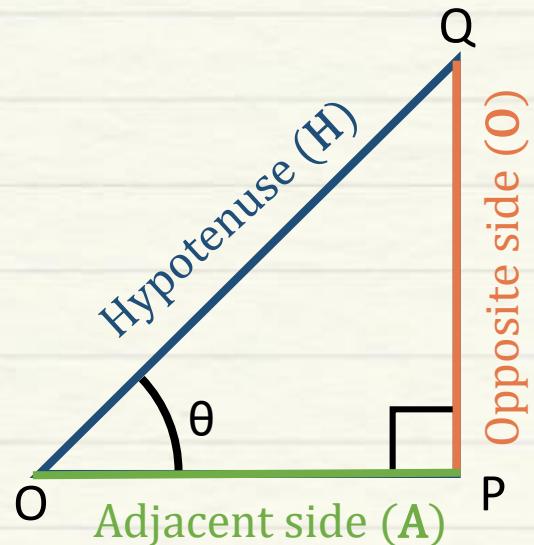
$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

CAH

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

TOA

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{Opposite}}{\text{Adjacent}}$$



i. $\sin \theta$

ii. $\cos \theta$

iii. $\tan \theta$

MULTIPLICATIVE

INVERSE

i. $\cosec \theta = \frac{1}{\sin \theta} = \frac{\text{Hypotenuse}}{\text{Opposite}}$

ii. $\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$

iii. $\cot \theta = \frac{1}{\tan \theta} = \frac{\text{Adjacent}}{\text{Opposite}}$



Trigonometric Ratios of Standard Angles

- With just the values of $\sin \theta$, we can calculate all other trigonometric ratios for standard angles.



An idea to learn the sin values

θ	0°	30°	45°	60°	90°
1. Write numbers from 0 to 4 in order.	0	1	2	3	4
2. Divide every number by 4	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
3. Take the square root of every number	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
4. Simplify	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\sin \theta$	$\sin 0^\circ$	$\sin 30^\circ$	$\sin 45^\circ$	$\sin 60^\circ$	$\sin 90^\circ$



Trigonometric Ratios of Standard Angles

Angles Ratios	Logic	0°	30°	45°	60°	90°
$\sin\theta$	$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	Reverse $\sin\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	$\frac{\sin\theta}{\cos\theta}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	$\frac{1}{\sin\theta}$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	$\frac{1}{\cos\theta}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	$\frac{1}{\tan\theta}$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0



Trigonometric ratios of complementary angles

$$\sin(90^\circ - \theta) = \cos\theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

$$\tan(90^\circ - \theta) = \cot\theta$$

$$\cot(90^\circ - \theta) = \tan\theta$$

$$\sec(90^\circ - \theta) = \cosec\theta$$

$$\cosec(90^\circ - \theta) = \sec\theta$$

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{PQ}{OQ}$$

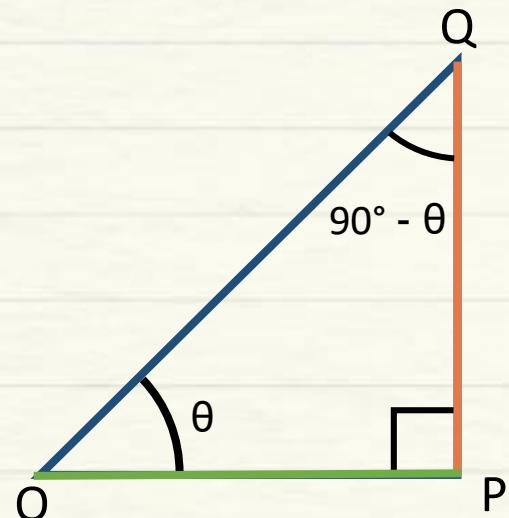
$$\cos(90^\circ - \theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{PQ}{OQ}$$

$$\cos(90^\circ - \theta) = \sin\theta$$

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{OP}{OQ}$$

$$\sin(90^\circ - \theta) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{OP}{OQ}$$

$$\sin(90^\circ - \theta) = \cos\theta$$



$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{PQ}{OP}$$

$$\cot(90^\circ - \theta) = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{PQ}{OP}$$

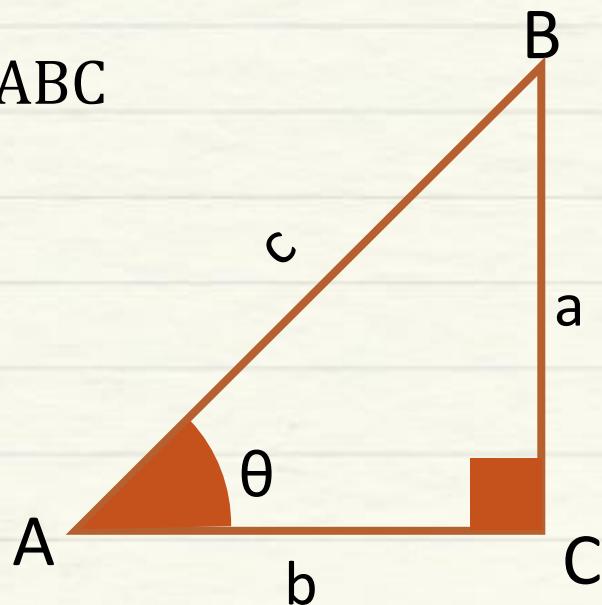
$$\cot(90^\circ - \theta) = \tan\theta$$



Proof of Trigonometric Identities

In a right - Angled Triangle Δ ABC

In $\triangle ABC$ we know that



By Pythagoras Theorem,

$$a^2 + b^2 = c^2$$

Dividing both sides by c^2

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$

From 1 and 2,

$$\sin^2 \theta + \cos^2 \theta = 1$$



Proof of $1 + \tan^2\theta = \sec^2\theta$

We know that

$$\sin^2\theta + \cos^2\theta = 1$$

Dividing both the sides by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$


$$\tan^2\theta + 1 = \sec^2\theta$$

Proof of $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

We know that

$$\sin^2\theta + \cos^2\theta = 1$$

Dividing both the sides by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$


$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

Three Basic Trigonometric Identities

1

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1 \\ \sin^2\theta &= 1 - \cos^2\theta \\ \cos^2\theta &= 1 - \sin^2\theta\end{aligned}$$

2

$$\begin{aligned}\sec^2\theta - \tan^2\theta &= 1 \\ 1 + \tan^2\theta &= \sec^2\theta \\ \sec^2\theta - 1 &= \tan^2\theta\end{aligned}$$

3

$$\begin{aligned}\operatorname{cosec}^2\theta - \cot^2\theta &= 1 \\ 1 + \cot^2\theta &= \operatorname{cosec}^2\theta \\ \operatorname{cosec}^2\theta - 1 &= \cot^2\theta\end{aligned}$$



Mind Map

