Pair of Linear Equations in Two variables
1. General Form of a Linear Equation

2. Types of Pairs of Linear Equations

3. Methods of Solving Pairs of Linear Equations

4. Solving non-linear pair of equations

\[ a_1x + b_1y + c_1 = 0 \]
\[ a_2x + b_2y + c_2 = 0 \]
1. Linear Equations in Two Variables

**General Form**

\[ ax + by + c = 0 \]

- **Coefficients**: \(a, b\)
- **Variables**: \(x, y\)
- **Constant**: \(c\)

where, \(a\) and \(b\) are non-zero real numbers

**Pair of Linear Equations in Two Variables**

Consider two different equations in \(x\) and \(y\),

\[ 2x + 7y + 5 = 0 \]
\[ 8x + 3y + 3 = 0 \]

These two combined are known as pair of linear equations in two variables.

**General Form of Pair of Linear Equations in Two Variables**

\[ a_1x + b_1y + c_1 = 0 \]
\[ a_2x + b_2y + c_2 = 0 \]
2. Types of Pairs of Linear Equations

- **Parallel Lines**
  \[ \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \]
  - Inconsistent equations
  - No solution

- **Intersecting Lines**
  \[ \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \]
  - Consistent equations
  - At least one solution

- **Coinciding Lines**
  \[ \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \]

3. Methods of Solving Pairs of Linear Equations

- **Methods of Solving**
  - Graphical Method
  - Algebraic Methods
    - Substitution Method
    - Elimination Method
    - Cross-Multiplication Method
3.1 Graphical Method

Find points to construct lines on a graph paper for the two given equations.

To construct a line, we need at least two points of the line, we find the value by substituting the values of \(x\) and \(y\) in the two equations.

\[
2x - 1y = -1, \quad 3x + 2y = 9
\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>(-1/2)</th>
<th>1</th>
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<tbody>
<tr>
<td>(y)</td>
<td>1</td>
<td>0</td>
<td>3</td>
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<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>3</th>
<th>1</th>
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<tbody>
<tr>
<td>(y)</td>
<td>(9/2)</td>
<td>0</td>
<td>3</td>
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Draw the two lines on a graph and mark the points at which they intersect.

The \(x\)-coordinate and the \(y\)-coordinate of the point at which the two lines intersect is the solution(s) of the pair of equations.

(1, 3)
3.2 Substitution Method

\[ x + y = 4 \quad , \quad x - y = 2 \]

Take one of the equations and move \( y \) to LHS and the rest to RHS to get the value of \( y \) in terms of \( x \).

\[ y = 4 - x \]

Substitute the obtained value of \( y \) in the other equation to get the numerical value of \( x \).

\[ x - y = 2 \]
\[ x - (4 - x) = 2 \]
\[ 2x - 4 = 2 \]
\[ x = 3 \]

Now, substitute the obtained value of \( x \) in either of the equations to get the value of \( y \).

\[ x + y = 4 \]
\[ 3 + y = 4 \]
\[ y = 1 \]
3.3 Elimination Method

\[3x + 2y = 18, \quad 5x + 4y = 32\]

1. Note down equations aligned to respective variables as shown.

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>+3x</td>
<td>+2y</td>
<td>=</td>
</tr>
<tr>
<td>+5x</td>
<td>+4y</td>
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2. Pick the variable which will be easier to eliminate.

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<tr>
<td>+5x</td>
<td>+4y</td>
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3. Equalise the coefficients of the variable to be eliminated by multiplying every term of the equation with the same number.

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<tbody>
<tr>
<td>+3x \times 2 &amp; +2y \times 2 &amp; = +18 \times 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+5x &amp; +4y &amp; = +32</td>
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4. Subtract the second equation from the first equation by reversing all the signs.

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<tbody>
<tr>
<td>+6x &amp; +4y &amp; = +36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5x &amp; -4y &amp; = -32</td>
<td></td>
<td></td>
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<tr>
<td>+x &amp; +0y &amp; = +4</td>
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5. Substitute the value of the now known variable into the simpler equation to get the value of the other variable.

We know that,
\[x = 4\]
And,
\[3x + 2y = 18\]
\[\Rightarrow 3 \times 4 + 2y = 18\]
\[\Rightarrow 12 + 2 = 18\]
\[\Rightarrow 2y = 6\]
\[\Rightarrow y = 3\]

6. Verify the values obtained for \(x\) and \(y\) by putting them in the given equations.

\[3x + 2y = 18,\]
\[\text{LHS} = 3x + 2y = 3 \times 4 + 2 \times 3 = \text{RHS}\]
\[5x + 4y = 32,\]
\[\text{LHS} = 5x + 4y = 5 \times 4 + 4 \times 3 = \text{RHS}\]

From the above, \(x = 4\) and \(y = 3\). Therefore, \((4, 3)\) is the solution of the simultaneous equations “\(3x + 2y = 18\)” and “\(5x + 4y = 32\)”.
3.4 Cross-Multiplication Method

Write the two equations in the general form:

\[ a_1x + b_1y + c_1 = 0 \]
\[ a_2x + b_2y + c_2 = 0 \]

Now write the coefficients, variables and constants in the pattern shown. After that multiply and subtract in the direction of the arrows as shown:

\[
\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}
\]

Get \(x\) and \(y\) in the LHS and substitute the respective values to get the answer.

\[
x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}
\]
Solving Non-Linear Pair of Equations

Solve for \( x \) and \( y \):

\[
\frac{5}{x-1} + \frac{1}{y} = 2, \quad \frac{6}{x-1} - \frac{3}{y} = 1
\]

Identify what part can be substituted with some other variables to make the equation linear:

\[
5 \left( \frac{1}{x-1} \right) + 1 \left( \frac{1}{y} \right) = 2 \\
6 \left( \frac{1}{x-1} \right) - 3 \left( \frac{1}{y} \right) = 1
\]

\[
p = \frac{1}{x-1} \\
q = \frac{1}{y}
\]

Solve the obtained pair of linear equations using any method:

\[
5p + q = 2 \\
6p - 3q = 1
\]

\[
p = \frac{1}{3} \\
q = \frac{1}{3}
\]

Now solve for \( x \) and \( y \):

\[
x = \frac{1}{p} + 1 \\
y = \frac{1}{q}
\]

\[
x = \frac{4}{3} \\
y = 3
\]
Pair of Linear equations in two variables

Types of Linear equations
- Parallel Lines
- Intersecting lines
- Coincident lines

Solving the Pairs of Non-Linear Equations

Reducing the Equations into Linear Form
- Elimination method
- Cross-multiplication method
- Substitution Method
- Graphical method

Solving the Pairs of Linear Equations

Mind Map