MATHEMATICS

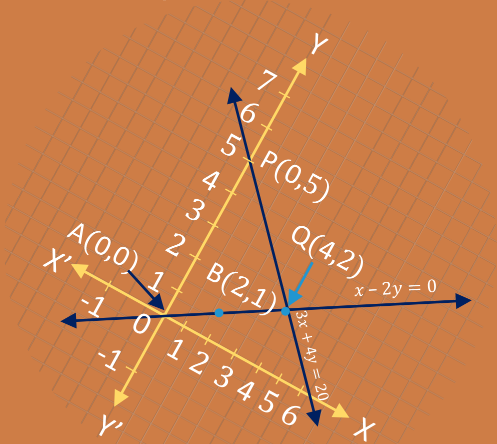
BYJU'S

POST CLASS NOTES

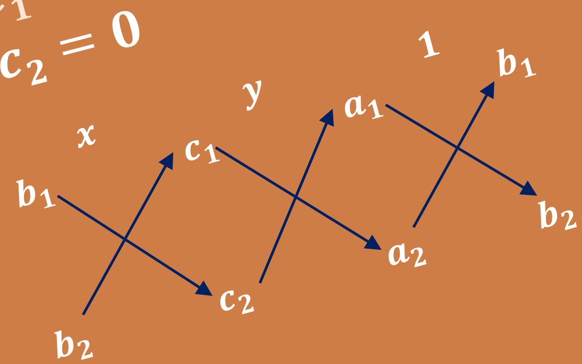
# Pair of Linear Equations in Two variables

# Topics

1. General Form of a Linear Equation
2. Types of Pairs of Linear Equations
3. Methods of Solving Pairs of Linear Equations
4. Solving non-linear pair of equations




$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$



# 1. Linear Equations in Two Variables

## General Form

Coefficients

Constant

$$ax + by + c = 0$$

Variables

where, **a and b** are **non-zero** real numbers

## Pair of Linear Equations in Two Variables

Consider two different equations in $x$ and $y$,

$$2x + 7y + 5 = 0$$

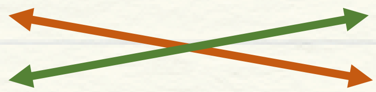
$$8x + 3y + 3 = 0$$

These two combined are known as pair of **linear equations** in two variables.

## General Form of Pair of Linear Equations in Two Variables

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

## 2. Types of Pairs of Linear Equations

| Parallel Lines | Intersecting Lines | Coinciding Lines |
| --- | --- | --- |
|  |  |  |
| $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ | $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ | $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ |

Inconsistent equations
No solution

Consistent equations
At least one solution

## 3. Methods of Solving Pairs of Linear Equations

Methods of Solving

- Graphical Method
- Algebraic Methods
  - Substitution Method
  - Elimination Method
  - Cross-Multiplication Method

# 3.1 Graphical Method

$2x - 1y = -1$ , $3x + 2y = 9$

**Find points to construct lines on a graph paper for the two given equations**

To construct a line, we need at least two point of the line, we find the value substituting values of x and y in the two equations.

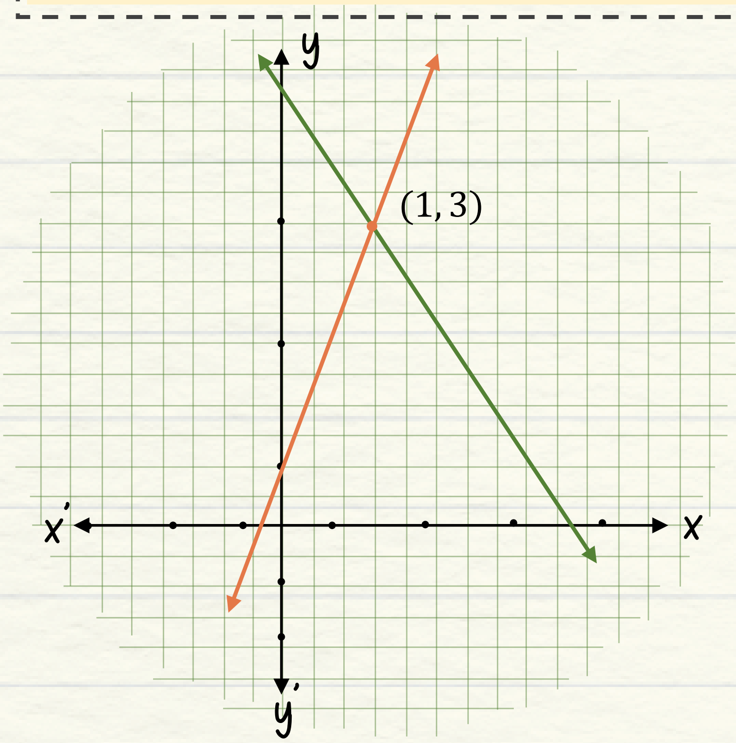
$2x - 1y = -1$

| $x$ | 0 | $-\frac{1}{2}$ | 1 |
|---|---|---|---|
| $y$ | 1 | 0 | 3 |

$3x + 2y = 9$

| $x$ | 0 | 3 | 1 |
|---|---|---|---|
| $y$ | $\frac{9}{2}$ | 0 | 3 |

**Draw the two line on a graph and mark the points at which they intersect.**




The x-coordinate and the y-coordinate of the point at which the two lines intersect is the solution(s) of the pair of equations.

## 3.2 Substitution Method

$$x + y = 4 \quad , \quad x - y = 2$$

Take one of the equations and move 'y' to LHS and the rest to RHS to get the value of 'y' in terms of 'x'.

$$y = 4 - x$$

Substitute the obtained value of 'y' in the other equation to get the numerical value of 'x'.

$$x - y = 2$$
$$x - (4 - x) = 2$$
$$2x - 4 = 2$$
$$x = 3$$

Now, substitute the obtained value of 'x' in either of the equations to get the value of 'y'.

$$x + y = 4$$
$$3 + y = 4$$
$$y = 1$$

# 3.3 Elimination Method

$3x + 2y = 18$ , $5x + 4y = 32$

**1** Note down equations aligned to respective variables as shown.

| | | | |
|---|---|---|---|
| $+3x$ | $+2y$ | $=$ | $+18$ |
| $+5x$ | $+4y$ | $=$ | $+32$ |

**2** Pick the variable which will be easier to eliminate.

| | | | |
|---|---|---|---|
| $+3x$ | $+2y$ | $=$ | $+18$ |
| $+5x$ | $+4y$ | $=$ | $+32$ |

**3** Equalise the coefficients of the variable to be eliminated by multiplying every term of the equation with the same number.

| | | | |
|---|---|---|---|
| $+3x$ $\times 2$ | $+2y$ $\times 2$ | $=$ | $+18$ $\times 2$ |
| $+5x$ | $+4y$ | $=$ | $+32$ |

**4** Subtract the second equation from the first equation by reversing all the signs.

| | | | |
|---|---|---|---|
| $+6x$ | $+4y$ | $=$ | $+36$ |
| $-5x$ | $-4y$ | $=$ | $-32$ |
| $+x$ | $+0y$ | $=$ | $+4$ |

**5** Substitute the value of the now known variable into the simpler equation to get the value of the other variable.

We know that,

$x = 4$

And, $3x + 2y = 18$

$\Rightarrow 3 \times 4 + 2y = 18$

$\Rightarrow 12 + 2 = 18$

$\Rightarrow 2y = 6$

$\Rightarrow y = 3$

From the above, x = 4 and y = 3.
Therefore, (4,3) is the solution of the simultaneous equations
"3x + 2y = 18" and
"5x + 4y = 32".

**6** Verify the values obtained for $x$ and $y$ by putting them in the given equations

$3x + 2y = 18$,

LHS $= 3x + 2y$

$= 3 \times 4 + 2 \times 3$

$=$ RHS

$5x + 4y = 32$

LHS $= 5x + 4y$

$= 5 \times 4 + 4 \times 3$

$=$ RHS

# 3.4 Cross-Multiplication Method

Write the **two equations** in the **general form:**

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Now write the **coefficients, variables and constants in the pattern shown**. After that multiply and subtract in the direction of the arrows as shown:

$$\begin{array}{ccccccc} & x & & y & & 1 & \\ b_1 & & c_1 & & a_1 & & b_1 \\ b_2 & & c_2 & & a_2 & & b_2 \end{array}$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Get $x$ and $y$ in the LHS and **substitute the respective values** to get the answer.

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \qquad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

# Solving Non-Linear Pair of Equations

Solve for x and y.

$$\frac{5}{x-1} + \frac{1}{y} = 2 \quad , \quad \frac{6}{x-1} - \frac{3}{y} = 1$$

Identify what part can be substituted with some other variables to make the equation linear.

$$5 \times \left(\frac{1}{x-1}\right) + 1 \times \left(\frac{1}{y}\right) = 2$$

$$6 \times \left(\frac{1}{x-1}\right) - 3 \times \left(\frac{1}{y}\right) = 1$$

$$p = \frac{1}{x-1}$$

$$q = \frac{1}{y}$$

Solve the obtained pair of linear equations using any method

$$5p + q = 2$$

$$6p - 3q = 1$$

$$p = \frac{1}{3}$$

$$q = \frac{1}{3}$$

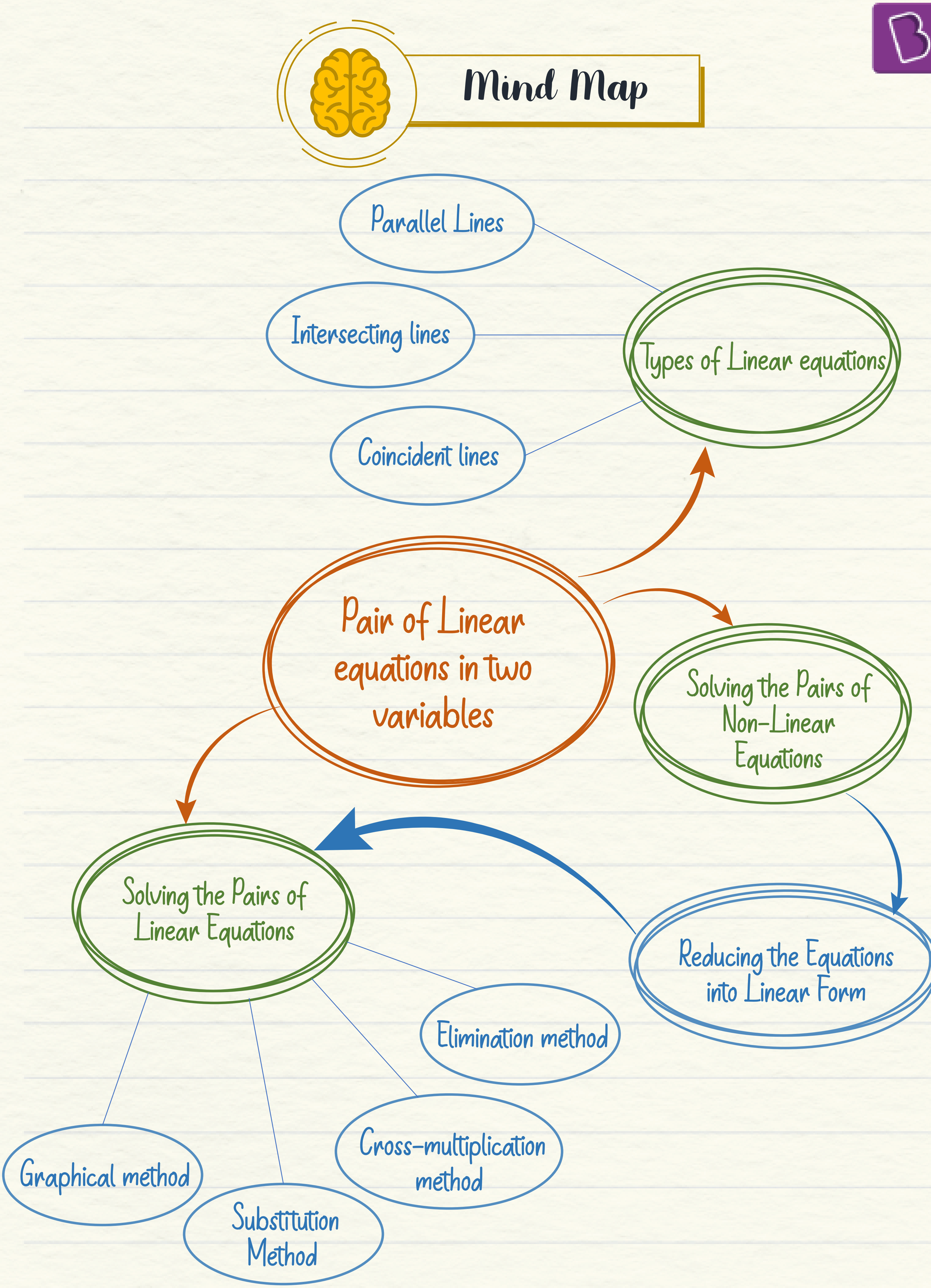
Now solve for x and y.

$$x = \frac{1}{p} + 1$$

$$\boxed{x = \frac{4}{3}}$$

$$y = \frac{1}{q}$$

$$\boxed{y = 3}$$


Mind Map
Parallel Lines
Intersecting lines
Coincident lines
Types of Linear equations
Pair of Linear equations in two variables
Solving the Pairs of Non-Linear Equations
Solving the Pairs of Linear Equations
Reducing the Equations into Linear Form
Elimination method
Cross-multiplication method
Graphical method
Substitution Method