MATHEMATICS

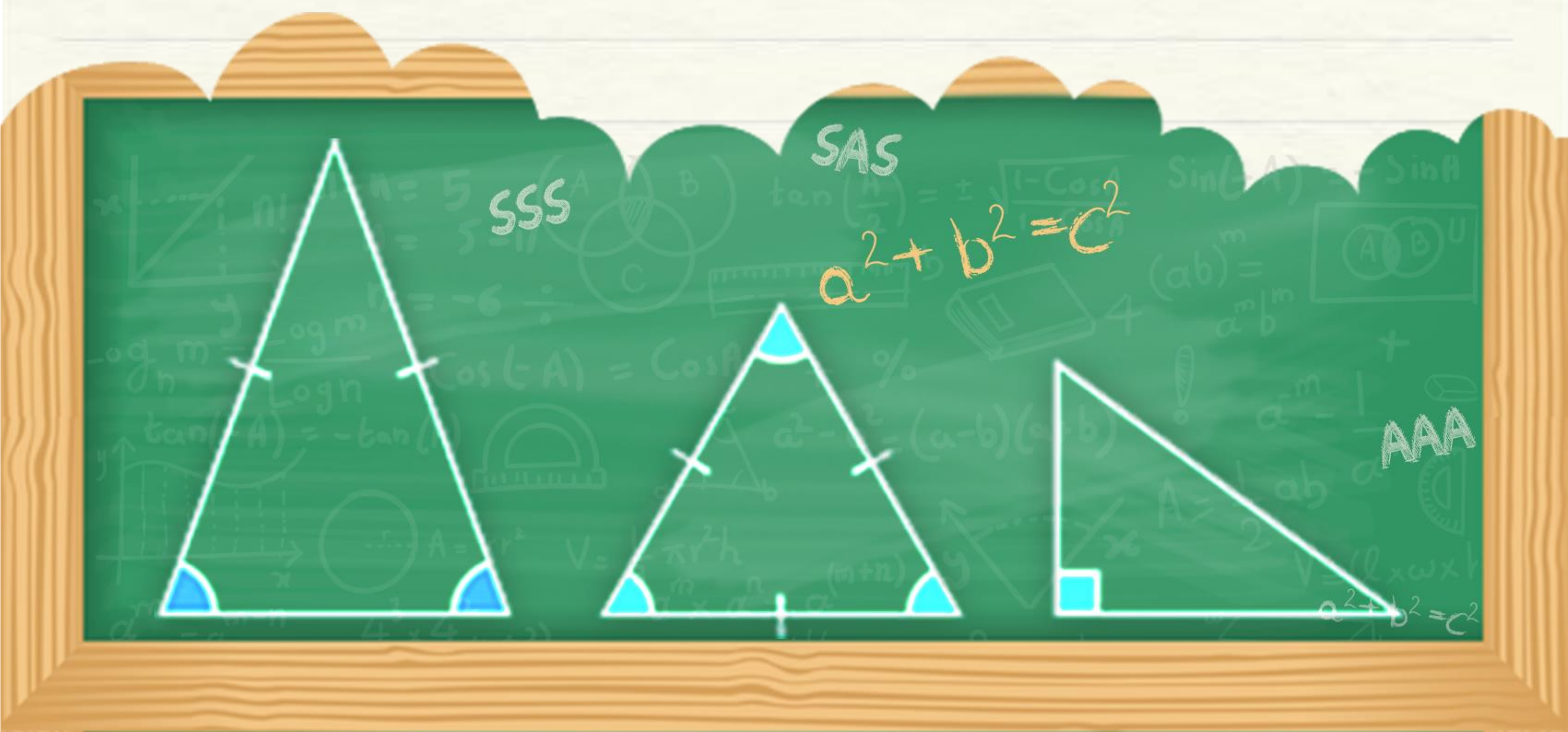
BYJU'S

POST CLASS NOTES

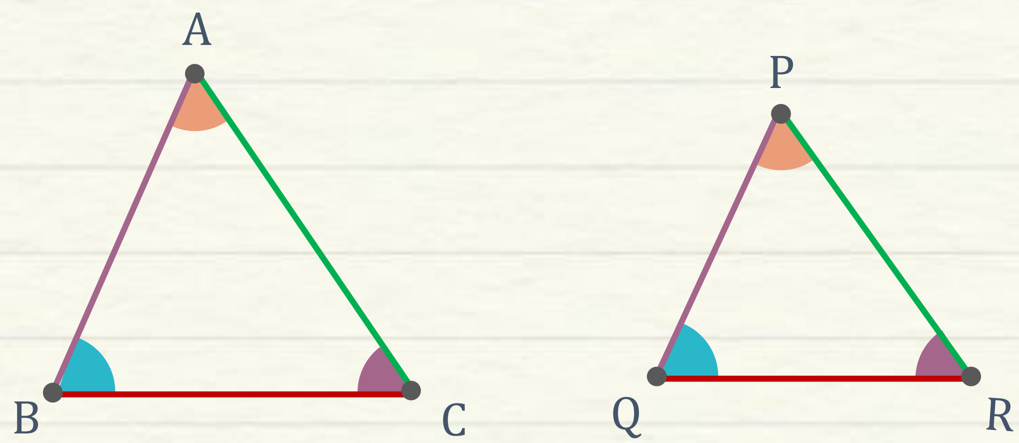
Triangles

Topics

1. Similar Triangles
2. Criteria of Similarity of Triangles
3. Pythagoras Theorem
4. Basic Proportionality Theorem



Similar Triangles



Similar Triangles → Same Shape

Similar Triangles → Different or Same size

Relation between Corresponding Sides and Angles

★ Two triangles are similar, if

- Their corresponding angles are equal.

$$\angle A = \angle P$$

$$\angle B = \angle Q$$

$$\angle C = \angle R$$

- Their corresponding sides are in the same ratio.

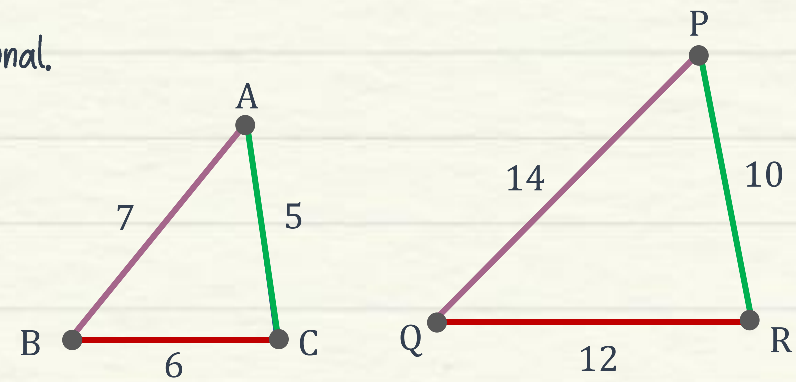
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = k$$

Criteria for Similarity of Triangles

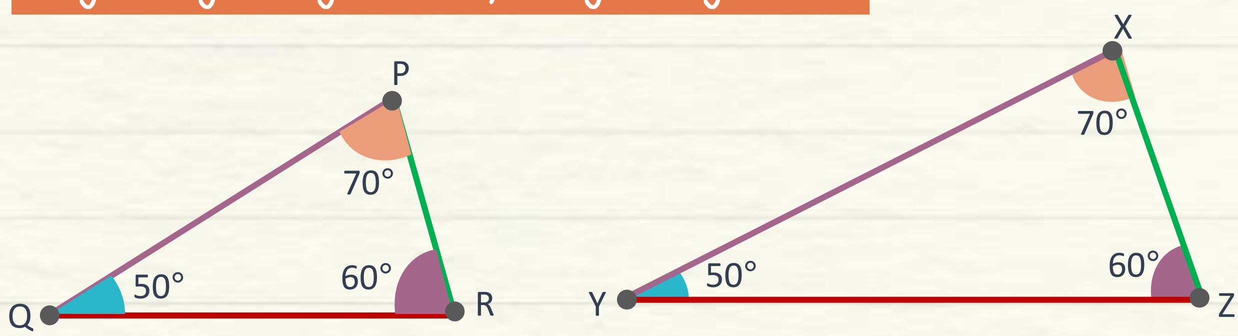
Side-Side-Side (SSS)

- Corresponding sides are proportional.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$



Angle-Angle-Angle (AAA) / Angle-Angle (AA)



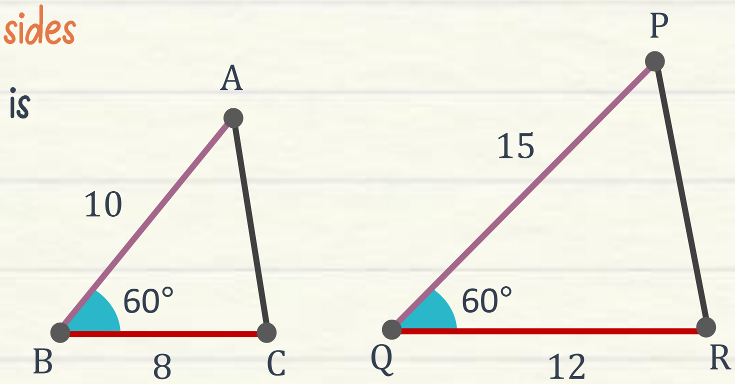
- Corresponding angles are equal.
- Triangles are similar even if a pair of corresponding angles are equal.

Side-Angle-Side (SAS)

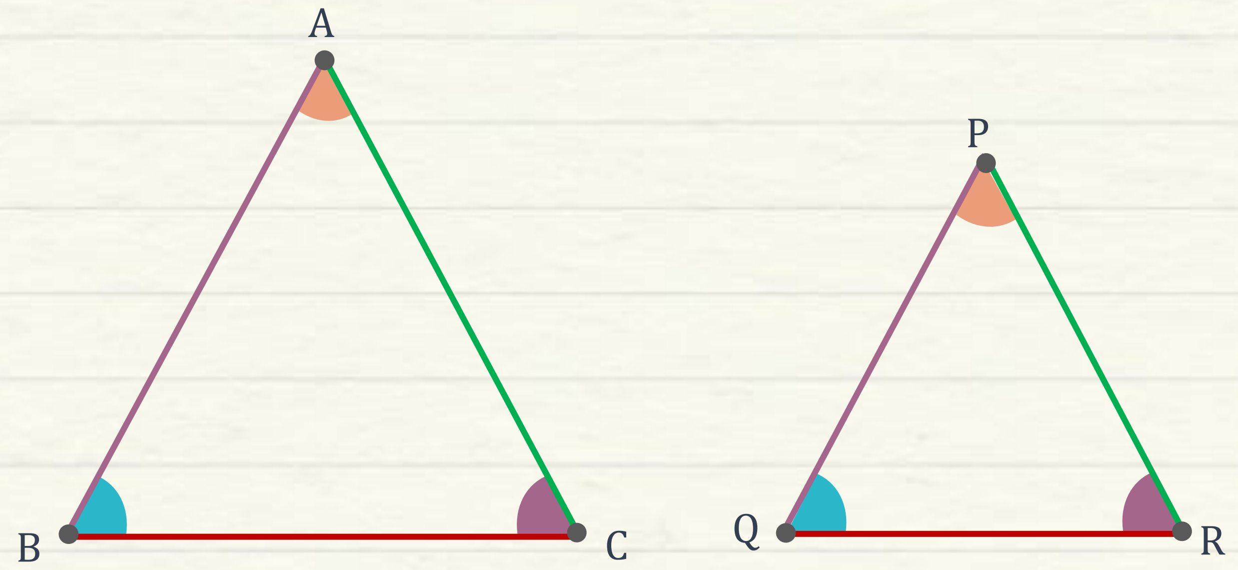
- Pair of adjacent corresponding sides are proportional and one angle is equal.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{2}{3}$$

$\angle B = \angle Q$



Ratio of Areas of Similar Triangles

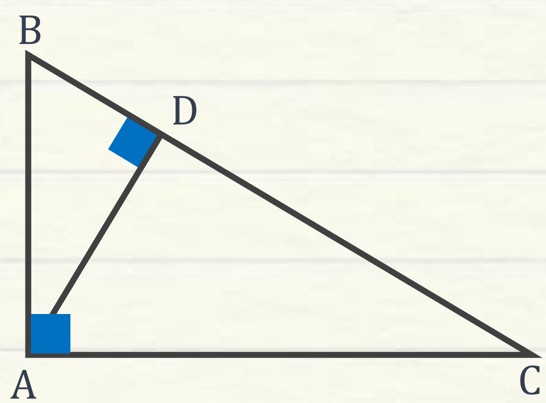


Ratio of Area of Similar Triangles

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Properties of Right-Angled Triangles

Similarity of triangles when a perpendicular is drawn from the vertex of the right angle.



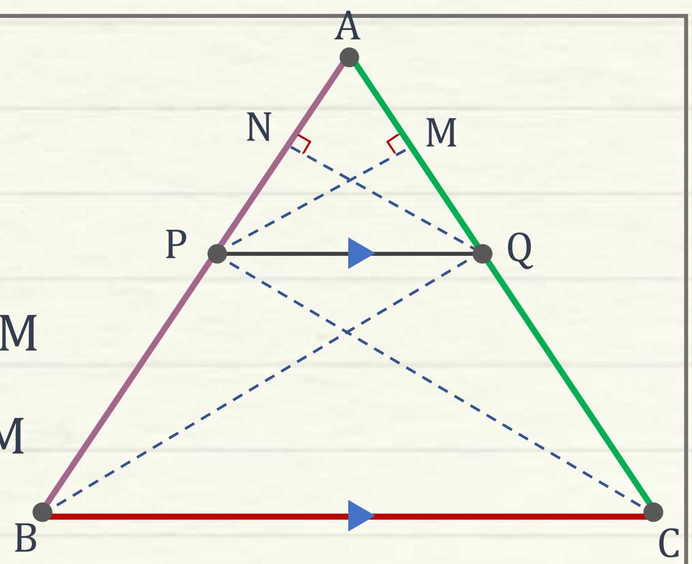
$\Delta ABC \sim \Delta ADC \sim \Delta ADB$ (AA Similarity)

All the three triangles have:

- A right-angle.
- A common angle.

Basic Proportionality Theorem

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.



Proof:

Area of $\Delta APQ = \frac{1}{2} \times AP \times QN$

Area of $\Delta PBQ = \frac{1}{2} \times PB \times QN$

Area of $\Delta APQ = \frac{1}{2} \times AQ \times PM$

Area of $\Delta QCP = \frac{1}{2} \times QC \times PM$

Now,

$$\frac{\text{Area of } \Delta\, APQ}{\text{Area of } \Delta\, PBQ} = \frac{\frac{1}{2} \times AP \times QN}{\frac{1}{2} \times PB \times QN} = \frac{AP}{PB} \quad \ldots\ldots\ldots\ldots(1)$$

Similarly,

$$\frac{\text{Area of } \Delta\, APQ}{\text{Area of } \Delta\, QCP} = \frac{\frac{1}{2} \times AQ \times PM}{\frac{1}{2} \times QC \times PM} = \frac{AQ}{QC} \quad \ldots\ldots\ldots(2)$$

The triangles drawn between the same parallel lines and on the same base have equal areas.

$\therefore$ Area of $\Delta PBQ =$ Area of ΔQCP ....(3)

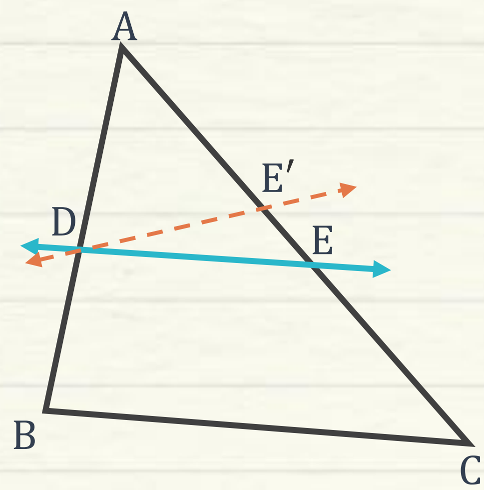
From (1), (2) and (3)

$$\boxed{\frac{AP}{PB} = \frac{AQ}{QC}}$$

Converse of Basic Proportionality Theorem

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Proof:



If $\frac{AD}{DB} = \frac{AE}{EC}$, then DE || BC.

Suppose a line DE, intersects the two sides of a triangle AB and AC at D and E, such that:

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \ldots\ldots(1)$$

Assume DE is not parallel to BC. Now, draw a line DE' parallel to BC.

Hence, by Basic Proportionality Theorem,

$$\frac{AD}{DB} = \frac{AE'}{E'C} \quad \ldots\ldots(2)$$

From eq. 1 and 2, we get

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

Adding 1 on both the sides

$$\frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1 \Rightarrow \frac{AE+EC}{EC} = \frac{AE'+E'C}{E'C}$$

$$\frac{AC}{EC} = \frac{AC}{E'C} \Rightarrow \text{So, } EC = E'C$$

This is possible only when E and E' coincides.

But DE' || BC

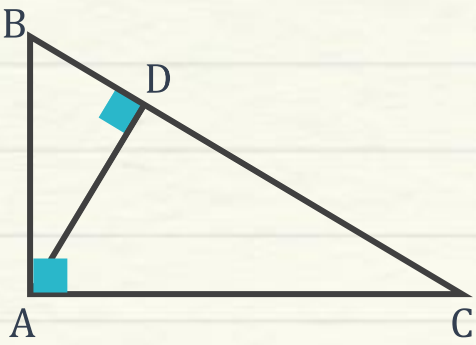
∴ DE || BC.

Properties of Right-Angled Triangles

Pythagoras Theorem

In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides.

Proof:



$\triangle ADB \sim \triangle ABC$

$\therefore \dfrac{AD}{AB} = \dfrac{AB}{AC}$ (corresponding sides of similar triangles)

$AB^2 = AD \times AC$ ....(1)

Also, $\triangle ADC \sim \triangle ABC$

$\therefore \dfrac{CD}{BC} = \dfrac{BC}{AC}$ (corresponding sides of similar triangles)

$BC^2 = CD \times AC$ ....(2)

(1) + (2)

$AB^2 + BC^2 = AD \times AC + CD \times AC$

$AB^2 + BC^2 = AC\,(AD + CD)$

Since, $AD + CD = AC$

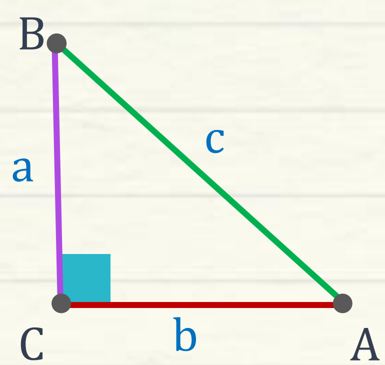
$$\boxed{\therefore AC^2 = AB^2 + BC^2}$$

Converse of Pythagoras Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

Proof:

Construct another triangle, $\triangle EGF$, such as $AC = EG$ and $BC = FG$.



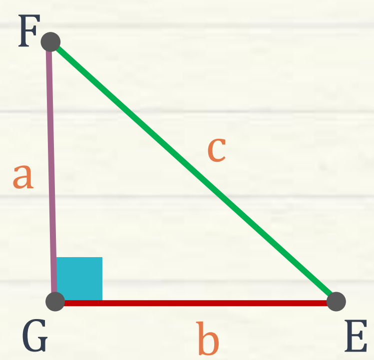
In $\triangle EGF$, by Pythagoras Theorem:

$EF^2 = EG^2 + FG^2 = b^2 + a^2$ ......(1)

In $\triangle ABC$, by Pythagoras Theorem:

$AB^2 = AC^2 + BC^2 = b^2 + a^2$ ......(2)

From (1) and (2)



$EF^2 = AB^2$

$EF = AB$

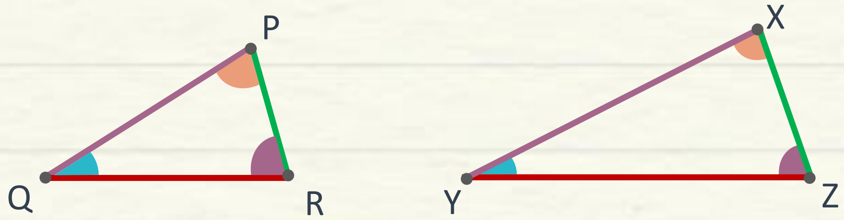
$\Rightarrow \triangle ACB \cong \triangle EGF$ (By SSS)

$\Rightarrow \angle C$ is right angle

$\therefore$ $\triangle ABC$ is a right triangle.

Important Theorems and Formulae

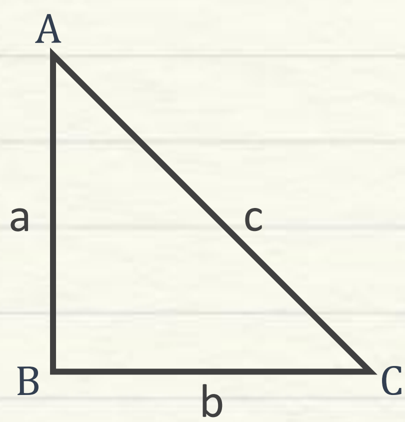
Similarity of Triangles

SSS	
AAA/AA	
SAS	

Pythagoras Theorem

- In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides.

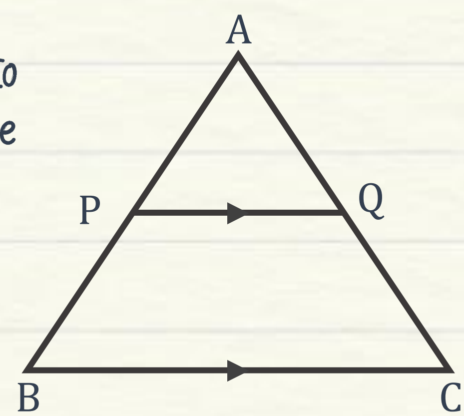
$$\boxed{a^2 + b^2 = c^2}$$



Basic Proportionality Theorem

- If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$\boxed{PQ \parallel BC, \quad \frac{AP}{PB} = \frac{AQ}{QC}}$$



Mind Map


SAS
AAA
Basic Proportionality Theorem
SSS
Converse of Basic Proportionality Theorem
Criterion for Similarity
Similarity of Triangles
Ratios of Areas of Similar Triangles
Properties of right triangles
Pythagoras theorem
Converse of Pythagoras Theorem