Triangles
1. Similar Triangles
2. Criteria of Similarity of Triangles
3. Pythagoras Theorem
4. Basic Proportionality Theorem
**Similar Triangles**

Two triangles are similar, if

- **Their corresponding angles are equal.**

\[ \angle A = \angle P \]
\[ \angle B = \angle Q \]
\[ \angle C = \angle R \]

- **Their corresponding sides are in the same ratio.**

\[ \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = k \]
Criteria for Similarity of Triangles

**Side-Side-Side (SSS)**

- Corresponding sides are proportional,

\[
\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}
\]

**Angle-Angle-Angle (AAA)/Angle-Angle (AA)**

- Corresponding angles are equal.
- Triangles are similar even if a pair of corresponding angles are equal.

**Side-Angle-Side (SAS)**

- Pair of adjacent corresponding sides are proportional and one angle is equal,

\[
\frac{AB}{PQ} = \frac{BC}{QR} = \frac{2}{3}
\]

\[\angle B = \angle Q\]
**Ratio of Areas of Similar Triangles**

\[
\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2
\]

**Properties of Right-Angled Triangles**

Similarity of triangles when a perpendicular is drawn from the vertex of the right angle.

\(\triangle ABC \sim \triangle ADC \sim \triangle ADB\) (AA Similarity)

All the three triangles have:
- A right-angle.
- A common angle.
Basic Proportionality Theorem

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

**Proof:**

Area of $\triangle APQ = \frac{1}{2} \times AP \times QN$
Area of $\triangle PBQ = \frac{1}{2} \times PB \times QN$
Area of $\triangle APQ = \frac{1}{2} \times AQ \times PM$
Area of $\triangle QCP = \frac{1}{2} \times QC \times PM$

Now,

\[
\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle PBQ} = \frac{\frac{1}{2} \times AP \times QN}{\frac{1}{2} \times PB \times QN} = \frac{AP}{PB} \quad \text{(1)}
\]

Similarly,

\[
\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle QCP} = \frac{\frac{1}{2} \times AQ \times PM}{\frac{1}{2} \times QC \times PM} = \frac{AQ}{QC} \quad \text{(2)}
\]

The triangles drawn between the same parallel lines and on the same base have equal areas.

\[\therefore \text{Area of } \triangle PBQ = \text{Area of } \triangle QCP \quad \text{(3)}\]

From (1), (2) and (3)

\[
\frac{AP}{PB} = \frac{AQ}{QC}
\]
Converse of Basic Proportionality Theorem

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

**Proof:**

If \( \frac{AD}{DB} = \frac{AE}{EC} \), then \( DE \parallel BC \).

Suppose a line \( DE \), intersects the two sides of a triangle \( AB \) and \( AC \) at \( D \) and \( E \), such that:

\[ \frac{AD}{DB} = \frac{AE}{EC} \tag{1} \]

Assume \( DE \) is not parallel to \( BC \). Now, draw a line \( DE' \) parallel to \( BC \).

Hence, by Basic Proportionality Theorem,

\[ \frac{AD}{DB} = \frac{AE'}{E'C} \tag{2} \]

From eq. \( 1 \) and \( 2 \), we get

\[ \frac{AE}{EC} = \frac{AE'}{E'C} \]

Adding 1 on both the sides

\[ \frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1 \quad \Rightarrow \quad \frac{AE+EC}{EC} = \frac{AE'+E'C}{E'C} \]

\[ \frac{AC}{EC} = \frac{AC}{E'C} \quad \Rightarrow \quad \text{So, } EC = E'C \]

This is possible only when \( E \) and \( E' \) coincides.

But \( DE' \parallel BC \)

\[ \therefore \quad DE \parallel BC. \]
Properties of Right-Angled Triangles

Pythagoras Theorem

In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides.

Proof:

\( \triangle ADB \sim \triangle ABC \)

\[ \frac{AD}{AB} = \frac{AB}{AC} \quad (\text{corresponding sides of similar triangles}) \]

\[ AB^2 = AD \times AC \quad (1) \]

Also, \( \triangle ADC \sim \triangle ABC \)

\[ \frac{CD}{BC} = \frac{BC}{AC} \quad (\text{corresponding sides of similar triangles}) \]

\[ BC^2 = CD \times AC \quad (2) \]

\((1) + (2)\)

\[ AB^2 + BC^2 = AD \times AC + CD \times AC \]

\[ AB^2 + BC^2 = AC (AD + CD) \]

Since, \( AD + CD = AC \)

\[ \therefore AC^2 = AB^2 + BC^2 \]
Converse of Pythagoras Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

Proof:

Construct another triangle, \( \triangle EGF \), such as \( AC = EG \) and \( BC = FG \).

In \( \triangle EGF \), by Pythagoras Theorem:
\[
EF^2 = EG^2 + FG^2 = b^2 + a^2 \quad \ldots \quad (1)
\]

In \( \triangle ABC \), by Pythagoras Theorem:
\[
AB^2 = AC^2 + BC^2 = b^2 + a^2 \quad \ldots \quad (2)
\]

From (1) and (2)

\[
EF^2 = AB^2
\]

\[ EF = AB \]

\[ \Rightarrow \triangle ACB \cong \triangle EGF \ (By \ SSS) \]

\[ \Rightarrow \angle C \text{ is right angle} \]

\[ \therefore \triangle ABC \text{ is a right triangle.} \]
### Similarity of Triangles

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<th>AAA/AA</th>
<th>SAS</th>
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#### Pythagoras Theorem

In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides.

\[ a^2 + b^2 = c^2 \]

#### Basic Proportionality Theorem

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

\[ PQ \parallel BC, \quad \frac{AP}{PB} = \frac{AQ}{QC} \]
Similarity of Triangles

- Basic Proportionality Theorem
- Converse of Basic Proportionality Theorem
- SSS
- SAS
- AAA

Properties of right triangles
- Pythagoras theorem
- Converse of Pythagoras Theorem

Ratios of Areas of Similar Triangles