

Date: 11/11/2021 Subject: Mathematics Topic : Polynomials

Class: X

1. Identify the cubic polynomials among the following.

- 1. $2x^3 + 3x^2 + 2x + 1$ 2. $x^3 + 2x + 3$ 3. $\sqrt{3}x + 5$ 4. $y + \sqrt{2}$ **X** A. 1,2 and 3 **X** B. 3 and 4
- **x C**. 2 and 3

D. 1 and 2

For a cubic polynomial, the degree of the variable in the polynomial should be 3.

1. $2x^3 + 3x^2 + 2x + 1$ – Degree of x is 3. So, cubic polynomial

- 2. $x^3 + 2x + 3$ Degree of x is 3. So, cubic polynomial.
- 3. $\sqrt{3}x + 5$ Degree of x is 1 Linear polynomial.
- 4. $y + \sqrt{2}$ Degree of y is 1 Linear polynomial.





Zero of a polynomial is that value of x for which the value of the polynomial becomes zero. The maximum number of zeroes of a polynomial is equal to the degree of the polynomial. The zeroes may be real (equal or unequal) or unreal.

In this question, the graph is not intersecting the x-axis. So, the polynomial has no zeros.





3. Which of the following graph represents the quadratic polynomial $-x^2 + 5x - 6$?



D. Cannot be represented on a graph.

One way of solving this question is to find the zeroes of given polynomial by conventional factorization. But we will solve it in a smart way. We just find the sum of zeroes from polynomial and compare it from the graphs, which satisfies the sum of zeroes.

From polynomial,

Sum of zeroes = $\frac{-b}{a} = \frac{-5}{-1} = 5$

From the graphs given , we look for graph whose sum of zeroes is 5. Thus graph having 2,3 as zeroes is the required graph.



4. The zeros of the polynomial $x^2 - \sqrt{2}x - 12$ are A. $\sqrt{2}, -\sqrt{2}$ (X) \checkmark B. $_{3\sqrt{2},-2\sqrt{2}}$ **x** C. $_{3-\sqrt{2},2\sqrt{2}}$ **x** D. $_{3\sqrt{2},2\sqrt{2}}$ Given, $x^2 - \sqrt{2}x - 12$ Let's factorise using formula $x=rac{-b\pm\sqrt{b^2-4ac}}{2a}$ $a=1; \;\; b=-\sqrt{2}; \;\; c=-12$ $x=rac{-(-\sqrt{2})\pm\sqrt{(-\sqrt{2})^2-4 imes1 imes(-12)}}{2 imes1}$ $=\frac{\sqrt{2}\pm\sqrt{2+48}}{2}=\frac{\sqrt{2}\pm\sqrt{50}}{2}$ $= \frac{\sqrt{2} \pm \sqrt{2 \times 5 \times 5}}{2} = \frac{\sqrt{2} \pm 5 \sqrt{2}}{2}$ $x=rac{\sqrt{2}+5\sqrt{2}}{2};\;\;rac{\sqrt{2}-5\sqrt{2}}{2}$ $=rac{6\sqrt{2}}{2}; \ rac{-4\sqrt{2}}{2}$ $x=3\sqrt{2};~-2\sqrt{2}$ Zeroes are $3\sqrt{2} \& -2\sqrt{2}$



5. If a and b are the zeroes of a polynomial $px^2 - 5x + q$, then find the values of p and q, if a + b = ab = 10.

5 and $\frac{1}{2}$ × Α. Β. 5 and 2C. $\frac{1}{2}$ and 5 X D. 10 and 1We know that, for a quadratic equation $ax^2 + bx + c = 0$, Sum of roots = $\alpha + \beta$ and product of roots $= \alpha \beta$ where α and β are the roots of the equation. Also, $\alpha + \beta = \frac{-b}{a}$ and $\alpha \beta = \frac{c}{a}$ So, $a+b=rac{5}{p} ext{and}\ ab=rac{q}{p}$ It is given that a + b = ab = 10 $\Rightarrow 10 = \frac{5}{p}$ Hence, $p = \frac{1}{2}$ Also, $ab = \frac{q}{p}$ $egin{array}{ll} \Rightarrow 10 = rac{q}{p} \ \Rightarrow q = 10p \end{array}$

 $\Rightarrow q = 10 imes rac{1}{2}$

 $\Rightarrow q = 5$



6. What is the maximum number of times the graph of the polynomial $y = px^3 + qx^2 + rx + s$ intersects the x axis?



The graph of a polynomial of degree 'n' can cut the x-axis at a maximum of 'n' values.

Since the given polynomial has a degree 3, it can cut the x axis at most 3 times.

 According to the graph below, the product of the zeroes of the polynomial will be





In the graph, the polynomial curve intersects the X – axis at 1 point. So number of zeroes is 1

8.



Practice Questions - Term I







In a quadratic polynomial, if the coefficient of 'a' is negative, then the graph is upside down, i.e., the two hands of the graph point downwards.

^{10.} The graph of y = p(x) is given. How many zeroes can exist? Assume that the p(x) is always increasing beyond x = 10.



In the graph, the line intersects the X – axis at no point. So number of zeroes is 0

X

D. 3



11. If x = 2 and x = 1 are the zeroes of the quadratic polynomial $ax^2 - 3x + b$, then find the value of a - b.

X A. 0
X B. 1
C. -1
X D. 2
Let
$$f(x) = ax^2 - 3x + b$$
.
Since $x = 2$ is a zero of $f(x)$, $f(2) = 0$.
 $\Rightarrow 4a - 6 + b = 0$(*i*)
Also, since $x = 1$ is a zero $f(x)$, $f(1) = 0$,
 $\Rightarrow a - 3 + b = 0$(*ii*)
(*i*) - (*ii*) $\Rightarrow 3a = 3$
 $\Rightarrow a = 1$

 $\Rightarrow a = 1$ Thus, (ii) $\Rightarrow b = 2$

Hence, (a-b) = 1 - 2 = -1.

12. If the sum of the zeroes of the polynomial $9x^2 - kx + 2$ is $\frac{11}{9}$ find the value of k.



Given polynomial: $9x^2 - kx + 2$ On dividing the poynomial by 9, we get $x^2 - \frac{k}{9} + \frac{2}{9}$.

We know that, the general quadratic equation can be written as $x^2 - (\text{Sum of zeroes})x + (\text{Product of zeroes})$

The sum of the zeroes is given as $\frac{11}{9}$. $\Rightarrow \frac{k}{9} = \frac{11}{9}$

$$\Rightarrow \frac{1}{9} = \frac{1}{9}$$

 $\Rightarrow k = 11$



13. Find a cubic polynomial whose zeroes are 2, - 3 and 4.



^{14.} Find the quadratic polynomial whose sum of its zeroes (roots) is $-\frac{8}{5}$ and the product of the zeroes (roots) is $\frac{7}{5}$.

X A. $14x^2 + 7x + 5$ **B**. $5x^2 + 8x + 7$ **X** C. $2x^2 - 8x + 7$ **X** D. $5x^2 - 8x + 7$

Given that,

Sum of zeroes = $\frac{-8}{5}$

Product of zeroes = $\frac{7}{5}$

Required quadratic polynomial is,

 $f(x) = [(x^2 - (sum \ of \ roots)x + (product \ of \ roots)]$

Substituting the given values we get,

$$f(x) = [x^2 - rac{(-8)}{5}x + rac{7}{5}]$$

 $f(x)=[x^2+rac{8}{5}x+rac{7}{5}]$

multiplying by 5 we get

$$f(x) = 5x^2 + 8x + 7$$

 \therefore Required polynomial is $5x^2 + 8x + 7$.



15. If α, β and γ are the zeros of the polynomial $2x^3 - 6x^2 - 4x + 30$, then the value of $(\alpha\beta + \beta\gamma + \gamma\alpha)$ is





Based on the given information, answer the following question.

If the equation of the bridge is prepersented by the following graph y = p(x), then name the type of the polynomial it traces.





A. Linear
B. Quadratic
C. Cubic
D. Bi-quadratic

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The graph here, cuts the x-axis at two different points. Hence, the number of zeroes for the given graph is 2.

We know that, the degree of the polynomial indicates the maximum number of zeroes it can have.

Here, the maximum number of zeroes is 2. Thus, y = p(x) has to be a quadratic equation. IBYJU'S

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B



Based on the given information, answer the following question.

If the hills are represented by the cubic polynomial $t(x) = px^3 + qx^2 + rx + s$, then which of the following is always true?





$$\checkmark$$
 D. $p \neq 0$

Given: Hills are represented by the cubic polynomial $t(x) = px^3 + qx^2 + rx + s$.

If s = 0, then $t(x) = px^3 + qx^2 + rx$ r = 0, then $t(x) = px^3 + qx^2 + s$ q = 0, then $t(x) = px^3 + rx + s$ All these are cubic polynomials since the highest degree is 3.

But when p = 0, then $t(x) = qx^2 + rx + s$ which is a quadratic equation. Hence, p cannot be equal to 0.



Based on the given information, answer the following question.

If the path traced by the hills is represented by the graph y = p(x) below,



find the number of zeroes.



In the given figure, the graph of a polynomial p(x) cuts the x-axis at three distinct points. i.e the value of polynomial is equal to zero at these three points.

 \therefore The number of zeroes of p(x)=3



Based on the given information, answer the following question.

Find a quadratic polynomial for the bridge if 6 is the sum and 8 is the product of its zeroes.





To find: A quadratic polynomial whose sum is6 and the product is 8.

We know that, the general quadratic equation can be written as $x^2 - (\text{Sum of zeroes})x + (\text{Product of zeroes})$

Given: Sum of zeroes = 6 Product of zeroes = 8

Hence, the required polynomial $= x^2 - 6x + 8$



Based on the given information, answer the following question.

If the hills are prepresented by the cubic polynomial $t(x) = 2x^3 + 8x^2 + 9x + 16$, then the product of the zeroes is:





To find: The product of the zeroes of the polynomial $t(x) = 2x^3 + 8x^2 + 9x + 16$

We know that, a general cubic polynomial can be written in the form of $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$ where α, β, γ are the zeroes of the polynomial.

On comparing the given polynomial to its general form, $t(x) = x^3 + 4x^2 + rac{9}{2}x + 8$



Here, the negative of the constant term will be the product of the zeroes. Hence, the required answer is -8.