Topics

1. Polynomial and Terms Related to it
2. Special Types of Polynomials
3. Geometrical Meaning of Zeroes of a Polynomial
4. Relationship between Zeroes and Coefficients of a Polynomial
5. Division Algorithm for Polynomials

$p(x) = x^2 - 1$
Definition of a Polynomial

An algebraic expression in which the variable(s) is/are raised to non-negative integral exponents is called a polynomial.

Standard Form of a Polynomial in \( x \) of Degree \( n \)

An algebraic expression of the form

\[
p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0,
\]

where \( a_0, a_1, a_2, \ldots, a_n \) are real numbers and \( a_n \neq 0 \),

is the standard form of a polynomial in \( x \) of degree \( n \).
The Degree of a Polynomial $p(x)$ is the highest exponent to which $x$ is raised.

The Value of a Polynomial $p(x)$ at $x = k$ is obtained by replacing $x = k$ in the polynomial expression.

A real number '$a$' is a Zero of a Polynomial $p(x)$ if $p(a) = 0$.

**Example**

$\text{Degree} = 2.$

$\text{Value of } p(x) \text{ at } x = 1 \text{ is } p(1) = 4(1)^2 - 1 = 3.$

$p(x) = 4x^2 - 1$

Zeroes of $p(x)$ are $\pm \frac{1}{2}$, since $p\left(\frac{1}{2}\right) = p\left(-\frac{1}{2}\right) = 0.$
Special Types of Polynomials

Based on Number of Terms

1 term → Monomial
Ex: $x, -5y$

2 terms → Binomial
Ex: $2x - 5, 6y + 8$

3 terms → Trinomial
Ex: $x^2 - 3x + 2$

Based on Degree

Degree = 1 → Linear
Ex: $2y - 3$

Degree = 2 → Quadratic
Ex: $4x^2 + 5x - 2$

Degree = 3 → Cubic
Ex: $8x^3 - 5$
A zero of a polynomial $p(x)$ represents the x-coordinate of the point where the graph of $p(x)$ intersects the x-axis.
Relationship between Zeros and Coefficients of a Polynomial

**Quadratic Polynomial**

General form: \( p(x) = ax^2 + bx + c \)

Sum of zeroes \( \alpha + \beta = \frac{-b}{a} \)

Product of zeroes \( \alpha \beta = \frac{c}{a} \)

**Cubic Polynomial**

General form: \( p(x) = ax^3 + bx^2 + cx + d \)

Sum of zeroes \( \alpha + \beta + \gamma = \frac{-b}{a} \)

Sum of products of zeroes taken two at a time \( \alpha \beta + \beta \gamma + \gamma \alpha = \frac{c}{a} \)

Product of zeroes \( \alpha \beta \gamma = \frac{-d}{a} \)
Division Algorithm for Polynomials

Dividend = Quotient × Divisor + Remainder

\[ p(x) = q(x) \times g(x) + r(x) \]