MATHEMATICS

BYJU'S

POST CLASS NOTES

Polynomials

Topics

1. Polynomial and Terms Related to it
2. Special Types of Polynomials
3. Geometrical Meaning of Zeroes of a Polynomial
4. Relationship between Zeroes and Coefficients of a Polynomial
5. Division Algorithm for Polynomials

Polynomials

Polynomials

"Poly" means many

"nomials" means terms

So, polynomials means many terms

Definition of a Polynomial

An algebraic expression in which the variable(s) is / are raised to non-negative integral exponents is called a polynomial.

Standard Form of a Polynomial in x of Degree n

An algebraic expression of the form

$$\mathbf{p}(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where $a_0, a_1, a_2, \ldots, a_n$ are real numbers and $a_n \neq 0$,

is the standard form of a polynomial in x of degree n.

Terms Related to Polynomials

The Degree of a Polynomial $p(x)$ is the highest exponent to which x is raised.

The Value of a Polynomial $p(x)$ at $\mathrm{x} = \mathrm{k}$ is obtained by replacing $\mathrm{x} = \mathrm{k}$ in the polynomial expression.

A real number $'a'$ is a Zero of a Polynomial $p(x)$ if $p(a) = 0$.

Example

Degree = 2.

Value of $p(x)$ at $\boldsymbol{x} = \mathbf{1}$ is $p(1) = 4(1)^2 - 1 = 3$.

$$\boldsymbol{p(x) = 4x^2 - 1}$$

Zeroes of $p(x)$ are $\pm\frac{1}{2}$, since $p\left(\frac{1}{2}\right) = p\left(-\frac{1}{2}\right) = 0$.

Special Types of Polynomials

Based on Number of Terms

1 term → Monomial

Ex: x, $-5y$

2 terms → Binomial

Ex: $2x - 5$, $6y + 8$

3 terms → Trinomial

Ex: $x^2 - 3x + 2$

Based on Degree

Degree = 1 → Linear

Ex: $2y - 3$

Degree = 2 → Quadratic

Ex: $4x^2 + 5x - 2$

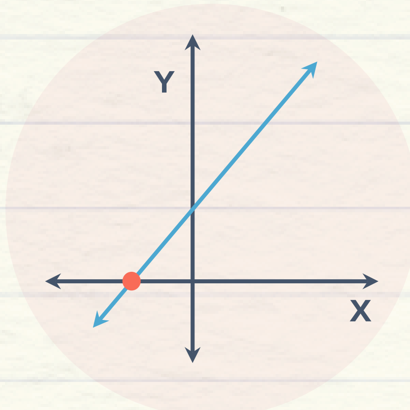
Degree = 3 → Cubic

Ex: $8x^3 - 5$

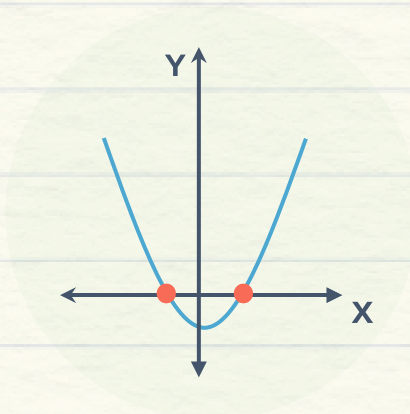
Geometrical Meaning of Zeroes of a Polynomial

A zero of a polynomial $p(x)$ represents the x-coordinate of the point where the graph of $p(x)$ intersects the x-axis.

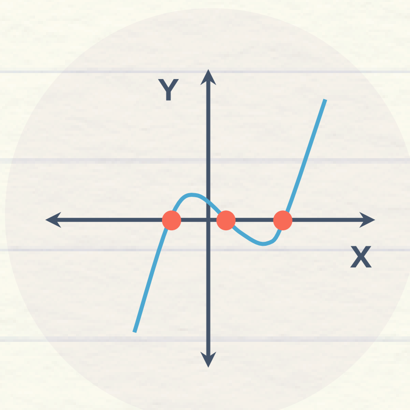
1 Zero



2 Zeroes



3 Zeroes



Relationship between Zeroes and Coefficients of a Polynomial

Quadratic Polynomial

General form: $p(x) = ax^2 + bx + c$

Sum of zeroes $= \alpha + \beta = \dfrac{-b}{a}$

Product of zeroes $= \alpha\beta = \dfrac{c}{a}$

Cubic Polynomial

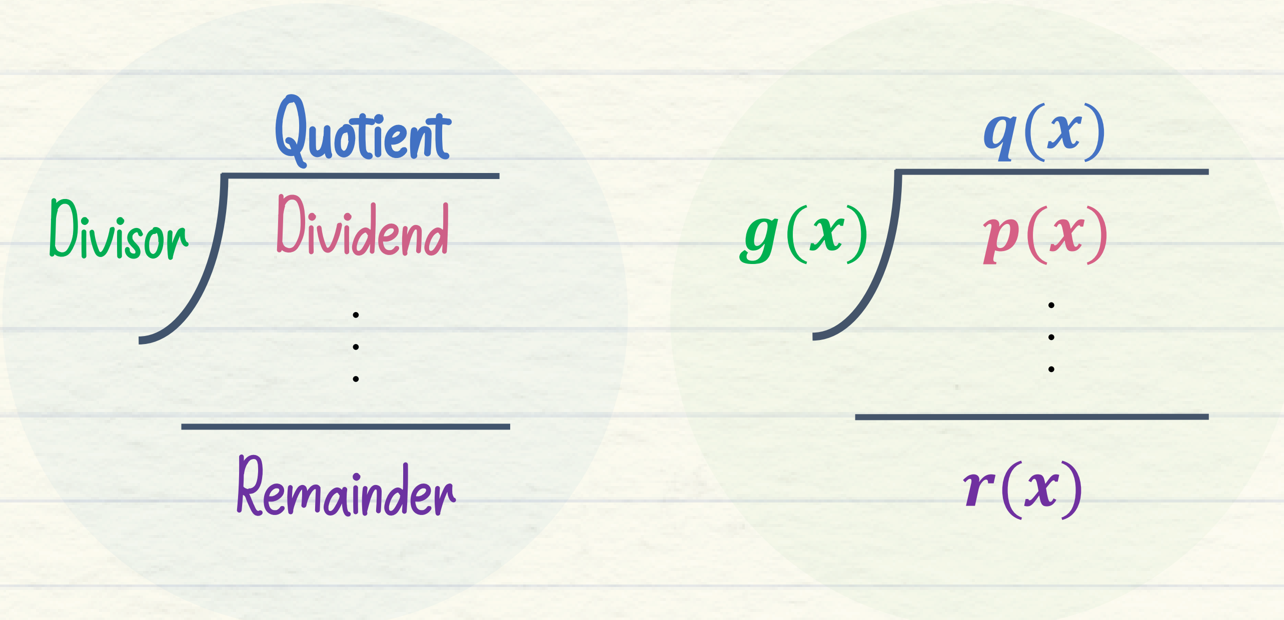
General form: $p(x) = ax^3 + bx^2 + cx + d$

Sum of zeroes $= \alpha + \beta + \gamma = \dfrac{-b}{a}$

Sum of product of zeroes taken two at a time $= \alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{c}{a}$

Product of zeroes $= \alpha\beta\gamma = \dfrac{-d}{a}$

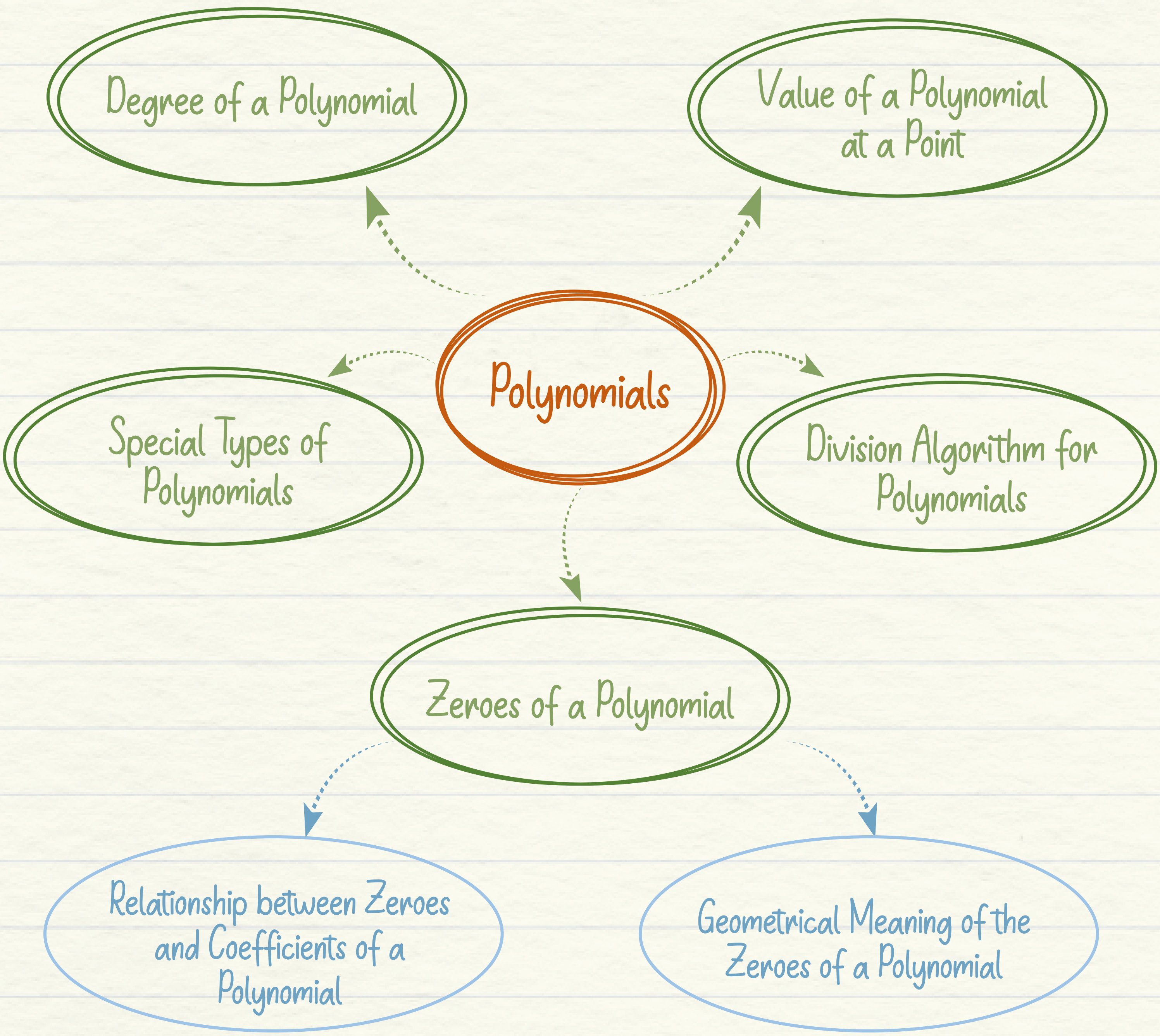
Division Algorithm for Polynomials



Dividend = Quotient × Divisor + Remainder

$$p(x) = q(x) \times g(x) + r(x)$$

Mind Map



Degree of a Polynomial
Value of a Polynomial at a Point
Polynomials
Special Types of Polynomials
Division Algorithm for Polynomials
Zeroes of a Polynomial
Relationship between Zeroes and Coefficients of a Polynomial
Geometrical Meaning of the Zeroes of a Polynomial