



POST CLASS NOTES

# Polynomials





# Topics



1. Polynomial and Terms Related to it
2. Special Types of Polynomials
3. Geometrical Meaning of Zeroes of a Polynomial
4. Relationship between Zeroes and Coefficients of a Polynomial
5. Division Algorithm for Polynomials



# Polynomials

## Polynomials

"Poly" means many

"nomials" means terms

So, polynomials means many terms

### Definition of a Polynomial

An algebraic expression in which the variable(s) is/are raised to non-negative integral exponents is called a polynomial.

### Standard Form of a Polynomial in $x$ of Degree $n$

An algebraic expression of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0,$$

where  $a_0, a_1, a_2, \dots, a_n$  are real numbers and  $a_n \neq 0$ ,

is the standard form of a polynomial in  $x$  of degree  $n$ .



# Terms Related to Polynomials

The **Degree of a Polynomial**  $p(x)$  is the **highest exponent** to which  $x$  is raised.

The **Value of a Polynomial**  $p(x)$  at  $x = k$  is obtained by replacing  $x = k$  in the polynomial expression.

A real number ' $a$ ' is a **Zero of a Polynomial**  $p(x)$  if  $p(a) = 0$ .

Example

Degree = 2.

Value of  $p(x)$  at  $x = 1$  is  
 $p(1) = 4(1)^2 - 1 = 3$ .

$$p(x) = 4x^2 - 1$$

**Zeros of  $p(x)$**  are  $\pm \frac{1}{2}$ , since  
 $p\left(\frac{1}{2}\right) = p\left(-\frac{1}{2}\right) = 0$ .

# Special Types of Polynomials

## Based on Number of Terms

1 term  $\rightarrow$  **Monomial**

Ex:  $x$ ,  $-5y$

2 terms  $\rightarrow$  **Binomial**

Ex:  $2x - 5$ ,  $6y + 8$

3 terms  $\rightarrow$  **Trinomial**

Ex:  $x^2 - 3x + 2$

## Based on Degree

Degree = 1  $\rightarrow$  **Linear**

Ex:  $2y - 3$

Degree = 2  $\rightarrow$  **Quadratic**

Ex:  $4x^2 + 5x - 2$

Degree = 3  $\rightarrow$  **Cubic**

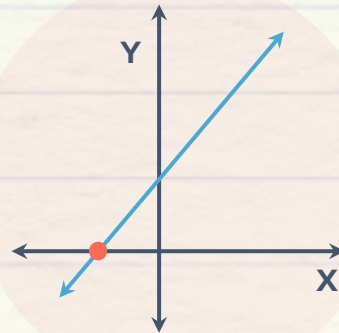
Ex:  $8x^3 - 5$



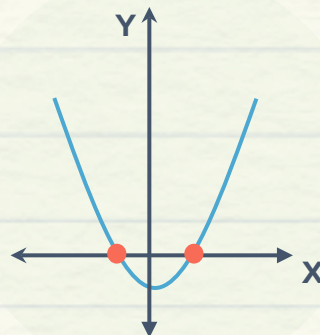
# Geometrical Meaning of Zeros of a Polynomial

A zero of a polynomial  $p(x)$  represents the  $x$ -coordinate of the point where the graph of  $p(x)$  intersects the  $x$ -axis.

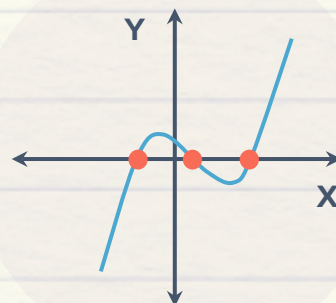
1 Zero



2 Zeros



3 Zeros



# Relationship between Zeroes and Coefficients of a Polynomial

## Quadratic Polynomial

General form:  $p(x) = ax^2 + bx + c$

$$\text{Sum of zeroes} = \alpha + \beta = \frac{-b}{a}$$

$$\text{Product of zeroes} = \alpha\beta = \frac{c}{a}$$

## Cubic Polynomial

General form:  $p(x) = ax^3 + bx^2 + cx + d$

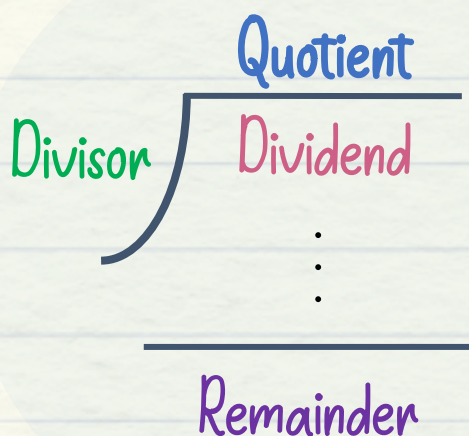
$$\text{Sum of zeroes} = \alpha + \beta + \gamma = \frac{-b}{a}$$

$$\text{Sum of product of zeroes taken two at a time} = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\text{Product of zeroes} = \alpha\beta\gamma = \frac{-d}{a}$$

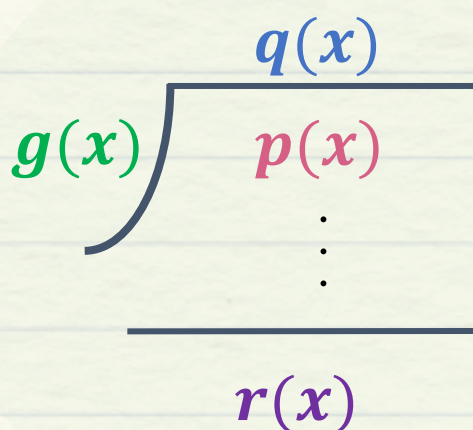


# Division Algorithm for Polynomials



A diagram illustrating the components of polynomial division. It features a large light blue circle containing a division symbol. The word "Divisor" is written in green to the left of the symbol. The word "Quotient" is written in blue above the horizontal line of the symbol. The word "Dividend" is written in pink below the horizontal line. Three vertical dots are positioned below the dividend. A horizontal line is drawn below the dots, and the word "Remainder" is written in purple below this line.

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor} \overline{) \text{Dividend}} \\ \vdots \\ \hline \text{Remainder} \end{array}$$



A diagram illustrating the components of polynomial division. It features a large light green circle containing a division symbol. The expression  $g(x)$  is written in green to the left of the symbol. The expression  $q(x)$  is written in blue above the horizontal line of the symbol. The expression  $p(x)$  is written in pink below the horizontal line. Three vertical dots are positioned below the dividend. A horizontal line is drawn below the dots, and the expression  $r(x)$  is written in purple below this line.

$$\begin{array}{r} q(x) \\ g(x) \overline{) p(x)} \\ \vdots \\ \hline r(x) \end{array}$$

$$\text{Dividend} = \text{Quotient} \times \text{Divisor} + \text{Remainder}$$

$$p(x) = q(x) \times g(x) + r(x)$$





## Mind Map

