MATHEMATICS

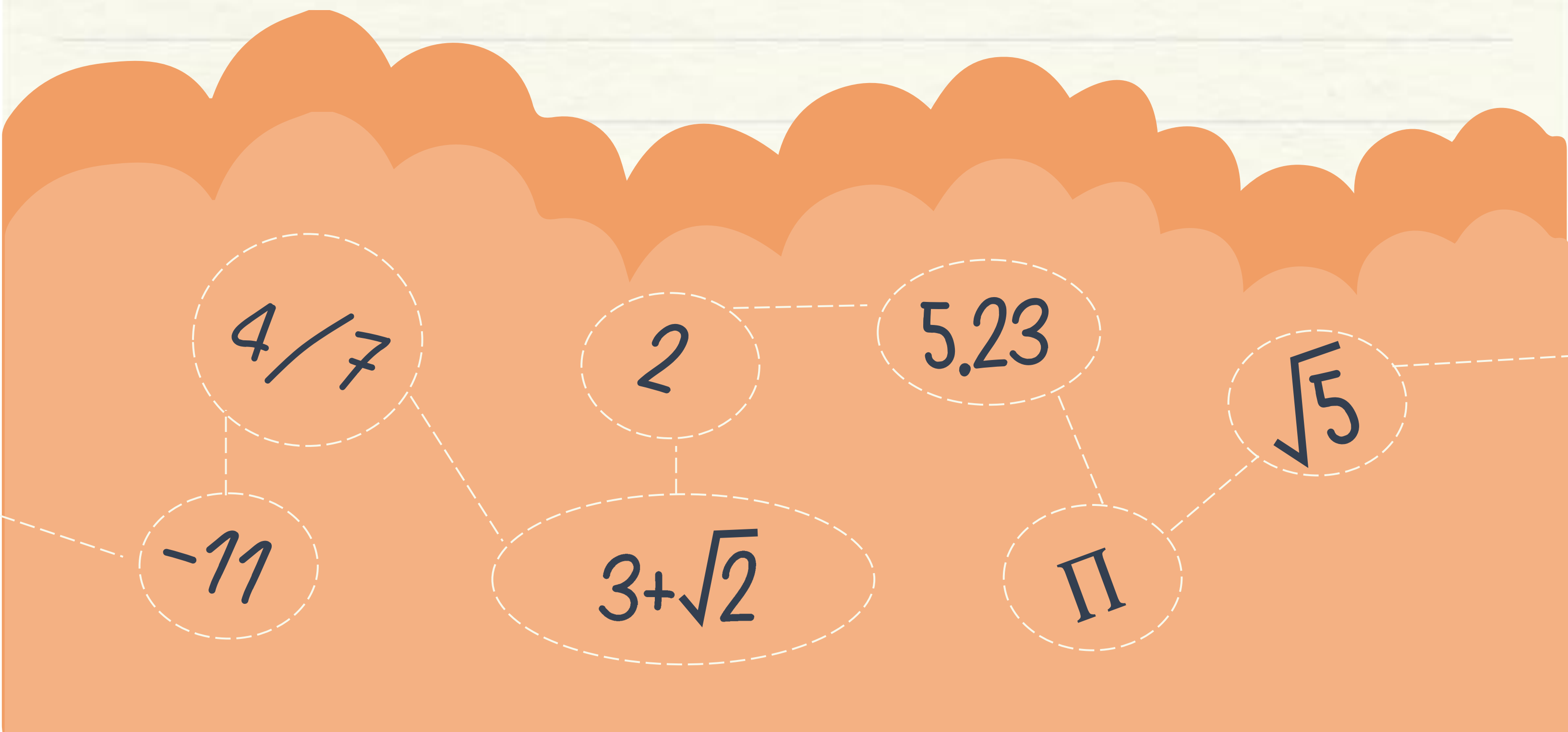
BYJU'S

POST CLASS NOTES

Real Numbers

Topics

1. Algorithm and Lemma
2. Euclid's Division Lemma
3. Euclid's Division Algorithm
4. Fundamental Theorem of Arithmetic
5. Rational Numbers and Their Decimal Expansions
6. Irrational Numbers



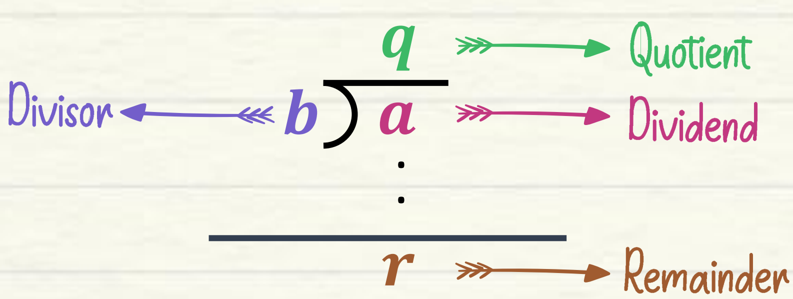
1. Algorithm and Lemma

- An algorithm is a series of well-defined steps which gives a procedure for solving a type of problem.
- A lemma is a proven statement used for proving another statement.

2. Euclid's Division Lemma

If we have two positive integers a and b, then there exist unique integers q and r which satisfy the condition:

$$a = bq + r, \qquad 0 \leq r < b$$



$$\textit{Dividend} = \textit{Divisor} \times \textit{Quotient} + \textit{Remainder}$$

3 Euclid's Division Algorithm

Based on Euclid's division lemma.

To obtain the HCF of two positive integers, say a and b, with a > b, follow the algorithm below:

Consider positive integers a and b

Apply the division lemma to find q and r such that

$$\boldsymbol{a} = \boldsymbol{b}q + r, 0 \leq r < b$$

If $\boldsymbol{r} \neq \mathbf{0}$ → Replace a with b, Replace b with r (and apply the division lemma again)

If $\boldsymbol{r} = \mathbf{0}$ → b is the required HCF.

Example

Use Euclid's algorithm to find the HCF of 441 and 567.

Solution: By Euclid's algorithm, we have

$$567 = \mathbf{441} \times 1 + \mathbf{126}$$

$$\mathbf{441} = \mathbf{126} \times 3 + \mathbf{63}$$

$$\mathbf{126} = \mathbf{63} \times 2 + \mathbf{0}$$

$$\therefore \boldsymbol{HCF}(\mathbf{441}, \mathbf{567}) = \mathbf{63}.$$

4. Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorisation is unique (apart from the order).

Example

The prime factorisation of the number 8190 is:

$$8190 = 2^1 \times 3^2 \times 5^1 \times 7^1 \times 13^1$$

4.1 Theorem Based on Fundamental Theorem of Arithmetic

If a prime number p divides a^2, then p divides a, where a is a positive integer.

Example

Let us consider, p = 3, a = 9

3 divides 9^2

3 divides 9.

4.2 Relation between HCF and LCM

For any two positive integers a and b,

$$HCF\ (a, b) \times LCM\ (a, b) = a \times b$$

Note: This relationship only holds good for two numbers.

5. Rational Numbers and Their Decimal Expansions

Rational number, $Q = \frac{p}{q}$

Decimal Expansion

Terminating	Non-terminating repeating
$q = 2^m \times 5^n$	$q \neq 2^m \times 5^n$

Note: Here **m** and **n** are non-negative integers.

Example

Examine whether $\frac{17}{210}$ is a terminating decimal.

Solution: Here, $q = 210 = 2 \times 3 \times 5 \times 7$

Since denominator is not in the form of $2^m \times 5^n$, the rational number has non-terminating repeating decimal expansion.

6. Irrational Numbers

A number 's' is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Example

Prove that, $\sqrt{2}$ is irrational.

Proof: By using method of contradiction

Assume $\sqrt{2}$ is a rational number.

$\sqrt{2} = \frac{a}{b}$ (a and b are co-primes and $b \neq 0$)

$\Rightarrow b\sqrt{2} = a$

Squaring both the sides

$(b\sqrt{2})^2 = a^2$

$\Rightarrow 2b^2 = a^2$ (a is an even number)

Let a = 2k (k is an integer)

$2b^2 = 4k^2$

$\Rightarrow b^2 = 2k^2$ (b is an even number)

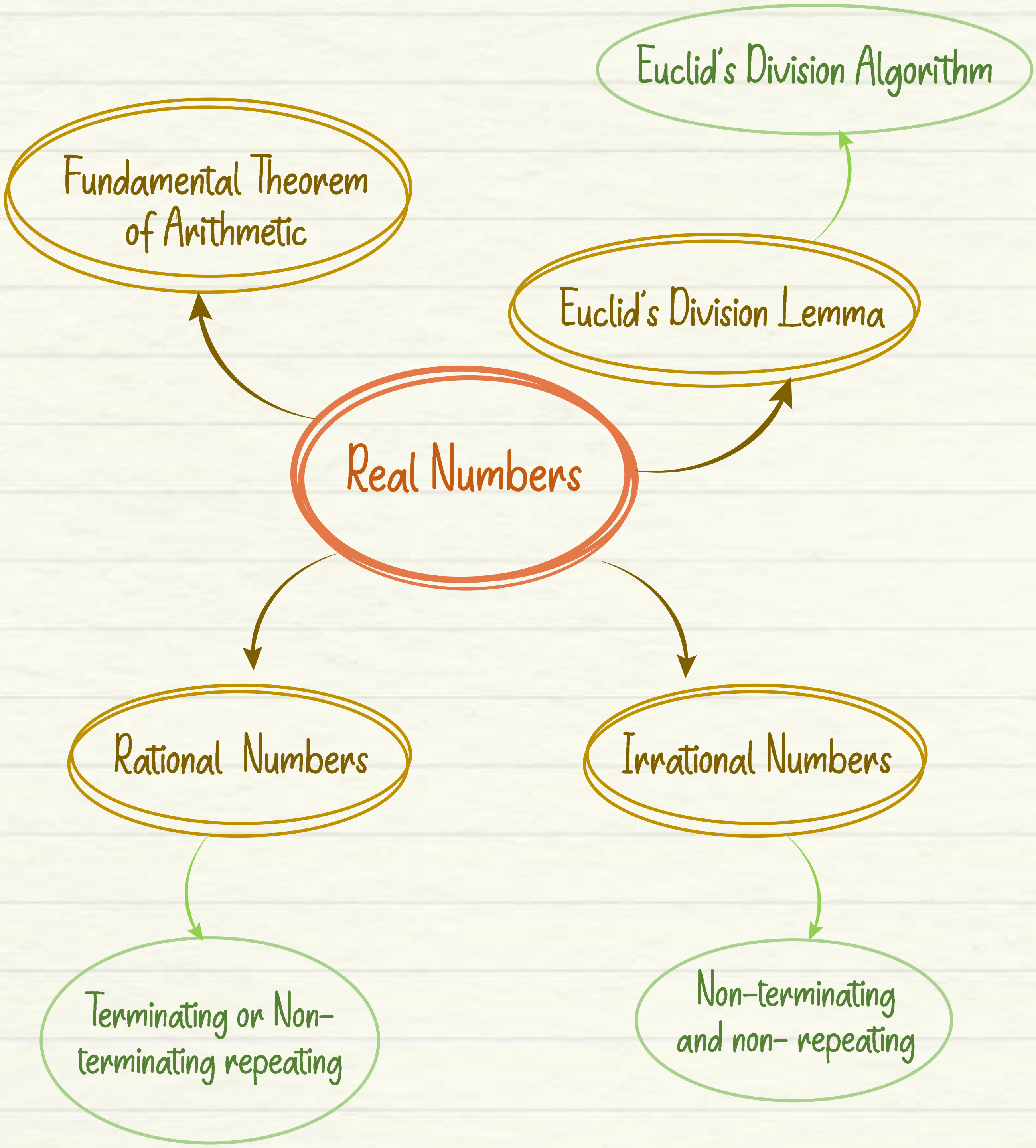
∴ a and b have 2 as a common factor.

But this contradicts the fact that a and b are co-primes.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

Hence $\sqrt{2}$ is irrational.

Mind Map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