BYJU'S Classes

PHYSICS

ELECTROSTATICS

INTRODUCTION TO ELECTROSTATICS

What you already know

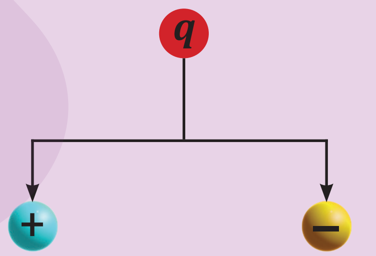
- Reflection by plane and spherical mirrors
- Refraction through spherical surfaces and prism

What you will learn

- Charge and its origin
- Measurement of charge
- Properties of charge
- Methods of charging a body

Charge

- Charge is an intrinsic property of matter.
- A charged body exerts a force on other charged bodies near it.
- There are two types of forces: attraction and repulsion.
- There are two types of charges: positive and negative.
- SI unit of charge: Coulomb
- Standard symbol: C
- Charge of an electron, $e^- = -1e = -1.6 \times 10^{-19}\ C$
- Charge of a proton, $p^+ = +1e = 1.6 \times 10^{-19}\ C$

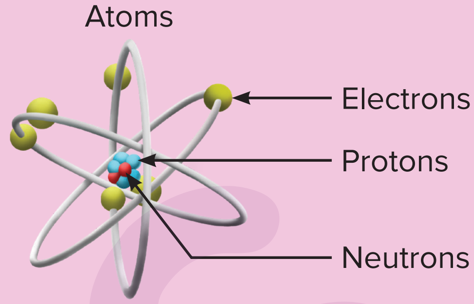


Opposite charges attract one another, while similar charges repel.

When we rub a glass rod with a cloth, the glass rod gets a positive charge. However, if we rub a plastic rod with the cloth, the plastic rod acquires a negative charge.

Origin of Charge

Atoms are the basic building blocks of matter. We know that the atom consists of three subatomic particles: electrons, protons, and neutrons. The protons and neutrons are present in the nucleus of an atom, while the electrons revolve around the nucleus in a defined path. The electrons and protons are negatively charged and positively charged, respectively. However, the neutrons have no charge, and they are neutral.



Electrons

Electrons have the smallest unit of negative charge in them. They are represented by e^-.

Protons

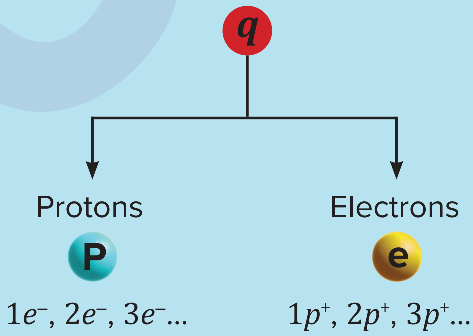
Protons have the smallest unit of positive charge in them. They are represented by p^+.

BOARDS

Properties of Charge

(i) Quantisation of charge

- The charge on one e^- is $-1.6 \times 10^{-19}\ C$. It is the smallest charge that can exist independently.
- The charge on e^- is also known as the **elementary charge or fundamental charge**.
- Charge on any object is an integral multiple of the fundamental charge, i.e., charge of an electron. A body cannot have a charge in fractions.
  $q = \pm ne$



(ii) Charge is additive in nature (Measuring charge)

For measuring the charge, let us consider the following two objects:

In the first object, there are three protons and five electrons as shown in the figure.

We have,

$n_p = 3$

$n_e = 5$

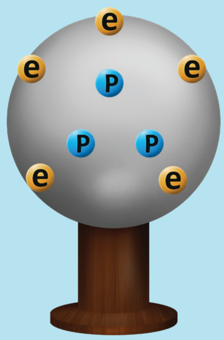
Net charge, q = Charge of protons + Charge of electrons

$q = n_p(+e) + n_e(-e)$

$q = 3e + (-5e)$

$q = -2e$

Here, n_p and n_e are the number of protons and number of electrons, respectively. Also, $(+e)$ and $(-e)$ are the charges of a proton and an electron, respectively.



Object 1

In the second object, there are three protons and one electron as shown in the figure.

We have,

$n_p = 3$

$n_e = 1$

Net charge, q = Charge of protons + Charge of electrons

$q = n_p(+e) + n_e(-e)$

$q = 3e + (-e)$

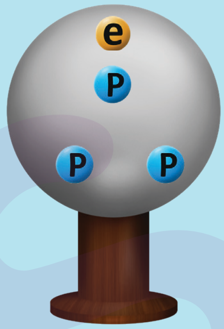
$q = +2e$

In general, the net charge of a body is given by,

$q_{net} = (n_p - n_e)e$

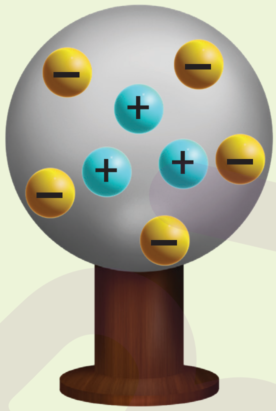
Or, $q_{net} = (n_p - n_e)1.6 \times 10^{-19}\ C$

This shows that charge is additive in nature.

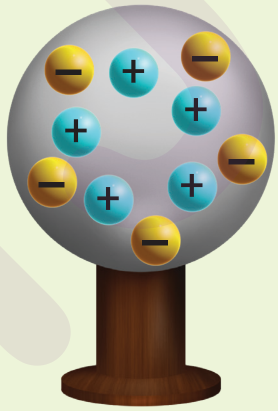


Object 2

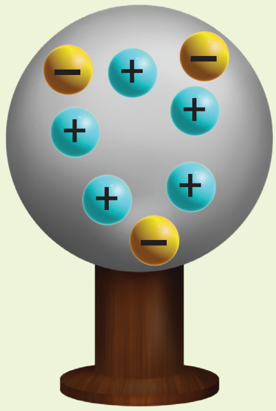
- In general, atoms are electrically neutral, i.e., atoms contain equal numbers of protons and electrons. On the other hand, if an atom has an unequal number of protons and electrons, then the atom is known as electrically charged.
- Objects with an excess of electrons are known as negatively charged objects. Those with a deficiency of electrons are known as positively charged objects.



Negatively charged body



Neutral body



Positively charged body

A body acquires a charge of 8 mC after it was struck by lightning during a thunderstorm. What is the difference between the number of protons and electrons on the body?

(A) 5×10^{-16}	(B) 10×10^{16}	(C) 10×10^{-16}	(D) 5×10^{16}

Solution

Given,

Net charge on the body, $q_{net} = 8\ mC = 8 \times 10^{-3}\ C$

Also, the net charge on the body is given by,

$q_{net} = (n_p - n_e)e$

$q_{net} = (n)e$

Where,

n = Difference between number of protons and electrons

$$\Rightarrow n = \frac{q_{net}}{e}$$

$$\Rightarrow n = \frac{8 \times 10^{-3}}{1.6 \times 10^{-19}}$$

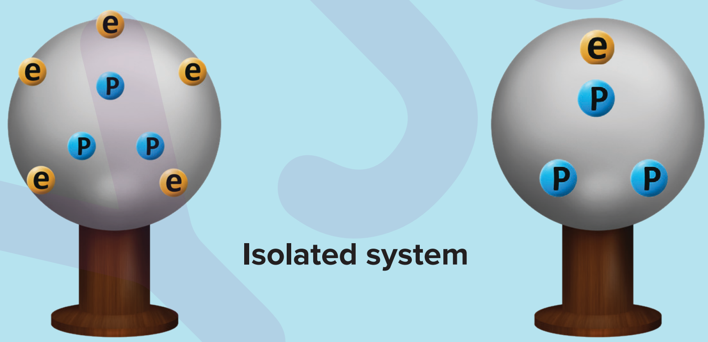
$$\Rightarrow n = 5 \times 10^{16}$$

Thus, option (D) is the correct answer.

(iii) Conservation of charge

A charge can neither be created nor destroyed but can only be transferred from one body to another.

Consider two bodies with some charges which are isolated and separated from each other.



Isolated system

From the figure, we can observe that the charge on the first body is given by,

$q_1 = -2e$

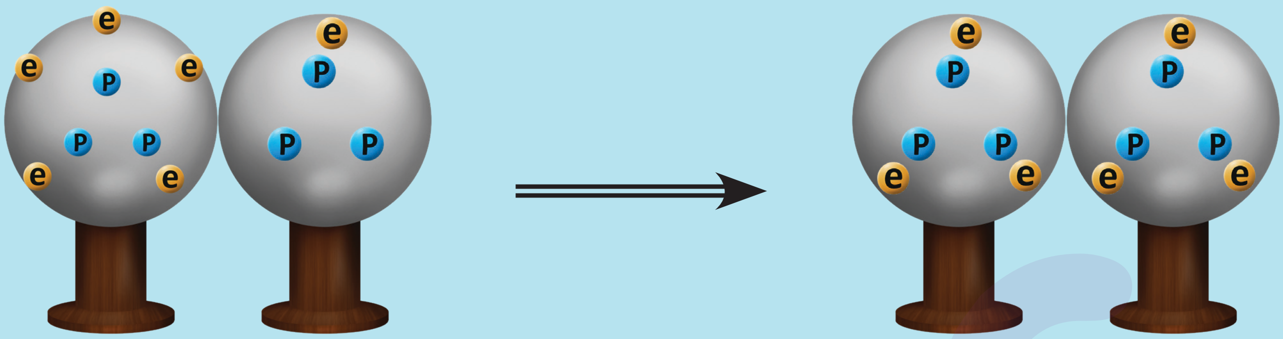
The charge on the second body is given by,

$q_2 = +2e$

Initially, the net charge of the system is given by,

$q_{net} = q_1 + q_2 = -2e + 2e = 0$

When the objects are made to interact (touch each other), then the electrons from the first body move to the second body.



Interactive system Isolated system

Therefore, the net charge of the system is given by,

$q'_{net} = (3e - 3e) + (3e - 3e) = 0$

(iv) Charge is always associated with mass; Charge cannot exist without mass

Some similarities between charge and mass:

1. Both are intrinsic properties of matter, i.e., without mass, a charge cannot exist. However, mass can exist without a charge.
2. Both of them exerts force on other bodies/chagres.
3. Both of them are scalar quantities

Matter	Charge	Mass
e	✓	✓
P	✓	✓

Differences between charge and mass

Charge	Mass
Charges are of two kinds: positive and negative.	Masses are always positive.
They can attract and repel each other.	They always attract.
Charge does not vary with velocity. It is relativistically invariant.	Mass can vary when the velocity is very very high (comparable to the velocity of light).
Charge is always conserved.	Mass can be converted into energy.
Charge cannot exist without mass.	Mass can exist with zero net charge.

Dimension of Electric Charge

We know that the electric current is the amount of charge flowing per unit time.

$$I = \frac{Q}{t}$$

$$Q = I \times t$$

Therefore,

Coulomb = Ampere × Second

$$[Q] = \left[A^1 T^1\right]$$

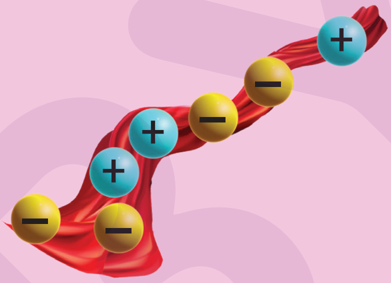
BOARDS

Methods of Charging a Body

Charging by friction (Valid for insulators)

Suppose that there are two substances with different charge affinity. For example, let us take a silk cloth and a glass object. We know that the silk cloth has a good affinity towards negative charge, and the glass does not. It means that the silk cloth can attract more and more negative charges, whereas the glass objects lose their electrons very easily.

Therefore, when we rub the silk cloth with the glass, the silk cloth attracts the negative charge from the glass and becomes negatively charged. At the same time, by losing the negative charges, the glass becomes positively charged.



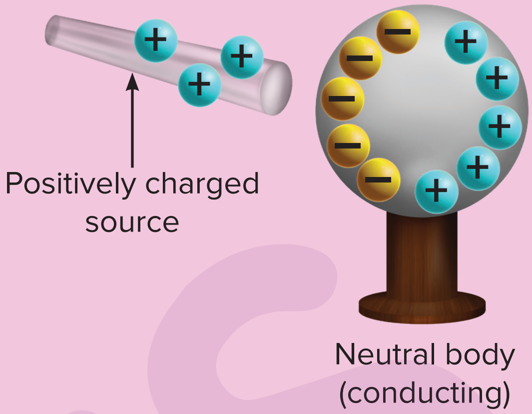
Negatively charged
Silk (High affinity to negative charge)

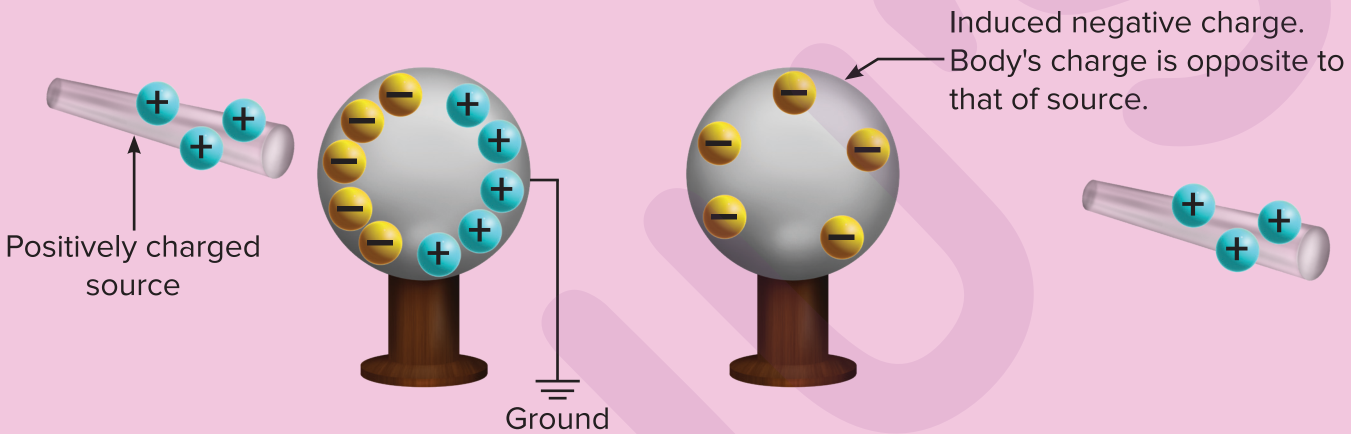
Positively charged
Glass (Low affinity to negative charge)

Charging by induction

In this case, let us consider two objects, one is a positively charged object, and the other is a neutral object. These objects are placed very close to each other but not in contact. Let us assume that a glass rod is positively charged. When the glass rod is brought near the neutral object, due to electrostatic attraction and repulsion from the neutral object, the glass rod attracts all the negative charge towards itself and pushes all the positive charges in the opposite direction as shown in the figure on the next page.

Due to this separation of the charges, the polarisation of the charges takes place. After this, the positive side is connected to the ground. The ground is the infinite source, i.e., it can take or give any amount of charge. So, the positive charges of the body are neutralised by the negative charges. On removing the ground and the source (glass rod), the body now only contains excess negative charges that spread on the object and become negatively charged.

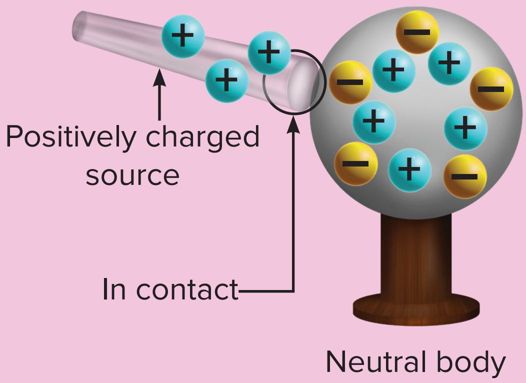




By this method, we can charge a body in the opposite nature to that of the source.

Charging by conduction

Unlike charging by induction, the charged source is brought in contact to the neutral object. When the source is brought in contact with the neutral object, it attracts all the opposite charges and takes away the charges from the body. Due to this, when the source is taken away, it makes the object deficient in the charge opposite to that of the source. Now, the object gets charged in the same charge as that of the source. This method of charging is known as charging by conduction.



Which of the following methods of charging a body can lead to a charge of $+2.4 \times 10^{-19}\ C$ on a body?

(A) Charging by induction

(B) Charging by conduction

(C) Charging by friction

(D) None of the above

Solution

The net charge on a body is given by,

$$Q = (n_p - n_e)e$$

$$\Rightarrow Q = ne$$

$$\Rightarrow n = \frac{Q}{e}$$

$$\Rightarrow n = \frac{2.4 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$\Rightarrow n = 1.5$$

Since the value of n cannot be a decimal or fraction, the transfer of charge is not possible.

Thus, option (D) is the correct answer.

PHYSICS

ELECTROSTATICS

COULOMB'S LAW

NOTES

What you already know

- Charge and its origin
- Properties of charge
- Method of charging a body

What you will learn

- Coulomb's torsion balance
- Coulomb's law
- Vector form of Coulomb's law

Coulomb's Torsion Balance and Coulomb's Law

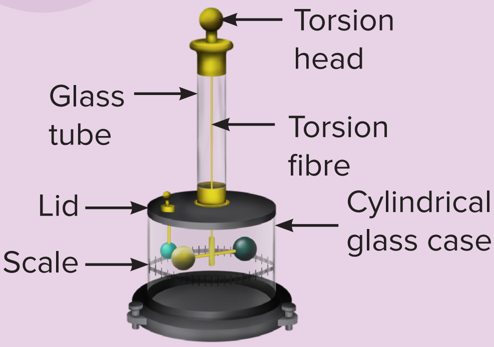
Charles-Augustin de Coulomb invented an instrument known as the Coulomb's torsion balance. Using this instrument, he measured the charge on a body.

Concept

- The torsion balance experiment helps to measure small forces.
- It is based on the principle that a wire or thread resists twisting with a force that is proportional to the stress applied on it.
- Torsion balances are used to measure small electric, magnetic, and gravitational forces.

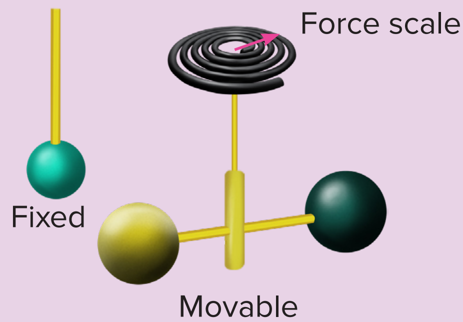
Methodology

Torsion balance consists of a cylindrical glass case. A glass tube is attached to it and the tube ends with a piece of metal. A torsion fibre (metal or thread) runs through this metal that ends with a metal rod at one end and a sphere at the other. The ends of the metal rod are connected with two spheres. It can swing freely due to its suspended state by the torsional string . A scale encircling the glass case is shown in the figure. Another fixed sphere is present in the glass case.



Procedure and working

The fixed sphere and the spheres connected with a straight rod, which can swing, are given the same nature of charge. Thus, they repel each other and start to rotate. On rotating, the torsional string gets twisted and the twisting shows reading on the force scale. Hence, we can measure the force exerted by one charged sphere on another charged sphere.



Experimentally, let two charges be placed at distance D from each other. Let the charges be q_1 and q_2 and the force exerted by them be F. Now, on changing the magnitude of charges q_1 and q_2 or on changing the distance between the charges, we observe some changes. The changes are summarised in the following table:

D	q_1	q_2	Force	Relation
$4r$	–1	4	F	$F \propto q_1$
$4r$	–2	4	$2F$	$F \propto q_2$
$4r$	–2	2	F	$F \propto q_1 q_2$
$2r$	–1	4	$4F$	$F \propto \frac{1}{D^2}$

From the table, we can observe that the magnitude of force acting on the charge is proportional to the magnitude of product of charges q_1 and q_2. Also, the force is inversely proportional to the square of the distance between the charges.

If two point charges are present at distance r from each other, then by Coulomb's law, the force is given by,

$$F \propto \frac{q_1 q_2}{r^2}$$

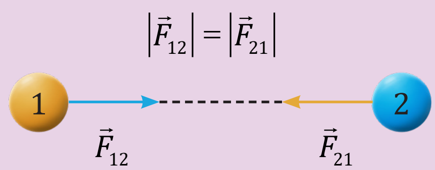
$$\Rightarrow F = K\frac{q_1 q_2}{r^2}$$

Where,

$K = 9 \times 10^9\ Nm^2C^{-2}$

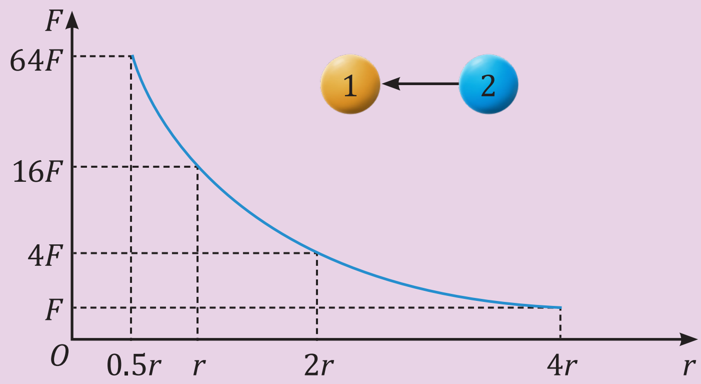
It is the proportionality constant.

We know that if the first charge applies an attractive force on the second charge, i.e., $\vec{F}_{21}$, then the second charge applies an attractive force on the first charge, i.e., $\vec{F}_{12}$. Where, $\vec{F}_{12}$ and $\vec{F}_{21}$ are equal in magnitude but opposite in direction.



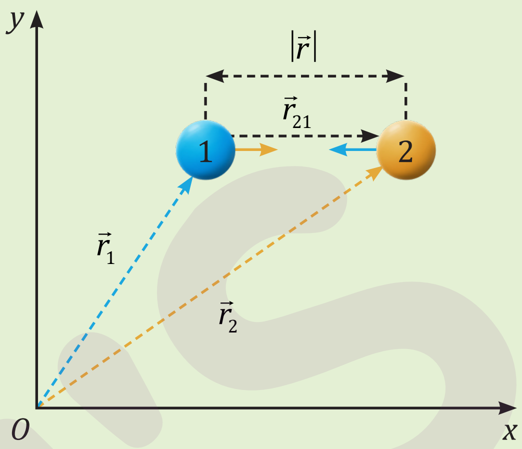
An experiment is done in which there are two charged bodies. One is fixed and the other is brought near it. A graph of F vs r is shown.

If the charges are separated by a distance of $4r$, then the force is F. When we bring the charge from $4r$ to $2r$, then by applying Coulomb's law, the force is $4F$. Similarly, when the charge is brought to a distance of $0.5r$, then the force increases to $64F$.



Coulomb's Law in Vector Form

Let us consider that two charges of opposite nature are placed somewhere in space. The position vector of the first charge is $\vec{r}_1$. The position vector of the second charge is $\vec{r}_2$. According to Coulomb's law, the forces act along the direction of line joining the two charges. Since we have taken the charges of the opposite nature, there must be an attractive force acting between them.



There is a force on the first charge due to the second charge, i.e., $\vec{F}_{12}$. Similarly, there is a force on the second charge due to the first charge, i.e., $\vec{F}_{21}$. The displacement vector between the two charges is $\vec{r}_{21}$. Let the distance between the two charges be $|\vec{r}|$.

By applying Coulomb's law, we get the following:

$$F = K\frac{q_1 q_2}{r^2}$$

Also, the displacement vector is given by,

$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1 = |\vec{r}|\hat{r}$$

Therefore, force acting on the first charge due

to the second charge is given by,

$$\vec{F}_{12} = \frac{Kq_1q_2}{|\vec{r}_2 - \vec{r}_1|^2}\hat{r}_{21}$$

$$\vec{F}_{12} = \frac{Kq_1q_2}{|\vec{r}_{21}|^2}\hat{r}_{21} \quad .....(i)$$

The direction vector is given by,

$$\hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|}$$

By substituting $\hat{r}_{21}$ in equation (i), we get,

$$\vec{F}_{12} = \frac{Kq_1q_2}{|\vec{r}_{21}|^2} \times \frac{\vec{r}_{21}}{|\vec{r}_{21}|}$$

$$\Rightarrow \vec{F}_{12} = \frac{Kq_1q_2}{|\vec{r}_{21}|^3}\vec{r}_{21} \quad .....(ii)$$

Also, the force on the second charge due

to the first charge is given by,

$$\Rightarrow \vec{F}_{21} = \frac{Kq_1q_2}{|\vec{r}_{21}|^3}(-\vec{r}_{21})$$

$$\Rightarrow \vec{F}_{21} = -\frac{Kq_1q_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

Comparing this with equation (ii), we get,

$$\vec{F}_{21} = -\vec{F}_{12}$$

Thus, Coulomb's law agrees with Newton's third law of motion.

Two point charges A and B have charges $+Q$ and $-Q$, respectively. They are placed at a certain distance. The force acting between them is F. If 25% of charge A is transferred to B, then find the force between the charges.

(A) $\frac{16F}{9}$ (B) $\frac{4F}{3}$ (C) F (D) $\frac{9F}{16}$

Solution NEET

In the first case, the force due to the charges is given by,

$$F = K\frac{q_1 q_2}{r^2} = -\frac{KQ^2}{r^2} \quad .....(i)$$

Now, 25% of charge A is transferred to charge B.

The charge A becomes $= +Q - \frac{Q}{4} = +\frac{3Q}{4}$

The charge B becomes $= -Q + \frac{Q}{4} = -\frac{3Q}{4}$

By applying Coulomb's law, the new force is given by,

$$F' = \frac{K\left(\frac{3}{4}Q\right)\left(-\frac{3}{4}Q\right)}{r^2}$$

$$\Rightarrow F' = -\frac{9}{16}\frac{KQ^2}{r^2}$$

From equation (i), we get,

$$\Rightarrow F' = \frac{9}{16}F$$

Thus, option (D) is the correct answer.

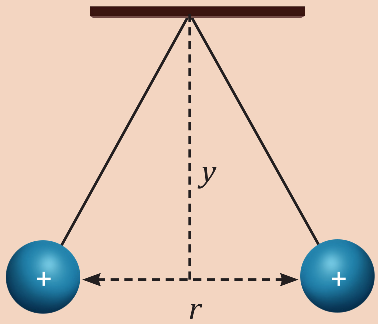
Two pith balls carry equal charges. They are suspended from a common point by strings of equal lengths. The equilibrium separation between them is r. Now, the strings are rigidly clamped at half the height. Find the equilibrium separation between the balls.

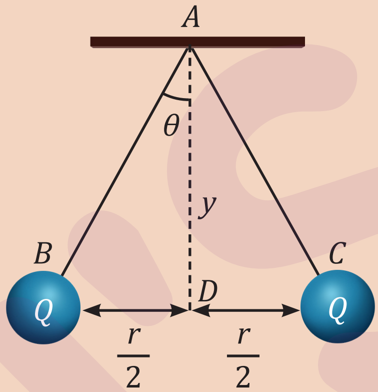
(A) $\left(\frac{r}{\sqrt{2}}\right)^2$ (B) $\frac{r}{2^{\frac{1}{3}}}$ (C) $\frac{2r}{\sqrt{3}}$ (D) $\frac{2r}{3}$

Solution NEET

In the first case,

The distance between the two pith balls is *r*. The distance from the point of suspension is *y*.





The FBD of the first charge is given, where,

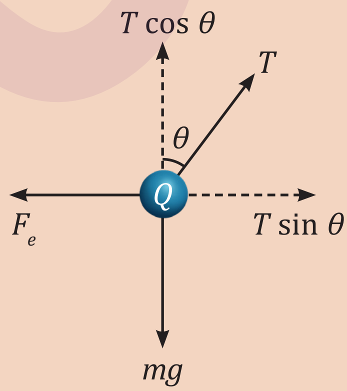
F_e = Electrostatic force

T = Tension force

In the equilibrium condition, we get,

$T \sin\theta = F_e$ ...(*i*)

$T \cos\theta = mg$ ...(*ii*)



On dividing equation (*i*) by equation (*ii*), we get,

$$\tan\theta = \frac{F_e}{mg} = \frac{KQ^2}{r^2 \times mg} \quad ...(iii)$$

Also, in triangle ABD, we get,

$$\tan\theta = \frac{r}{2y} \quad ...(iv)$$

From equations (*iii*) and (*iv*), we get,

$$\frac{r}{2y} = \frac{KQ^2}{r^2 \times mg}$$

$$\Rightarrow y = \alpha r^3 \left(\text{Where, } \alpha = \frac{mg}{2KQ^2} = \text{Constant}\right) ...(v)$$

Similarly, when $y = \frac{y}{2}$, the relation changes to the following:

$$\frac{y}{2} = \alpha (r')^3 .....(vi)$$

By dividing equation (v) by equation (vi), we get the following:

$$\frac{y}{\left(\frac{y}{2}\right)} = \frac{\alpha r^3}{\alpha (r')^3}$$

$$\Rightarrow 2 = \left(\frac{r}{r'}\right)^3$$

$$\Rightarrow 2^{\frac{1}{3}} = \frac{r}{r'}$$

$$\Rightarrow r' = \frac{r}{2^{\frac{1}{3}}}$$

Thus, option (B) is the correct answer.

Two identical charged spheres are suspended from a common point by two massless strings, each of length l. Initially, they are apart at a distance of d ($d << l$) because of their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with velocity v. How does velocity v vary as a function of distance x between the spheres?

(A) $v \propto (x)^{-\frac{1}{2}}$ (B) $v \propto x^{-1}$ (C) $v \propto x^{\frac{1}{2}}$ (D) $v \propto x$

Solution NEET

At some instant of time t,

Consider that two spheres are present at distance x from each other. The charge is leaking at a constant rate.

The FBD of the first (left) sphere at time t is given below, where,

F_e = Electrostatic force

T = Tension force

In the equilibrium condition, we get the following:

$T \sin\theta = F_e$ .....(i)

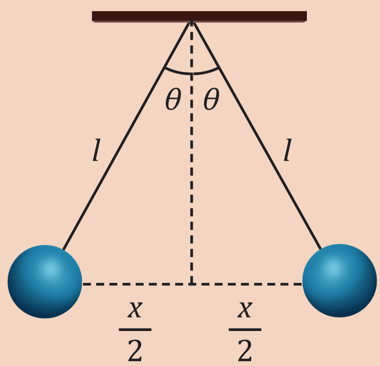
$T \cos\theta = mg$ .....(ii)

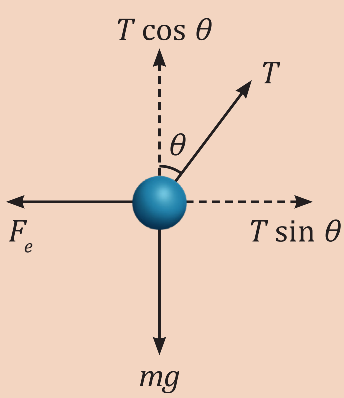
On dividing equation (i) by equation (ii), we get,

$$\tan\theta = \frac{F_e}{mg} = \frac{KQ^2}{x^2 \times mg} \quad .....(iii)$$

From the triangle, we get,

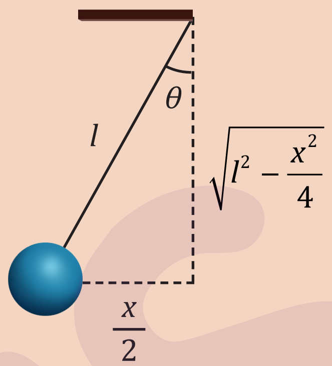
$$\tan\theta = \frac{x}{2\sqrt{l^2 - \left(\frac{x^2}{4}\right)}}$$





Since $x << l$, the term changes to,

$$\tan\theta = \frac{x}{2\sqrt{l^2}}$$



$$\Rightarrow \tan\theta = \frac{x}{2l} \quad .....(iv)$$

From equations (iii) and (iv), we get the following:

$$\frac{F_e}{mg} = \frac{x}{2l}$$

$$\Rightarrow \frac{KQ^2}{x^2 mg} = \frac{x}{2l}$$

$$\Rightarrow \frac{KQ^2}{mg} = \frac{x^3}{2l}$$

Therefore, we get the following,

$$Q^2 = Ax^3 \left(\text{Where, } A = \frac{mg}{2Kl}\right)$$

$$\Rightarrow Q \propto x^{\frac{3}{2}}$$

Also, it is given that the charge is leaking at a constant rate.

$$\frac{dQ}{dt} = C = \text{Constant}$$

Also,

$$Q = Ax^{\frac{3}{2}}$$

By differentiating both the sides with respect to time, we get,

$$\frac{dQ}{dt} = A\frac{d}{dt}\left(x^{\frac{3}{2}}\right)$$

$$\Rightarrow C = A\frac{3}{2}\left(x^{\frac{3}{2}-1}\right)\frac{dx}{dt}$$

$$\Rightarrow C' = x^{\frac{1}{2}}v \left(\text{Where, } C' = \frac{2C}{3A} = \text{ Constant}\right)$$

$$\Rightarrow v = \frac{C'}{x^{\frac{1}{2}}}$$

$$\Rightarrow v \propto x^{-\frac{1}{2}}$$

Thus, option (A) is the correct answer.

PHYSICS

ELECTROSTATICS

PRINCIPAL OF SUPERPOSITION

NOTES

What you already know

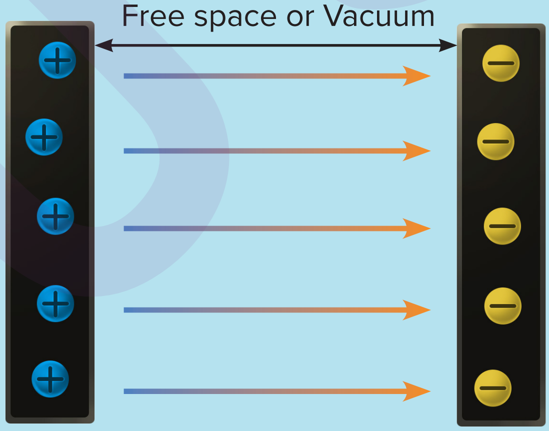
- Origin of charge
- Coulomb's torsion balance
- Coulomb's law
- Vector form of Coulomb's law

What you will learn

- Permittivity of free space
- Permittivity of medium
- Relative permittivity
- Limitations of Coulomb's law
- Principle of superposition

Permittivity of Free Space (ε_o)

Let us consider that a positively charged plate and a negatively charged plate are separated by some distance and placed in a vacuum, i.e., there is no medium in between the charges.



When there are charges that are present close to each other, they have their influence (they apply force on other charges) up to a certain distance. The region where they have the influence is known as the field of the charge. We also know that when two charges are present close to each other, they apply force on each other. The force given by Coulomb's law is as follows:

$$F = \frac{kq_1q_2}{r^2}$$

Thus, the magnitude of the force depends on the value of k. The value of constant k is given by,

$$k = \frac{1}{4\pi\varepsilon_o}$$

Here, ε_o is the permittivity of the free space.

$\varepsilon_o = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

Permittivity: Permittivity is the property of a medium or a material that measures the opposition offered by a medium or material to an external electric field.

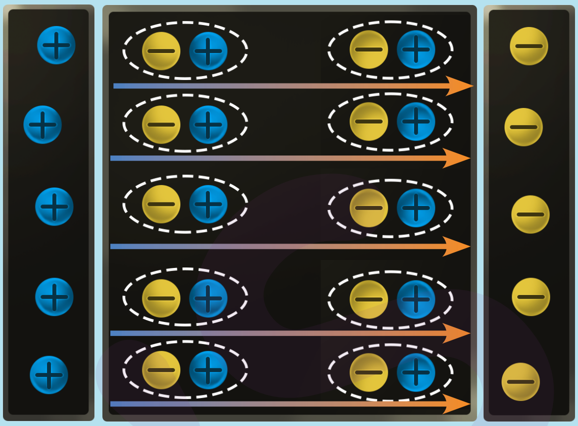
If the permittivity of a medium or material is high, it means that the opposition offered by the medium or material to an external electric field is high and vice versa.

Note

The permittivity of free space is minimum, i.e., the opposition to an electric field in free space is minimum.

Permittivity of Medium (ε_m)

Let us consider that a positively charged plate and a negatively charged plate are separated by some distance. Now, a medium is introduced in between the charged plates. The medium has some positive charges and some negative charges. Due to the influence of the charged plates, the charged particles of the medium get attracted to the opposite charges, and the medium gets polarised.



As the medium gets polarised, and due to its net dipole moment, it will generate a field (internal field) opposite to the direction of the field produced by the two oppositely charged plates (external field).

Note

If the medium gets highly polarised, the magnitude of the net internal field produced by the medium becomes high. Therefore, the opposition offered by the medium of the material to an external electric field is high and vice versa.

Thus, the force between the actual charges (charged plates) gets influenced by the field generated by the medium.

The equation of Coulomb's law is given as follows:

$$F = \frac{kq_1q_2}{r^2}$$

The value of k for this case is given by,

$$k = \frac{1}{4\pi\varepsilon_m}$$

Here, ε_m is the permittivity of the medium.

Relative Permittivity (ε_r)

Relative permittivity is the ratio of the permittivity of the medium to the permittivity of the free space.

$$\varepsilon_r = \frac{\varepsilon_m}{\varepsilon_o}$$

So, the permittivity of any medium can be written as follows:

$$\varepsilon_m = \varepsilon_r \times \varepsilon_o$$

The value of k can be written as follows:

$$k = \frac{1}{4\pi\varepsilon_m} = \frac{1}{4\pi\varepsilon_r\varepsilon_o}$$

Relative permittivity is also known as the **dielectric constant**.

When air is replaced by a dielectric medium of dielectric constant K, what happens to the maximum force of attraction between the two charges that are separated by a distance?

(A) It decreases K times.

(B) It remains unchanged.

(C) It increases K times.

(D) It increases K^{-2} times.

Solution

Consider that two opposite charges, $+Q$ and $-Q$, are placed at a distance of r from each other.

Case I: When air is present between the charges

The force acting between the two charges is given by,

$$F = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{Q^2}{r^2}$$

$$\Rightarrow F = \frac{1}{4\pi\varepsilon_o}\frac{Q^2}{r^2}\ (\text{For air, value of } \varepsilon_r \text{ is } 1)$$

Case II: When air is replaced with a dielectric medium

The force acting between the two charges is given by,

$$F' = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{Q^2}{r^2}$$

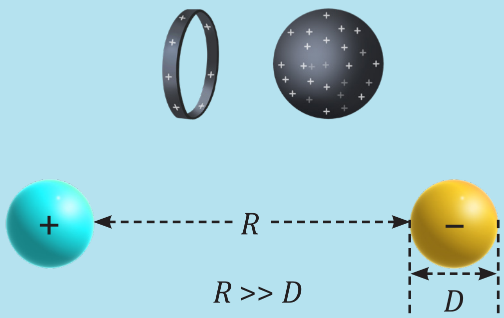
$$\Rightarrow F' = \frac{1}{4\pi\varepsilon_o K}\frac{Q^2}{r^2}\ (\text{Since } \varepsilon_r \text{ is } K \text{ for the dielectric medium})$$

$$\Rightarrow F' = \frac{F}{K}\left(\text{Since } F = \frac{1}{4\pi\varepsilon_o}\frac{Q^2}{r^2}\right)$$

Thus, option (A) is the correct answer.

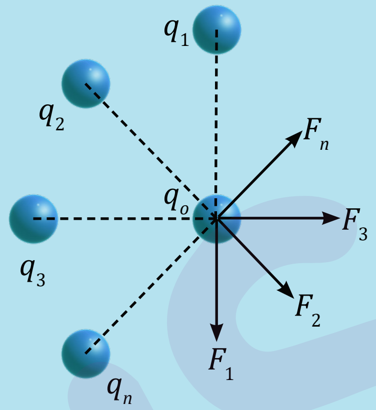
Limitations of Coulomb's Law

1. It is difficult to apply Coulomb's law when charges are in arbitrary shape.
2. Coulomb's law is not valid for charges in motion (relative motion should be zero).
3. Charges must be point charges, i.e., the extension of the charges must be smaller than the separation between the charges.
4. The separation must be greater than the nuclear size.



Principle of Superposition

Consider a system with n number of charges present as shown in the figure.



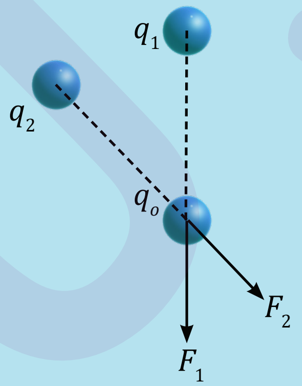
Let us assume that the charges are of the same polarity. For finding the net force on any charge, we have to find the forces by each charge present in the vicinity.

Let us consider charge q_o and analyze all the forces acting on it. The forces acting on q_o are shown in the figure.

The net force acting on q_o is given by,

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n$$

The force applied by one charge does not affect the force by other charges. They have their individual effects, but the net force acting on the charge changes.



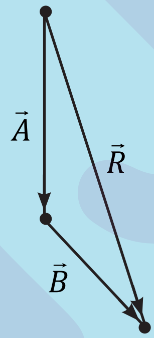
Suppose q_1 and q_2 are applying a force on charge q_o, then the net force is obtained by applying the triangle law of vector addition.

By applying the triangle law of vector addition, we get,

$$\vec{R} = \vec{A} + \vec{B}$$

The magnitude of the resultant vector is given by,

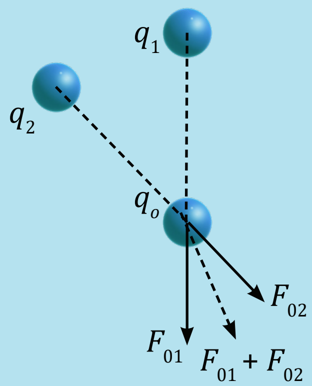
$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$



For forces $\vec{F}_{01}$ and $\vec{F}_{02}$, the resultant vector is given by,

$$\vec{F}_{net} = \vec{F}_{01} + \vec{F}_{02}$$

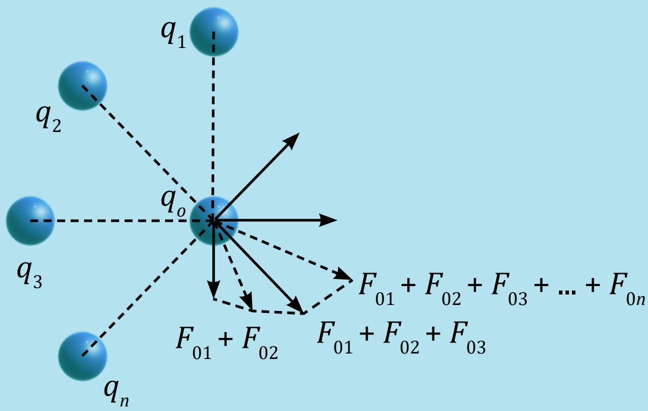
$$F_{net} = \sqrt{F_{01}^2 + F_{02}^2 + 2F_{01}F_{02}\cos\theta}$$



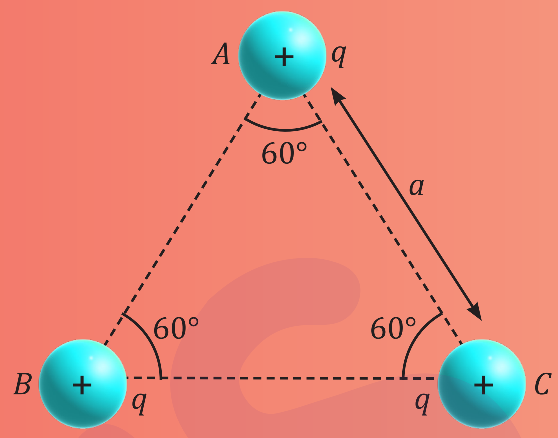
Similarly, for n charges, the net force is given by,

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n$$

The **principle of superposition** states that in a system of n charges, the resultant force on a charge is the **vector sum of forces due to all the remaining charges**.



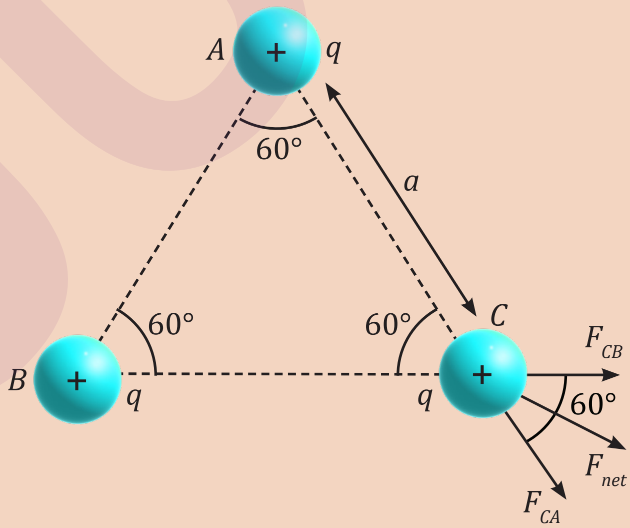
Three identical charged particles A, B, and C, with a charge of $+q$, are present on the vertices of an equilateral triangle having sides of length a as shown in the figure. Find the resultant force on particle C.



(A) $\frac{\sqrt{3}kq^2}{a^2}$ (B) $\frac{kq^2}{a^2}$ (C) $\frac{2kq^2}{a^2}$ (D) $\frac{\sqrt{3}kq^2}{a}$

Solution

For the charge at C, both the charges at A and B apply force. Since the charges are the same, repulsion forces act in the opposite directions as shown in the figure.



So, the magnitude of the force acting on C due to A is given by,

$$\left|\vec{F}_{CA}\right| = \frac{kq^2}{a^2} \quad \text{......}(i)$$

Similarly, the magnitude of the force on C due to B is given by,

$$\left|\vec{F}_{CB}\right| = \frac{kq^2}{a^2} \quad \text{......}(ii)$$

By comparing equations (i) and (ii), we get,

$$\left|\vec{F}_{CA}\right| = \left|\vec{F}_{CB}\right| = \left|\vec{F}\right|$$

The net force acting on the charge at C is given by,

$$\left|\vec{F}_{net}\right| = \sqrt{F^2 + F^2 + 2F^2\cos 60^\circ}$$

$$\Rightarrow \left|\vec{F}_{net}\right| = \sqrt{F^2 + F^2 + 2F^2 \times \frac{1}{2}}$$

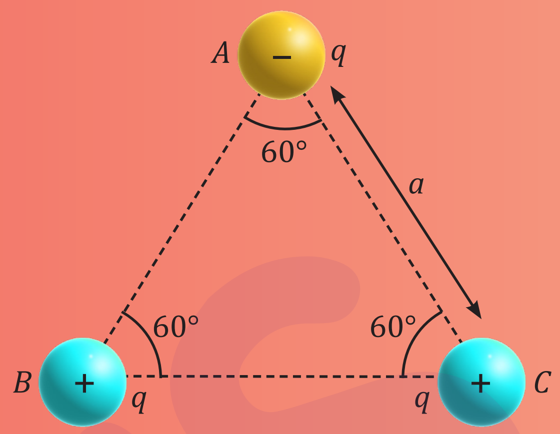
$$\Rightarrow \left|\vec{F}_{net}\right| = \sqrt{3F^2}$$

$$\Rightarrow \left|\vec{F}_{net}\right| = \sqrt{3}F$$

$$\Rightarrow \left|\vec{F}_{net}\right| = \sqrt{3}\frac{kq^2}{a^2}$$

Thus, option (A) is the correct answer.

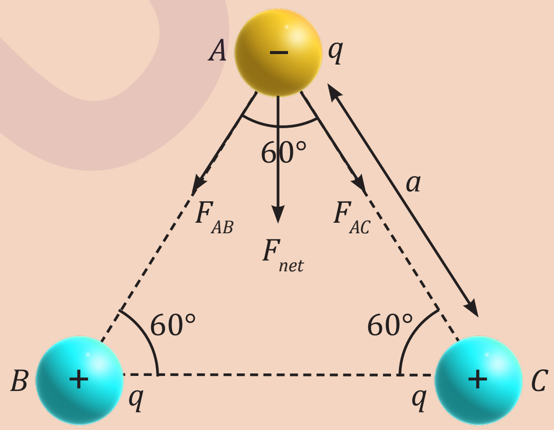
Three charged particles, A, B, and C with respective charges of $-q$, $+q$, and $+q$, are present on the vertices of an equilateral triangle with sides of length a as shown in the figure. Find the resultant force on particle A.



(A) $\frac{\sqrt{3}kq^2}{a^2}$ (B) $\frac{kq^2}{a^2}$ (C) $\frac{2kq^2}{a^2}$ (D) $\frac{\sqrt{3}kq^2}{a}$

Solution

Since A has a negative q charge and the charges at B and C are positive, there will be forces of attraction between A and B and A and C.



So, the magnitude of the force acting on A due to B is given by,

$$\left|\vec{F}_{AB}\right| = \frac{kq^2}{a^2} \quad ......(i)$$

Similarly, the magnitude of the force on A due to C is given by,

$$\left|\vec{F}_{AC}\right| = \frac{kq^2}{a^2} \quad ......(ii)$$

By comparing equations (i) and (ii), we get,

$$\left|\vec{F}_{AB}\right| = \left|\vec{F}_{AC}\right| = \left|\vec{F}\right|$$

The net force acting on the charge at A is given by,

$$\left|\vec{F}_{net}\right| = \sqrt{F^2 + F^2 + 2F^2\cos 60^\circ}$$

$$\Rightarrow \left|\vec{F}_{net}\right| = \sqrt{F^2 + F^2 + 2F^2 \times \frac{1}{2}}$$

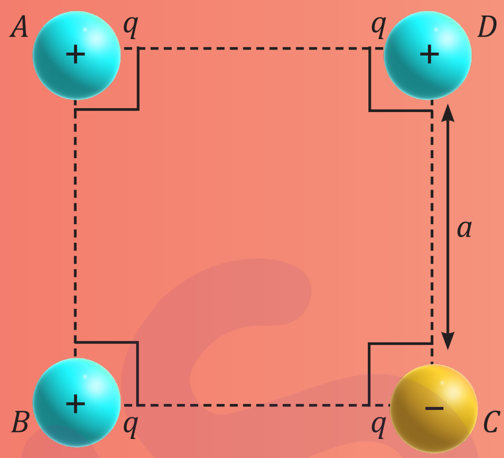
$$\Rightarrow \left|\vec{F}_{net}\right| = \sqrt{3F^2}$$

$$\Rightarrow \left|\vec{F}_{net}\right| = \sqrt{3}F$$

$$\Rightarrow \left|\vec{F}_{net}\right| = \sqrt{3}\frac{kq^2}{a^2}$$

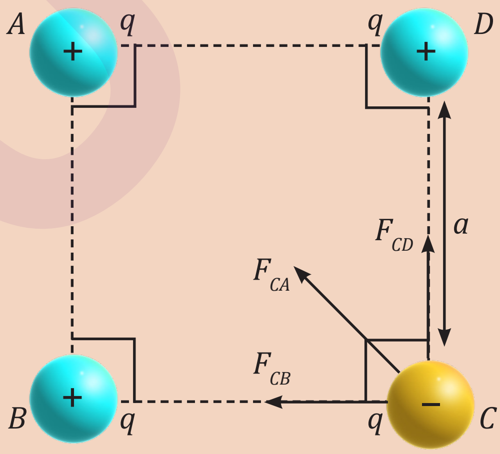
Thus, option (A) is the correct answer.

Four particles, A, B, C, and D with respective charges of $+q$, $+q$, $-q$, and $+q$, are placed on the vertices of a square with sides of length a. Find the resultant force acting on particle C.



(A) $\left(\sqrt{2}+\frac{1}{2}\right)\frac{kq^2}{a^2}$ (B) $\frac{\sqrt{3}kq^2}{a^2}$ (C) $\frac{\sqrt{2}kq^2}{a^2}$ (D) $\left(1+\sqrt{2}\right)\frac{kq^2}{a^2}$

Solution NEET



Since the charge at C is $-q$, the acting net force becomes an attractive force, as the charge at A, B, and D is $+q$.

So, the magnitude of the force acting on C due to B is given by,

$$\left|\vec{F}_{CB}\right| = \frac{kq^2}{a^2} \quad ......(i)$$

Similarly, the magnitude of the force on C due to D is given by,

$$\left|\vec{F}_{CD}\right| = \frac{kq^2}{a^2} \quad ......(ii)$$

By comparing equations (i) and (ii), we get,

$$\left|\vec{F}_{CB}\right| = \left|\vec{F}_{CD}\right| = \frac{kq^2}{a^2}$$

The magnitude of the force on C due to A is given by,

$$\left|\vec{F}_{CA}\right| = \frac{kq^2}{\left(\sqrt{2}a\right)^2} \quad \left(\text{The distance between } A \text{ and } C \text{ is } \sqrt{2}a\right)$$

$$\Rightarrow \left|\vec{F}_{CA}\right| = \frac{kq^2}{2a^2}$$

The resultant force due to the charge at B and D is given by,

$$\left|\vec{F}_{net_{BD}}\right| = \sqrt{F_{CB}^2 + F_{CD}^2 + 2F_{CB}F_{CD}\cos 90^\circ}$$

$$\Rightarrow \left|\vec{F}_{net_{BD}}\right| = \sqrt{F_{CB}^2 + F_{CD}^2}$$

$$\Rightarrow \left|\vec{F}_{net_{BD}}\right| = \sqrt{2}\frac{kq^2}{a^2}$$

The net force is given by,

$$\left|\vec{F}_{net}\right| = \left|\vec{F}_{CA}\right| + \left|\vec{F}_{net_{BD}}\right| \qquad \left(\because \vec{F}_{CA} \text{ and } \vec{F}_{net_{BD}} \text{ are in the same direction}\right)$$

$$\Rightarrow \left|\vec{F}_{net}\right| = \frac{kq^2}{2a^2} + \sqrt{2}\frac{kq^2}{a^2}$$

$$\Rightarrow \left|\vec{F}_{net}\right| = \left(\frac{1}{2} + \sqrt{2}\right)\frac{kq^2}{a^2}$$

Alternative method

We have,

$$\left|\vec{F}_{CB}\right| = \left|\vec{F}_{CD}\right| = \frac{kq^2}{a^2}$$

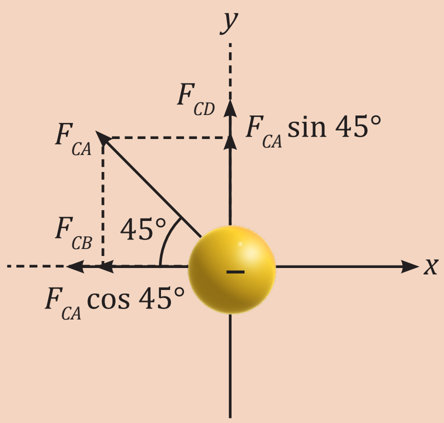
And,

$$\left|\vec{F}_{CA}\right| = \frac{kq^2}{\left(\sqrt{2}a\right)^2} \qquad \left(\text{The distance between } A \text{ and } C \text{ is } \sqrt{2}a\right)$$

$$\Rightarrow \left|\vec{F}_{CA}\right| = \frac{kq^2}{2a^2}$$

Now, by resolving the force on C due to A into its two components, we get,

The net force along the x-axis is given by,



$$\vec{F}_x = -\vec{F}_{CB}\,\hat{i} - \left(\vec{F}_{CA}\cos 45°\right)\hat{i}$$

$$\Rightarrow \vec{F}_x = -\frac{kq^2}{a^2}\hat{i} - \left(\frac{kq^2}{2a^2} \times \frac{1}{\sqrt{2}}\right)\hat{i}$$

$$\Rightarrow \vec{F}_x = -\frac{kq^2}{a^2}\hat{i} - \left(\frac{kq^2}{2\sqrt{2}a^2}\right)\hat{i}$$

Similarly, along the y-axis, the net force is given by,

$$\vec{F}_y = +\vec{F}_{CD}\,\hat{j} + \left(\vec{F}_{CA}\sin 45°\right)\hat{j}$$

$$\Rightarrow \vec{F}_y = +\frac{kq^2}{a^2}\hat{j} + \left(\frac{kq^2}{2a^2} \times \frac{1}{\sqrt{2}}\right)\hat{j}$$

$$\Rightarrow \vec{F}_y = +\frac{kq^2}{a^2}\hat{j} + \left(\frac{kq^2}{2\sqrt{2}a^2}\right)\hat{j}$$

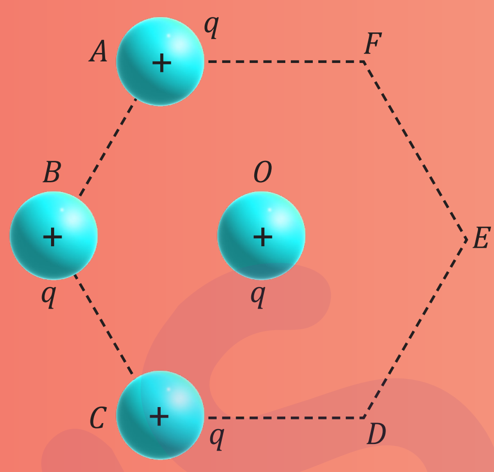
The net force is given by,

$$\left|\vec{F}_{net}\right| = \left|\vec{F}_x\right| + \left|\vec{F}_y\right| = \sqrt{\left|\vec{F}_x\right|^2 + \left|\vec{F}_y\right|^2 + 2\left|\vec{F}_x\right|\left|\vec{F}_y\right|\cos 90°}$$

$$\Rightarrow \left|\vec{F}_{net}\right| = \left(\sqrt{2} + \frac{1}{2}\right)\frac{kq^2}{a^2}$$

Thus, option (A) is the correct answer.

Three equal charges of $+q$ are placed at the corners of a regular hexagon of side a as shown in the figure. Find the force on $+q$, which is placed at centroid O.



(A) $\frac{3kq^2}{a^2}$ (B) $\frac{kq^2}{a^2}$ (C) $\frac{2kq^2}{a^2}$ (D) $\frac{\sqrt{3}kq^2}{a}$

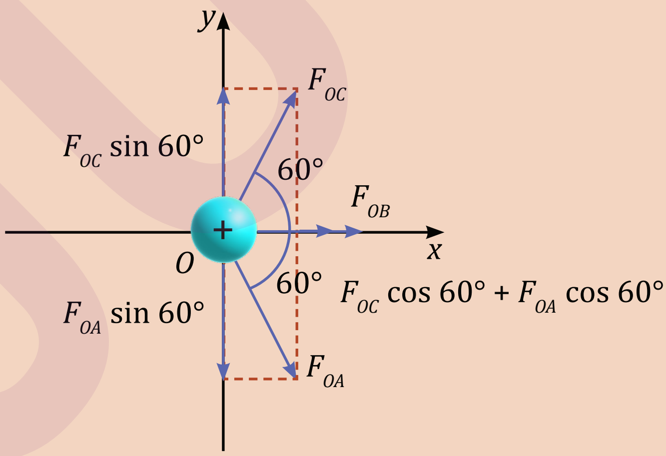
Solution NEET

Since all the charges are of the same polarity, there will be repulsion between all the charges.

The angle between the forces is given by,

$$\theta = \frac{360°}{n} = 60°$$

Therefore, the magnitude of all three forces is the same, as they are at the same distance from O.



$$\left|\vec{F}_{OA}\right| = \left|\vec{F}_{OB}\right| = \left|\vec{F}_{OC}\right| = \frac{kq^2}{a^2}$$

The resultant force due to the charge at A and C is given by,

$$\left|\vec{F}_{net_{AC}}\right| = \sqrt{F_{OA}^2 + F_{OC}^2 + 2F_{OA}F_{OC}\cos 120°}$$

$$\Rightarrow \left|\vec{F}_{net_{AC}}\right| = \sqrt{F_{OA}^2 + F_{OC}^2 - F_{OA}F_{OC}}$$

$$\Rightarrow \left|\vec{F}_{net_{AC}}\right| = \sqrt{\left(\frac{kq^2}{a^2}\right)^2 + \left(\frac{kq^2}{a^2}\right)^2 - \left(\frac{kq^2}{a^2}\right)^2}$$

$$\Rightarrow \left|\vec{F}_{net_{AC}}\right| = \frac{kq^2}{a^2}$$

The direction of the vector will be along the direction of $\vec{F}_{OB}$.

The resultant of the forces $\vec{F}_{OB}$ and $\vec{F}_{net_{AC}}$ is given by,

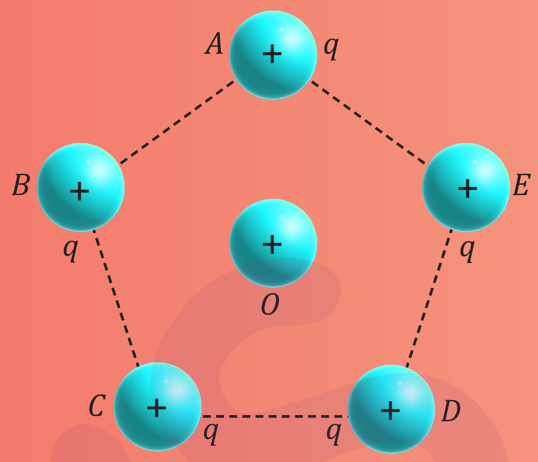
$$\left|\vec{F}_{net}\right| = \left|\vec{F}_{OB}\right| + \left|\vec{F}_{net_{AC}}\right|$$

$$\Rightarrow \left|\vec{F}_{net}\right| = \frac{kq^2}{a^2} + \frac{kq^2}{a^2}$$

$$\Rightarrow \left|\vec{F}_{net}\right| = 2\frac{kq^2}{a^2}$$

Thus, option (C) is the correct answer.

Five equal charges of $+q$ are placed at the corners of a regular pentagon of side a. Find the force on $+q$, which is placed at centroid O.



(A) Zero

(B) $\frac{\sqrt{2}kq^2}{a^2}$

(C) $\frac{2kq^2}{a^2}$

(D) $\frac{kq^2}{3a}$

Solution

All the charges are of the same polarity and are placed at the vertices of a regular pentagon, i.e., the charges are placed in a symmetrical manner.

Therefore, the net force applied by all the charges on a charge that is placed at the center of the symmetric figure is zero.

Thus, option (A) is the correct answer.

PHYSICS

ELECTROSTATICS

EQUILIBRIUM OF CHARGES

NOTES

What you already know

- Permittivity
- Limitations of Coulomb's law
- Principle of superposition

What you will learn

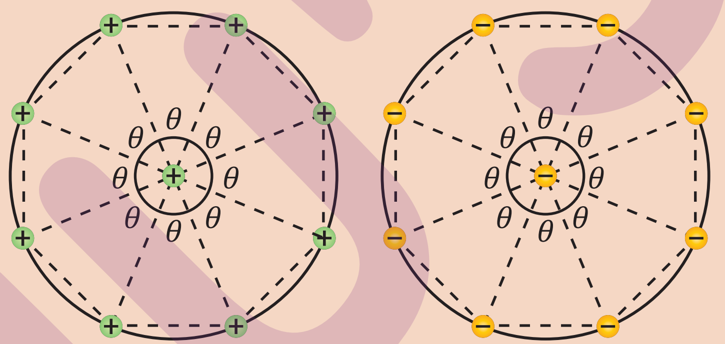
- Analysis of equilibrium
- Third charge in equilibrium

Recap

For n-sided polygon

The net force at the centre of a regular n-sided polygon due to n similar charge placed symmetrically at its vertices is **zero**. The angle subtended by the force due to the charges on the charge present at the centre of the polygon is given by,

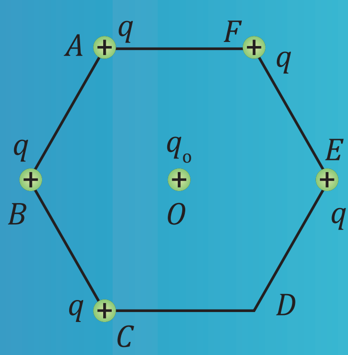
$$\theta = \frac{360°}{n}$$



If one charge is removed from a regular n-sided polygon

On removal of a charge from the vertex of the n-sided polygon, the resultant force becomes the same as the charge but in the opposite direction.

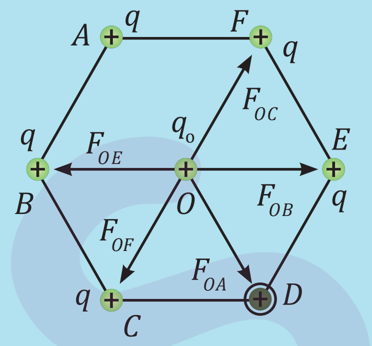
Five charges of equal charge $+q$ are placed at corners of a regular hexagon of side a. What is the force on $+q_o$ at centroid O?



(A) $\frac{3kqq_o}{a^2}$ (B) $\left(1+\sqrt{3}\right)\frac{kqq_o}{a^2}$ (C) $\frac{2kqq_o}{a^2}$ (D) $\frac{kqq_o}{a^2}$

Solution

Let us introduce a similar charge $+q$ in the empty place at D. By doing this, the charges in the system become symmetrical, and the net force on $+q_o$ becomes zero. We can see that the force applied on charge $+q_o$ by the charges at B and E are equal in magnitude but opposite in direction. Thus, they cancel each other. Similarly, the force applied on charge $+q_o$ by the charges at C and F are equal in magnitude but opposite in direction. Thus, they cancel each other. However, in reality, there is no charge at D. So, the repulsive force due to charge at A is not canceled by the charge at D. Thus, this unbalanced force acting on charge $+q_o$ is the net resultant force acting on it.



The force acting on q_o is given by,

$$F = \frac{kqq_o}{a^2}$$

Thus, option (D) is the correct answer.

BOARDS

Analysis of Equilibrium

A charge is said to be in equilibrium if the net electrostatic force acting on that charge is zero. Based on the magnitude and location of the charges, there are two different types of equilibriums:

(a) Stable equilibrium

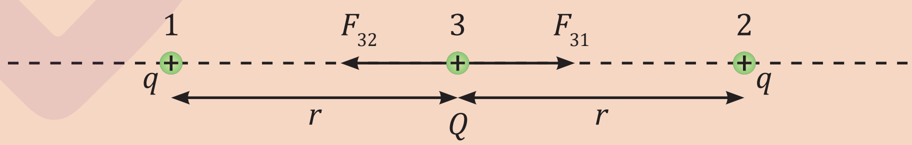
(b) Unstable equilibrium

Here, we will discuss stable equilibrium and unstable equilibrium in detail.

Stable equilibrium

When a particle is displaced slightly from an equilibrium position and the net force acting on it brings it back to the initial position, it is said to be in stable equilibrium.

For example, let us consider three positive charges namely 1, 2, and 3 of magnitudes q, q, and Q, respectively. Charges 1 and 2 are placed at a finite distance and they are fixed. The third charge Q is placed at the midpoint of the two charges along the line joining charges 1 and 2. It is not fixed. Thus, it can move from its position.

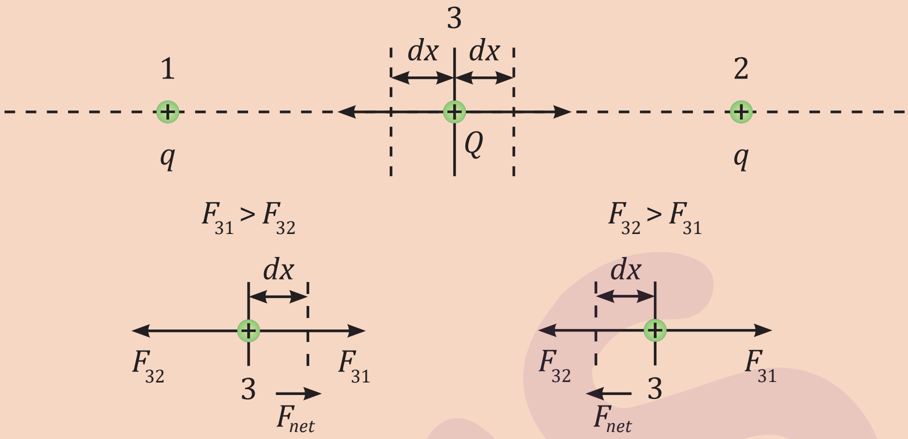


Let the repulsive force applied by charges 1 and 2 on the third charge be $\vec{F}_{31}$ and $\vec{F}_{32}$, respectively. $\vec{F}_{31}$ and $\vec{F}_{32}$ are opposite in direction. So, from Coulomb's law, the magnitude is given by,

$$\frac{kqQ}{r^2} = \frac{kqQ}{r^2} \Rightarrow F_1 = F_2$$

Case 1: In this case, if we shift the charge at the middle (third charge) towards the right by dx, then the repulsive force $\vec{F}_{32}$ acting on Q due to charge 2 increases as the distance decreases, and the force $\vec{F}_{31}$ acting on Q due to charge 1 decreases as the distance increases.

Due to this, the net force on Q acts towards the left (initial position of Q). Thus, we can say that charge 3 is in a stable equilibrium.

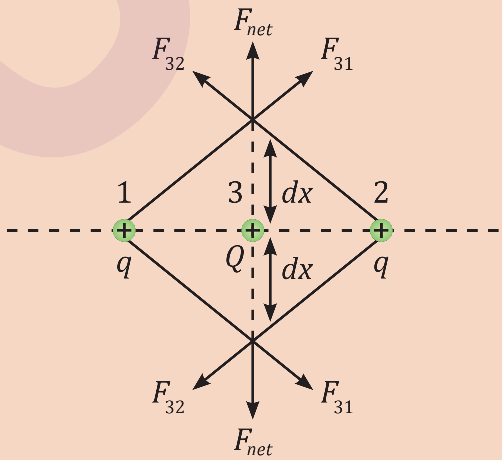


Case 2: In this case, if the charge at the middle (third charge) is displaced towards the left by dx, then the repulsive force $\vec{F}_{31}$ acting on Q due to charge 1 increases as the distance decreases, and force $\vec{F}_{32}$ acting on Q due to charge 2 decreases as the distance increases. Due to this, the net force on Q acts towards the right (initial position of Q). Thus, we can say that charge 3 is in a stable equilibrium.

Unstable equilibrium

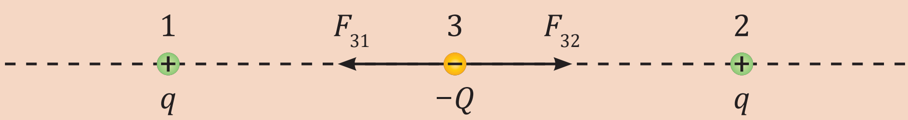
When a particle is displaced slightly from an equilibrium position and the net force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in an unstable equilibrium.

In the same example (discussed above), the third charge Q placed at the middle along the line joining charges 1 and 2 is displaced by a small distance dx either upwards or downwards from the equilibrium position in the direction perpendicular to the line joining the two fixed charges. In both cases, the separation distance of charge 3 from charges 1 and 2 varies in the same proportion. Thus, the magnitude of the repulsive forces acting on charge 3 by charges 1 and 2 are $\vec{F}_{31}$ and $\vec{F}_{32}$. They are equal in magnitude and along the line joining the charges as shown in the figure. Hence, the net force acts in the direction of displacement of charge 3. Due to the net force, charge 3 moves further away from the initial position. Therefore, it can be said that charge 3 is in an unstable equilibrium.



Stable and Unstable Equilibrium (For Negative Centre Charge)

Consider the same system we discussed above (three positive charges namely 1, 2, and 3 of magnitudes q, q, and Q, respectively). However, in this case, the third charge Q is replaced by $-Q$, and other than that everything is the same. Charges 1 and 2 are placed at a finite distance and are fixed. The third charge ($-Q$) is placed at the midpoint of the two charges along the line joining charges 1 and 2, and it is not fixed. Thus, it can move from its position as shown in the figure.



Let the attractive forces applied by charges 1 and 2 on the third charge be $\vec{F}_{31}$ and $\vec{F}_{32}$, respectively. $\vec{F}_{31}$ and $\vec{F}_{32}$ are opposite in direction and equal in magnitude.

Stable equilibrium

In this case, the third charge ($-Q$) placed at the middle along the line joining charges 1 and 2 is displaced by a small distance dx either upwards or downwards from the equilibrium position in the direction perpendicular to the line joining the two fixed charges. In both cases, the separation distance of charge 3 from charges 1 and 2 varies in the same proportion. Thus, the magnitude of the attractive force acting on charge 3 by charges 1 and 2 are F_{31} and F_{32}. They are equal in magnitude and along the line joining the charges as shown in the figure. Hence, the net force acts in the opposite direction of displacement of charge 3. Due to the net force, charge 3 moves towards the initial position. Therefore, it can be said that charge 3 is in a stable equilibrium.

Unstable equilibrium

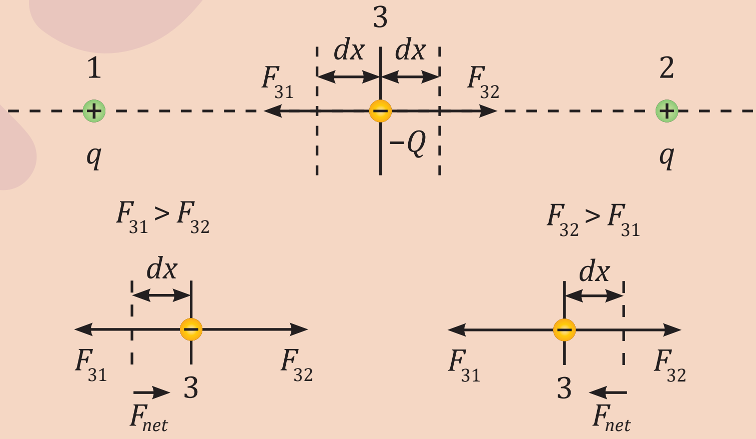
In the same example (discussed above), the third charge ($-Q$) placed at the middle along the line joining charges 1 and 2 is displaced by a small distance dx either towards the left or right from the equilibrium position along the line joining the two fixed charges. Two cases are possible where the third charge ($-Q$) can move towards the left or right, depending upon the direction of the net force.

Case 1:

In this case, if the charge at the middle (third charge) is displaced towards the left by a small distance dx, then the attractive force $\vec{F}_{31}$ acting on $-Q$ due to charge 1 increases as the distance decreases, and the attractive force $\vec{F}_{32}$ acting on $-Q$ due to charge 2 decreases as the distance increases. Due to this, the net force on $-Q$ acts towards the left (away from the initial position of Q). Therefore, we can say that charge 3 is in an unstable equilibrium.

Case 2:

In this case, if the charge at the middle (third charge) is displaced towards the right by a small distance dx, then the attractive force $\vec{F}_{32}$ acting on $-Q$ due to charge 2 increases as the distance decreases, and the attractive force $\vec{F}_{31}$ acting on $-Q$ due to charge 1 decreases as the distance increases. Due to this, the net force on $-Q$ acts towards the right (away from the initial position of Q). Therefore, we can say that charge 3 is in an unstable equilibrium.



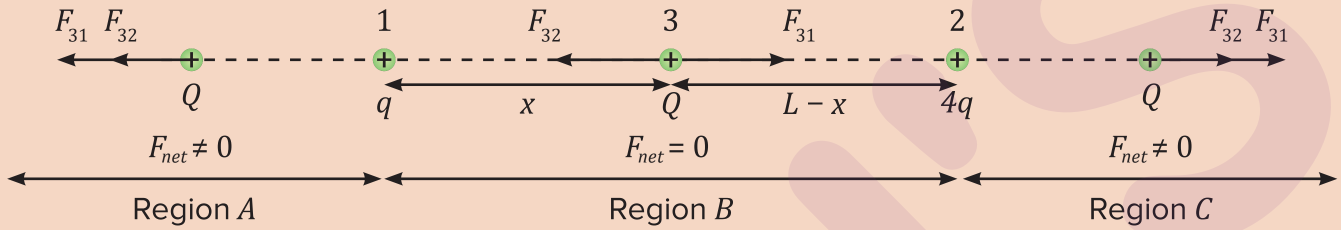
The conditions for the equilibrium of the system (with no charge fixed) of all the three charged particles (discussed above) are as follows:

(1) The three charges must be collinear.
(2) The three charges must not be of the same sign.
(3) The three charges must not be of the same magnitude.
(4) On obeying the mentioned conditions, the equilibrium of the system will always be unstable in nature.

Third Charge in Equilibrium

Case 1: Like charges

Let us consider three positive charges namely 1, 2, and 3 of magnitudes q, $4q$, and Q, respectively. Charges 1 and 2 or charges q and $4q$ are fixed at distance L from each other. Let us divide the vicinity of space near charges q and $4q$ into three regions A, B, and C as shown in the figure. If a third charge Q has to be placed in this system such that it should be in equilibrium, then to find the equilibrium of the third charge Q in the given system, we have to draw the force diagram for charge Q in all three regions A, B, and C.



Region A

If the third charge Q is placed in region A, the repulsive forces acting on charge 3 by charges 1 and 2 are $\vec{F}_{31}$ and $\vec{F}_{32}$. They have different magnitudes and are in the same direction (towards the left) along the line joining as shown in the figure. Hence, the net force cannot be zero in this region and it acts towards the left. Due to the net force, charge 3 moves towards the left. Therefore, there is no possibility for the third charge Q to attain equilibrium in this region.

Region C

Similar to region A, if we place the third charge Q in region C, the repulsive forces acting on charge 3 by charges 1 and 2 are $\vec{F}_{31}$ and $\vec{F}_{32}$. They have different magnitudes and are in the same direction (towards the right) along the line joining as shown in the figure. Hence, the net force cannot be zero in this region and it acts towards the right. Due to the net force, charge 3 moves towards the right. Therefore, there is no possibility for the third charge Q to attain equilibrium in this region.

Region B

If we place the third charge Q in region B, the repulsive forces acting on charge 3 by charges 1 and 2 are $\vec{F}_{31}$ and $\vec{F}_{32}$. They have different magnitudes and are in the opposite direction of the line joining as shown in the figure. Since $\vec{F}_{31}$ and $\vec{F}_{32}$ act in the opposite directions, there is a possibility that forces $\vec{F}_{31}$ and $\vec{F}_{32}$ can cancel each other. It is possible if the third charge is placed near the smaller charge q. Let the distance be x. Thus, the distance between $4q$ and the third charge becomes $L - x$.

Region	Direction of $\vec{F}_{31}$	Direction of $\vec{F}_{32}$	Possibility of the equilibrium position
Region A	Acting towards left	Acting towards left	Not possible as both the forces are acting in the same direction
Region B	Acting towards right	Acting towards left	Possible as the forces are in the opposite direction
Region C	Acting towards right	Acting towards right	Not possible as both the forces are acting in the same direction

For $\vec{F}_{net} = 0$, $\vec{F}_{31} = \vec{F}_{32}$

$$\frac{kQq}{x^2} = \frac{kQ(4q)}{(L-x)^2}$$

$$\Rightarrow \frac{(L-x)^2}{x^2} = 4$$

$$\Rightarrow \frac{L-x}{x} = \pm 2$$

$$\Rightarrow x = -L \text{ or } x = \frac{L}{3}$$

Since x cannot be $-L$, we get, $x = \frac{L}{3}$.

Shortcut method

The distance of the third charge from the smaller charge is given by,

$$x = \left(\frac{\sqrt{q_1}}{\sqrt{q_2} + \sqrt{q_1}}\right) L \qquad \cdots (i)$$

Where,
L = Distance of separation between the charges
q_1 = Smaller charge
q_2 = Bigger charge
For the given example, if we substitute the charges in equation (i), we get,

$$x = \left(\frac{\sqrt{q}}{\sqrt{4q} + \sqrt{q}}\right) L$$

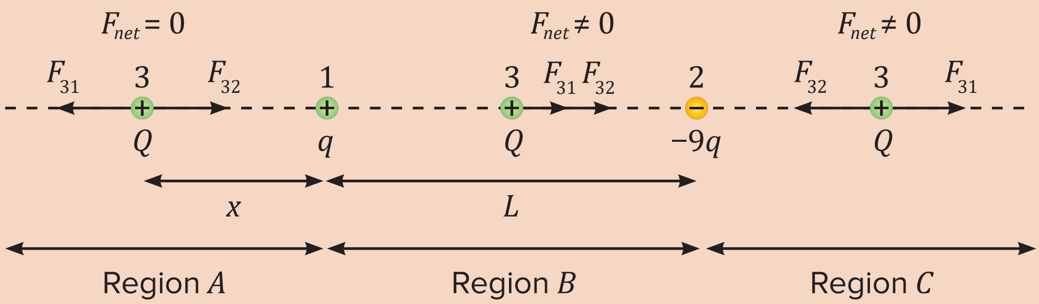
$$\Rightarrow x = \left(\frac{\sqrt{q}}{2\sqrt{q} + \sqrt{q}}\right) L$$

$$\Rightarrow x = \frac{L}{3}$$

This is similar to what we have obtained by the previous method.

Case 2: Unlike charges

Similar to the above case (like charges), let us consider three charges namely 1, 2, and 3 of magnitudes q, $-9q$, and Q, respectively. Charges 1 and 2 or charges q and $-9q$ are fixed at distance L from each other. Let us divide the vicinity of space near charges q and $-9q$ into three regions A, B, and C as shown in the figure. If a third charge Q has to be placed in this system such that it should be in equilibrium, then to find the equilibrium of the third charge Q in the given system, we have to draw the force diagram for charge Q in all three regions A, B, and C.



Region B

If the third charge Q is placed in region B, the forces acting on charge 3 by charges 1 and 2 are $\vec{F}_{31}$ and $\vec{F}_{32}$. They have different magnitudes and are in the same direction along the line joining as shown in the figure. $\vec{F}_{31}$ is a repulsive force and $\vec{F}_{32}$ is an attractive force. Thus, both act in the same direction. Hence, the net force cannot be zero in this region and it acts towards the right. Due to the net force, charge 3 moves towards the right. Therefore, there is no possibility for the third charge Q to attain equilibrium in this region.

Region C

If we place the third charge Q in region C, the forces acting on charge 3 by charges 1 and 2 are $\vec{F}_{31}$ and $\vec{F}_{32}$. They have different magnitudes and are in the opposite direction along the line joining as shown in the figure. The separation between charges 1 and 3 is higher than the separation between charges 2 and 3. The magnitude of charge 2 is greater than charge 1. Thus, $\vec{F}_{32}$ is greater in magnitude when compared with $\vec{F}_{31}$. Hence, the net force cannot be zero in this region and it acts towards the left. Due to the net force, charge 3 moves towards the left. Therefore, there is no possibility for the third charge Q to attain equilibrium in this region.

Region A

If we place the third charge Q in region A, the forces acting on charge 3 by charges 1 and 2 are $\vec{F}_{31}$ and $\vec{F}_{32}$. They have different magnitudes and are in the opposite direction along the line joining as shown in the figure. Since $\vec{F}_{31}$ and $\vec{F}_{32}$ act in opposite directions, there is a possibility that forces $\vec{F}_{31}$ and $\vec{F}_{32}$ cancel each other. It is possible if the third charge is placed near the smaller charge q. Let the distance be x. Thus, the distance between $-9q$ and the third charge becomes $L + x$.

Region	Direction of $\vec{F}_{31}$	Direction of $\vec{F}_{32}$	Possibility of the equilibrium position
Region A	Acting towards left	Acting towards right	Possible as the forces are in opposite direction.
Region B	Acting towards right	Acting towards right	Not possible as both are along the same direction
Region C	Acting towards right	Acting towards left	Not possible as the negative charge is bigger and distance is less.

For $\vec{F}_{net} = \vec{0}$, $\vec{F}_{31} = \vec{F}_{32}$

$$\frac{kQq}{x^2} = \frac{kQ(9q)}{(L+x)^2}$$

$$\Rightarrow \frac{(L+x)^2}{x^2} = 9$$

$$\Rightarrow \frac{L+x}{x} = \pm 3$$

$$\Rightarrow x = -\frac{L}{4} \text{ or } x = \frac{L}{2}$$

Since x cannot be $-\frac{L}{4}$, we get, $x = \frac{L}{2}$.

Shortcut method

The distance of the third charge from the smaller charge is given by,

$$x = \left(\frac{\sqrt{q_1}}{\sqrt{q_2} - \sqrt{q_1}}\right)L \qquad \cdots(i)$$

Where,

L = Distance of separation between the charges

q_1 = Smaller charge

q_2 = Bigger charge

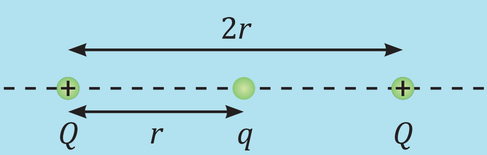
We will get the same value of x if we substitute the values of charge and distance as we did in the case of like charges.

A charge q is placed at the center of the line joining two equal positive charges Q. For the system of the three charges to be in equilibrium, what is the value of q?

(A) $-Q$ (B) $+\frac{Q}{2}$ (C) $-\frac{Q}{4}$ (D) Q

Solution NEET

We have two positive charges (Q) and a charge q is placed in between the two positive charges and the system is required to be in equilibrium. We know that for a system of charges to be in equilibrium, the charges must not be of the same sign. Thus, charge q should be negative in nature.



For the first charge to be in equilibrium, the net force acting on it will be 0.

Therefore,

$$F_{12} + F_{23} = 0$$

$$\Rightarrow \frac{kQ^2}{(2r)^2} + \frac{kQq}{r^2} = 0$$

$$\Rightarrow \frac{kQ}{r^2}\left[\frac{Q}{4} + q\right] = 0$$

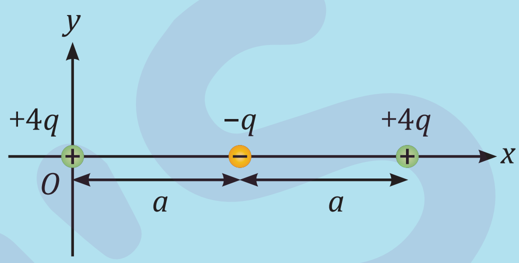
$$\Rightarrow q = -\frac{Q}{4}$$

Thus, option (C) is the correct answer.

If point charges $+4q$, $-q$, and $+4q$ are kept on the x-axis at points $x = 0$, $x = a$, and $x = 2a$, respectively, then which of the following statements is correct?

(A) Only $-q$ is in a stable equilibrium.
(B) None of the charges are in equilibrium.
(C) All the charges are in an unstable equilibrium.
(D) All the charges are in a stable equilibrium.

Solution



The net force acting on $-q$ charge is given as follows:

$$F_{-q} = -\frac{k(q)(4q)}{a^2} + \frac{k(q)(4q)}{a^2} = 0$$

Also, the net force acting on both the charges $+4q$ is given by,

$$F_{4q} = -\frac{k(q)(4q)}{a^2} + \frac{k(4q)(4q)}{(2a)^2}$$

$$\Rightarrow F_{4q} = -\frac{4kq^2}{a^2} + \frac{4kq^2}{a^2} = 0$$

Hence, we can clearly conclude that the system is in equilibrium as the net force on all the charges is 0. However, the charge in the middle is a negative charge and the other two charges are positive. So, on the slight movement of $-q$ charge, the system will no longer be in a state of equilibrium. Therefore, all three particles are in an unstable equilibrium.

Thus, option (C) is the correct answer.

To find the equilibrium, draw the force diagram in each possible region and check the regions in which the forces can cancel each other.

BYJU'S Classes

PHYSICS

ELECTROSTATICS

ELECTRIC FIELD INTENSITY

NOTES

What you already know

- Coulomb's law
- Third charge in equilibrium

What you will learn

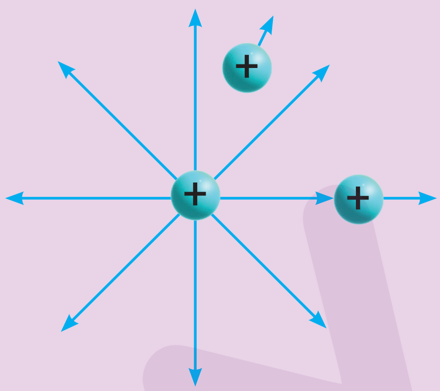
- Electric field
- Electric field intensity

BOARDS

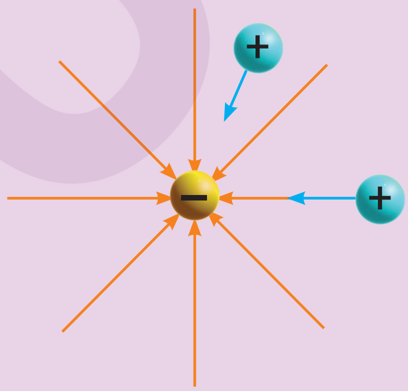
Electric Field

Electric field exists in a region where an electric charge experiences an electrostatic force.

When a charge is placed in an electric field, it experiences an electrostatic force. In the electric field of a positive charge, if another unit positive charge (test charge) is placed, then it experiences a repulsive force in that field. On the other hand, if a unit positive charge is placed in the electric field of a negative charge, then the unit positive charge experiences an attractive force towards the negative charge.

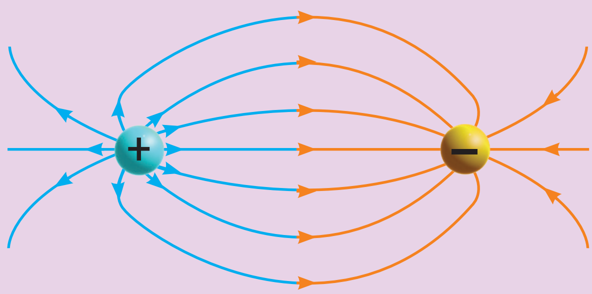


Radially outwards



Radially inwards

To visualize the electric field geometrically, Michael Faraday introduced electric field lines or electric lines of force. From a positive charge, the electric field lines emerge radially outward. However, in a negative charge, the electric field lines goes radially inward.
If one positive and one negative charge are placed closer to each other, then the electric field lines appear to be coming out from the positive charge and going into the negative charge.

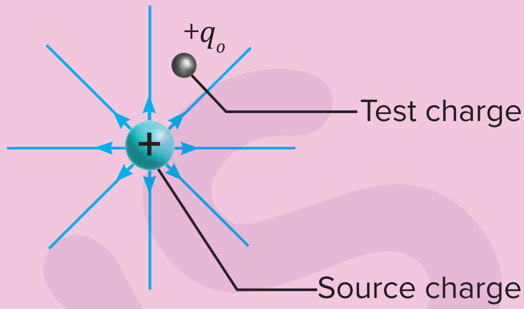


NEET

Electric Field Intensity

At a point, the electric field intensity is the force experienced by a unit positive charge placed in the electric field.

$$|\vec{E}| = \frac{|\vec{F}_e|}{q_o}$$



Let us consider a positive charge that is the source charge, and a positive charge $+q_o$ which is so small that it cannot produce its own electric field. This small charge is known as the test charge. When the positive test charge is brought to the region of the electric field of the source charge, it experiences an electrostatic force. The direction of force experienced by the positive test charge gives the direction of the electric field.

When the source charge is positive

Let us consider a positive test charge q_o that is placed in the electric field of charge q at distance r as shown in the figure.

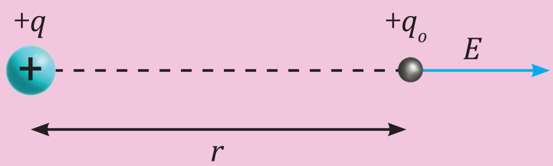
As the source charge is positive, the direction of the electric field is radially outward.
The intensity of the electric field is given by,

$$|\vec{E}| = \frac{|\vec{F}_e|}{q_o} \quad ...(i)$$

Where, F_e is the electrostatic force.

$$|\vec{F}_e| = \frac{kqq_o}{r^2}$$

By substituting $|\vec{F}_e|$ in equation (i), we get,



$$|\vec{E}| = \frac{\frac{kqq_o}{r^2}}{q_o}$$

$$\Rightarrow |\vec{E}| = \frac{kq}{r^2}$$

$$\boxed{\Rightarrow E = \frac{kq}{r^2}}$$

From the equation, we can observe that the electric field intensity is inversely proportional to the square of the distance between them.

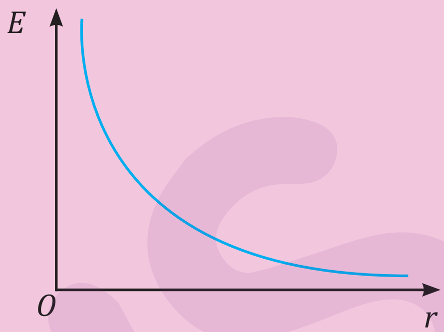
$$E \propto \frac{1}{r^2}$$

The electric field is independent of the test charge but depends on the source charge.

$E \propto q$

If we plot a graph between the electric field intensity and distance r for a point charge, then the obtained graph is as shown.

r	E
∞	0
0	Undefined

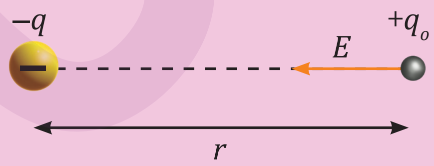


When the source charge is negative

Let us consider a positive test charge $+q_o$, which is placed at distance of r from the source charge $-q$.

The electric field intensity is given by,

$$|\vec{E}| = \frac{|\vec{F}_e|}{q_o} \quad ...(i)$$



Where, F_e is the electrostatic force.

$$|\vec{F}_e| = \frac{kqq_o}{r^2}$$

By substituting $|\vec{F}_e|$ in equation (i), we get,

$$|\vec{E}| = \frac{\frac{kqq_o}{r^2}}{q_o}$$

$$\Rightarrow |\vec{E}| = \frac{kq}{r^2}$$

$$\Rightarrow E = \frac{kq}{r^2}$$

The electric field intensity in the vector form is given by,

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$ (We use q with proper sign)

Where, $\vec{r} = |r|\hat{r}$

$$\boxed{\vec{E} = \frac{kq}{|r|^3}\vec{r}}$$

The dimension of electric field intensity is given by,

$$E = \frac{F}{q_o} = \frac{N}{C}$$

$$F = \left[M^1 L^1 T^{-2}\right]$$

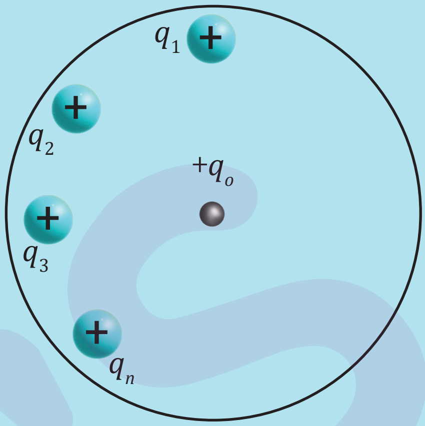
$$q_o = I \times t = \left[A^1 T^1\right]$$

Dimensional formula for $E = \left[M^1 L^1 A^{-1} T^{-3}\right]$

Principle of Superposition

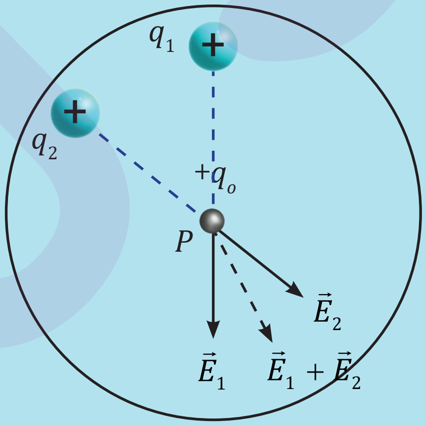
The principle of superposition states that every charge in space creates an electric field at a point independent of the presence of other charges in that medium. The resultant electric field is a vector sum of the electric field due to individual charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n$$



Consider that n number of charges are present in a system and a positive test charge is brought to the field as shown in the figure. The charges in the body are positive and the test charge is also positive.

The electric field intensity at point P in space due to two point charges q_1 and q_2 is E_1 and E_2, respectively (as shown in the figure).

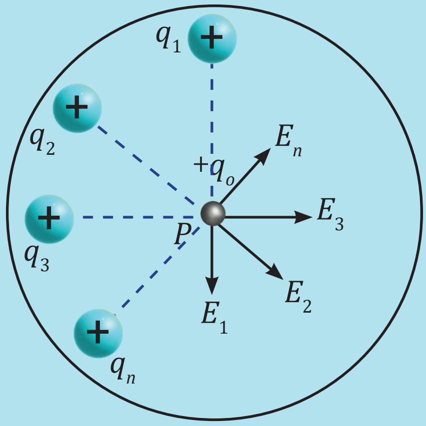


To obtain the resultant electric field intensity, we have to apply the triangle law of vector addition.

$$\left|\vec{E}_{net}\right| = \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos\theta}$$

Similarly, the net electric field at point P in space due to a system of n charges is given by,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n$$

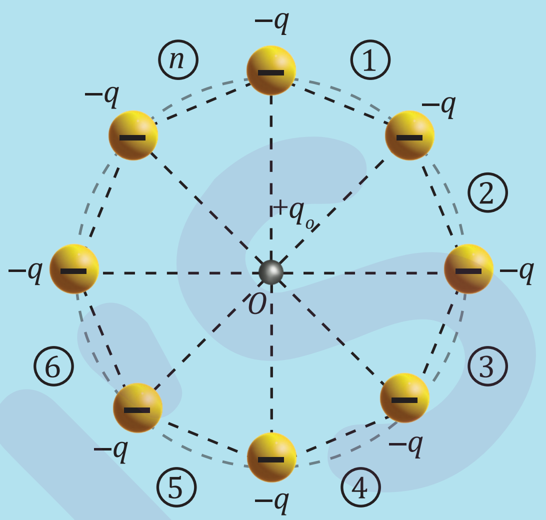


Therefore, in a system of n charges, the resultant electric field at a point in space is the vector sum of the electric field due to all individual charges.

Analysis of electric field

Regular polygon arrangement

Due to the symmetrical arrangement of charges, the net electric field is zero at the centre of an n-sided polygon.



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n = \vec{0}$$

The angle subtended by an n-sided polygon

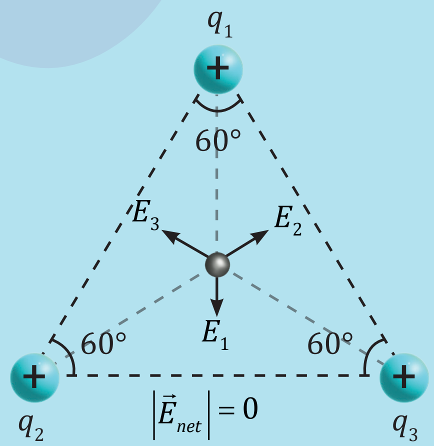
The value of the angle subtended is given by,

$$\theta = \frac{2\pi}{n}$$

Where, n is the number of sides.

Example:

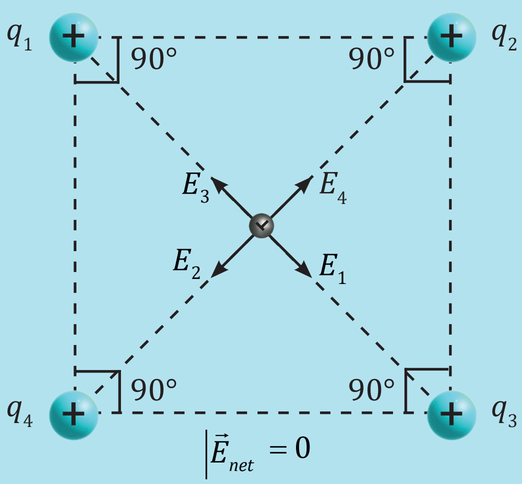
For a triangle, $n = 3$



So, at the center of the triangle, the angle between the electric field vector is,

$$\theta = \frac{2\pi}{n} = \frac{2\pi}{3} = 120°$$

For a square, $n = 4$



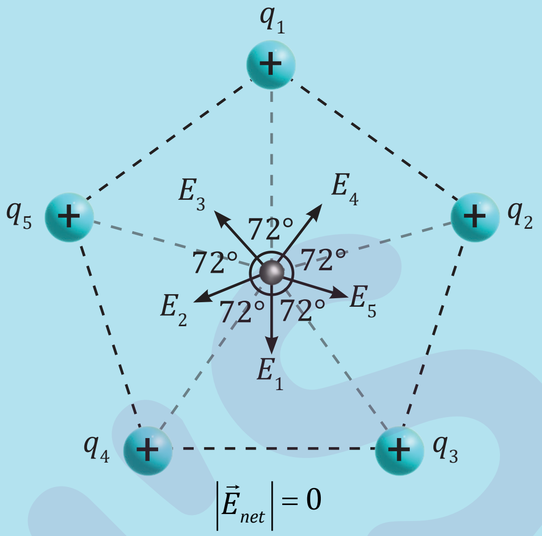
So, at the center of the square, the angle between the electric field vector is,

$$\theta = \frac{2\pi}{n} = \frac{2\pi}{4} = 90°$$

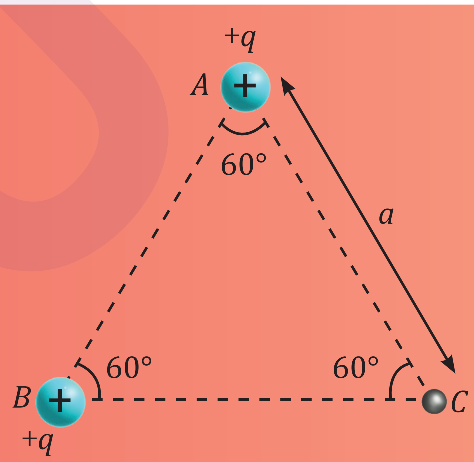
For a pentagon, $n = 5$

So, at the center of the pentagon, the angle between the electric field vector is,

$$\theta = \frac{2\pi}{n} = \frac{2\pi}{5} = 72°$$



Two identical charged particles A and B with charge $+q$ are present on the vertices of an equilateral triangle having sides of length a as shown in the figure. Find the magnitude of the resultant electric field at point C.



(A) $\frac{\sqrt{3}kq}{a^2}$ (B) $\frac{kq}{a^2}$ (C) $\frac{2kq}{a^2}$ (D) $\frac{\sqrt{3}kq}{a}$

Solution

The electric field at point C due to the charges at A and B is given by E_A and E_B , respectively, as shown in the figure.

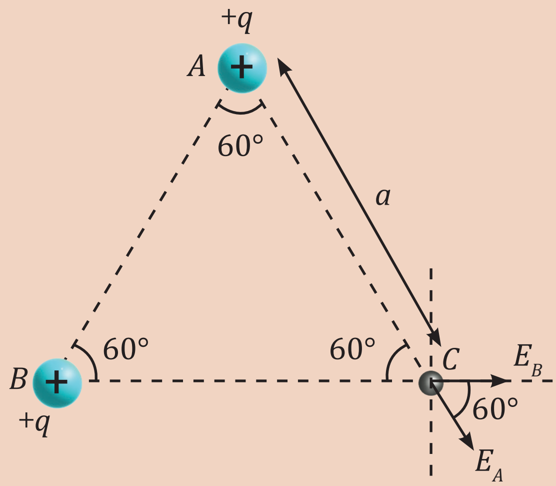
$$\left|\vec{E}_A\right| = \frac{kq}{a^2} = \left|\vec{E}_B\right| = E$$

The magnitude of the resultant electric field is given by,

$$\left|\vec{E}_{net}\right| = \sqrt{E_A^2 + E_B^2 + 2E_AE_B \cos\theta}$$

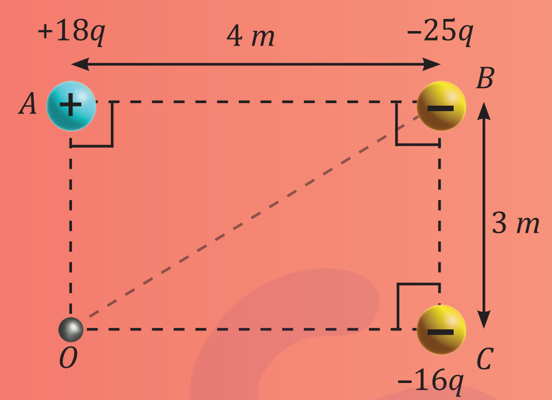
$$\Rightarrow \left|\vec{E}_{net}\right| = \sqrt{E^2 + E^2 + 2E^2 \cos 60°}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \sqrt{3}\,\frac{kq}{a^2}$$



Thus, option (A) is the correct answer.

For the given arrangement of charges, what will be the net electric field at point O?



(A) $\frac{9kq}{5}\hat{i}+\frac{7kq}{5}\hat{j}$ (B) $\frac{9kq}{5}\hat{i}-\frac{7kq}{5}\hat{j}$ (C) $\frac{11kq}{5}\hat{i}-\frac{9kq}{5}\hat{j}$ (D) $\frac{11kq}{5}\hat{i}+\frac{9kq}{5}\hat{j}$

Solution NEET

The electric fields at point O due to the charges at A, B, and C are given by $\vec{E}_A$, $\vec{E}_B$, and $\vec{E}_C$ respectively, as shown in the figure.

By applying the Pythagoras theorem in the right angled ΔBOC, we get,

$BO = 5\ m$

Also, in ΔBOC,

$$\sin\theta = \frac{3}{5}$$

$$\theta = 37°$$

To obtain the net electric field at point O, we have to calculate the individual electric field intensities due to the charges at A, B, and C.

The magnitude of the electric field at point O by the charges at A, B, and C are given as follows:

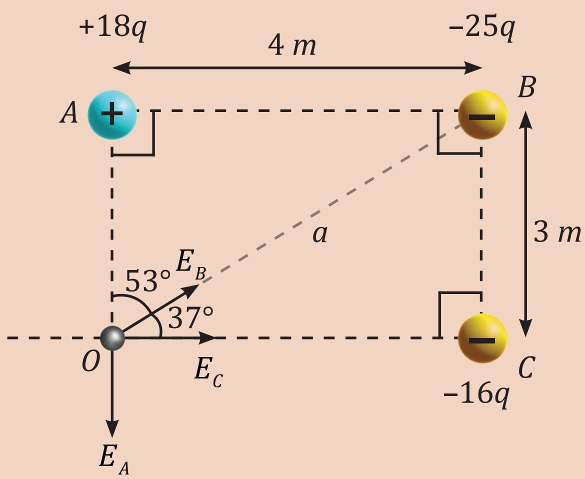
$$\left|\vec{E}_A\right| = \frac{k(18q)}{9}$$

$$\Rightarrow \left|\vec{E}_A\right| = 2kq$$

Similarly,

$$\left|\vec{E}_B\right| = \frac{k(16q)}{16} = kq$$

$$\left|\vec{E}_B\right| = \frac{k(25q)}{25} = kq$$



In the question, the net electric field is given in terms of unit vectors. Thus, we have to resolve the electric field along the x-axis and y-axis.

Along the x-axis, the component of the electric field is given by,

$$E_x = E_B \cos 37° + E_C$$

$$\Rightarrow E_x = \frac{4}{5}kq + kq$$

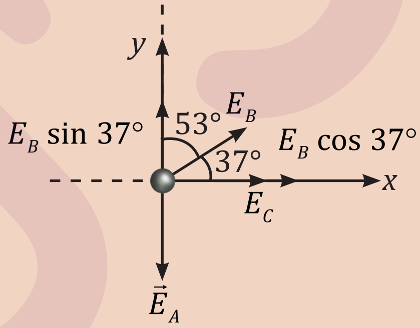
$$\Rightarrow E_x = \frac{9}{5}kq$$

Along y-axis, the component of net electric field is given by,

$$E_y = E_B \sin 37° - E_A$$

$$\Rightarrow E_y = \frac{3}{5}kq - 2kq$$

$$\Rightarrow E_y = -\frac{7}{5}kq$$



Therefore, the net electric field is given by,

$$\vec{E}_{net} = E_x\hat{i} + E_y\hat{j}$$

$$\vec{E}_{net} = \frac{9kq}{5}\hat{i} - \frac{7kq}{5}\hat{j}$$

Thus, option (B) is the correct answer.

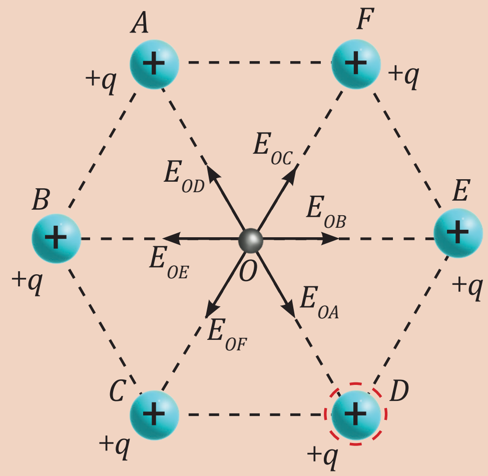
Five charges with equal magnitudes of $+q$ are placed at the corners of a regular hexagon of side a. What is the magnitude of the electric field at centroid O?

(A) $\frac{3kq}{a^2}$ (B) $\left(1+\sqrt{3}\right)\frac{kq}{a^2}$ (C) $\frac{2kq}{a^2}$ (D) $\frac{kq}{a^2}$

Solution

In such problems, where a charge is omitted from an n-sided polygon, we first assume that the empty space has a charge similar to the other vertices of the polygon.

Due to this, the net electric field will be zero at the centroid. Now, if we remove the charge at D, which was our original arrangement, then the net electric field will be along the direction of OD due to the charge at A.



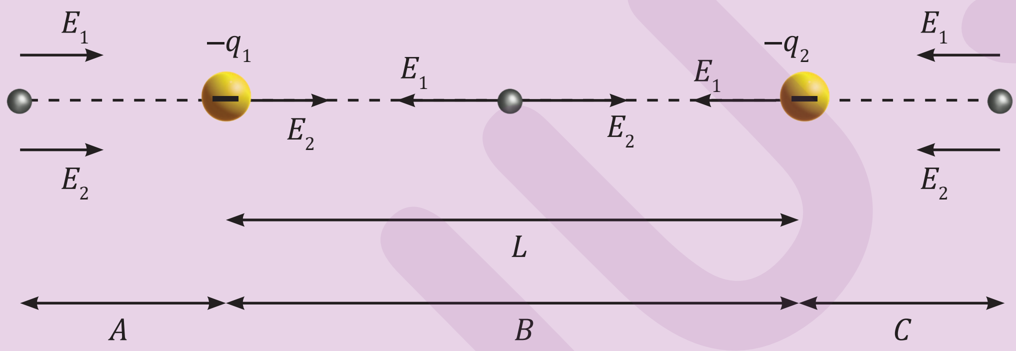
$$\left|\vec{E}_{OA}\right| = \frac{kq}{r^2} = \frac{kq}{a^2}$$

Thus, option (D) is the correct answer.

Analysis of Electric Field

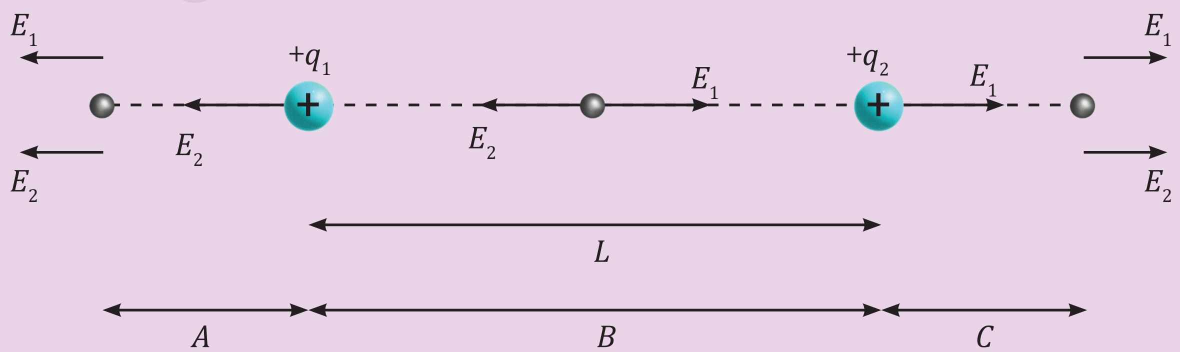
Case: Like charges

Consider two fixed like charges, $-q_1$ and $-q_2$, which are present at distance L from each other. Now, we divide the whole region into three parts, A, B, and C, to search for the neutral point (where the net electric field is zero).



Both charges are negative			
	Direction of E_1	**Direction of E_2**	**Possibility of the equilibrium position**
Region A	Acting towards right	Acting towards right	Not possible as both the fields are acting in the same direction
Region B	Acting towards left	Acting towards right	Possible as the electric fields are in the opposite direction
Region C	Acting towards left	Acting towards left	Not possible as both the fields are acting in the same direction

For both positive charges:

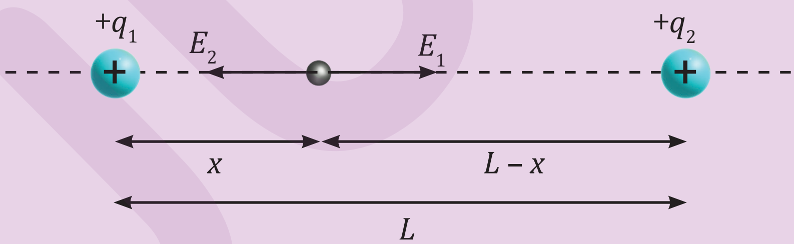


Both charges are positive			
	Direction of E_1	**Direction of E_2**	**Possibility of the equilibrium position**
Region A	Acting towards left	Acting towards left	Not possible as both the fields are acting in the same direction
Region B	Acting towards right	Acting towards left	Possible as the electric fields are in the opposite direction
Region C	Acting towards right	Acting towards right	Not possible as both the fields are acting in the same direction

Let the test charge be at distance x from charge $+q_1$. The distance from charge $+q_2$ becomes $L - x$. We also assumed the following:

$|+q_1| \le |+q_2|$

For the net electric field to be zero,



$$\left|\vec{E}_1\right| = \left|\vec{E}_2\right|$$

$$\left|\vec{E}_1\right| = \frac{kq_1}{x^2} \ldots(i)$$

$$\left|\vec{E}_2\right| = \frac{kq_2}{(L-x)^2} \ldots(ii)$$

By substituting the values in equations (i) and (ii), we get,

$$\frac{kq_1}{x^2} = \frac{kq_2}{(L-x)^2}$$

$$\Rightarrow \frac{q_1}{q_2} = \left(\frac{x}{(L-x)}\right)^2$$

$$\Rightarrow \pm\sqrt{\frac{q_1}{q_2}} = \frac{x}{(L-x)}$$

$$\Rightarrow \frac{L}{x} = 1 \pm \sqrt{\frac{q_2}{q_1}}$$

We know, $\frac{L}{x} > 1 \; (\because x < L)$

$$x = \left(\frac{\sqrt{q_1}}{\sqrt{q_2} + \sqrt{q_1}}\right)L$$

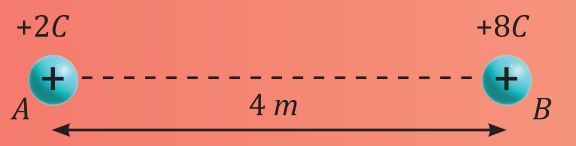
Where,

x = Distance from the smaller charge

q_1 = Smaller charge

q_2 = Bigger charge

Find the distance of the point from A where the net electric field is zero for the given configuration.

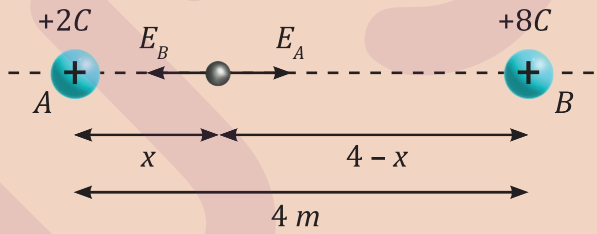


(A) 1.5 m towards the left
(B) 1.33 m towards the left
(C) 1.5 m towards the right
(D) 1.33 m towards the right

Solution

Let us assume the point where the net field is zero is at distance x from the smaller charge.

By applying the shortcut method, we get,



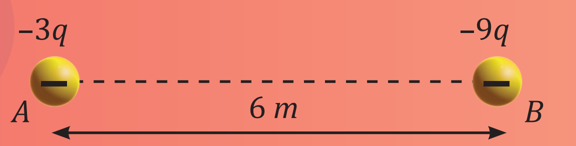
$$x = \left(\frac{\sqrt{q_1}}{\sqrt{q_2}+\sqrt{q_1}}\right)L$$

$$\Rightarrow x = \left(\frac{\sqrt{2}}{\sqrt{8}+\sqrt{2}}\right)4 = \frac{4}{3} = 1.33\ m$$

The electric field is zero at 1.33 m towards the right from A.

Thus, option (D) is the correct answer.

Find the distance of the point from A where the net electric field is zero for the given configuration.

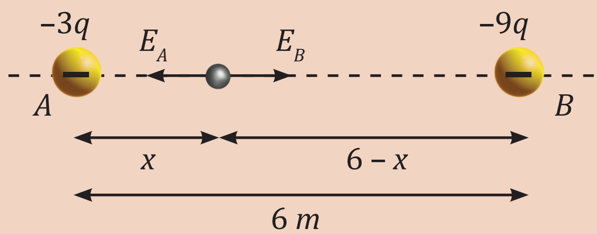


(A) 4.2 m towards the left
(B) 2.2 m towards the left
(C) 2.2 m towards the right
(D) 4.2 m towards the right

Solution

Let us assume the point where the net field is zero is at distance x from the smaller charge.
In this case, $|-3q| < |-9q|$

By applying the shortcut method, we get,



$$x = \left(\frac{\sqrt{q_1}}{\sqrt{q_2}+\sqrt{q_1}}\right)L$$

$$\Rightarrow x = \left(\frac{\sqrt{3q}}{\sqrt{9q}+\sqrt{3q}}\right)6 = \frac{6}{1+\sqrt{3}} \approx 2.2\,m$$

The electric field is zero at 2.2 m towards the right from A.

Thus, option (C) is the correct answer.

PHYSICS

ELECTROSTATICS

ELECTRIC FIELD DUE TO CONTINUOUS CHARGE-1

NOTES

What you already know

- Electric field
- Electric field intensity
- Principle of superposition

What you will learn

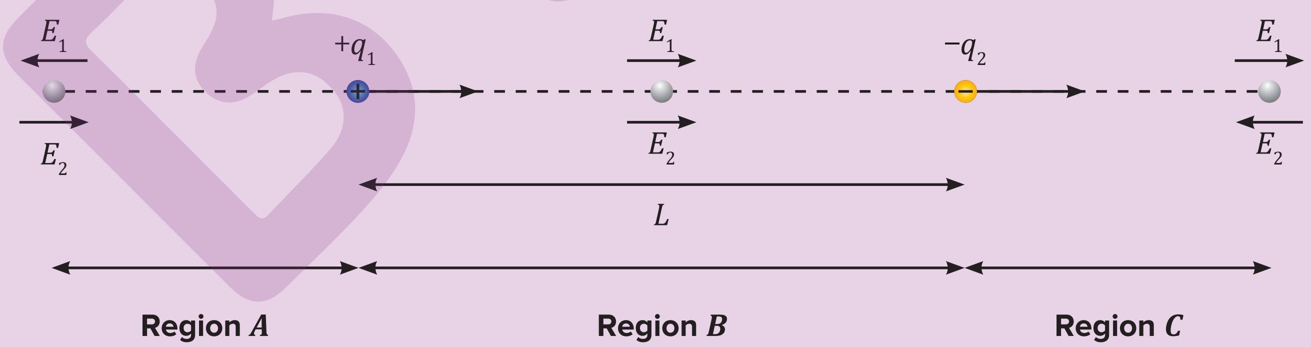
- Analysis of electric field
- Electric field vs position curve
- Electric field due to continuous charge distribution

Analysis of Electric Field

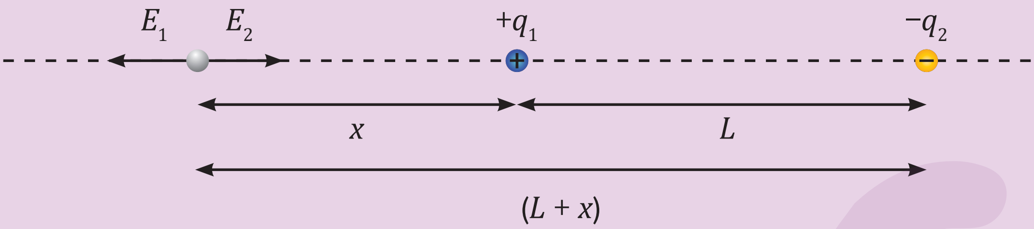
Case: Unlike charges

Consider that two fixed unlike charges $+q_1$ and $-q_2$ are present at distance L from each other. Now, divide the whole region into three regions A, B, and C to find the neutral point (the net electric field is zero). Let the electric fields due to charges q_1 and $-q_2$ be E_1 and E_2 respectively. We know that the electric field due to positive charge is away from it and due to negative charge is towards itself. Directions of electric fields in the three regions is shown in the figure.

Region	Direction of $\vec{E}_1$	Direction of $\vec{E}_2$	Possibility of the equilibrium position
Region A	Acting towards left	Acting towards right	Possible as the electric fields are in the opposite direction
Region B	Acting towards right	Acting towards right	Not Possible as the electric fields are in the same direction
Region C	Acting towards right	Acting towards left	Possible as the electric fields are in the opposite direction



The net electric field becomes zero in a position that is near the charge with less magnitude because it has to compensate for the electric field of a bigger charge.



Let us assume that $|+q_1| < |-q_2|$. Let the test charge be at distance x from charge $+q_1$ so that the distance from charge $-q_2$ becomes $L + x$.
For the net electric field to be zero,

$$E_1 = E_2$$

$$\frac{kq_1}{x^2} = \frac{kq_2}{(L+x)^2}$$

$$\Rightarrow \frac{q_1}{q_2} = \left(\frac{x}{(L+x)}\right)^2$$

$$\Rightarrow \pm\sqrt{\frac{q_1}{q_2}} = \left(\frac{x}{L+x}\right)$$

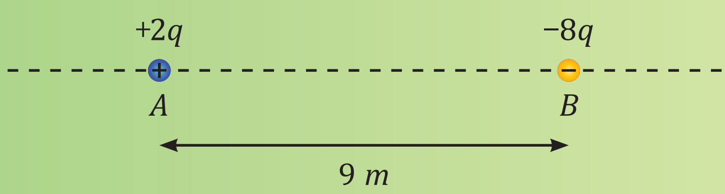
$$\Rightarrow \frac{L}{x} = \pm\sqrt{\frac{q_2}{q_1}} - 1$$

$$\Rightarrow \frac{L}{x} = \sqrt{\frac{q_2}{q_1}} - 1 \qquad \left(\because \frac{L}{x} \text{ cannot be negative}\right)$$

$$\boxed{x = \left(\frac{\sqrt{q_1}}{\sqrt{q_2} - \sqrt{q_1}}\right)L}$$

Where,
x = Distance from the smaller charge
q_1 = Smaller charge
q_2 = Bigger charge

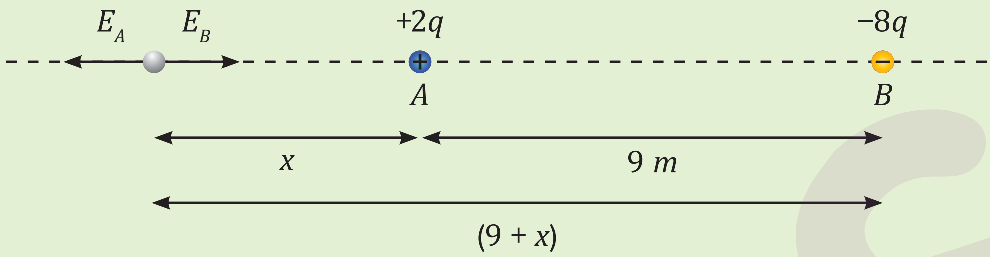
Find the distance of the point from A where the net electric field is zero for the given configuration.



(A) 6 m towards the left
(B) 9 m towards the left
(C) 6 m towards the right
(D) 9 m towards the right

Solution

In this case, $|+2q| < |-8q|$. So, the neutral point lies near charge $+2q$.



Let x be the distance from A where electric field will be zero.
We have,

$$x = \left(\frac{\sqrt{q_1}}{\sqrt{q_2} - \sqrt{q_1}}\right)L$$

$$\Rightarrow x = \left(\frac{\sqrt{2q}}{\sqrt{8q} - \sqrt{2q}}\right)9$$

$$\Rightarrow x = 9m$$

Thus, option (B) is the correct answer.

NEET

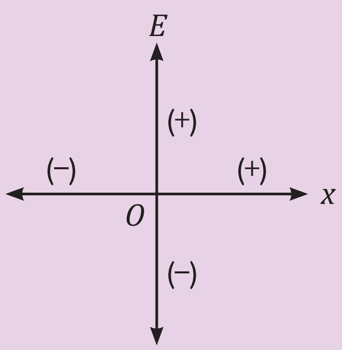
Electric Field vs Position Curve

To draw an electric field vs position (E-x) curve, we have to follow the following sign conventions:

Sign convention

The electric field (E) is taken along the y-axis, whereas the position (x) is taken along the x-axis.

Positive electric field	Negative electric field
At a point, the electric field **towards the positive x-axis** is considered as the positive electric field.	At a point, the electric field **towards the negative x-axis** is considered as the negative electric field.
The positive electric field is **plotted above the x-axis.**	The negative electric field is **plotted below the x-axis.**



For a positive point charge

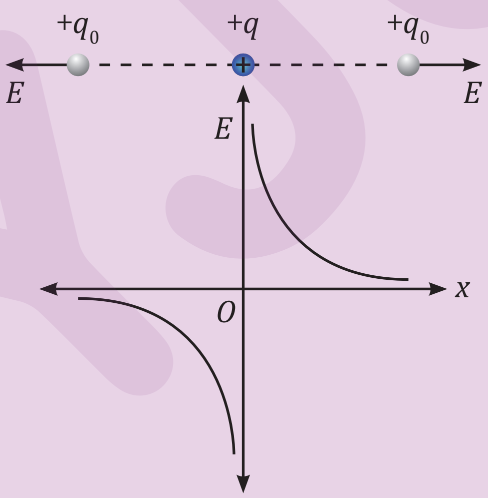
We know that $E \propto \dfrac{1}{x^2}$.

x	E
∞	0
0	Undefined
Approaches to zero	∞

For a positive charge, if we find the electric fields on the right and the left of the charge by placing a test charge, then we get the following:

Electric field of a positive charge			
	Direction of electric field	**Nature of electric field**	**Graph**
On the right side of the charge	Towards the positive x-axis	Positive	Plotted above the x-axis
On the left side of the charge	Towards the negative x-axis	Negative	Plotted below the x-axis

Hence, the E-x graph is given as follows:

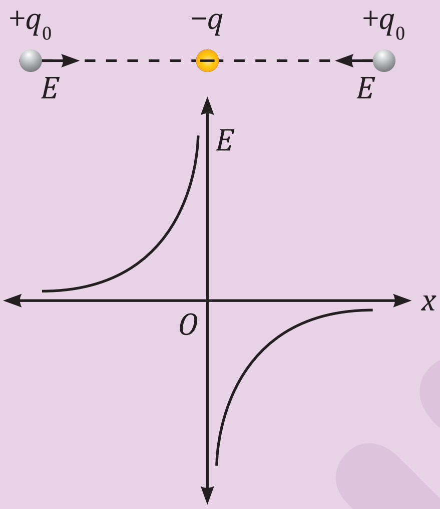


For a negative point charge

For a negative charge, if we find the electric fields on the right and the left of the charge by placing a test charge, then we get the following:

Electric field of a negative charge			
	Direction of electric field	**Nature of electric field**	**Graph**
On the right side of the charge	Towards the negative x-axis	Negative	Plotted below the x-axis
On the left side of the charge	Towards the positive x-axis	Positive	Plotted above the x-axis

Hence, the E-x graph is given as follows:



For two positive point charges

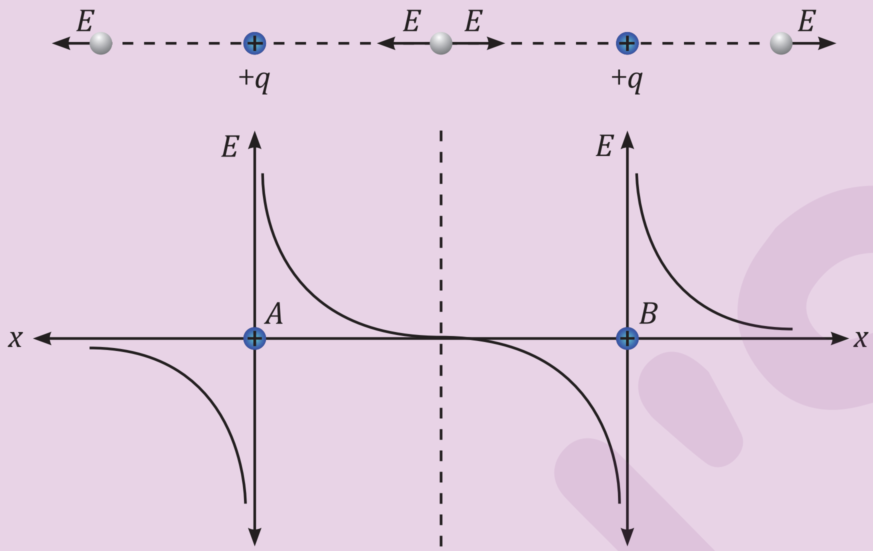
Here, the two like charges of the same magnitude are separated by some distance. So, there is a neutral point at the midpoint between the two charges (the net electric field is zero). Since $E \propto \frac{1}{x^2}$, the electric field intensity of a charge is maximum near the vicinity of the charge. As the distance increases, the electric field intensity decreases. Here, let us consider two positive point charges A and B of the same magnitude separated by some distance. Now, identify the electric field near the vicinity of charges A and B as follows:

Electric field near the vicinity of charge A			
	Direction of electric field	**Nature of electric field**	**Graph**
On the right side of the charge A	Towards the positive x-axis	Positive	Plotted above the x-axis
On the left side of the charge A	Towards the negative x-axis	Negative	Plotted below the x-axis

Electric field near the vicinity of charge B			
	Direction of electric field	**Nature of electric field**	**Graph**
On the right side of the charge B	Towards the positive x-axis	Positive	Plotted above the x-axis
On the left side of the charge B	Towards the negative x-axis	Negative	Plotted below the x-axis

In between A and B, there is a point where the net electric field is zero.

Hence, the E-x graph is given as follows:



For two negative point charges

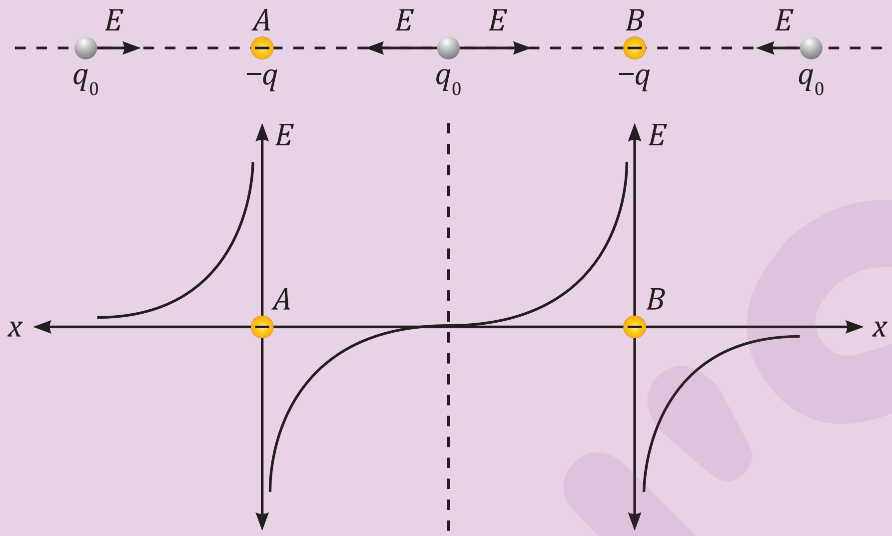
Similar to two positive charges, in this case also, let us consider two negative point charges A and B of the same magnitude separated by some distance. Now, identify the electric field near the vicinity of charges A and B.

Electric field near the vicinity of charge A			
	Direction of electric field	**Nature of electric field**	**Graph**
On the right side of the charge A	Towards the negative x-axis	Negative	Plotted below the x-axis
On the left side of the charge A	Towards the positive x-axis	Positive	Plotted above the x-axis

Electric field near the vicinity of charge B			
	Direction of electric field	**Nature of electric field**	**Graph**
On the right side of the charge B	Towards the negative x-axis	Negative	Plotted below the x-axis
On the left side of the charge B	Towards the positive x-axis	Positive	Plotted above the x-axis

In between A and B, there is a point where the net electric field is zero.

Hence, the E-x graph is given as follows:



For two unlike charges

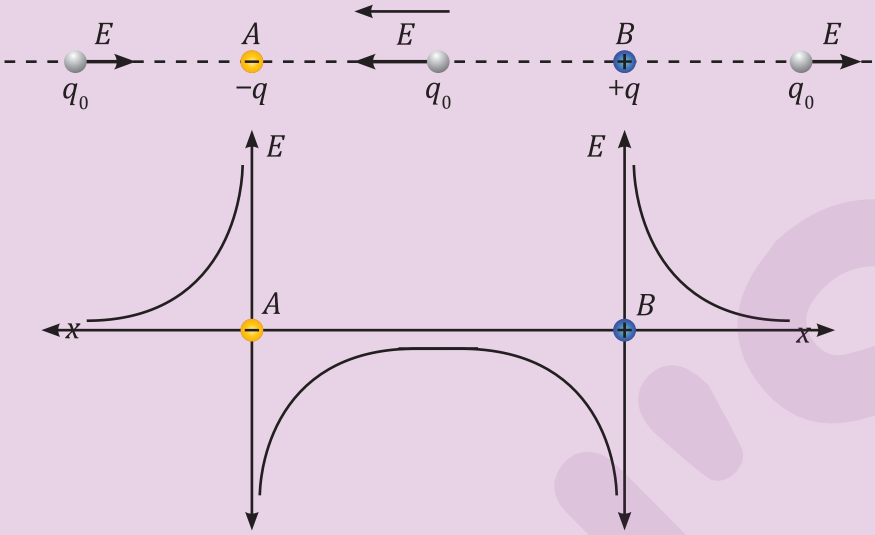
Here, the two unlike charges of the same magnitude are separated by some distance. There is no possibility of a neutral point (the net electric field is zero) between two unlike charges. Since $E \propto \frac{1}{x^2}$, the electric field intensity of a charge is maximum near the vicinity of the charge. As the distance increases, the electric field intensity decreases. Therefore, the net electric field is not zero at the midpoint between unlike charges. Here, let us consider two point charges A (negative charge) and B (positive charge) of the same magnitude separated by some distance. Now, identify the electric field near the vicinity of charges A and B.

Electric field near the vicinity of charge A			
	Direction of electric field	**Nature of electric field**	**Graph**
On the right side of the charge A	Towards the negative x-axis	Negative	Plotted below the x-axis
On the left side of the charge A	Towards the positive x-axis	Positive	Plotted above the x-axis

Electric field near the vicinity of charge B			
	Direction of electric field	**Nature of electric field**	**Graph**
On the right side of the charge B	Towards the positive x-axis	Positive	Plotted above the x-axis
On the left side of the charge B	Towards the negative x-axis	Negative	Plotted below the x-axis

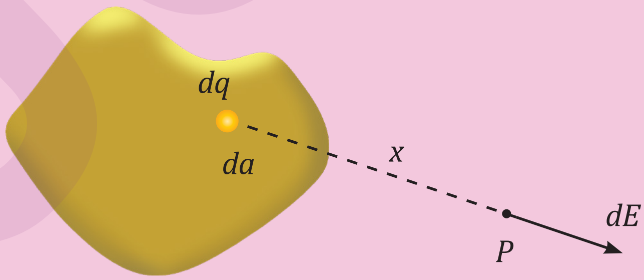
In between A and B, there is a point where the value of the electric field is minimum.

Hence, the E-x graph is given as follows:



Electric Field Due to Continuous Charge Distribution

Consider a continuous charge Q distributed uniformly throughout area A to obtain the electric field at some point P, which is at distance r from the body.
Consider a small elemental charge dq of area da. Due to this small elemental charge, the field is $d\vec{E}$.
The distance between dq and point P is x.
The electric field due to charge dq is given by,



$$d\vec{E} = \frac{k\,dq}{x^2}\hat{x} \quad \cdots(i)$$

The electric field for the whole body is given by,

$$\int d\vec{E} = \int \frac{k\,dq}{x^2}\hat{x}$$

The charge per unit area (σ) is given by,

$$\sigma = \frac{Q}{A}$$

So, the small elemental charge dq is given by,

$$\Rightarrow \sigma = \frac{dq}{da}$$

$$\Rightarrow dq = \sigma\, da$$

$$\Rightarrow dq = \frac{Q}{A} da$$

By substituting the value of dq in equation (i), we get,

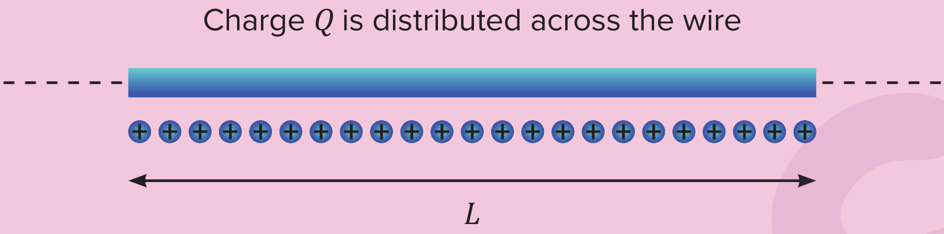
$$dE = \frac{k\,dq}{x^2}$$

$$dE = \frac{kQ}{A}\frac{da}{x^2}$$

On integrating the equation with proper limits, we can find the net electric field acting at point P.

Linear charge density (λ)

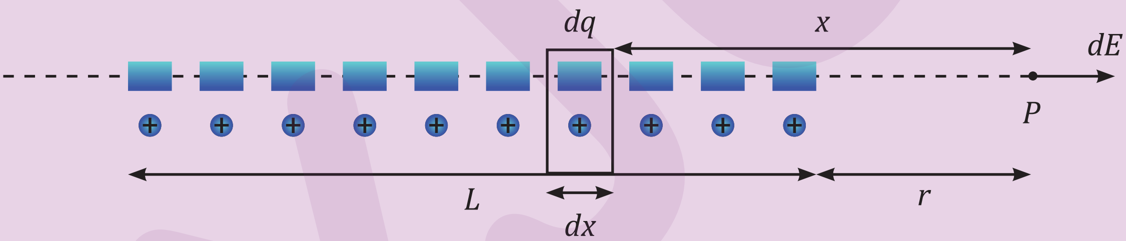
The amount of charge per unit length is known as linear charge density (λ).



$$\lambda = \frac{Q}{L}$$

Electric Field at an Axial Point

Consider a charged rod of length L and point P, which is at distance r from the rod. Consider a small charge dq of length dx that is at distance x from point P. Due to the small charge dq, the electric field dE is generated.



The electric field due to the small charge dq is given by,

$$dE = \frac{k\,dq}{x^2} \quad ...(i)$$

We know, $\lambda = \dfrac{Q}{L}$

$$dq = \lambda\, dx$$

$$\Rightarrow dq = \frac{Q}{L}dx$$

By substituting the value of dq in equation (i), we get,

$$dE = \frac{kQ}{L}\frac{dx}{x^2}$$

By integrating dE from $x = r$ to $x = r + L$, we get,

$$\int dE = \int_{r}^{r+L} \frac{kQ}{L}\frac{dx}{x^2}$$

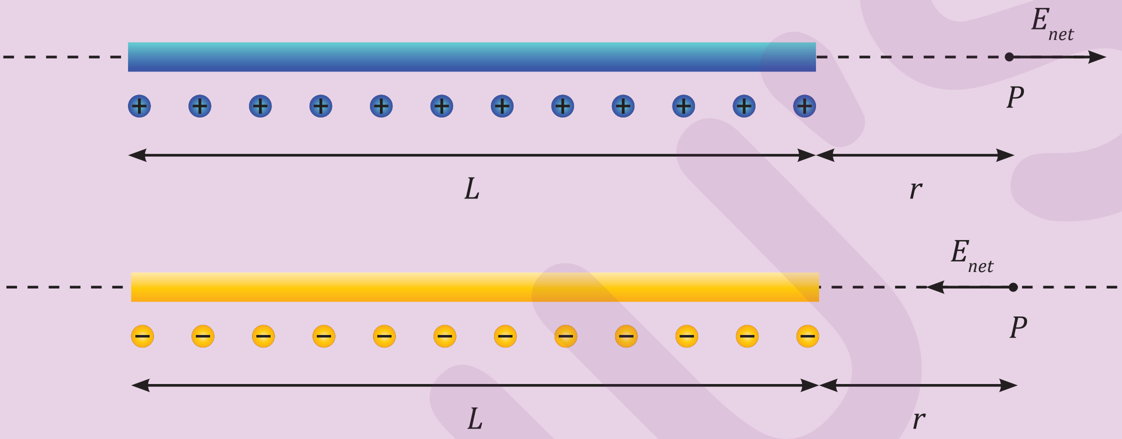
$$\Rightarrow \int dE = \frac{kQ}{L}\int_{r}^{r+L} x^{-2}dx$$

$$\Rightarrow \int dE = \frac{kQ}{L}\left[\frac{x^{-2+1}}{-2+1}\right]_r^{r+L}$$

$$\Rightarrow \int dE = \frac{kQ}{L}\left[-\frac{1}{x}\right]_r^{r+L}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{kQ}{L}\left[\frac{1}{r} - \frac{1}{r+L}\right]$$

The direction of the net electric field can be determined by the nature of the charge on the rod.



PHYSICS

ELECTROSTATICS

ELECTRIC FIELD DUE TO CONTINUOUS CHARGE: 2

What you already know

- Analysis of an electric field
- Electrified position curve
- Electric field due to continuous charge distribution

What you will learn

- Uniformly distributed line charge
- Special cases for uniformly distributed charges

NEET BOARDS

Uniformly Distributed Line Charge

Electric field at a non-axial point

Consider a wire of length L and charge Q is uniformly distributed along the length of the rod. Let us consider a non-axial point P that is at a distance of r from the wire. Now, let us drop a perpendicular to the wire from point P that meets the wire at point O. Since charge is uniformly distributed along the length, to find the electric field at point P, consider an element having a charge of dq with thickness dx at a distance of x from point O. Let the distance between the field point P and the element dx be a, and the angle subtended by element dq with the perpendicular dropped from the field point P be θ. For the extremes of the rod, the angle subtended by the left and right ends of the rod with the field point is α and β, respectively. Let us take the clockwise direction as positive and the anticlockwise as negative. So, the range of θ is from $+\alpha$ to $-\beta$. Let dE be the electric field at point P by the element charge dq. Thus, it is resolved into perpendicular and parallel component of the electric field as $dE\cos\theta$ and $dE\sin\theta$, respectively, as shown in the figure.

Electric field (dE) at point P by the element charge (dq) is given by,

$$dE = \frac{k\,dq}{a^2} \qquad \text{.......}(i)$$

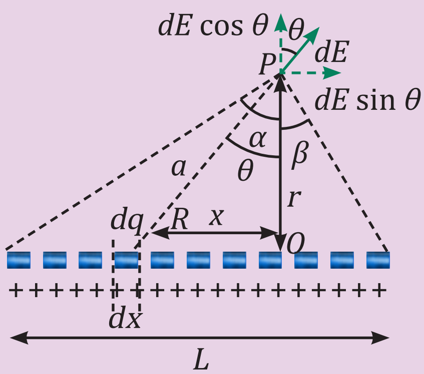
From ΔPOR,

$$\cos\theta = \frac{r}{a}$$

$$\Rightarrow a = \frac{r}{\cos\theta}$$

By squaring both sides, we get,

$$\Rightarrow a^2 = \frac{r^2}{\cos^2\theta}$$



By substituting the value of a^2 in equation (i), we get,

$$dE = \frac{k\,dq\,\cos^2\theta}{r^2} \quad ....(ii)$$

The linear charge density of the wire is given by,

$$\lambda = \frac{Q}{L}$$

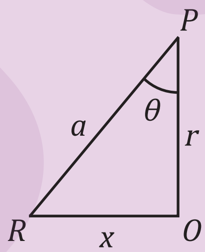
The elemental charge dq can be written as follows:

$$dq = \lambda\,dx$$

$$\Rightarrow dq = \frac{Q}{L}dx$$

By substituting the value of dq in equation (ii), we get,

$$dE = \frac{kQ}{L}\frac{dx\cos^2\theta}{r^2} \quad ......(iii)$$



In ΔPOR,

$$x = r\tan\theta$$

By differentiating both sides, we get,

$$dx = r\sec^2\theta\,d\theta$$

By substituting dx in equation (iii), we get,

$$dE = \frac{kQ}{L}\frac{r\sec^2\theta\cos^2\theta\,d\theta}{r^2}$$

$$\Rightarrow dE = \frac{kQ\,d\theta}{Lr} \quad ......(iv) \left(\because \sec^2\theta = \frac{1}{\cos^2\theta}\right)$$

The electric field at point P can be resolved into two components: one parallel to the length of the wire $E_{\parallel}$ and one perpendicular to the length of the wire $E_{\perp}$.

$$E_{\parallel} = \int dE\,\sin\theta = \int_{-\beta}^{\alpha}\frac{kQ\sin\theta\,d\theta}{Lr}$$

$$\Rightarrow E_{\parallel} = \frac{kQ}{Lr}\int_{-\beta}^{\alpha}\sin\theta\,d\theta$$

$$\Rightarrow E_{\parallel} = \frac{kQ}{Lr}\left[-\cos\theta\right]_{-\beta}^{\alpha}$$

$$\Rightarrow E_{\parallel} = \frac{kQ}{Lr}\left[\cos\beta - \cos\alpha\right] \quad ....(v)$$

Also,

$$E_{\perp} = \int dE \cos\theta = \int_{-\beta}^{\alpha} \frac{kQ}{Lr} \cos\theta \, d\theta$$

$$\Rightarrow E_{\perp} = \frac{kQ}{Lr}[\sin\theta]_{-\beta}^{\alpha}$$

$$\Rightarrow E_{\perp} = \frac{kQ}{Lr}[\sin\alpha + \sin\beta] \quad \text{.........}(vi)$$

Note

Equations (v) and (vi) are the generalised relations for the parallel and perpendicular components of the electric field at any arbitrary non-axial point due to linear charge distribution.

Special cases

1. The electric field at a point on the perpendicular bisector (equatorial):

In this case, let us find the electric field at point P, which lies along the perpendicular bisector at a distance of r from the wire as shown in the figure. Thus, the angle subtended by the left and right ends of the rod with the field point P is,

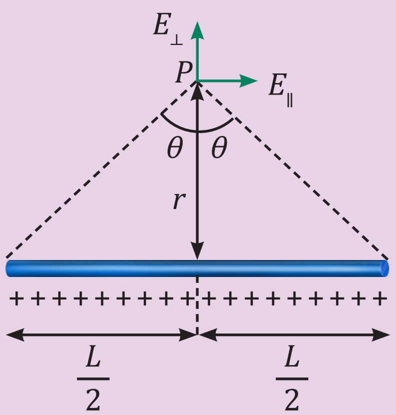
$\alpha = \beta = \theta$

The parallel component of the electric field at any non-axial point is given by,

$$E_{\parallel} = \frac{kQ}{Lr}[\cos\beta - \cos\alpha]$$

$$\Rightarrow E_{\parallel} = \frac{kQ}{Lr}[\cos\theta - \cos\theta] = 0$$

The perpendicular component of the electric field is given by,



$$E_{\perp} = \frac{kQ}{Lr}[\sin\alpha + \sin\beta]$$

$$\Rightarrow E_{\perp} = \frac{kQ}{Lr}[\sin\theta + \sin\theta]$$

$$\Rightarrow E_{\perp} = \frac{2kQ}{Lr}[\sin\theta]$$

The net electric field is given by,

$$|\vec{E}_{net}| = \sqrt{E_{\parallel}^2 + E_{\perp}^2}$$

$$\Rightarrow |\vec{E}_{net}| = \frac{2kQ\sin\theta}{Lr}$$

2. Electric field due to an infinite wire:

Consider an infinite wire with charge Q distributed uniformly along the length of the wire. Let us find the electric field at a non-axial point P, which is at a distance of r from the rod as shown in the figure.

The linear charge density of the wire is given by,

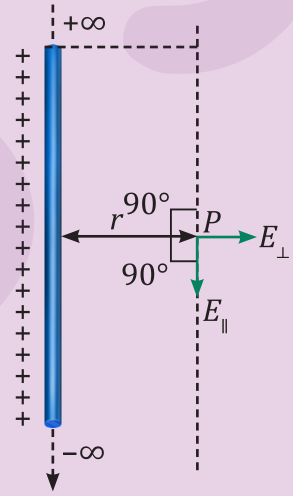
$$\lambda = \frac{Q}{L}$$

Since the wire is of infinite length, the angle subtended by the left and right ends of the wire with the field point P is, $\alpha = \beta = 90°$.

The parallel component of the electric field at any non-axial point is given by,

$$E_{\parallel} = \frac{kQ}{Lr}[\cos\beta - \cos\alpha]$$

$$\Rightarrow E_{\parallel} = \frac{kQ}{Lr}[\cos 90° - \cos 90°] = 0$$



The perpendicular component of the electric field is given by,

$$E_{\perp} = \frac{kQ}{Lr}[\sin 90° + \sin 90°]$$

$$\Rightarrow E_{\perp} = \frac{kQ}{Lr}[1 + 1]$$

$$\Rightarrow E_{\perp} = \frac{2k\lambda}{r}$$

The net electric field is given by,

$$\left|\vec{E}_{net}\right| = \sqrt{E_{\parallel}^2 + E_{\perp}^2}$$

$$\boxed{\Rightarrow \left|\vec{E}_{net}\right| = \frac{2k\lambda}{r}}$$

3. Electric field due to a semi-infinite wire:

In a semi-infinite and long wire, one end is finite and the other end is infinite. Charge Q is distributed uniformly along the length of the wire. Let us find the electric field at a non-axial point P, which is at a distance of r from the wire as shown in the figure.

The linear charge density of the wire is given by,

$$\lambda = \frac{Q}{L}$$

Since the wire is semi-infinitely long, the angle subtended by the left and right ends of the wire with the field point P is, $\alpha = 0$, $\beta = 90°$.

The parallel component of the electric field at any non-axial point is given by,

$$E_{\parallel} = \frac{kQ}{Lr}[\cos\beta - \cos\alpha]$$

$$\Rightarrow E_{\parallel} = \frac{kQ}{Lr}[\cos 90° - \cos 0°]$$

$$\Rightarrow E_{\parallel} = -\frac{k\lambda}{r}$$

Negative sign shows that the direction of the field is opposite to the conventional positive direction

The perpendicular component of the electric field is given by,

$$E_{\perp} = \frac{kQ}{Lr}[\sin 0° + \sin 90°]$$

$$\Rightarrow E_{\perp} = \frac{kQ}{Lr}[0+1]$$

$$\Rightarrow E_{\perp} = \frac{k\lambda}{r}$$

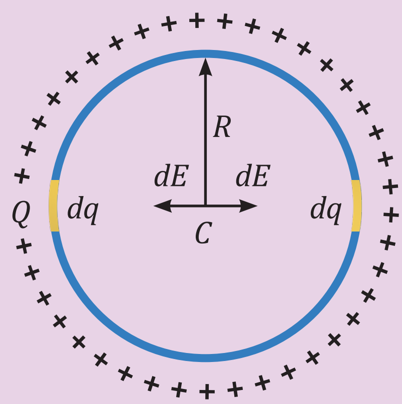
The net electric field is given by,

$$\left|\vec{E}_{net}\right| = \sqrt{E_{\parallel}^2 + E_{\perp}^2}$$

$$\boxed{\Rightarrow \left|\vec{E}_{net}\right| = \sqrt{2}\frac{k\lambda}{r}}$$

4. The electric field at the centre of a uniformly charged ring:

Consider a ring of radius R and charge Q is uniformly distributed along the length of a wire as shown in the figure.



Let us consider a small element of charge dq on the ring and an electric field dE on the centre due to this charge. On the contrary, to the small element of charge dq, there exists another element of charge dq on the diametrically opposite side. It creates an equal and opposite electric field at the centre. Similarly, for every element, there will be a counter element in the ring. Thus, the net electric field at the centre of the ring is zero.

$$\boxed{\left|\vec{E}_C\right| = 0}$$

ELECTROSTATICS

ELECTRIC FIELD DUE TO CONTINUOUS CHARGE: 3

What you already know

- Uniformly distributed line charges
- Special cases for uniformly distributed charges

What you will learn

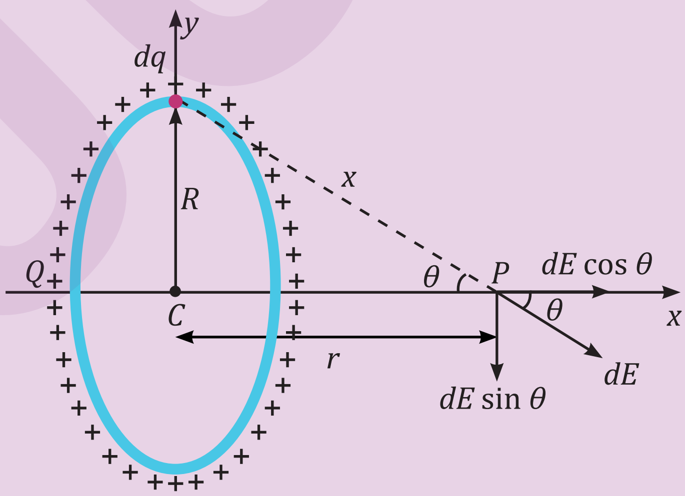
- Electric field due to:
  1. Uniformly charged ring
  2. Uniformly charged semicircular ring
  3. Uniformly charged quarter ring

BOARDS NEET

Uniformly Charged Ring

Electric field intensity at an axial point

Consider a circular ring of radius R. Charge Q is distributed uniformly along the circumference of the ring. To find the net electric field due to the uniformly charged ring at an axial point P which is at a distance r from the center of the ring, consider a small elemental charge dq on the ring. At point P, the electric field due to small elemental charge dq will be dE as shown in the figure. This field dE can be resolved into two components, one along the axis of the ring and the other one perpendicular to the axis of the ring. Thus, the component of the electric field along the x-axis is given by $dE \cos \theta$ and the component of the electric field along the y-axis is given by $dE \sin \theta$.



Let us consider another small elemental charge dq on the ring which is diametrically opposite to the small elemental charge dq considered earlier. Thus, at point P, the electric field due to small elemental charge dq will be dE and this electric field dE can be resolved into two components.The component along the x-axis is given by $dE \cos \theta$ and the component along the y-axis is given by $dE \sin \theta$.

We can observe that due to the diametrically opposite element charge, the components of the electric field along the y-axis cancel each other. Because of the circular symmetry of the ring, the net electric field along the y-axis is zero. Thus, the net electric field due to all the elemental charges will be along the axis of the ring only (x-axis).

The small elemental electric field dE due to small elemental charge dq at point P is given by,

$$dE = \frac{k\,dq}{x^2}$$

The net electric field along the x-axis is given by,

$$E_x = \int dE \cos\theta$$

$$\Rightarrow E_x = \int \frac{k\,dq}{x^2}\cos\theta$$

We know that for a ring about an axial point, k, x, and θ is same for every elements of the ring.

Therefore, $\dfrac{k\cos\theta}{x^2}$ is constant.

$$\Rightarrow E_x = \frac{k\cos\theta}{x^2}\int dq$$

$$\Rightarrow E_x = \frac{kQ\cos\theta}{x^2} \qquad ......(i)$$

From the figure on the previous page, we get,

$$\cos\theta = \frac{r}{x} = \frac{r}{\sqrt{r^2+R^2}}$$

By substituting the value of $\cos\theta$ in equation (i), we get,

$$E_x = \frac{kQ}{x^2}\frac{r}{x} = \frac{kQr}{x^3}$$

$$\Rightarrow E_x = \frac{kQr}{\left(r^2+R^2\right)^{\frac{3}{2}}}$$

Also, the electric field along y-axis is given by,

$$E_y = 0$$

The net electric field is given by,

$$\left|\vec{E}_{net}\right| = \sqrt{E_x^2 + E_y^2}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{kQr}{\left(r^2+R^2\right)^{\frac{3}{2}}} \qquad ......(ii)$$

Now, the linear charge density of the ring is given by,

$$\lambda = \frac{Q}{2\pi R}$$

$$\Rightarrow Q = 2\lambda\pi R$$

By substituting Q in equation (ii), we get,

$$\left|\vec{E}_{net}\right| = \frac{2\pi k\lambda Rr}{\left(r^2+R^2\right)^{\frac{3}{2}}}$$

Case 1: When $r >> R$

$r^2 + R^2 \approx r^2$

The net electric field is given by,

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{kQr}{\left(r^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{kQr}{r^3}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{kQ}{r^2}$$

The ring acts as a point charge when $r >> R$.

Case 2: When $r << R$

$r^2 + R^2 \approx R^2$

The net electric field is given by,

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{kQr}{\left(R^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{kQr}{R^3}$$

To get the location of the maximum value of the net electric field of the ring,

$$\frac{dE}{dr} = 0$$

$$\frac{d}{dr}\left[\frac{kQr}{\left(r^2 + R^2\right)^{\frac{3}{2}}}\right] = 0$$

$$\Rightarrow kQ\frac{d}{dr}\left[\frac{r}{\left(r^2 + R^2\right)^{\frac{3}{2}}}\right] = 0$$

Let $u = r$ and $v = \left(r^2 + R^2\right)^{\frac{3}{2}}$ and we know that,

$$\frac{d}{dr}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

Now,

$$u' = \frac{du}{dr} = \frac{dr}{dr} = 1$$

$$v' = \frac{dv}{dr} = \frac{d}{dr}\left(r^2 + R^2\right)^{\frac{3}{2}} = \frac{3}{2}\left(r^2 + R^2\right)^{\frac{1}{2}}(2r)$$

$$v^2 = \left(\left(r^2 + R^2\right)^{\frac{3}{2}}\right)^2 = \left(r^2 + R^2\right)^3$$

By substituting all the values, we get,

$$\frac{dE}{dr} = kQ\frac{d}{dr}\left[\frac{r}{\left(r^2 + R^2\right)^{\frac{3}{2}}}\right] = 0$$

$$\Rightarrow \frac{dE}{dr} = kQ\left[\frac{\left(r^2 + R^2\right)^{\frac{3}{2}}(1) - r\frac{3}{2}\left(r^2 + R^2\right)^{\frac{1}{2}}(2r)}{\left(r^2 + R^2\right)^3}\right] = 0$$

$$\Rightarrow \left(r^2+R^2\right)^{\frac{3}{2}} - 3r^2\left(r^2+R^2\right)^{\frac{1}{2}} = 0$$

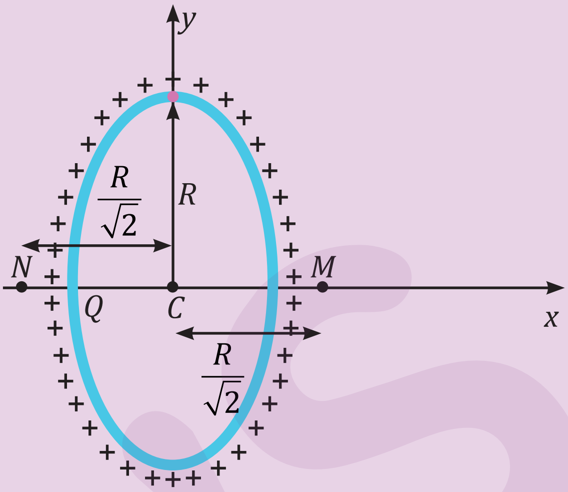
$$\Rightarrow \left(r^2+R^2\right)^1\left(r^2+R^2\right)^{\frac{1}{2}} = 3r^2\left(r^2+R^2\right)^{\frac{1}{2}}$$

$$\Rightarrow r^2+R^2 = 3r^2$$

$$\Rightarrow R^2 = 2r^2$$

$$\Rightarrow r = \pm\frac{R}{\sqrt{2}}$$

$\therefore E$ will be maximum when $r = \pm\frac{R}{\sqrt{2}}$. In the figure, M and N are these two points where the electric field due to the ring is maximum.

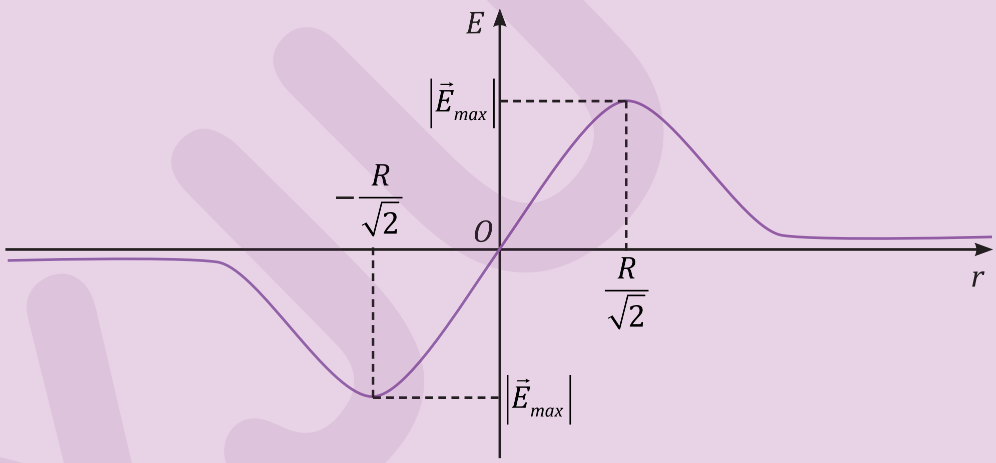


By substituting the magnitude of r in equation (ii), we get,

$$\left|\vec{E}_{max}\right| = \frac{kQ\left(\frac{R}{\sqrt{2}}\right)}{\left(\frac{R^2}{2}+R^2\right)^{\frac{3}{2}}}$$

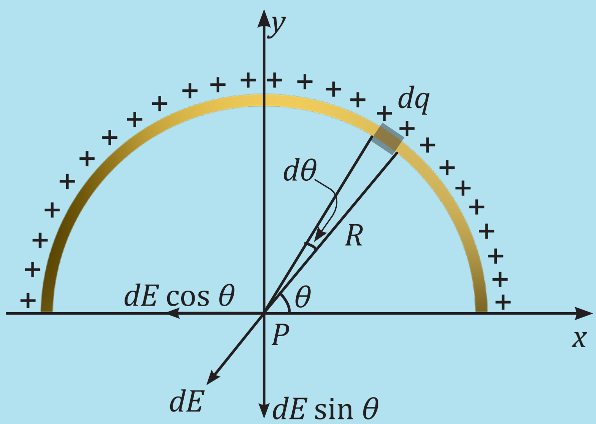
$$\Rightarrow \left|\vec{E}_{max}\right| = \frac{kQ\left(\frac{R}{\sqrt{2}}\right)}{\left(\frac{3\sqrt{3}}{2\sqrt{2}}R^3\right)}$$

$$\Rightarrow \left|\vec{E}_{max}\right| = \frac{2kQ}{3\sqrt{3}R^2}$$



Uniformly Charged Semicircular Ring

Consider a semicircular ring of radius R. Charge Q is uniformly distributed along the circumference of the semicircular ring. To find the net electric field due to the uniformly charged semicircular ring at its center, let us consider a small elemental charge dq on the ring that subtends an angle θ with the x-axis and the angular width of the elemental charge dq is $d\theta$. At the center, the electric field due to small elemental charge dq will be dE as shown in the figure. This field dE can be resolved into two components, the component of the electric field along the x-axis is given by $dE\cos\theta$ and the component of the electric field along the y-axis is given by $dE\sin\theta$.



Similarly, let's consider another elemental charge dq on the semicircular ring that resides at symmetrically opposite side to the small elemental charge dq considered earlier. Thus, at point P, the electric field due to the new small elemental charge dq will be dE and this electric field dE can also be resolved into two components along the x and y axes as $dE\cos\theta$ and $dE\sin\theta$, respectively.

Since for every elemental charge in the ring, there is a similar elemental charge in symmetrically opposite side of the ring. Hence, the net electric field along the x-axis is zero. Thus, the net electric field due to all the elemental charges of the semicircular ring will be along the y-axis only.

Let the small elemental charge of the semicircular ring be,

$dq = \lambda\, dx \quad ....(i)$

Here, dx is the small arc subtending angle $d\theta$.

$dx = R\, d\theta$

By substituting the value of dx in equation (i), we get,

$dq = \lambda R\, d\theta \quad ....(ii)$

Also, the linear charge density of the semicircular ring (λ) is given by,

$$\lambda = \frac{Q}{\pi R}$$

The small elemental electric field due to small elemental charge dq is given by,

$$dE = \frac{k\,dq}{R^2}$$

$$\Rightarrow dE = \frac{k\lambda R\, d\theta}{R^2} \quad (\because dq = \lambda R\, d\theta)$$

$$\Rightarrow dE = \frac{kQ}{\pi R^2} d\theta \quad \left(\because \lambda = \frac{Q}{\pi R}\right)$$

The net electric field along the x-axis is given by,

$$E_x = \int_0^{\pi} dE \cos\theta = 0$$

The net electric field along the y-axis is given by,

$$E_y = \int_0^{\pi} dE\ \sin\theta$$

$$\Rightarrow E_y = \frac{kQ}{\pi R^2}\int_0^{\pi} \sin\theta\, d\theta$$

$$\Rightarrow E_y = \frac{kQ}{\pi R^2}\left[-\cos\theta\right]_0^{\pi}$$

$$\Rightarrow E_y = \frac{kQ}{\pi R^2}\left[-\cos\pi + \cos 0\right]$$

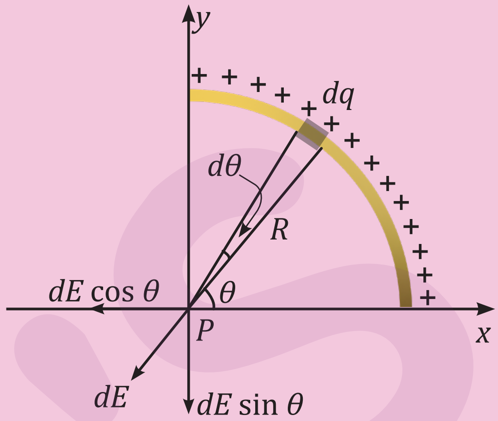
$$\Rightarrow E_y = \frac{2kQ}{\pi R^2}$$

The net electric field is given by,

$$\left|\vec{E}_{net}\right| = \left|\vec{E}_y\right| = \frac{2kQ}{\pi R^2} = \frac{2k\lambda}{R} \quad \left(\because \lambda = \frac{Q}{\pi R}\right)$$

Uniformly Charged Quarter Ring

Consider a quarter ring of radius R. Let us take a small charge dq at an angle θ from the x-axis. The angular width of the elemental ring is $d\theta$. Due to the elemental charge dq, there will be a small electric field dE at the centre of the ring and this field dE can be resolved into components along the x and y axes as $dE \cos\theta$ and $dE \sin\theta$, respectively.



Let the small elemental charge of the quarter ring be,

$dq = \lambda\, dx \quad ....(i)$

and, $dx = R\, d\theta$

$$\lambda = \frac{2Q}{\pi R}$$

The small electric field due to charge dq is given by,

$$dE = \frac{k\, dq}{R^2}$$

$$\Rightarrow dE = \frac{k2Q\, d\theta}{\pi R^2} \qquad \left(\because dq = \frac{2Q}{\pi} d\theta\right)$$

The net electric field along the x-axis is given by,

$$E_x = \int_0^{\frac{\pi}{2}} dE \cos\theta = \int_0^{\frac{\pi}{2}} \frac{2kQ\, d\theta}{\pi R^2} \cos\theta$$

$$\Rightarrow E_x = \frac{2kQ}{\pi R^2}[\sin\theta]_0^{\frac{\pi}{2}}$$

$$\Rightarrow E_x = \frac{2kQ}{\pi R^2}\left[\sin\frac{\pi}{2} - \sin 0\right]$$

$$\Rightarrow E_x = \frac{2kQ}{\pi R^2}$$

The net electric field along the y-axis is given by,

$$E_y = \int_0^{\frac{\pi}{2}} dE \sin\theta = \int_0^{\frac{\pi}{2}} \frac{2kQ\, d\theta}{\pi R^2} \sin\theta$$

$$\Rightarrow E_y = \frac{2kQ}{\pi R^2}[-\cos\theta]_0^{\frac{\pi}{2}}$$

$$\Rightarrow E_y = \frac{2kQ}{\pi R^2}\left[-\cos\frac{\pi}{2} + \cos 0\right]$$

$$\Rightarrow E_y = \frac{2kQ}{\pi R^2}$$

The net electric field is given by,

$$\vec{E}_{net} = E_x\left(-\hat{i}\right) + E_y\left(-\hat{j}\right)$$

$$\Rightarrow \vec{E}_{net} = \frac{2kQ}{\pi R^2}\left(-\hat{i}\right) + \frac{2kQ}{\pi R^2}\left(-\hat{j}\right)$$

$$\Rightarrow \vec{E}_{net} = -\frac{2kQ}{\pi R^2}\left(\hat{i}\right) - \frac{2kQ}{\pi R^2}\left(\hat{j}\right)$$

So,

$$\left|\vec{E}_{net}\right| = \sqrt{\left|\vec{E}_x\right|^2 + \left|\vec{E}_y\right|^2}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{2\sqrt{2}kQ}{\pi R^2} = \frac{\sqrt{2}k\lambda}{R}$$

PHYSICS

ELECTROSTATICS

MOTION OF CHARGED PARTICLE

What you already know

- Electric field due to a:
  1. Uniformly charged ring
  2. Uniformly charged semicircular ring
  3. Uniformly charged quarter ring

What you will learn

- Electric field due to a sector ring
- Motion of a charged particle in an external electric field

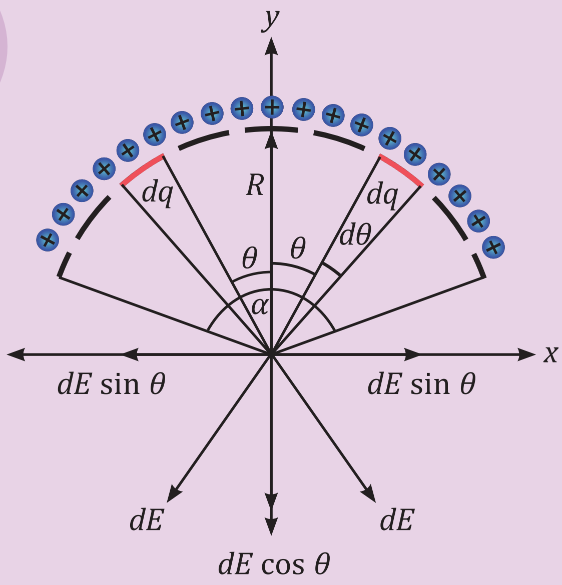
Uniformly Charged Ring Sector

Electric field at the centre of a sector ring

Consider a sector of a ring of radius R and charge Q is distributed uniformly along the circumference of the ring. The ring subtends an angle α at the centre P, and the y-axis divides the ring into two halves. To find the net electric field by a sector of a uniformly charged ring at its centre P, let us consider a small elemental charge dq of thickness dx at an angle θ from the vertical. The angular width of the small elemental charge is $d\theta$ and the electric field at point P due to the small elemental charge dq is dE as shown in the figure. The components of the electric field along the x-axis and y-axis are $dE \sin\theta$ and $dE \cos\theta$, respectively.

Similarly, consider another small elemental charge dq that is symmetrically opposite to the small elemental charge considered earlier. Thus, at point P, the electric field due to the small elemental charge dq is dE, and this electric field dE can be resolved into two components as shown in the figure.

We can observe that due to the symmetrically opposite charge element dq, the components of electric field along the x-axis of both the charges cancel each other. Similarly, for every element, there is a symmetrically opposite element. Thus, the net electric field due to all the elemental charges is along the y-axis of the ring.



The linear charge density of the ring is given by,

$$\lambda = \frac{Q}{L} = \frac{Q}{R\alpha}$$

The small charge is given by,

$$dq = \lambda\, dx$$

$$\Rightarrow dq = \frac{Q}{R\alpha} dx$$

$$\Rightarrow dq = \frac{Q}{R\alpha} R\, d\theta$$

The small elemental electric field dE due to small elemental charge dq at point P is given by,

$$dE = \frac{k\,dq}{R^2}$$

$$\Rightarrow dE = \frac{kQR\,d\theta}{LR^2}$$

$$\Rightarrow dE = \frac{kQ\,d\theta}{LR}$$

The component of the electric field along the x-axis is given by,

$$E_x = \int_{\frac{-\alpha}{2}}^{\frac{\alpha}{2}} dE \sin\theta = 0$$

The component of the electric field along the y-axis is given by,

$$E_y = \int_{\frac{-\alpha}{2}}^{\frac{\alpha}{2}} dE \cos\theta$$

$$\Rightarrow E_y = \frac{kQ}{LR}\int_{\frac{-\alpha}{2}}^{\frac{\alpha}{2}} \cos\theta\, d\theta$$

$$\Rightarrow E_y = \frac{kQ}{LR}[\sin\theta]_{\frac{-\alpha}{2}}^{\frac{\alpha}{2}}$$

$$\Rightarrow E_y = \frac{kQ}{LR}\left[\sin\frac{\alpha}{2} - \sin\frac{-\alpha}{2}\right]$$

$$\Rightarrow E_y = \frac{2kQ}{LR}\sin\frac{\alpha}{2}$$

The net electric field at the centre of the sector of the ring is given by,

$$\left|\vec{E}_{net}\right| = \left|\vec{E}_y\right| = \frac{2kQ}{LR}\sin\frac{\alpha}{2} = \frac{2k\lambda}{R}\sin\frac{\alpha}{2} \qquad .....(i)$$

Equation (*i*) is the general relation for finding the electric field at the centre of an arc. Using this result, we can find the net electric field at the centre of a quarter ring, semicircular ring, etc. by substituting the proper value of α.

Case 1: Electric field at the centre of a quarter ring

For a quarter ring, $\alpha = 90°$

The net electric field is given by,

$$\left|\vec{E}_{net}\right| = \frac{2k\lambda}{R}\sin\frac{\alpha}{2}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{2k\lambda}{R}\sin\frac{90°}{2} = \frac{2k\lambda}{R}\sin 45°$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{2k\lambda}{R}\times\frac{1}{\sqrt{2}} = \frac{\sqrt{2}k\lambda}{R}$$

Case 2: Electric field at the centre of a semicircular ring

For a semicircular ring, $\alpha = 180°$

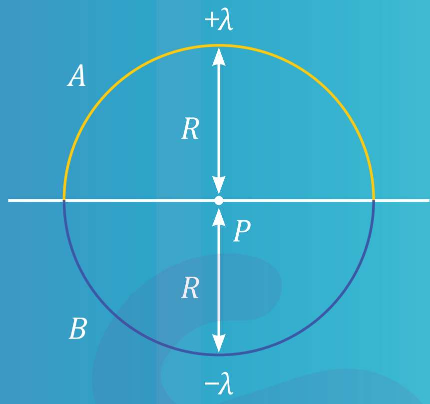
The net electric field is given by,

$$\left|\vec{E}_{net}\right| = \frac{2k\lambda}{R}\sin\frac{\alpha}{2}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{2k\lambda}{R}\sin\frac{180°}{2} = \frac{2k\lambda}{R}\sin 90°$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{2k\lambda}{R}\times 1 = \frac{2k\lambda}{R}$$

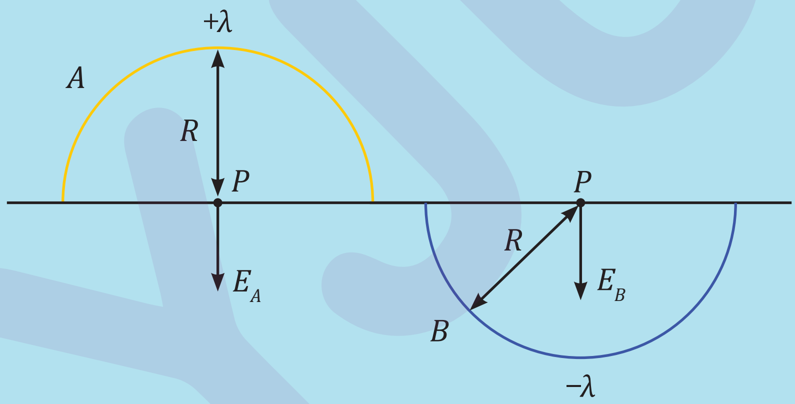
Find the electric field at the centre of the ring shown in the figure (where λ is the linear charge density).



(A) $\frac{4k\lambda}{R}$ (B) $\frac{2k\lambda}{R}$ (C) $\frac{k\lambda}{R}$ (D) Zero

Solution

The linear charge density of the upper ring is $+\lambda$ and that of the lower part is $-\lambda$. To solve this kind of a problem, let us divide the ring into two semicircular rings.



For the upper half of the ring, the electric field is acting away from the ring along vertically downward direction, and the magnitude of the electric field is given by,

$$E_A = \frac{2k\lambda}{R} \quad \text{[Since the ring is semicircular]}$$

For the lower half of the ring, the electric field is also acting vertically downwards as the ring has negative linear charge density, and the magnitude of the electric field is given by,

$$E_B = \frac{2k\lambda}{R}$$

The net electric field at point P is given by,

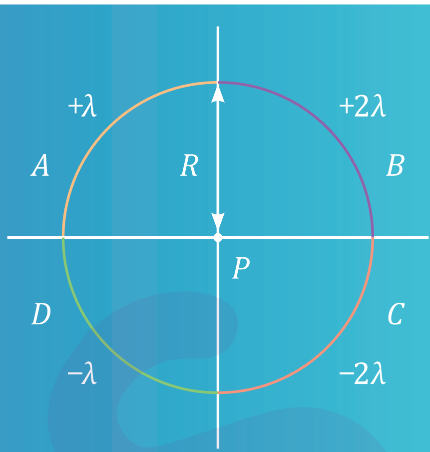
$$\left|\vec{E}_P\right| = \left|\vec{E}_A\right| + \left|\vec{E}_B\right|$$

$$\Rightarrow \left|\vec{E}_P\right| = \frac{2k\lambda}{R} + \frac{2k\lambda}{R}$$

$$\Rightarrow \left|\vec{E}_P\right| = \frac{4k\lambda}{R}$$

Thus, option (A) is the correct answer.

Find the electric field at the centre of the ring shown in the figure (where λ is the linear charge density).



(A) $\frac{k\lambda}{R}$ (B) $\frac{4k\lambda}{R}$ (C) $\frac{6k\lambda}{R}$ (D) Zero

Solution

For quadrant A, the electric field will be towards C (since the charge is positive in quadrant A), and the magnitude of the electric field is given by,

$$E_A = \frac{\sqrt{2}k\lambda}{R}$$

For quadrant B, the electric field is towards quadrant D (since the charge is positive in quadrant B), and the magnitude of the electric field is given by,

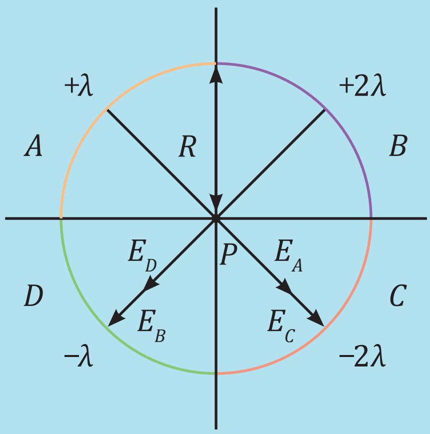
$$E_B = \frac{2\sqrt{2}k\lambda}{R}$$

For quadrant C, the electric field is towards C itself as it is negatively charged, and the magnitude of the electric field is given by,

$$E_C = \frac{2\sqrt{2}k\lambda}{R}$$

For quadrant D, the electric field is towards D itself, and the magnitude of the electric field is given by,

$$E_D = \frac{\sqrt{2}k\lambda}{R}$$



The net electric field along quadrant D is given by,

$$\left|\vec{E}_1\right| = \left|\vec{E}_B\right| + \left|\vec{E}_D\right|$$

$$\Rightarrow \left|\vec{E}_1\right| = \frac{3\sqrt{2}k\lambda}{R}$$

The net electric field along quadrant C is given by,

$$\left|\vec{E}_2\right| = \left|\vec{E}_A\right| + \left|\vec{E}_C\right|$$

$$\Rightarrow \left|\vec{E}_2\right| = \frac{3\sqrt{2}k\lambda}{R}$$

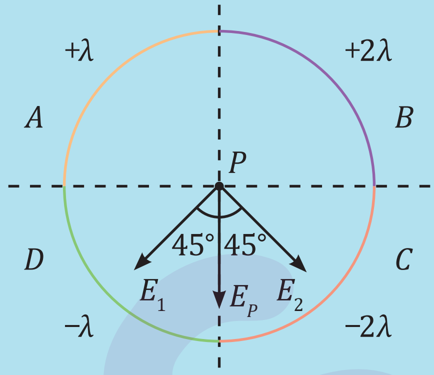
The net electric field is given by,

$$\left|\vec{E}_P\right| = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 90°}$$

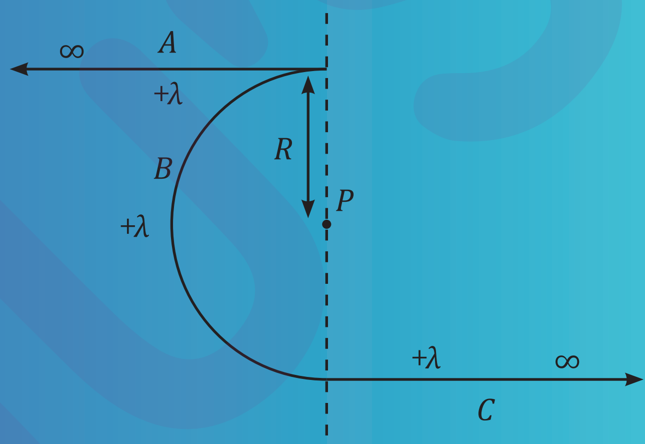
$$\Rightarrow \left|\vec{E}_P\right| = \sqrt{\left(\frac{3\sqrt{2}k\lambda}{R}\right)^2 + \left(\frac{3\sqrt{2}k\lambda}{R}\right)^2} = \sqrt{2}\left(\frac{3\sqrt{2}k\lambda}{R}\right)$$

$$\Rightarrow \left|\vec{E}_P\right| = \frac{6k\lambda}{R}$$

Thus, option (C) is the correct answer.



Find the electric field due to the combination at point P (where λ is the linear charge density).



(A) $\dfrac{4k\lambda}{R}$ (B) $\dfrac{2k\lambda}{R}$ (C) $\dfrac{6k\lambda}{R}$ (D) Zero

Solution

From the figure, it is clear that the given system is a combination of two semi-infinite wires and a semicircular ring. First, let us take the two semi-infinite wires. Due to wire A, the electric field at point P will have two components, one parallel to the rod and the other one perpendicular to the rod. Recall that the angle subtended by a semi infinite wire at the field point is given by,

$\alpha = 0°$ and $\beta = 90°$

The parallel component is given by,

$$E_{\parallel} = \frac{kQ}{Lr}[\cos\beta - \cos\alpha] = -\frac{kQ}{Lr} = -\frac{k\lambda}{r}$$

The perpendicular component is given by,

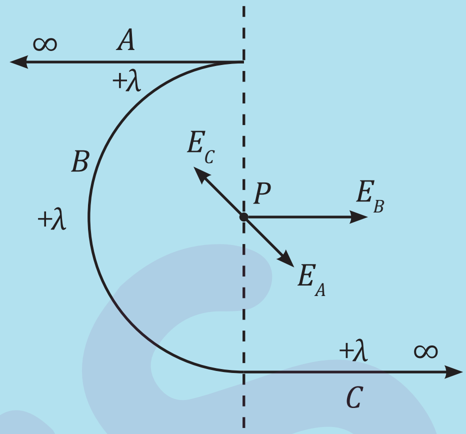
$$E_{\perp} = \frac{kQ}{Lr}[\sin\alpha + \sin\beta] = \frac{kQ}{Lr} = \frac{k\lambda}{r}$$

The net electric field is given by,

$$\left|\vec{E}_A\right| = \sqrt{E_{\parallel}^2 + E_{\perp}^2} = \frac{\sqrt{2}k\lambda}{r}$$

Similarly, due to wire C, the net electric field is given by,

$$\left|\vec{E}_C\right| = \sqrt{E_{\parallel}^2 + E_{\perp}^2} = \frac{\sqrt{2}k\lambda}{r}$$



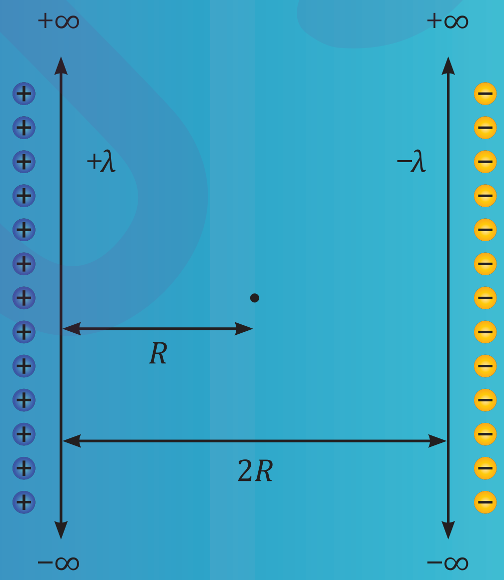
As the net electric fields due to both the rods are in the opposite directions, they cancel each other. The net electric field in this configuration is only due to the semicircular ring.

The net electric field due to the semicircular ring at its centre is given by, $\left|\vec{E}_B\right| = \dfrac{2k\lambda}{R}$, and it is directed away from the ring.

This is the net electric field due to the whole configuration.

Thus, option (B) is the correct answer.

Two parallel infinite line charges with linear charge densities $+\lambda$ and $-\lambda$ are placed at a distance of $2R$ in free space. What is the electric field midway between the two line charges?



(A) $\dfrac{\lambda}{\pi\varepsilon_0 R}$ (B) $\dfrac{\lambda}{2\pi\varepsilon_0 R}$ (C) Zero (D) $\dfrac{2\lambda}{\pi\varepsilon_0 R}$

Solution NEET

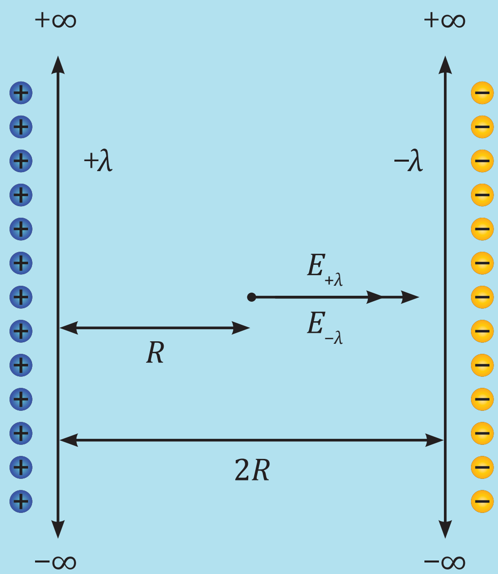
The electric field due to both the infinite line charges is along the same direction as they are oppositely charged.

The electric field due to wire having line charge density $+\lambda$ is given by,

$$E_{+\lambda} = \frac{2k\lambda}{R}$$

The electric field due to wire having line charge density $-\lambda$ is given by,

$$E_{-\lambda} = \frac{2k\lambda}{R}$$



The net electric field is given by,

$$\left|\vec{E}_{net}\right| = E_{+\lambda} + E_{-\lambda} = \frac{4k\lambda}{R}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{4\lambda}{4\pi\varepsilon_0 R} \qquad \left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$$

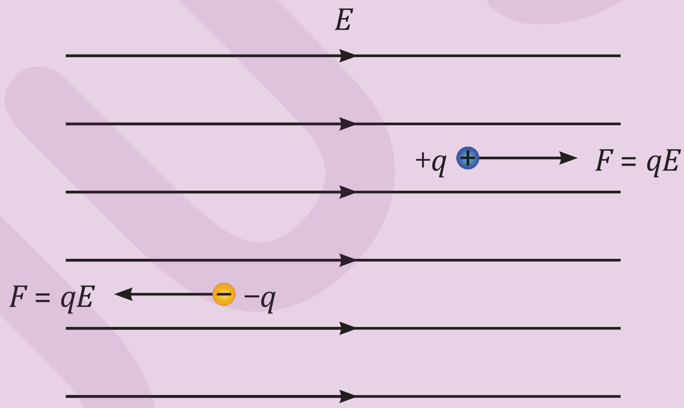
$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{\lambda}{\pi\varepsilon_0 R}$$

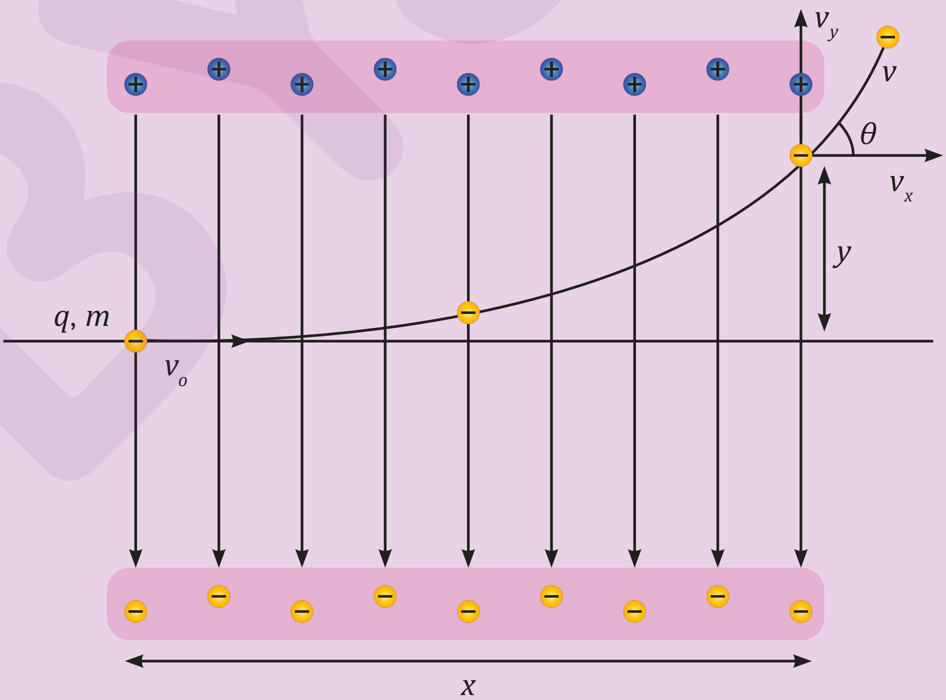
Thus, option (A) is the correct answer.

Motion of a Charged Particle

Consider a uniform electric field E in space (gravity-free space). Consider that two charges ($+q$ and $-q$) are placed in the electric field. The positively charged particle ($+q$) will experience an electrostatic force in the direction of the electric field, and the negatively charged particle ($-q$) will experience an electrostatic force opposite to the direction of the electric field as shown in the figure.

The magnitude of the electrostatic force is, $F = qE$.





To create a uniform electric field, let us take two oppositely charged plates of length x and place them parallelly with a small separation between them. One plate is positively charged and the other is negatively charged.

A constant electric field is generated in the region between the two plates and it is directed from the positively charged plate to the negatively charged plate. Now, let us consider that a charged particle of mass m having charge $-q$ enters perpendicularly into the uniform electric field with a velocity v_o as shown. As we know, when a charged particle moves through an electric field, it experiences an electrostatic force. Thus, charge $-q$ experiences an electrostatic force in the upward direction (Since unlike charges attract and like charges repel). Therefore, charge $-q$ gets deviated from its original path and let at a certain instant, the deviation is y. Let us assume that the velocity of the particle just before leaving the region of the uniform electric field is v.

Along the x-axis, the force acting on the particle is 0. Hence, the acceleration along the x-axis is 0.

$F_x = 0$ and $a_x = 0$

Force along the y-axis is, $F_y = qE$.

The acceleration along the y-axis is, $a_y = \frac{qE}{m}$.

Therefore, the net acceleration of the particle is given by, $a_{net} = a_y = \frac{qE}{m}$.

Along the x-axis,

$u_x = v_o$

$S_x = x$

Hence, by applying the second equation of motion in $1D$, we get,

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow x = v_o t$$

$$\Rightarrow t = \frac{x}{v_o} \quad \cdots (i)$$

Where, t is the time taken by the charge to travel through the electric field.

Along the y-axis,

$u_y = 0$

$S_y = y$

Again, by applying the second equation of motion in $1D$, we get,

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow y = \frac{1}{2} \frac{qE}{m} t^2$$

$$\Rightarrow y = \frac{1}{2} \frac{qE}{m} \frac{x^2}{v_o^2} \quad \left[\text{From equation } (i)\right]$$

This is the net deviation of the charged particle after passing through the electric field.

Velocity of the charged particle

The velocity of the particle along the x-axis is given by,

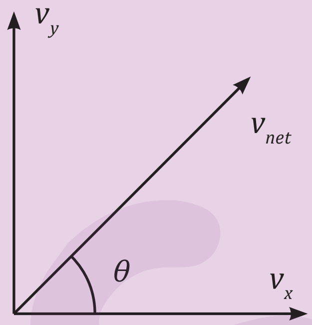
$v_x = u_x + a_x t$

$\Rightarrow v_x = v_o$

The velocity of the particle along the y-axis is given by,

$v_y = u_y + a_y t$

$$\Rightarrow v_y = \frac{qE}{m}\frac{x}{v_o}$$



The net velocity of the particle can be written as,

$$\vec{v}_{net} = v_x\hat{i} + v_y\hat{j}$$

$$\Rightarrow \vec{v}_{net} = v_o\hat{i} + \frac{qEx}{mv_o}\hat{j}$$

The angle subtended by the net velocity v_{net} is given by,

$$\tan\theta = \frac{v_y}{v_x} = \frac{\left(\frac{qE}{m}\frac{x}{v_o}\right)}{v_o} = \frac{qEx}{mv_o^2}$$

An electron falls from rest through a vertical distance h in a uniform and vertically upward directed electric field E. The direction of the electric field is now reversed, keeping its magnitude the same. A proton is allowed to fall from rest in it through the same vertical distance h. Find the time of fall of the electron in comparison to the time of fall of the proton.

(A) Smaller (B) 5 times greater (C) 10 times greater (D) Equal

Solution NEET

In the first case, the electron falls a distance of h and the initial velocity of the electron is 0.
Force acting on the electron is given by, $F = qE$.
The acceleration of the particle is given by, $a = \frac{qE}{m_e}$.

The time taken by the electron to fall is given by,

$$h = ut + \frac{1}{2}at_e^2$$

$$\Rightarrow t_e = \sqrt{\frac{2h}{a}}$$

$$\Rightarrow t_e = \sqrt{\frac{2hm_e}{qE}}$$

Where, t_e is the time taken by the electron to travel h distance.
For the second case, the proton travels a distance of h under the influence of the electric field. The time taken by the proton to travel distance h is given by,

$$t_p = \sqrt{\frac{2hm_p}{qE}}$$

We know that,
$m_p > m_e$
Time of fall of electron (t_e) < Time of fall of the proton (t_p)

Thus, option (A) is the correct answer.

PHYSICS

ELECTROSTATICS

ELECTRIC FIELD LINES AND THEIR PROPERTIES

What you already know

- Electric field intensity at the centre of ring sector
- Charged particle in an electric field and its parameters like acceleration, displacement, velocity, and angle of deflection

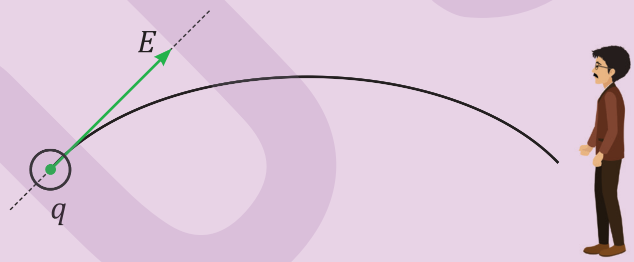
What you will learn

- Electric field lines and their representation
- Electric dipole
- Electric field at an axial point on a line joining a dipole

BOARDS

Electric Field Lines

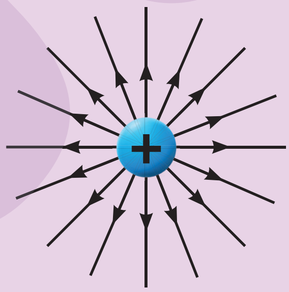
Michael Faraday first developed the idea of visualising electric field lines. They are also known as electric lines of force (ELOF), and they are imaginary lines.



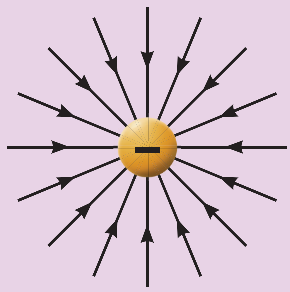
For an electric charge q, the electric field at a distance r is given by,

$$E = \frac{kq}{r^2}$$

For a positive charge, the electric field lines will be radially outwards (just like a bulb emits light). Whereas, for a negative charge, the electric field lines will be radially inwards.

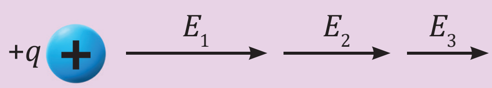


Radially outwards



Radially inwards

The following diagram shows the intensity of electric field lines of a positive charge; as the distance increases, the intensity of the electric field decreases (In the diagram the length of the arrow suggests the intensity of the field).

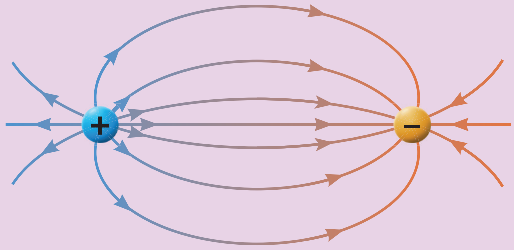


$E_1 > E_2 > E_3$

Properties of electric field lines

- Electric lines of forces originate at a positive charge and terminate at a negative charge.

- If two charges are close to each other, then the electric field lines will be as shown in the figure.



- For an isolated charge, the electric field lines terminate at infinity (∞).
- A tangent at any point on the electric line of force gives the direction of the net electric field. It is shown in the following diagram (Fig. 1) more precisely.
  So, if we take tangents at various points along an electric field line, they denote the direction of the electric field at each of those points, as shown in the figure (Fig. 2).

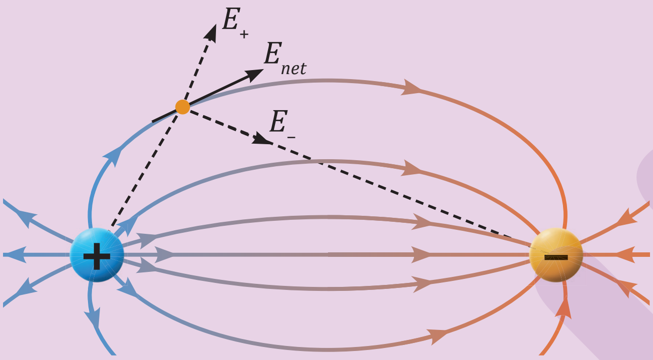


Fig. 1

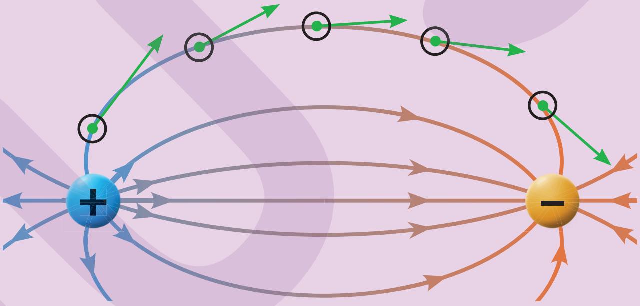
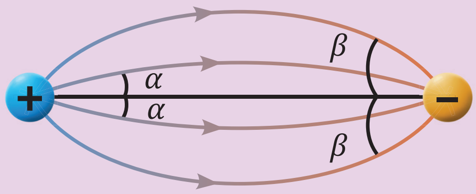
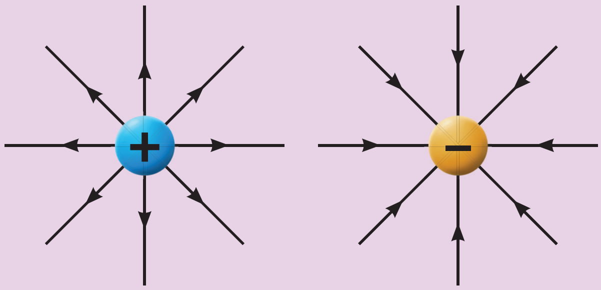


Fig. 2

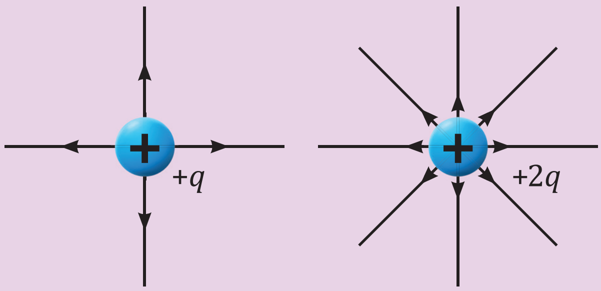
- Electric field lines do not form a closed loop. The reason is that the points of origination and termination should be different, as an electric field line originates from a positive charge and ends at a negative charge.
- For a system of two charges having equal magnitude and opposite nature, electric field lines are always symmetric about the line joining the two charges.



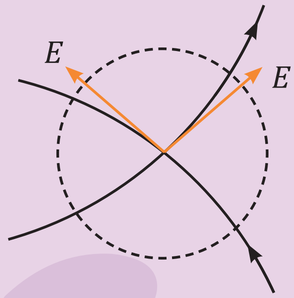
- Electric field lines are always perpendicular to the surface of the charged body.



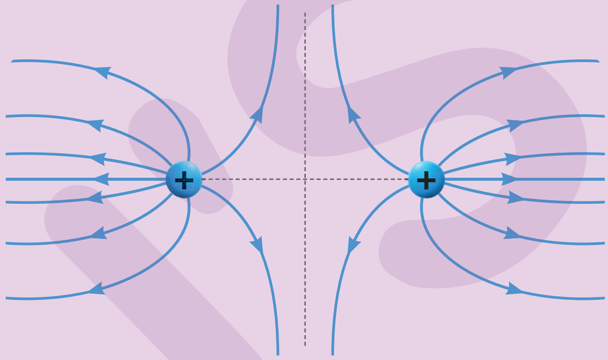
- For an individual charge, the number of electric lines of force is an independent choice. It means that for a given magnitude of the charge, the electric lines of forces can vary.
- The magnitude of the charge is directly proportional to the number of electric lines of force originating or terminating at the charge. It means that if for a '$+q$' charge, we take four lines, then for a '$+2q$' charge, the lines of forces should be double, i.e., eight.



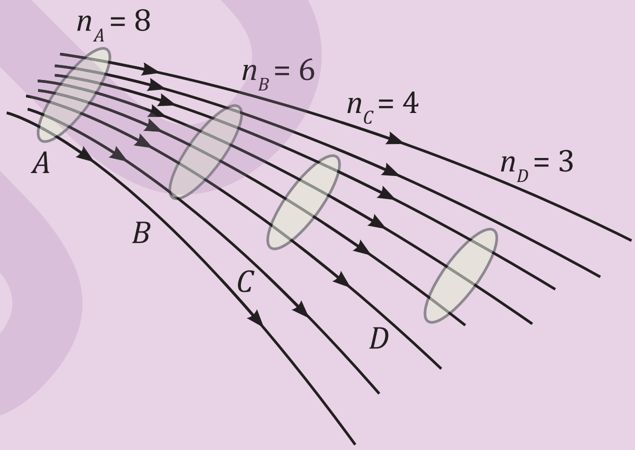
- No two electric field lines intersect, because the electric field is a vector quantity and it cannot have two different directions at the same point.



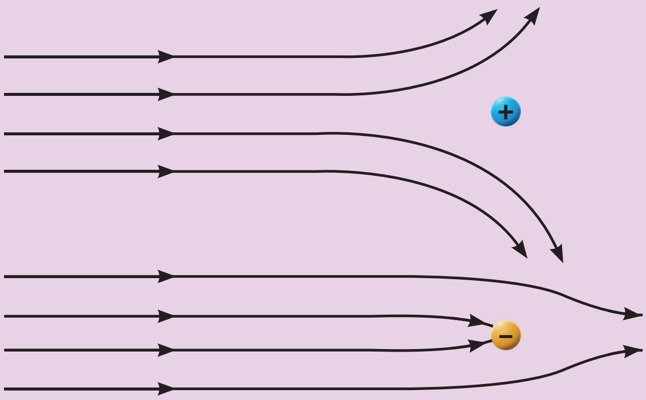
- The electric lines of force for two positive charges are pictorially depicted in the figure,



- The strength of the electric field (or electric field intensity) is represented as electric lines of force per unit area. For the given diagram, four equal surfaces A, B, C and D are considered, and the electric lines of force passing through them are also shown in the figure. The descending order of the number of electric lines of force passing through the surfaces (line density) is $A > B > C > D$. Thus, the the descending order of the electric field intensity is $E_A > E_B > E_C > E_D$. In other words, denser the electric lines of force, higher is the electric field intensity. Similarly, rarer the electric lines of force, lesser is the electric field intensity.

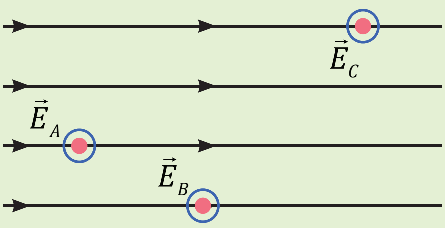


- The electric lines of force deviate from a positive charge and converge towards a negative charge.



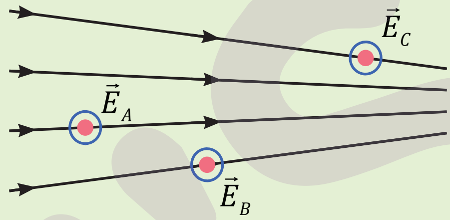
Uniform and Non-Uniform Electric Field

For a uniform electric field, all the electric field lines are equi-spaced and parallel to each other. This means that the intensity of the electric field at every point in the region is the same. While in a non-uniform electric field, the electric field lines are not equally spaced, i.e., they are converging or diverging, which means that the intensity of the electric field at each point in the region is not the same.



$$|\vec{E}_A| = |\vec{E}_B| = |\vec{E}_C|$$

Uniform

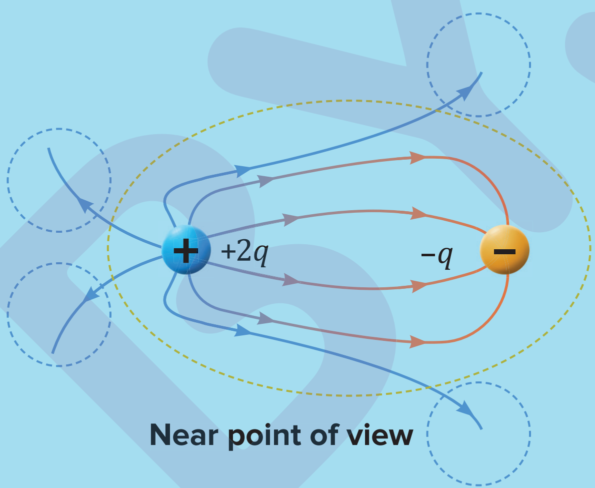


$$|\vec{E}_A| < |\vec{E}_B| < |\vec{E}_C|$$

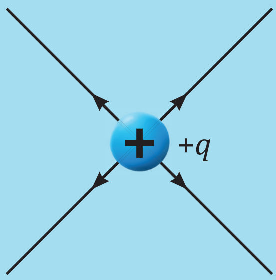
Non-Uniform

Far and Near Fields

As we know that the number of electric field lines emerges or terminates from a point charge is directly proportional to the magnitude of the charge, i.e., for the given system of $+2q$ and $-q$ charges, if there are eight electric lines of forces emerging from $+2q$ charge, then, four electric lines of force terminate at the $-q$ charge. For a near point of view, the electric field lines for the combination of $+2q$ and $-q$ charges look as given in the figure.



Near point of view

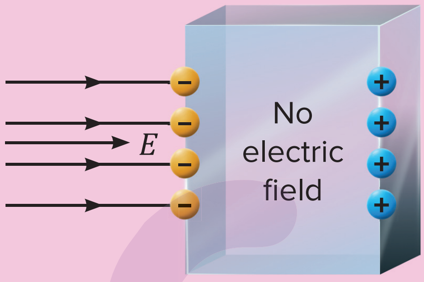


Far point of view

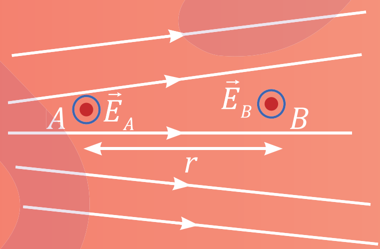
For a far point of view, the charges $+2q$ and $-q$ look like a point charge and its magnitude is the sum of the two charges, i.e., from the $+q$ charge, four electric field lines should emerge and they look as shown in the figure.

Electrostatic Shielding

Electric field lines of force never enter inside a conductor, i.e., when a conductor is placed in a uniform electric field, the free charges in the conductor move to the outer surface (net charge inside the conductor is zero), which ensures that the net electric field inside the conductor is zero. Whereas in an insulator, the charges cannot move freely. If the net charge density inside an insulator is non-zero, then the net electric field inside the insulator is non-zero. Thus, the electric field lines do exist inside an insulator. The given diagram explains how electrostatic shielding works in a conductor, i.e., how the charges inside a conductor oppose the external electric field.



The figure shows the electric lines of force emerging from a charged body. If the electric fields at points A and B are $\vec{E}_A$ and $\vec{E}_B$, respectively, and the separation between A and B is r, then which of the following is correct?



(A) $\left|\vec{E}_A\right| = \dfrac{\left|\vec{E}_B\right|}{r}$ (B) $\left|\vec{E}_A\right| = \dfrac{\left|\vec{E}_B\right|}{r^2}$ (C) $\left|\vec{E}_B\right| > \left|\vec{E}_A\right|$ (D) $\left|\vec{E}_A\right| > \left|\vec{E}_B\right|$

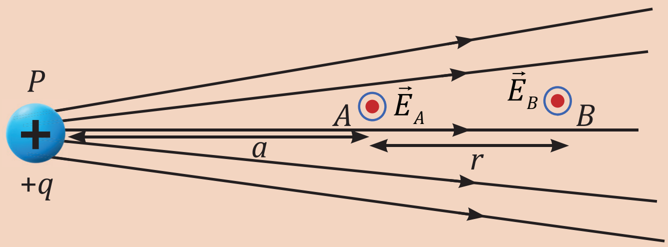
Solution

Let P be the charged body of charge q from which the electric field lines are emerging. Then,

$\left|\vec{E}_A\right| = \dfrac{kq}{a^2}$ and $\left|\vec{E}_B\right| = \dfrac{kq}{(a+r)^2}$

Therefore, $\dfrac{\left|\vec{E}_A\right|}{\left|\vec{E}_B\right|} = \dfrac{(a+r)^2}{a^2}$

$\Rightarrow \left|\vec{E}_A\right| > \left|\vec{E}_B\right|$



Thus, option (D) is the correct answer.

If electric field is uniform, then the electric lines of force are:

(A) Divergent (B) Convergent (C) Circular (D) Parallel

Solution

For a uniform electric field, the lines of forces are always equi-spaced and parallel to each other.

Thus, option (D) is the correct answer.

If the number of electric lines of force emerging from the charge $+4q$ are 28, then find out the number of electric lines of force emerging from the charge $+q$.

(A) 4 (B) 7 (C) 14 (D) 28

Solution

We know that the number of electric lines of force is proportional to the magnitude of the charge

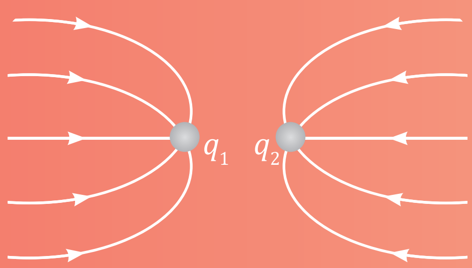
$$\Rightarrow \frac{q_1}{q_2} = \frac{x_1}{x_2}$$

$$\Rightarrow \frac{4q}{q} = \frac{28}{x_2}$$

Therefore, $x_2 = 7$

Thus, option (B) is the correct answer.

The given figure gives electric lines of force due to two charges, q_1 and q_2. What are the signs of the two charges?



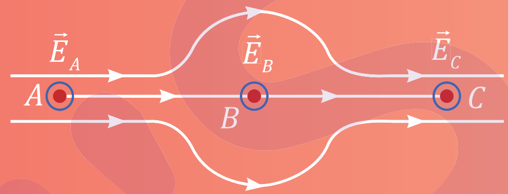
(A) $q_1 = +ve,\ q_2 = -ve$ (B) $q_1 = -ve,\ q_2 = +ve$ (C) $q_1 = -ve,\ q_2 = -ve$ (D) $q_1 = +ve,\ q_2 = +ve$

Solution

From the given figure, we can see the direction of the electric field lines that are converging at both the charges. Therefore, it is clear that both the charges carry a negative charge.

Thus, option (C) is the correct answer.

Determine the relation between the magnitude of the electric field at points A, B, and C.



(A) $|\vec{E}_A| = |\vec{E}_B| = |\vec{E}_C|$ (B) $|\vec{E}_A| \neq |\vec{E}_B| \neq |\vec{E}_C|$ (C) $|\vec{E}_A| = |\vec{E}_C| \neq |\vec{E}_B|$ (D) $|\vec{E}_A| \neq |\vec{E}_B| = |\vec{E}_C|$

Solution

We know that the closer the electric field lines (line density is high), the stronger will be the electric field. At points A and C, the electric field strength is equal in magnitude, while at point B, the field lines are far from each other, i.e., the line density at point B is less.

Thus, option (C) is the correct answer.

Electric Dipole

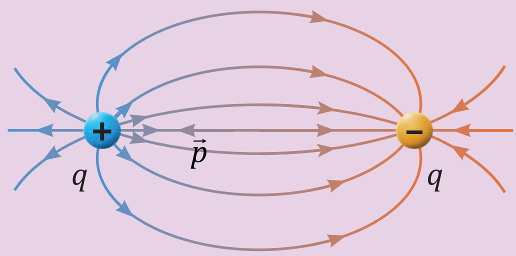
An electric dipole is a system consisting of two point charges that are equal in magnitude but opposite in nature, separated by an **infinitesimally small** distance.

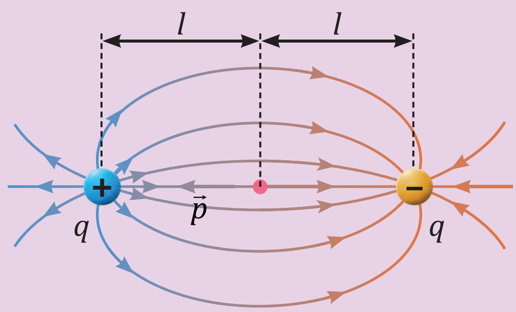
The centre of a dipole is considered to be the origin for all the measurements. Both the charges are at equal distance 'l' from the centre.

The dipole moment p is a vector and it is given by,

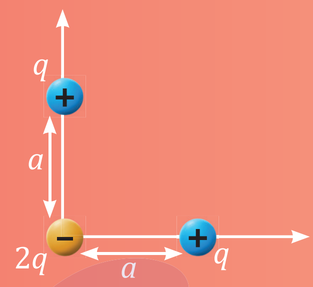
$\vec{p} = q2\vec{l}$

The direction of the dipole moment vector will be from $-q$ to $+q$. Thus, we define the dipole moment as the magnitude of the charge times the separation between the two charges. **The SI unit of the dipole moment vector is *Cm*.**





Find the magnitude of the net dipole moment for the given system.



(A) $\sqrt{2}qa$ (B) qa (C) $2qa$ (D) $\sqrt{3}qa$

Solution NEET

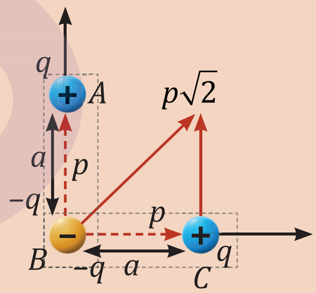
Suppose that the charge $-2q$ is made up of two charges of magnitude $-q$ as shown in the figure. Therefore, there will be two dipoles: one made up with $-q$ at B and q at A, and the other made up with $-q$ at B and q at C.

For the combination of two dipoles, the net dipole moment is given by,

$$|\vec{p}_{net}| = \sqrt{|\vec{p}|^2 + |\vec{p}|^2}$$

$$\Rightarrow |\vec{p}_{net}| = \sqrt{2}|\vec{p}| = \sqrt{2}\,qa$$

Thus, option (A) is the correct answer.

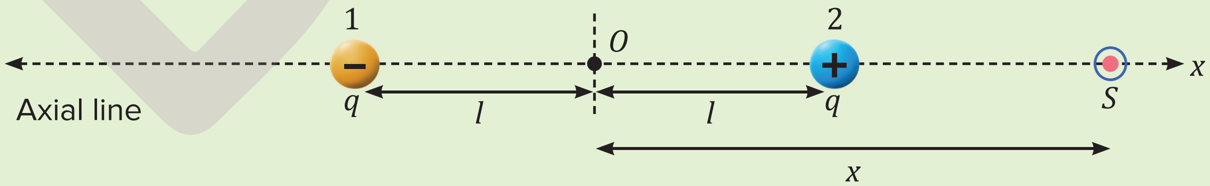


BOARDS

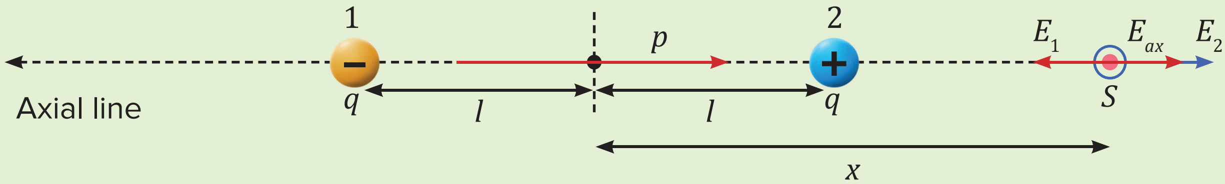
Electric Field Due to Dipole

Electric field at an axial point

Consider that an electric dipole is placed in such a way that its midpoint is at the origin as shown in the figure. The length of the dipole is $2l$, and the magnitude of the charges is q. Therefore, the magnitude of the dipole moment is, $p = 2ql$ and the direction of the dipole moment vector is along the positive x-axis (from negative charge to positive charge). Suppose that we want to find the electric field at point S, which is x distance away from the origin.



The following diagram shows the electric field direction at point S due to charges 1 and 2.



$$\left|\vec{E}_1\right| = \frac{kq}{(x+l)^2}, \left|\vec{E}_2\right| = \frac{kq}{(x-l)^2}$$

$$\left|E_{ax}\right| = \left|\vec{E}_2\right| - \left|\vec{E}_1\right|$$

$$\Rightarrow \left|E_{ax}\right| = \frac{kq}{(x-l)^2} - \frac{kq}{(x+l)^2} = kq\left[\frac{2xl+2xl}{\left(x^2-l^2\right)^2}\right]$$

Therefore, $\boxed{\left|E_{ax}\right| = \frac{2k(q2l)x}{\left(x^2-l^2\right)^2} = \frac{2k\left|\vec{p}\right|x}{\left(x^2-l^2\right)^2}}$ $\left[\text{Since, } \vec{p} = q2\vec{l}\right]$

For $x >> l$,

$$E_{ax} = \frac{2kpx}{x^4} = \frac{2kp}{x^3}$$

In vector notation,

$$\boxed{\vec{E}_{ax} = \frac{2k\vec{p}}{x^3}}$$

ELECTRIC FIELD DUE TO DIPOLE

What you already know

- Electric field lines and their representation
- Electric dipole
- Electric field at an axial point on a line joining a dipole

What you will learn

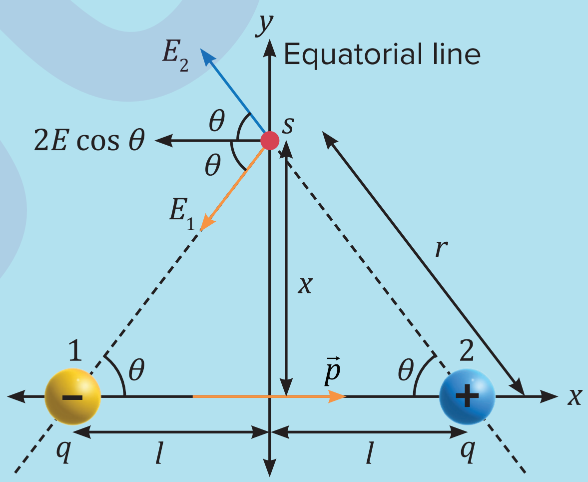
- Electric field at an equatorial point of a dipole
- Electric field at any point due to a dipole
- Dipole in a uniform electric field

Electric Field at an Equatorial point of a Dipole

An **equatorial line** is the perpendicular bisector of the axial line joining the charges of a dipole. Any point on the equatorial line is known as an **equatorial point**.

Consider an equatorial point s which is at distance x from the axis of the dipole. Let the length of the dipole be $2l$.

The electric field (E_1) due to charge $-q$ at point s is towards itself and the electric field (E_2) due to charge $+q$ at point s is away from itself as shown in the figure. Since the point s is on the equatorial line, let us assume the distance of point s from both the charges to be r.



By using Pythagoras's theorem on any of the right-angled triangles formed, we get the following:

$r = \sqrt{x^2 + l^2}$ ...(i)

Since the charges of the dipole are same in magnitude and equidistant from the equatorial point, the magnitude of the electric field of both the charges will also be the same.

Therefore,

$$\left|\vec{E}_1\right| = \left|\vec{E}_2\right| = \left|\vec{E}\right|$$

Let us divide the electric field vectors into their components along the x-axis and y-axis.

The net electric field along the x-axis is given by,

$$\left|\vec{E}_x\right| = E\cos\theta + E\cos\theta = 2E\cos\theta$$

Along the y-axis, the components of the electric field cancel each other. Therefore,

$$\left|\vec{E}_y\right| = 0$$

The net electric field due to the dipole on the equatorial point is given as follows:

$$\left|\vec{E}_{eq}\right| = \left|\vec{E}_x\right| = 2E\cos\theta \qquad ...(ii)$$

The magnitude of the electric field due to each charge of dipole at the equatorial point is given by,

$$\left|\vec{E}\right| = \frac{kq}{r^2}$$

By substituting the value of E in equation (ii), we get,

$$\left|\vec{E}_{eq}\right| = 2\frac{kq}{r^2}\cos\theta$$

$$\Rightarrow \left|\vec{E}_{eq}\right| = 2\frac{kq}{r^2}\times\frac{l}{r} \qquad \left(\because \cos\theta = \frac{l}{r}\right)$$

$$\Rightarrow \left|\vec{E}_{eq}\right| = \frac{k\left|\vec{p}\right|}{r^3} \qquad \left(\because \left|\vec{p}\right| = 2ql\right)......(iii)$$

By substituting the value of r in equation (iii), we get,

$$\Rightarrow \left|\vec{E}_{eq}\right| = \frac{k\left|\vec{p}\right|}{\left(x^2+l^2\right)^{\frac{3}{2}}}$$

If $x >> l$:

For any equatorial point which is far away from dipole, l becomes much smaller than x, and hence, l can be neglected. Therefore, the equation of the net electric field changes, which is given as follows:

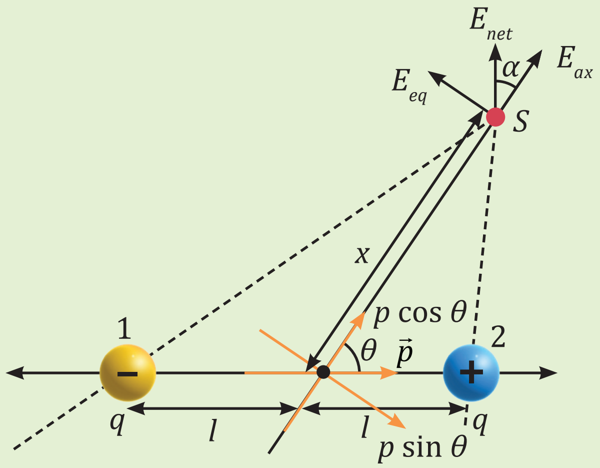
$$\left|\vec{E}_{eq}\right| = \frac{k\left|\vec{p}\right|}{\left(x^2+l^2\right)^{\frac{3}{2}}} = \frac{k\left|\vec{p}\right|}{x^3}$$

In this case, the dipole moment is in the opposite direction of the net electric field. So, the equation of the electric field at a far equatorial point can be written as follows:

$$\vec{E}_{eq} = \frac{-k\vec{p}}{x^3}$$

Electric Field at Any General Point in Space Due to a Dipole

Consider point S to be at a distance of x from the centre of the dipole, which subtends an angle θ with the dipole axis at the centre of the dipole. In this case, we can observe that point S is neither an equatorial point nor an axial point of the dipole. But we know the value of the electric field at an axial point and an equatorial point of a dipole. Now, we resolve the dipole moment vector into two components, one is along the line joining the centre of the dipole and field point S and the other one is perpendicular to the line joining the centre of the dipole and the field point S.



Since the dipole moment vector is resolved into its components, the components can be taken as two independent dipole moments, $p\cos\theta$ and $p\sin\theta$, respectively. Also, the field point S is an axial point for the dipole moment $p\cos\theta$ and an equatorial point for the dipole moment $p\sin\theta$.

Therefore, there are two electric fields acting on point S: one is the axial field and the other one is the equatorial field.

The electric field due to $p\cos\theta$ at an axial point is given by,

$$\left|\vec{E}_{ax}\right| = \frac{2kp\cos\theta}{x^3}$$

The electric field due to $p\sin\theta$ at an equatorial point is given by,

$$\left|\vec{E}_{eq}\right| = \frac{kp\sin\theta}{x^3}$$

The net electric field is given by,

$$\left|\vec{E}_{net}\right| = \sqrt{\left|\vec{E}_{ax}\right|^2 + \left|\vec{E}_{eq}\right|^2}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \sqrt{\left(\frac{2kp\cos\theta}{x^3}\right)^2 + \left(\frac{kp\sin\theta}{x^3}\right)^2}$$

$$\Rightarrow \left|\vec{E}_{net}\right| = \frac{kp}{x^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\boxed{\Rightarrow \left|\vec{E}_{net}\right| = \frac{kp}{x^3}\sqrt{1 + 3\cos^2\theta}} \quad \left(\because \cos^2\theta + \sin^2\theta = 1\right)$$

The angle subtended by the net electric field is given by,

$$\tan\alpha = \frac{\left|\vec{E}_{eq}\right|}{\left|\vec{E}_{ax}\right|}$$

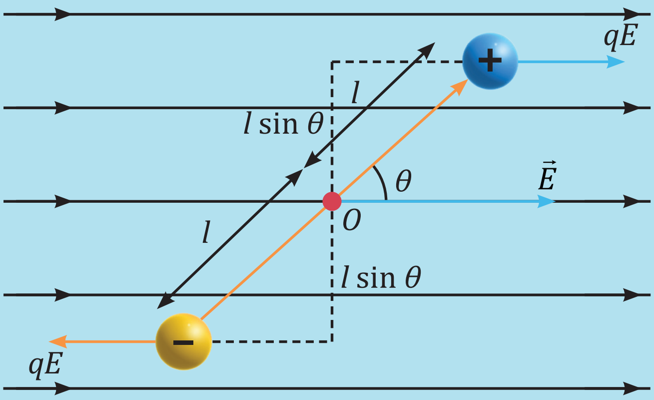
$$\Rightarrow \tan\alpha = \frac{\sin\theta}{2\cos\theta}$$

$$\boxed{\Rightarrow \tan\alpha = \frac{\tan\theta}{2}}$$

Dipole in a Uniform Electric Field

Consider a dipole placed in a uniform electric field. The dipole moment subtends an angle of θ with the electric field. The charges of the dipole experience a force equal in magnitude and opposite in direction due to the electric field as shown in the diagram.

Although the net force on the dipole is zero, since there are two equal and opposite forces with different lines of action, they form a couple. The couple generates a torque about the centre (O) of the dipole, which gives the tendency to rotate in the electric field until the dipole comes to an equilibrium state.



The net torque acting on the dipole is given by,

$\left|\vec{\tau}_{net}\right| = \left|\vec{\tau}_{+q} + \vec{\tau}_{-q}\right|$

The torque due to charge $+q$ is given by,

$\left|\vec{\tau}_{+q}\right| = qEl \sin\theta$

The torque due to charge $-q$ is given by,

$\left|\vec{\tau}_{-q}\right| = qEl \sin\theta$

Therefore, the net torque acting is given by,

$\left|\vec{\tau}_{net}\right| = 2qEl\sin\theta$

$\left|\vec{\tau}_{net}\right| = \left|\vec{p}\right|\left|\vec{E}\right|\sin\theta$

$\vec{\tau}_{net} = \vec{p} \times \vec{E}$

Maximum torque acting on a dipole

For $\left|\vec{\tau}_{net}\right|$ to be maximum,

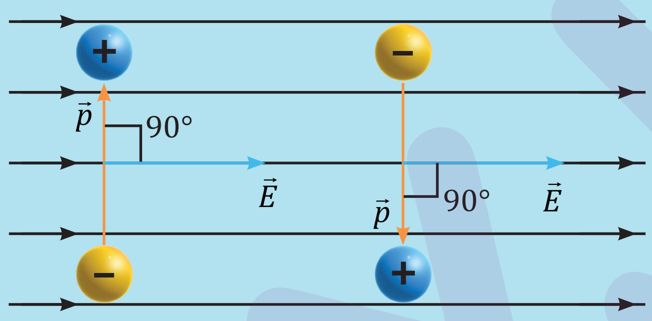
$\left|\vec{p}\right|\left|\vec{E}\right|\sin\theta$ needs to be maximum.

$\therefore\ \sin\theta = 1$

$\Rightarrow \theta = 90°$

Therefore,

$\boxed{\left|\vec{\tau}_{net}\right|_{max} = \left|\vec{p}\right|\left|\vec{E}\right|}$



The magnitude of torque is the maximum when the direction of the dipole moment is perpendicular to the direction of the electric field.

Minimum torque acting on the dipole

For $\left|\vec{\tau}_{net}\right|$ to be minimum,

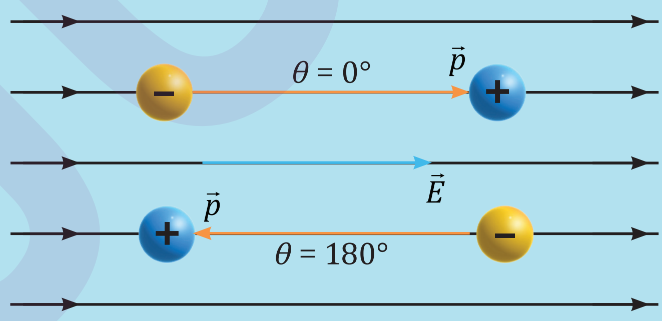
$\left|\vec{p}\right|\left|\vec{E}\right|\sin\theta$ needs to be minimum.

$\therefore\ \sin\theta = 0$

$\Rightarrow \theta = 0°$ and $180°$

Therefore,

$\boxed{\left|\vec{\tau}_{net}\right|_{min} = 0}$



The magnitude of torque is minimum (i.e., zero) when the direction of the dipole moment is opposite to or along the electric field.

A dipole of moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. Find the torque acting on it.

(A) $\vec{\tau} = \vec{p}\,.\,\vec{E}$ (B) $\vec{\tau} = \vec{p} \times \vec{E}$ (C) $\vec{\tau} = \vec{E} + \vec{p}$ (D) $\vec{\tau} = \vec{p} - \vec{E}$

Solution

For a dipole in a uniform electric field, the value of the torque is, $\tau_{net} = pE\sin\theta$.

Hence, this expression of the torque can only be achieved if we use the cross product of $\vec{p}$ and $\vec{E}$.

Therefore, the correct equation for finding the torque is given by,

$\vec{\tau} = \vec{p} \times \vec{E}$

Thus, option (B) is the correct answer.

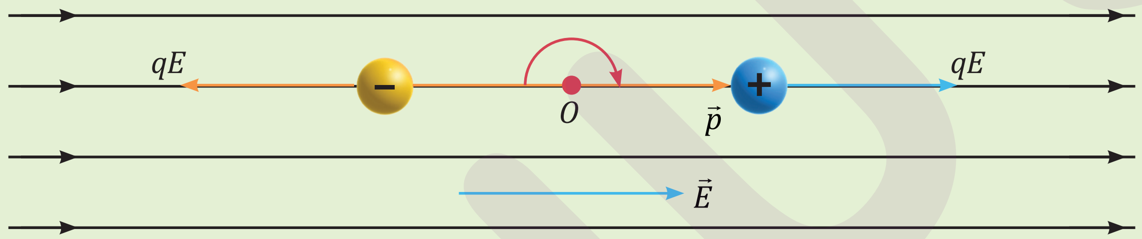
Analysis of the Equilibrium Position

When an electric dipole is placed in a uniform electric field, the net force on the dipole is zero and the net torque acting on the dipole can be zero or minimum at only two positions, i.e., $0°$ and $180°$. Hence, it can be said that at $\theta = 0°$ and $\theta = 180°$, the dipole is in equilibrium.

Stable equilibrium

When the dipole moment makes an angle of $0°$ with the electric field, the net torque ($\tau_{net} = pE \sin\theta$) on the dipole becomes zero ($\tau_{net} = 0$) and the net force on the dipole is already zero. Therefore, at $\theta = 0°$, the dipole is in equilibrium.

On slightly rotating the dipole from its position, there will be equal and opposite forces with different lines of action on the charges of the dipole, which will generate a torque about the centre (O) of the dipole. This torque will make sure that the dipole moment vector gets aligned along the direction of E and comes back to its initial state. Thus, at $\theta = 0°$, the dipole will be in a **stable equilibrium**.

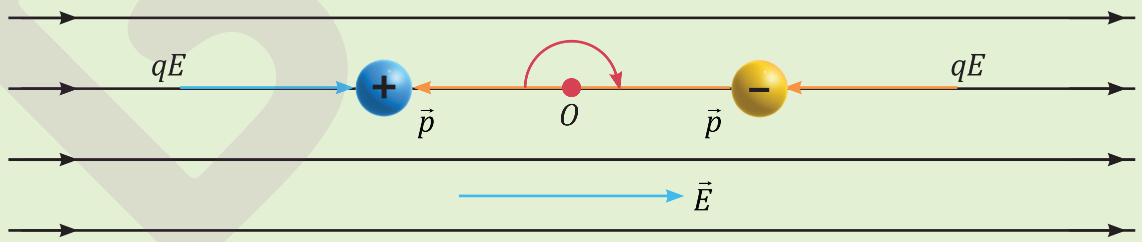


Stable equilibrium

Unstable equilibrium

When the dipole moment makes an angle of $180°$ with the electric field, the net torque ($\tau_{net} = pE \sin\theta$) on the dipole becomes zero ($\tau_{net} = 0$) and the net force on the dipole is already zero. Therefore, at $\theta = 180°$, the dipole is in equilibrium.

On slight rotation of the dipole from this position, there will be equal and opposite forces with different lines of action on the charges of the dipole, which will generate a torque about the centre (O) of the dipole. This torque will try to rotate the dipole even further away. It will not return to its initial state. Thus, at $\theta = 180°$, the dipole will be in an unstable equilibrium.



Unstable equilibrium

An electric dipole is placed at an angle of $30°$ with an electric field intensity of $2 \times 10^5\ NC^{-1}$. It experiences a torque equal to $4\ Nm$. Find the charge on the dipole if its length is $2\ cm$.

(A) $7\ mC$ (B) $8\ mC$ (C) $5\ mC$ (D) $2\ mC$

Solution

Given,

Angle subtended by the dipole, $\theta = 30°$

Electric field intensity, $|\vec{E}| = 2 \times 10^5\ NC^{-1}$

Torque experienced by the dipole, $|\vec{\tau}| = 4\ Nm$

Length of the dipole, $2l = 2\ cm$

The torque experienced by the dipole is given by,

$$|\vec{\tau}| = |\vec{p}||\vec{E}|\sin\theta$$

$$\Rightarrow |\vec{\tau}| = 2ql|\vec{E}|\sin\theta$$

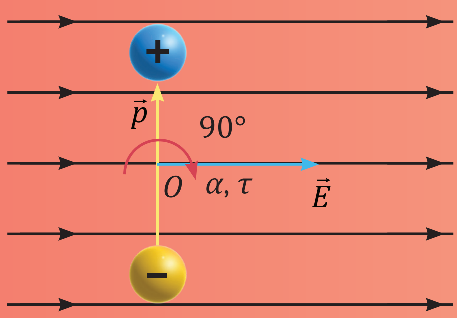
$$\Rightarrow q = \frac{|\vec{\tau}|}{2l|\vec{E}|\ \sin\theta}$$

$$\Rightarrow q = \frac{4}{2 \times 10^5 \times 0.5 \times 0.02}$$

$$\Rightarrow q = 2\ mC$$

Thus, option (D) is the correct answer.

A molecule having a dipole moment $\vec{p}$ and moment of inertia I about an axis passing through its centre is suddenly subjected to a uniform electric field $\vec{E}$ at a right angle to the direction of the molecule's dipole moment. Find the magnitude of the initial angular acceleration of the molecule.



(A) $\frac{|\vec{p}||\vec{E}|}{I}$ (B) $\frac{|\vec{p}|}{I|\vec{E}|}$ (C) $\frac{|\vec{E}|}{I|\vec{p}|}$ (D) $\frac{|\vec{p}||\vec{E}|}{2I}$

Solution

Given,

Angle subtended by the dipole, $\theta = 90°$

Torque is given by,

$$|\vec{\tau}| = |\vec{p}||\vec{E}|\sin\theta$$

$$\Rightarrow |\vec{\tau}| = |\vec{p}||\vec{E}|\sin 90°$$

$$\Rightarrow |\vec{\tau}| = |\vec{p}||\vec{E}| \quad ...(i)$$

Also, torque can be written as,

$$|\vec{\tau}| = I|\vec{\alpha}| \quad ...(ii)$$

Comparing equations (i) and (ii), we get,

$$I|\vec{\alpha}| = |\vec{p}||\vec{E}|$$

$$\Rightarrow |\vec{\alpha}| = \frac{|\vec{p}||\vec{E}|}{I}$$

Thus, option (A) is the correct answer.

ELECTROSTATICS

INTRODUCTION TO ELECTRIC FLUX

What you already know

- Electric field at an equatorial point of a dipole
- Electric field at any point due to a dipole
- Dipole in a uniform electric field

What you will learn

- Electric flux
- Measurement of electric flux
- Electric flux through a cube and a cylinder due to a uniform electric field
- Electric flux in a non-uniform electric field
- Electric flux through a sphere

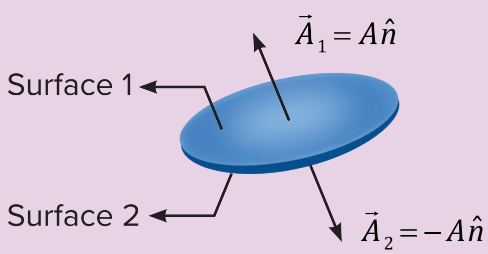
Electric Flux (ϕ)

The number of the electric field lines that intersect a given area normally is known as the electric flux. It is denoted by ϕ.

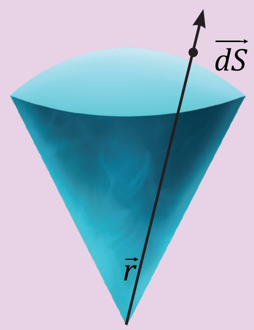
Area vector

1. Area is generally considered as a scalar quantity; however, for many cases, the area of a surface is taken as a vector.
2. The direction of the area vector is taken in the direction normal to the surface.

As $2D$ objects do not enclose a volume, their surfaces are known as **open surfaces**, (E.g., plane surfaces) . Consider a disc having two surfaces. For open surfaces, the unit normal or area vector can be drawn in either direction as shown in the figure.



As $3D$ objects enclose a volume, their surfaces are known as **closed surfaces**, (E.g., spheres) . Consider a part of a sphere as shown in the figure. For closed surfaces, the unit normal or area vector is drawn radially outward and normal to the surface.



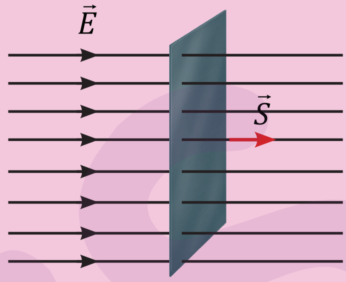
The value of the area is given by,

$$\vec{dS} = dS\,\hat{r}$$

$$\vec{S} = \int \vec{dS}$$

Measurement of Electric Flux

Electric flux is measured in terms of electric field strength. **The electric field strength at a particular point is defined as the electric flux passing through a unit normal area at that point**.

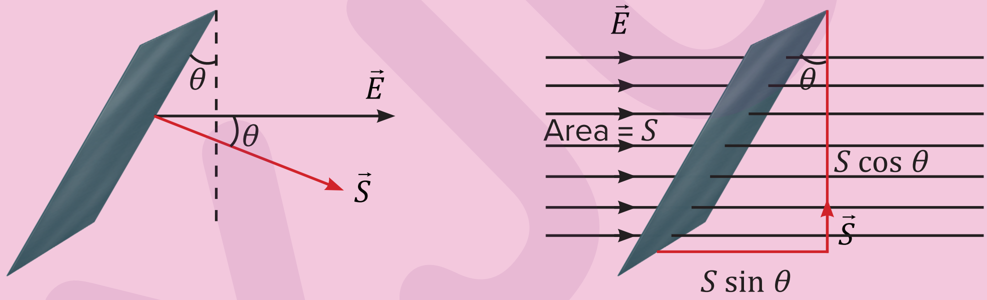


In a uniform electric field, let us consider that a plane surface is placed normally to the field. The area vector is also along the direction of the electric field as shown in the figure.

If ϕ is the flux passing through the surface area S, then the electric field intensity in terms of electric flux at any point in the area is given by,

$$E = \frac{\phi}{S}$$

Now, if the plane surface is placed in the uniform electric field at some angle θ with the electric field, then the area vector also subtends an angle θ with the electric field as shown in the diagram. By resolving the area into its two components, we get $S\cos\theta$ and $S\sin\theta$.



The component of the area $S\sin\theta$ is parallel to the electric field. Therefore, no electric field lines pass through it normally. Hence, the electric flux through the surface $S\sin\theta$ is zero. The component of the area $S\cos\theta$ is perpendicular to the electric field. Therefore, the electric field lines pass through it normally. Thus, the flux through the surface $S\cos\theta$ is non-zero.

The electric field intensity in terms of electric flux at any point in the area is given by,

$$E = \frac{\phi}{S}$$

$$E = \frac{\phi}{S\cos\theta}$$

$$\Rightarrow \phi = E.S\cos\theta$$

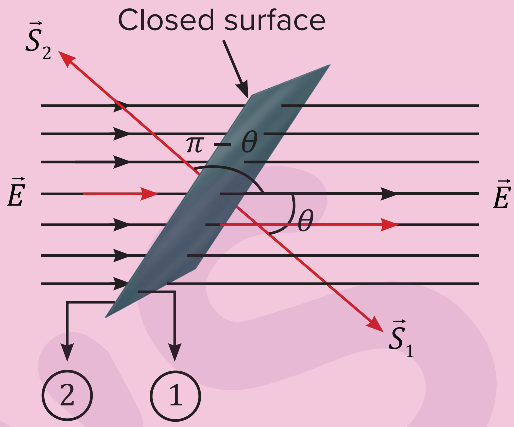
$$\boxed{\Rightarrow \phi = \vec{E}.\vec{S}}$$

Where θ is the angle between the area and the electric field.

Therefore, electric flux is the dot product of the electric field and the area vector and hence, is a scalar quantity.

Positive and negative flux

Consider a $3D$ object placed in a uniform electric field. We know that the surface of a $3D$ object is considered as a closed surface and the area vector is taken along the radially outward normal as shown in the figure. Let us consider that there are two surfaces for the given closed surface, and the electric field lines are entering into the surface S_2 and coming out from the surface S_1. The area vector of surface 1, $\vec{S}_1$, subtends an angle θ with the electric field. The electric flux through surface 1 is given by,



$$\phi_1 = E.S_1 \cos\theta$$

As θ is an acute angle, $\cos\theta =$ Positive

$\therefore\ \phi_1 =$ Positive

Therefore, **when the electric field lines are coming out from a $3D$ object or a closed surface, it is considered as positive flux**.

Now, for surface 2, the electric field lines enter into the closed surface and the area vector $\vec{S}_2$ subtends an angle $\pi - \theta$ with the electric field as shown in the figure. The electric flux through surface 2 is given by,

$$\phi_2 = E.S_2 \cos(\pi - \theta)$$

As $\pi - \theta$ is an obtuse angle, $\cos\theta =$ Negative

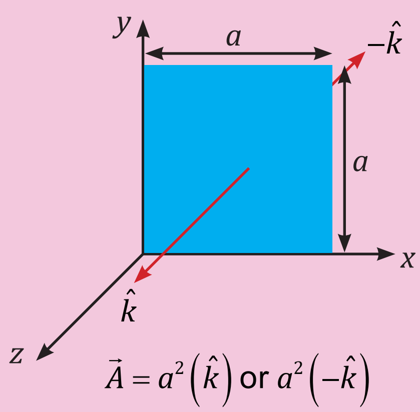
$\therefore\ \phi_2 =$ Negative

Therefore, **when the electric field lines enter into a $3D$ object or a closed surface, it is considered as negative flux**.

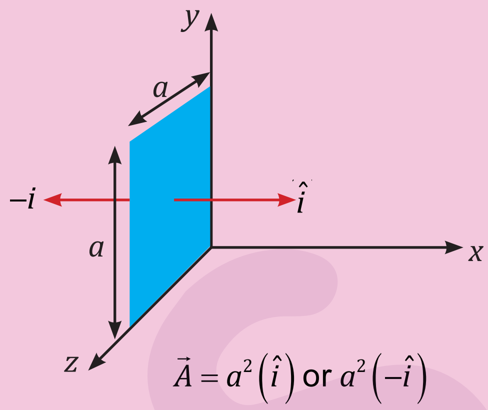
Area vector for $2D$ geometry

Let us find the area vector for the following surfaces:

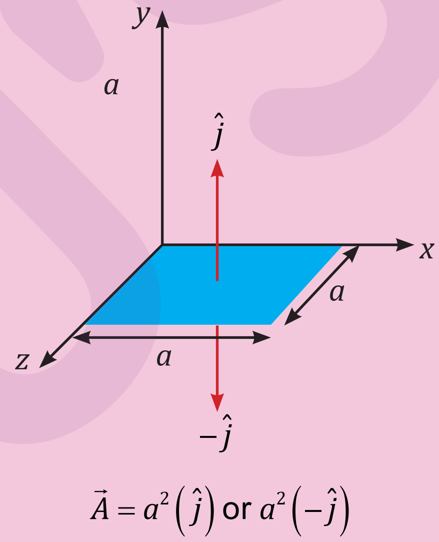
1. A plane surface is placed in the xy-plane. For this plane surface, the area vector will be either along $\hat{k}$ or $-\hat{k}$ as shown in the figure.



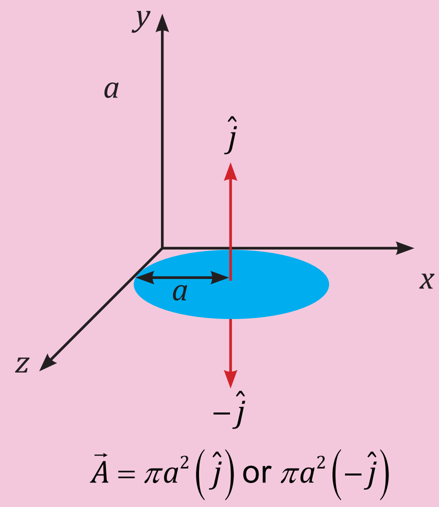
2. A plane surface is placed in the yz-plane. For this plane surface, the area vector will be either along $\hat{i}$ or $-\hat{i}$ as shown in the figure.



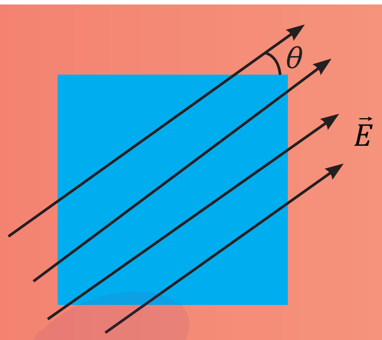
3. A plane surface is placed in the xz-plane. For this plane surface, the area vector will be either along $\hat{j}$ or $-\hat{j}$ as shown in the figure.



4. A disc is placed in the xz-plane. For this plane surface, the area vector will be either along $\hat{j}$ or $-\hat{j}$ as shown in the figure.



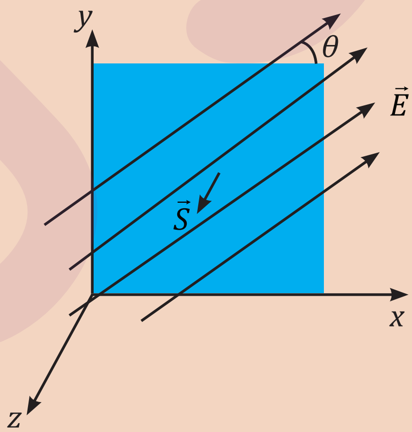
A square surface of side L (m) is placed in a uniform electric field E $(volt\ m^{-1})$ acting along the same plane at an angle θ with the horizontal side of the square. Find the electric flux linked to the surface in $volt\ m$.



(A) EL^2 (B) $EL\cos\theta$ (C) $EL^2\cos\theta$ (D) Zero

Solution

In this case, let us consider the square surface to be in the xy-plane. So, the area vector $\vec{S}$ is along the z-axis. It is also given that the electric field is acting along the xy-plane only. Thus, the area vector and the electric field are perpendicular to each other.



The flux through the square surface is given by,

$$\phi = E.S\cos 90°$$

$$\because \cos 90° = 0$$

$$\therefore\ \phi = 0$$

Thus, option (D) is the correct answer.

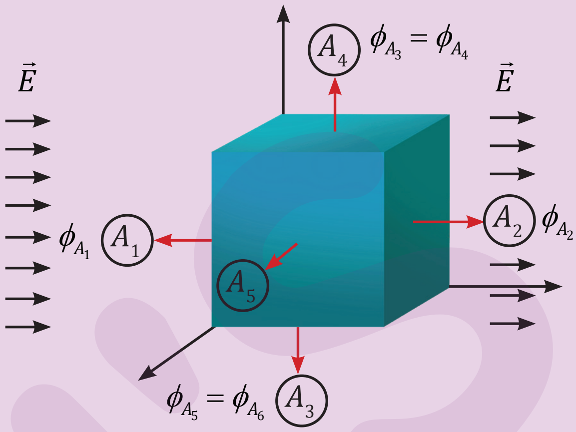
Summary sheet

For a $3D$ object or a closed surface

1. Only the outward normal is considered as an area vector.
2. The flux entering the surface is taken as negative and the flux leaving the surface is taken as positive.
3. If a $3D$ object or a closed object is placed in a uniform electric field, the net flux through the closed object will be zero, given that no net charge is enclosed by it. In other words, **if a closed object does not enclose any charge, then the net electric flux through the closed object will be zero**.

Electric Flux through a Cube Due to a Uniform Electric Field

Consider that a cube of side a is placed in a uniform electric field. The area of all the surfaces are named A_1, A_2, A_3, A_4, A_5, and A_6 as shown in the figure. (A_6 is the surface parallel to A_5)



The electric flux through surface area A_1 is given by,

$$\phi_1 = E.A_1 \cos 180°$$

$$\Rightarrow \phi_1 = Ea^2(-1) = -Ea^2$$

The electric flux through surface area A_2 is given by,

$$\phi_2 = E.A_2 \cos 0°$$

$$\Rightarrow \phi_2 = Ea^2(1) = Ea^2$$

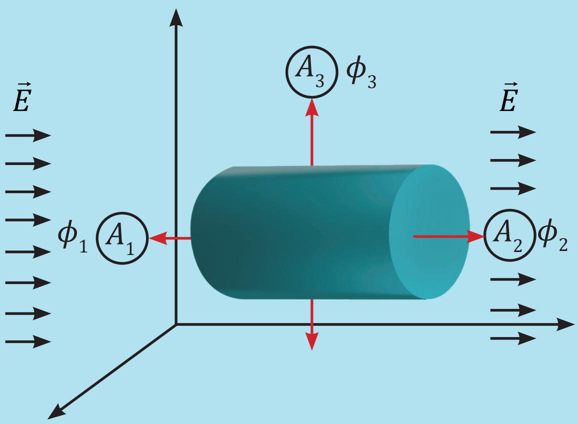
Other than A_1 and A_2, for all the other surfaces, the angle between the electric field and the area vector is 90°. So, the flux through surface areas A_3, A_4, A_5, and A_6 is zero.
Thus, the net flux is given by,

$$\phi_{net} = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6$$
$$= -Ea^2 + Ea^2 + 0 + 0 + 0 + 0$$
$$= 0$$

Therefore, the net flux through the cube is zero until any charge is placed inside the cube.

Electric Flux through a Cylinder Due to a Uniform Electric Field

Consider that a cylinder with a cross-sectional area of A is placed in a uniform electric field as shown in the figure.



The electric flux through surface A_1 is given by,

$$\phi_1 = E.A_1 \cos 180°$$

$$\Rightarrow \phi_1 = EA(-1) = -EA$$

The electric flux through surface A_2 is given by,

$$\phi_2 = E.A_2 \cos 0°$$

$$\Rightarrow \phi_2 = EA(1) = EA$$

For the curved surface A_3, the angle between the electric field and the area vector is 90°. So, the flux through surface area A_3 is zero.

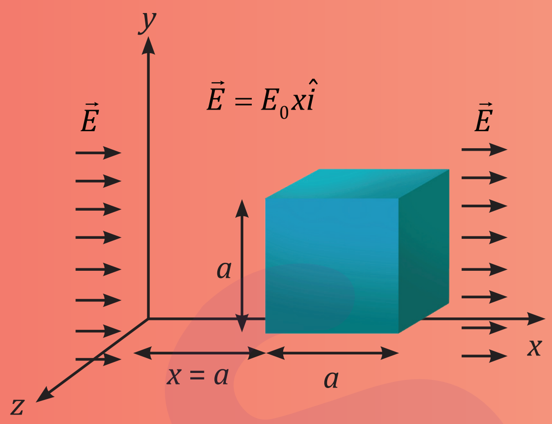
$$\phi_3 = 0$$

The net electric flux is given by,

$$\phi_{net} = \phi_1 + \phi_2 + \phi_3$$
$$= -Ea + Ea + 0$$
$$= 0$$

Therefore, the net flux through the cylinder is zero until and unless there is a charge enclosed in it.

What is the total electric flux passing through the cube in the given situation?



(A) E_0a^3 (B) $-E_0a^3$ (C) $2E_0a^3$ (D) Zero

Solution

Given,

The electric field, $\vec{E} = E_0 x\hat{i}$

The cube has six faces. They are named A_1, A_2, A_3, A_4, A_5, and A_6 as shown in the figure.

The electric flux through surface area A_1 is given by,

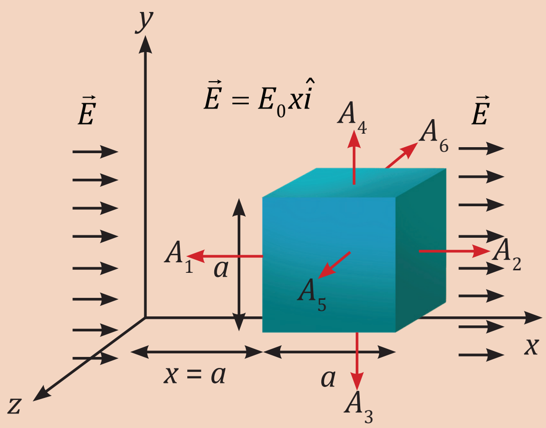
$$\phi_1 = \vec{E}.\vec{A}_1$$

$$\Rightarrow \phi_1 = EA_1 \cos\theta$$

$$\Rightarrow \phi_1 = EA_1 \cos 180°$$

$$\Rightarrow \phi_1 = -E_0xa^2 \left(\because E = E_0x\ ,\ A_1 = a^2\right)$$

$\Rightarrow \phi_1 = -E_0a^3$ (Since $x = a$ at the location of A_1)



The electric flux through surface area A_2 is given by,

$$\phi_2 = EA_2 \cos 0°$$

$$\Rightarrow \phi_2 = E_0xa^2 \left(\because E = E_0x\ ,\ A_2 = a^2\right)$$

$$\Rightarrow \phi_2 = E_0(2a)a^2 \ (\because x = 2a)$$

$$\Rightarrow \phi_2 = 2E_0a^3$$

Other than A_1 and A_2, for all the other surfaces, the angle between the electric field and the area vector is 90°. So, the flux through surface areas A_3, A_4, A_5, and A_6 is zero.

So, the net electric flux is given by,

$$\phi_{net} = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6$$
$$= -E_0a^3 + 2E_0a^3 + 0 + 0 + 0 + 0$$
$$= E_0a^3$$

Thus, option (A) is the correct answer.

NEET

Electric Flux in a Non-uniform Electric Field

Consider a $2D$ surface M placed in a non-uniform electric field. To find the net flux passing through surface M, let us consider an elemental area on the given surface and the area vector of that element $d\vec{S}$ that subtends an angle θ with the electric field. Let the electric flux passing through the elemental area be $d\phi$.

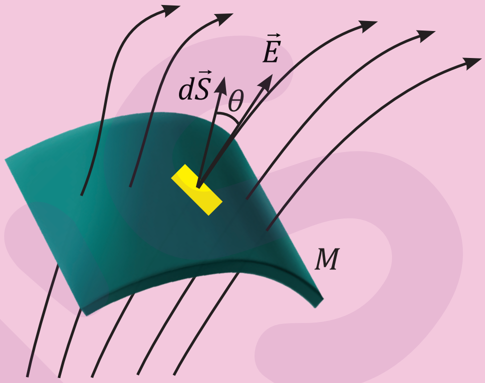
The small elemental flux can be written as,

$$d\phi = \vec{E} \,.\, d\vec{S}$$

Total flux through the surface M is given by,

$$\phi = \int \vec{E} \,.\, d\vec{S}$$

$$\Rightarrow \phi = \int EdS \cos\theta$$



NEET

Electric Flux Through a Sphere

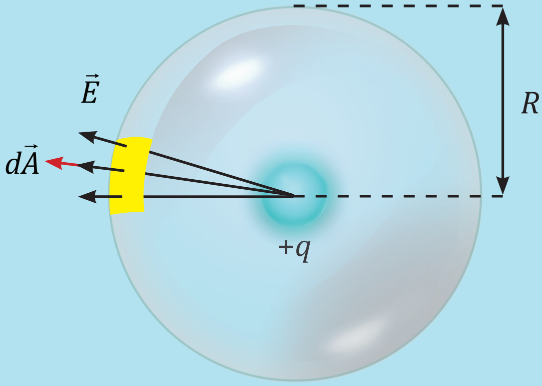
Let us consider that a small positive charge $+q$ is placed at the centre of a sphere of radius R. To find the net flux passing through the sphere, let us consider an elemental area on the concerned sphere, and the area vector of the element $d\vec{A}$ is directed radially outward to the surface. Also, the electric field due to the small charge $+q$ present inside the sphere is directed radially outward to the surface of the sphere.

The angle between the area vector and the electric field is 0°.

The elemental flux passing through $d\vec{A}$ is given by,

$$d\phi = \vec{E} \,.\, d\vec{A} = EdA\cos\theta$$

$$d\phi = EdA$$



The total flux through the sphere is given by,

$$\phi_{sphere} = \int d\phi = \int EdA \quad ....(i)$$

The value of electric field at a distance R from the charge is given by,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

By substituting the value of E in equation (i), we get,

$$\phi_{sphere} = \int \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} dA$$

$$\Rightarrow \phi_{sphere} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\int dA$$

$$\Rightarrow \phi_{sphere} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \times 4\pi R^2 \; (\because \text{Surface area of sphere, } A = 4\pi R^2)$$

$$\boxed{\Rightarrow \phi_{sphere} = \frac{q}{\varepsilon_0}}$$

PHYSICS

ELECTROSTATICS

GAUSS'S LAW

What you already know

- Electric flux
- Measurement of electric flux
- Electric flux through a cube and a cylinder due to a uniform electric field
- Electric flux in a non-uniform electric field
- Electric flux through a sphere

What you will learn

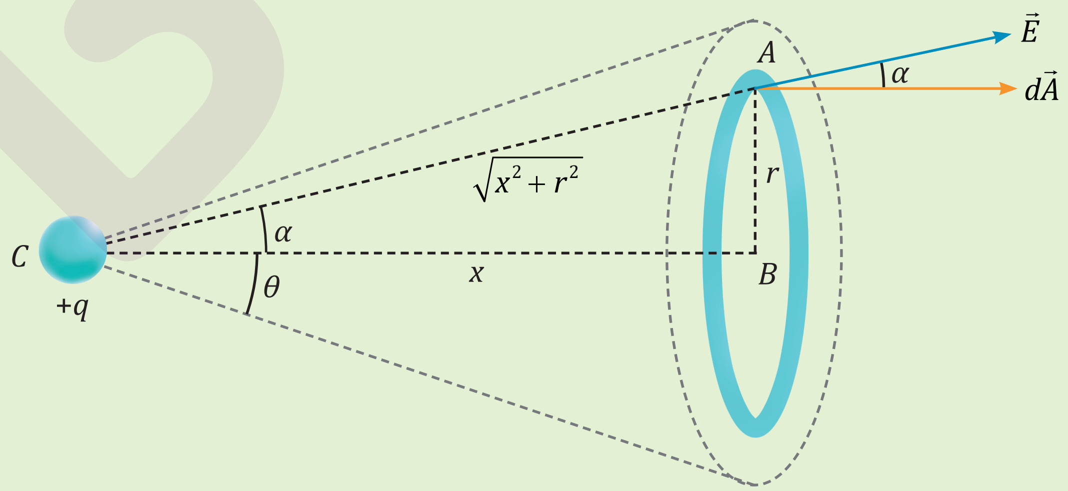
- Electric flux through the base of a cone
- Electric flux through a closed surface
- Gauss's law

NEET

Electric Flux through the Base of a Cone

Let us consider a charge $+q$ placed on the vertex of a cone of radius R and height x. The charge has its own electric field directed outwards. As we can see, the base of the cone is a disc, and a disc is a combination of infinite coaxial thin rings. To calculate the electric flux through the base of the cone, first divide the surface into small elementary areas, $d\vec{A}$. Let us consider an element of the disc which is nothing but a ring of radius r and thickness dr at the base of the cone. The area of the elemental ring is given by,

$$\left|d\vec{A}\right| = 2\pi r\, dr$$



The electric field on the ring subtends an angle α with the area vector and the most outward electric field lines intercepting the base of the cone or disc subtend an angle 2θ at the vertex of the cone.

The magnitude of the electric field on the small element of the ring at distance x from the charge $+q$ is given by,

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_o}\frac{q}{x^2 + r^2}$$

The electric flux through the small element of the ring is given by,

$$d\phi = \vec{E}\,.\,d\vec{A}$$

$$\Rightarrow d\phi = E\ dA \cos\alpha \quad ...(i)$$

In ΔABC,

$$\cos\alpha = \frac{x}{\sqrt{x^2 + r^2}}$$

By substituting E and $\cos\alpha$ in equation (i), we get,

$$d\phi = E\ dA \cos\alpha$$

$$\Rightarrow d\phi = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{x^2 + r^2} \times 2\pi r\ dr \times \frac{x}{\sqrt{x^2 + r^2}}$$

$$\Rightarrow d\phi = \frac{qx}{2\varepsilon_o} \times \frac{r\ dr}{\left(x^2 + r^2\right)^{\frac{3}{2}}}$$

To get the net flux through the base of the cone or disc, we have to integrate $d\phi$ from $r = 0$ to $r = R$.

$$\int_0^\phi d\phi = \int_0^R \frac{qx}{2\varepsilon_o} \times \frac{r\ dr}{\left(x^2 + r^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \phi = \frac{qx}{2\varepsilon_o}\int_0^R \frac{r\ dr}{\left(x^2 + r^2\right)^{\frac{3}{2}}} \quad ...(i)$$

Let $x^2 + r^2 = t^2$

By partially differentiating we get,

$$\Rightarrow 2r\ dr = 2t\ dt$$

$$\Rightarrow r\ dr = t\ dt$$

By substituting the value in equation (i), we get,

$$\Rightarrow \phi = \frac{qx}{2\varepsilon_o}\int_x^{\sqrt{x^2 + R^2}} \frac{t\ dt}{\left(t^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \phi = \frac{qx}{2\varepsilon_o}\int_x^{\sqrt{x^2 + R^2}} \frac{t\ dt}{t^3}$$

$$\Rightarrow \phi = \frac{qx}{2\varepsilon_o}\int_x^{\sqrt{x^2 + R^2}} t^{-2}\,dt$$

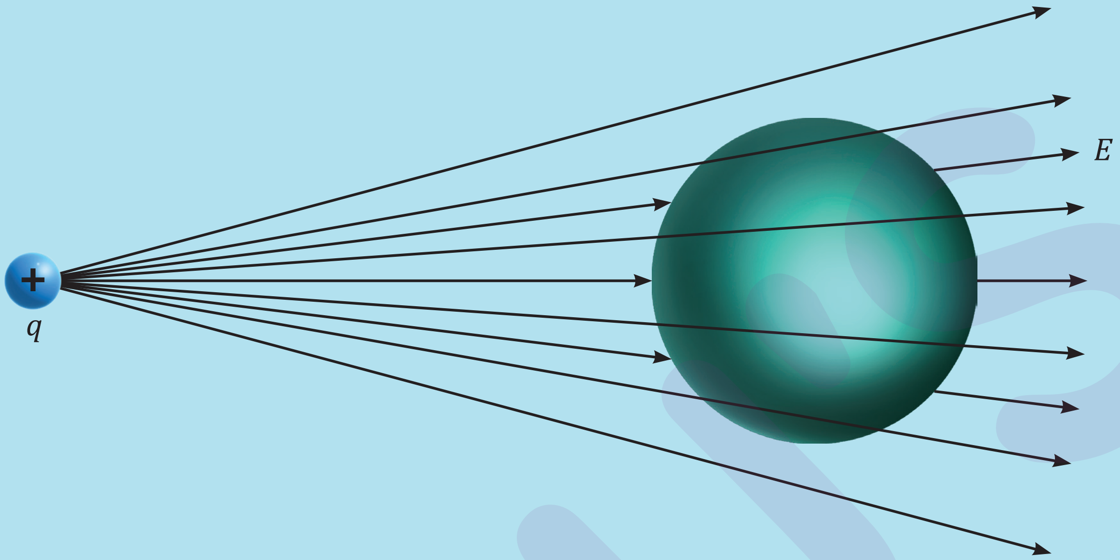
$$\Rightarrow \phi = \frac{qx}{2\varepsilon_o}\left[-\frac{1}{t}\right]_x^{\sqrt{x^2 + R^2}}$$

$$\Rightarrow \phi = \frac{q}{2\varepsilon_o}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

$$\boxed{\Rightarrow \phi = \frac{q}{2\varepsilon_o}(1 - \cos\theta)} \quad \left(\because \cos\theta = \frac{x}{\sqrt{x^2 + R^2}}\right)$$

Electric Flux through a Closed Surface

Consider a sphere ($3D$ or closed object) placed near a point charge $+q$. The electric flux from the point charge passes through the closed object.



We can observe that the flux entering the closed object is the same as the flux leaving the closed object. In other words, if a closed object does not enclose any charge, then the net electric flux through the closed object is zero.

So, the net flux is given by,

$\phi_{net} = \phi_{in} + \phi_{out} = 0$

The net flux, in this case, is zero.

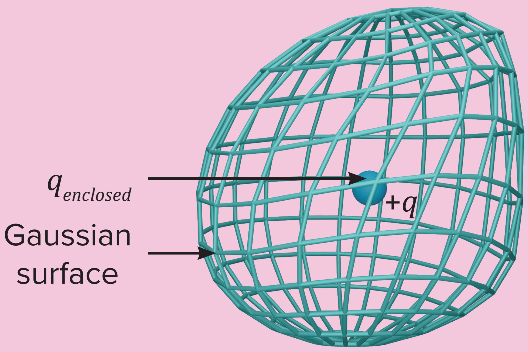
NEET

Gauss's Law

Gauss's law states that net flux through a closed surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

This law gives the following:

1. It gives the analysis of electric flux through a closed surface and its relation with the enclosed charge.
2. The total electric flux associated with a closed surface or a Gaussian surface is equal to the product of the sum of all the charges enclosed by the surface and the constant $\frac{1}{\varepsilon_0}$.

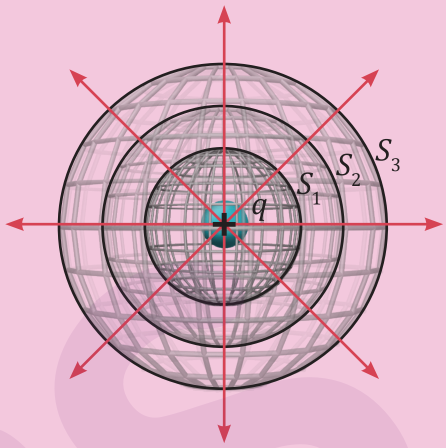


Important points about Gauss's law:

- While using Gauss's law, the considered closed or $3D$ surface is known as a Gaussian surface.
- The total electric flux through a closed surface is independent of the shape of the surface and the position of charge inside the closed surface.

Consider a point charge $+q$ that is inside three Gaussian surfaces S_1, S_2, and S_3 as shown in the figure. The net flux through each surface is the same.

- If a closed surface does not enclose any charge, then the net electric flux through the surface is zero.



Gaussian surface

A Gaussian surface is defined as a closed surface or the periphery of a volume on which Gauss's law is applied. Using a Gaussian surface (closed surface) or a $3D$ surface in a three-dimensional space, the flux of any vector field can be determined. A Gaussian surface can be real or imaginary, and its shape is dependent upon the type of charge or charge distribution inside the surface.

The mathematical form of Gauss's law

From Gauss's law, the net flux through a surface is given by,

$$\phi_{net} = \frac{q_{en}}{\varepsilon_o} \quad ...(i)$$

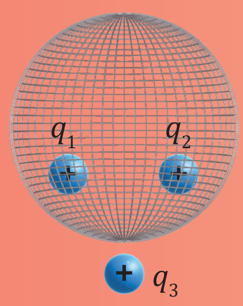
Also, the net flux through a surface is given by,

$$\phi_{net} = \oint \vec{E}.d\vec{A} \quad ...(ii)$$

From equations (i) and (ii), we get,

$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_o}$$

There are three positive charges, q_1, q_2, and q_3, present in space, and the Gaussian surface encloses the charges q_1 and q_2, and the third charge q_3 is outside the surface. Find the net flux using Gauss's law.



Solution

According to Gauss's law,

$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_o}$$

The net flux through the Gaussian surface does not depend on the charge placed outside the Gaussian surface. So, the net charge enclosed by the surface is given as follows:

$q_{en} = q_1 + q_2$

As we know, if a closed surface or object does not enclose any charge, then the net electric flux through the closed object is zero. Thus, the net flux by charge q_3 through the given surface is zero. So, the net electric field is given as follows:

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

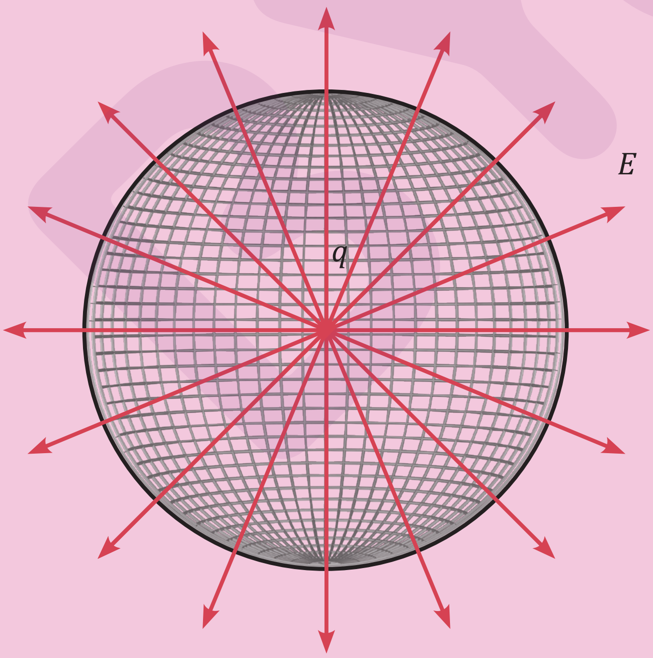
The net flux is given by,

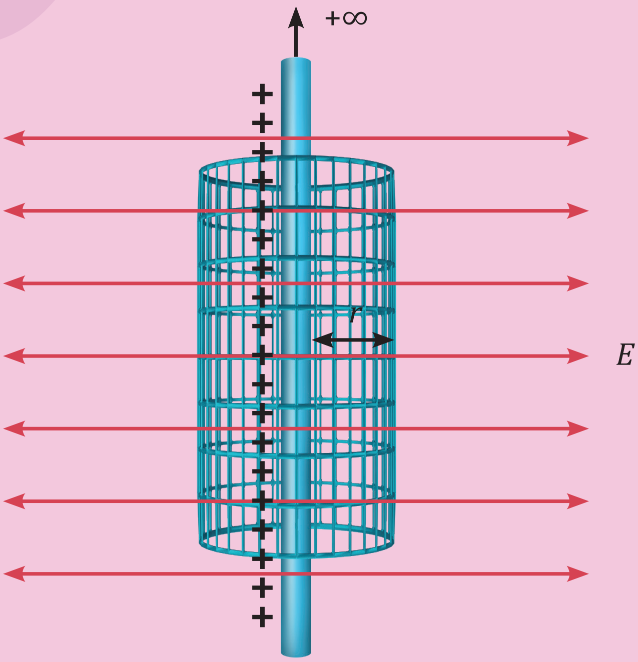
$$\phi_{net} = \oint\left(\vec{E}_1 + \vec{E}_2 + \vec{E}_3\right).d\vec{A} = \frac{q_1 + q_2}{\varepsilon_o}$$

Using Gauss's law, we can find the electric field due to some symmetric charge distributions.

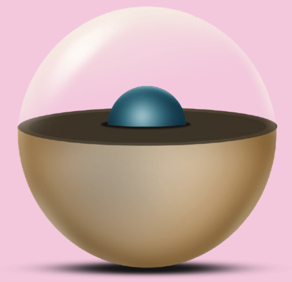
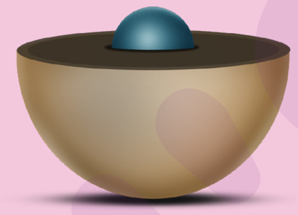
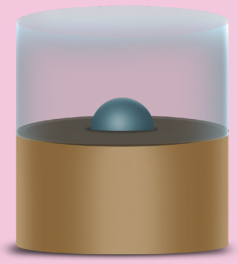
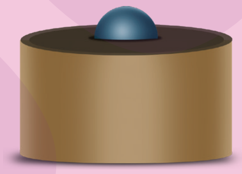
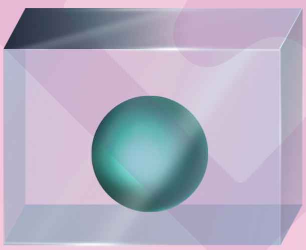
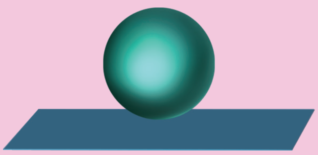
Essential properties of a useful Gaussian surface

- Gauss's law is applicable to any closed surface, i.e., a Gaussian surface must be a closed or $3D$.
- The electric field must be symmetric and equal at all the points on the Gaussian surface.
- The angle between $\vec{E}$ and $d\vec{A}$ must be the same at all the points of the Gaussian surface (preferably, $\theta = 0°$ or $90°$).
- For a point charge and line charge distribution, the Gaussian surface will be a sphere and a cylinder, respectively.





The net flux for different surfaces can be obtained by using symmetry. Some of the shapes are given in the table.

Shape	Net flux	Shape	Net flux
Sphere  A charge is placed symmetrically at the centre of the sphere.	$\phi_{net} = \dfrac{q}{\varepsilon_o}$	**Hemisphere**  A charge is placed symmetrically at the centre of a hemisphere.	$\phi_{net} = \dfrac{q}{2\varepsilon_o}$
Cylinder  A charge is placed symmetrically at the centre of a cylinder.	$\phi_{net} = \dfrac{q}{\varepsilon_o}$	**Semi-cylinder**  A charge is placed symmetrically at the centre of a semi-cylinder.	$\phi_{net} = \dfrac{q}{2\varepsilon_o}$
Cube  A charge is placed symmetrically at the centre of a cube.	$\phi_{net} = \dfrac{q}{\varepsilon_o}$	**One face of cube**  A charge is placed symmetrically at the centre of a square surface.	$\phi_{net} = \dfrac{q}{6\varepsilon_o}$

ELECTROSTATICS

APPLICATION OF GAUSS'S LAW

What you already know

- Electric flux through the base of a cone
- Electric flux through a closed surface
- Gauss's law

What you will learn

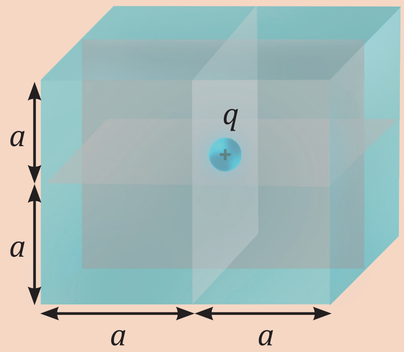
- Electric flux through the face of a cube
- Electric flux through the curved surface of a cylinder and a container
- Applications of Gauss's law

What is the flux through a cube of side a if a point charge q is at one of its corners?

(A) $\frac{2q}{\varepsilon_o}$ (B) $\frac{q}{8\varepsilon_o}$ (C) $\frac{q}{\varepsilon_o}$ (D) $\frac{q}{2\varepsilon_o}6a^2$

Solution NEET

It is given that the charge is placed at one corner of the cube. To enclose the charge inside a closed surface or a Gaussian surface such that the charge is symmetrical to the closed surface, take seven more cubes of the same dimensions as that of the original cube. All the cubes are arranged in such a manner that the charge is at the centre of the bigger cube as shown in the figure.



We know that the total flux due to a point charge is, $\phi = \frac{q}{\varepsilon_o}$.

Therefore, the total flux through the bigger cube (closed surface) is given by,

$$\phi_{total} = \frac{q}{\varepsilon_o}$$

The flux through each cube is given by,

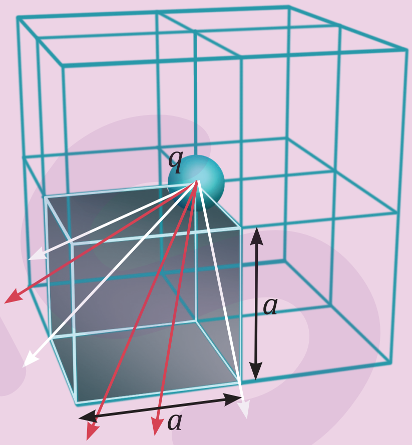
$$\phi_{each\ cube} = \frac{\phi_{total}}{8} = \frac{q}{8\varepsilon_o}$$

Thus, option (B) is the correct answer.

Flux through Each Face of a Cube

Consider a charge placed at one corner of the cube. We know that if a charge is placed at a corner of a cube, then the total flux through the cube is given by,

$$\phi_{net} = \frac{q}{8\varepsilon_o}$$



In a cube, there are six faces. The flux due to the charge does not pass through all the faces of the cube.

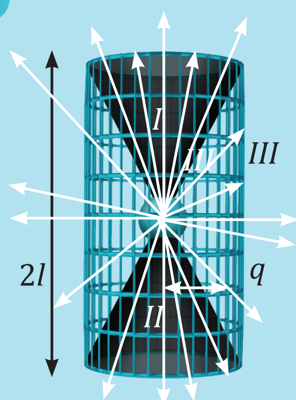
From the diagram, we can observe that the electric field lines are parallel to the three adjacent faces that are connected to the charge denoted by the white arrows. Thus, the total flux associated with the cube passes through the three other remaining faces denoted by the red arrows.

Therefore, the net flux through each of the surfaces is given by,

$$\phi_{each\ face} = \frac{\phi_{net}}{3} = \frac{q}{24\varepsilon_o}$$

Flux through the Curved Surface of a Cylinder

Consider a charge q placed symmetrically at the middle of a cylinder of radius R and height $2l$ as shown in the figure.



We know that the total flux due to a point charge is, $\phi = \frac{q}{\varepsilon_o}$.

Therefore, the total flux through the cylinder is given by, $\phi_{net} = \frac{q}{\varepsilon_o}$.

Also, the net flux is shared by three faces, two circular faces (I, II) and one curved face (III) as shown in the figure.

$\phi_{net} = \phi_I + \phi_{II} + \phi_{III}$

From symmetry, the flux from the surface I is equal to that of surface II.

$\therefore\ \phi_I = \phi_{II}$

So, the net flux through the curved surface is given by,

$\phi_{III} = \phi_{net} - (\phi_I + \phi_{II})$ ...(*i*)

From the figure, for the flux through surfaces I and II, we can assume that the charge is over the apex of the cones so that surfaces I and II become the base of the cones.

We know that the net flux through the base of a cone is given by,

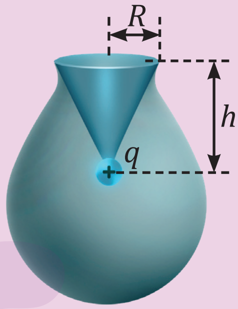
$$\phi_I = \frac{q}{2\varepsilon_o}\left(1 - \frac{l}{\sqrt{l^2 + R^2}}\right) = \phi_{II}$$

By substituting the values of ϕ_I and ϕ_{II} in equation (i), we get,

$$\phi_{III} = \frac{q}{\varepsilon_o} - \left(2 \times \frac{q}{2\varepsilon_o}\left(1 - \frac{l}{\sqrt{l^2 + R^2}}\right)\right)$$

$$\Rightarrow \phi_{III} = \frac{q}{\varepsilon_o}\left(\frac{l}{\sqrt{l^2 + R^2}}\right)$$

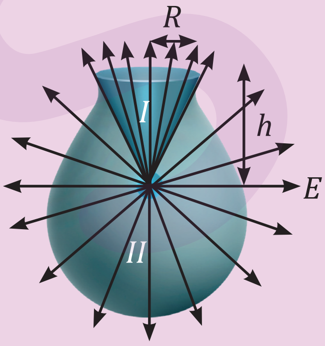
Flux through the Curved Surface of a Container



Consider a charge q placed inside a container with a mouth of radius R and the distance of the charge from the mouth is h.

We know that the total flux due to a point charge is, $\phi = \frac{q}{\varepsilon_o}$.

Therefore, the total flux through the container is given by, $\phi_{net} = \frac{q}{\varepsilon_o}$.



Also, the net flux is shared by two faces, a circular face or the mouth of the container (I), and the other one is the curved face of the container (II), as shown in the figure.

$\phi_{net} = \phi_I + \phi_{II}$

So, the net flux through the curved surface is given by,

$\phi_{II} = \phi_{net} - \phi_I$ ...(i)

From the figure, for the flux through surface I, we can assume that the charge is over the apex of the cone so that surface I becomes the base of the cone.

We know that the net flux through the base of a cone is given by,

$$\phi_I = \frac{q}{2\varepsilon_o}\left(1 - \frac{l}{\sqrt{h^2 + R^2}}\right)$$

By substituting the value of ϕ_I in equation (i), we get,

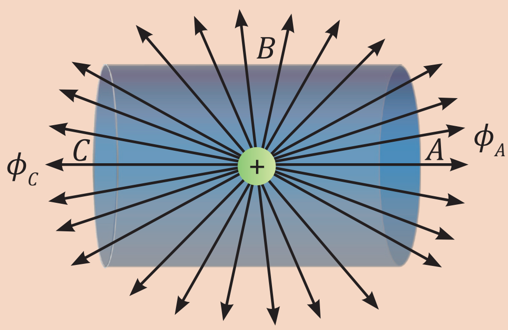
$$\phi_{II} = \frac{q}{\varepsilon_o} - \left(\frac{q}{2\varepsilon_o}\left(1 - \frac{l}{\sqrt{h^2 + R^2}}\right)\right)$$

This is the net flux through the curved surface of the container.

A hollow cylinder has a charge of q coulomb inside it. If ϕ is the electric flux in Vm associated with the curved surface, then what will be the flux linked with the plane surface in Vm?

(A) $\frac{1}{2}\left(\frac{q}{\varepsilon_o} - \phi\right)$ (B) $\frac{q}{2\varepsilon_o}$ (C) $\frac{\phi}{3}$ (D) $\frac{q}{\varepsilon_o} - \phi$

Solution NEET



The flux through the curved surface is given as ϕ.

We know that the total flux due to a point charge is, $\phi = \frac{q}{\varepsilon_o}$.

Therefore, the total flux through the cylinder is given by,

$$\phi_{net} = \frac{q}{\varepsilon_o}$$

Also, the net flux is shared by three faces, two circular faces (A, C) and one curved face (B), as shown in the figure.

$\phi_{net} = \phi_A + \phi_B + \phi_C$

The flux through surface A is equal to that of surface C due to symmetry.

$\therefore \phi_A = \phi_C$

So, the net flux through the curved surface is given by,

$\phi_B = \phi_{net} - (\phi_A + \phi_C)$

$\Rightarrow \phi = \phi_{net} - 2\phi_A$

$$\Rightarrow \frac{1}{2}(\phi_{net} - \phi) = \phi_A$$

$$\Rightarrow \frac{1}{2}\left(\frac{q}{\varepsilon_o} - \phi\right) = \phi_A$$

Thus, option (A) is the correct answer.

Calculation of Electric Field Using Gauss's Law

To find the electric field at a point in the vicinity of the charge configuration using Gauss's law, we need the following steps: We consider a closed surface in the surrounding of the charges such that the point is on the surface. The electric field is either parallel or perpendicular to this surface at every point of the surface.

We apply Gauss's law for this surface as follows:

$$\oint \vec{E}.d\vec{A} = \frac{\Sigma q_{enclosed}}{\varepsilon_o}$$

Applications of Gauss's Law

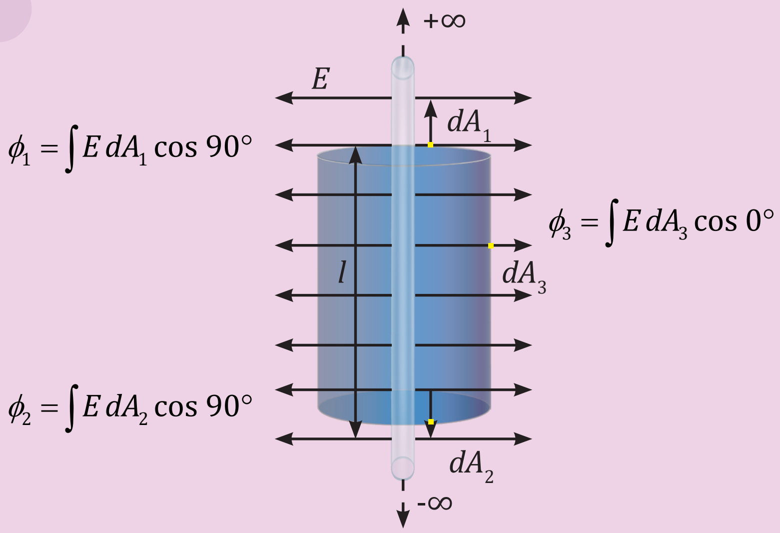
1. **Electric field due to infinitely long uniformly charged wire**

Consider an infinitely long uniformly charged wire. Let λ be the linear charge density.

Let us find the electric field at a distance r from the infinitely long uniformly charged wire by using Gauss's law. We have derived the field at r distance from the infinitely long uniformly charged wire as follows:

$$E = \frac{2k\lambda}{r}$$

Now, to obtain the electric field intensity at a distance r from the rod using Gauss's law, we have to take a Gaussian surface such that the electric field at every point on the surface is either parallel or perpendicular to this surface. In this case, we can take a cylinder of radius r and length l as the Gaussian surface as shown in the figure.



The cylinder (Gaussian surface) can be divided into three surfaces, two circular surfaces and one curved surface, A_1, A_2, and A_3. Their elements are marked as dA_1, dA_2, and dA_3, respectively, as shown in the figure.

The net charge enclosed by the Gaussian surface is, $q_{en} = \lambda l.$

Using Gauss's law,

$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_o}$$

$$\int \vec{E}.d\vec{A}_1 + \int \vec{E}.d\vec{A}_2 + \int \vec{E}.d\vec{A}_3 = \frac{\lambda l}{\varepsilon_o} \quad ....(i)$$

The area vectors $d\vec{A}_1$ and $d\vec{A}_2$ are perpendicular to the electric field.

$$\vec{E}.d\vec{A}_1 = E\,dA_1 \cos 90° = 0$$

Similarly, $\vec{E}.d\vec{A}_2 = E\,dA_2 \cos 90° = 0$

Therefore, equation (i) becomes,

$$\phi_3 = \int \vec{E}.d\vec{A}_3 = \frac{\lambda l}{\varepsilon_o} \quad ....(ii)$$

Now, the flux through surface 3 is given by,

$$\phi_3 = \int E\,dA_3 \cos 0° = \int E\,dA_3$$

$$\Rightarrow \phi_3 = E2\pi rl$$

The net flux through the cylinder is given by,

$$\phi_{cylinder} = \phi_1 + \phi_2 + \phi_3$$

$$\Rightarrow \phi_{cylinder} = 0 + 0 + E2\pi rl = E2\pi rl \quad ....(iii)$$

From equations (*ii*) and (*iii*), we get,

$$\frac{\lambda l}{\varepsilon_o} = E2\pi rl$$

$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_o r}$$

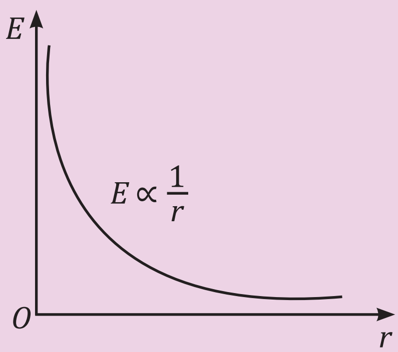
$$\Rightarrow E = \frac{2k\lambda}{r} \quad \left(\because k = \frac{1}{4\pi\varepsilon_o}\right)$$

Electric field (E) vs distance (r) graph

The electric field due to an infinitely long uniformly charged wire is inversely proportional to r.

For an infinite line charge,

$r \to \infty$	$E = 0$
$r \to 0$	E = Undefined



2. Electric field due to an infinitely large uniformly charged sheet

Consider a single layer sheet in which the charge is uniformly distributed. Let σ be the uniform surface charge density of the sheet. To obtain the electric field at a point x distance away from the sheet, we have to take a Gaussian surface such that the electric field at every point on the surface is either parallel or perpendicular to this surface. In this case, we can take a cylinder as the Gaussian surface as shown in the figure. The Gaussian surface (cylinder) can be divided into three surfaces, two circular surfaces and one curved surface, A_1, A_2, and A_3. Their elements are marked as dA_1, dA_2, and dA_3, respectively, as shown in the figure on the next page.

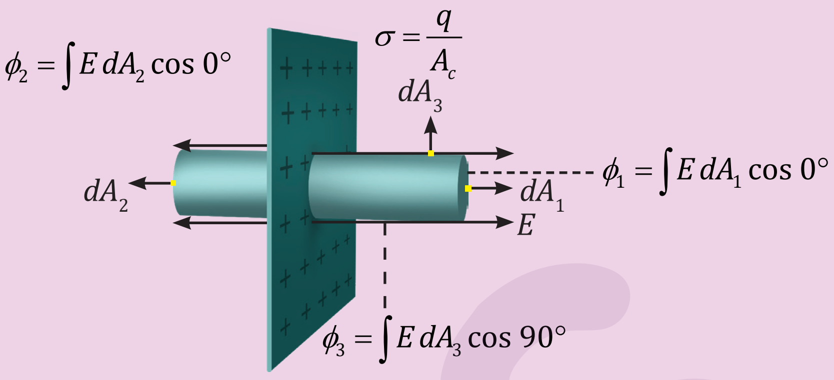
The flux through the curved surface of the cylinder is zero as the area vector of the curved surface is perpendicular to the electric field.

$\phi_3 = 0$

Due to symmetry, the flux through surface 1 is equal to that of surface 2.

$\phi_1 = \phi_2 = \int \vec{E}.d\vec{A}_c = EA_c$ ...(i)

Where, A_c is the circular surface area.



The net flux through the cylinder is given by,

$\phi_{cylinder} = \phi_1 + \phi_2 + \phi_3$

$\Rightarrow \phi_{cylinder} = EA_c + EA_c + 0$

$\Rightarrow \phi_{cylinder} = 2EA_c$

The net charge enclosed by the Gaussian surface is, $q_{en} = \sigma A_c$.

By applying Gauss's law, we get,

$$\phi_{cylinder} = \frac{q_{en}}{\varepsilon_o}$$

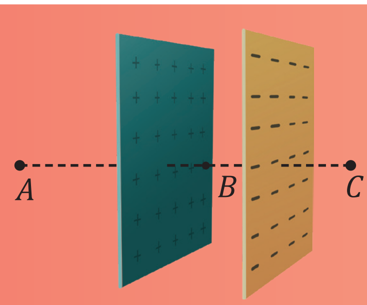
By substituting the values of $\phi_{cylinder}$ and q_{en} in the above equation, we get,

$$2EA_c = \frac{\sigma A_c}{\varepsilon_o}$$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_o}$$

Hence, the electric field due to a uniformly charged infinite sheet is constant everywhere.

What is the net electric field at points A, B, and C if two infinitely long, oppositely charged plane sheets having the same charge density σ are placed parallel to each other?



(A) $E_A = \frac{\sigma}{\varepsilon_o}, E_B = 0, E_C = \frac{\sigma}{\varepsilon_o}$

(B) $E_A = 0, E_B = \frac{\sigma}{\varepsilon_o}, E_C = \frac{\sigma}{\varepsilon_o}$

(C) $E_A = 0, E_B = \frac{\sigma}{2\varepsilon_o}, E_C = 0$

(D) $E_A = 0, E_B = \frac{\sigma}{\varepsilon_o}, E_C = 0$

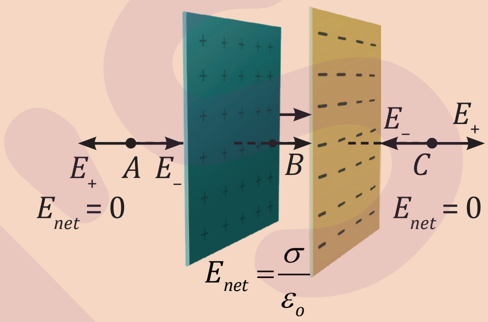
Solution

The electric field due to the positively charged infinite sheet is, $E_+ = \dfrac{\sigma}{2\varepsilon_o}$.

And the electric field due to the negatively charge infinite sheet is, $E_- = \dfrac{\sigma}{2\varepsilon_o}$.

At point A

The electric field due to the negatively charged sheet is towards the sheet, whereas the electric field due to the positively charged sheet is away from the sheet. So, the net electric field at A is zero.



$$\vec{E}_A = \vec{E}_+ + \vec{E}_-$$

$$\Rightarrow E_A = \frac{\sigma}{2\varepsilon_o} - \frac{\sigma}{2\varepsilon_o} = 0$$

At point B

The electric field due to the positively charged sheet and the negatively charged sheet is directed in the same direction. So, the net field at B is given by,

$$\vec{E}_B = \vec{E}_+ + \vec{E}_-$$

$$\Rightarrow E_A = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o}$$

At point C

The electric field due to the positively charged sheet is away from the sheet, whereas the electric field due to the negatively charged sheet is towards the sheet. So, the net electric field at C is zero.

$$\vec{E}_C = \vec{E}_+ + \vec{E}_-$$

$$\Rightarrow E_C = \frac{\sigma}{2\varepsilon_o} - \frac{\sigma}{2\varepsilon_o} = 0$$

Thus, option (D) is the correct answer.

PHYSICS

ELECTROSTATICS

APPLICATIONS OF GAUSS'S LAW-2

What you already know

- Electric flux through the face of a cube
- Electric flux through curved surface of cylinder and container

What you will learn

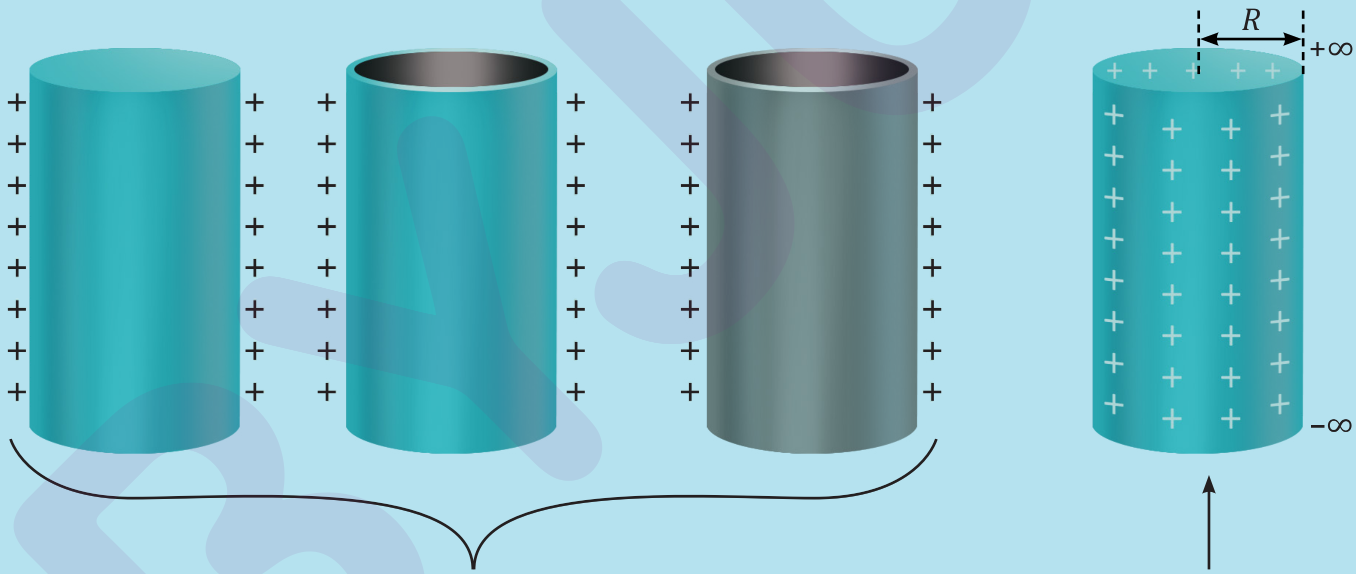
- Applications of Gauss's law: Charged cylinders, sheet and sphere

NEET

Application of Gauss's Law: Charged Cylinders

Different types of charged cylinders

- Solid conducting cylinder
- Hollow conducting cylinder
- Hollow non-conducting cylinder
- Solid non-conducting cylinder



Charge resides on surface only

Charge is distributed over the whole volume

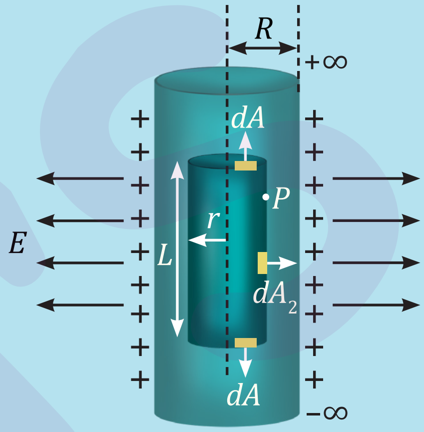
The solid conducting, hollow conducting, or hollow non-conducting cylinders have similar charge distribution, i.e., the charge can only reside over the surface, so we can treat them as a single case. Due to similar charge distribution, at equal distances, they have the same electric fields.

Electric field due to solid conducting cylinder/ hollow conducting cylinder/ thin non-conducting cylinder

Here, we can take any of the three types of cylinders, i.e. solid conducting, hollow conducting, or hollow non-conducting. For our study, we are taking a hollow cylinder.

a. The electric field at a distance r inside the cylinder ($r < R$)

Consider an infinitely long, uniformly charged hollow cylinder of radius R, with surface charge density σ. Consider a point P, which is present inside the cylinder. For this case, the Gaussian surface will be a cylinder of radius r and length L, as shown in the figure.



By applying Gauss's law, we get,

$$\oint \vec{E}_{inside} \cdot d\vec{A} = \frac{q_{en}}{\varepsilon_0}$$

As the charge enclosed is 0,

$$q_{en} = 0$$

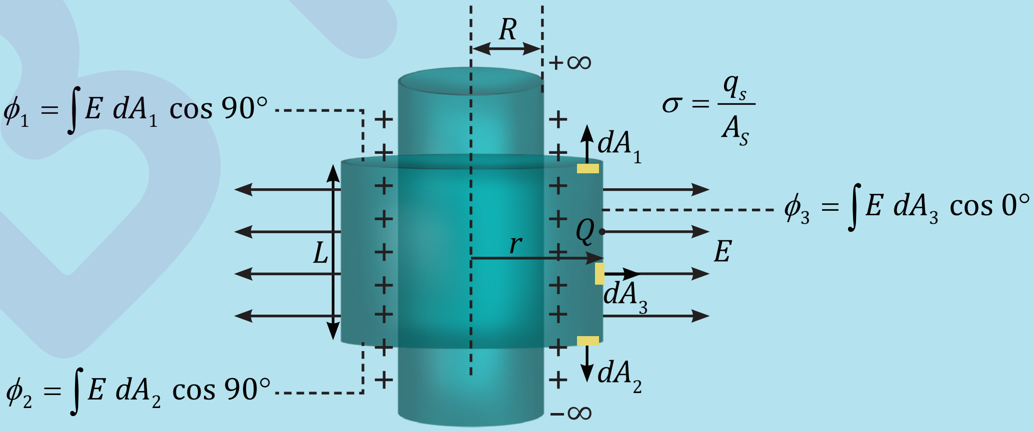
$$\therefore \vec{E}_{inside} = 0$$

Since the surface area of the gaussian surface is a non-zero quantity, thus, to hold the equality, the net electric field inside the hollow cylinder must be equal to zero. Hence, the net electric field inside the solid conducting, hollow conducting, or hollow non-conducting infinitely long uniformly charged cylinders is zero.

---

b. The electric field at a distance r outside the cylinder ($r > R$)

Consider an infinitely long, uniformly charged hollow cylinder of radius R, with surface charge density σ. Consider a point Q, which is present outside the cylinder. For this case, the gaussian surface will be a cylinder of radius r and length L. Now, the cylinder (gaussian surface) can be divided into three surfaces, two circular surfaces, and one curved surface, A_1, A_2, and A_3, and their elements are marked as dA_1, dA_2, and dA_3, respectively, as shown in the figure.



For surface areas, A_1 and A_2, the angle between area vector and the electric field is 90°, so $\phi_1 = \phi_2 = 0$.

Therefore, the net flux is given by,

$$\phi_{net} = \phi_1 + \phi_2 + \phi_3$$

$$\phi_{net} = \phi_3 = \oint \vec{E}.d\vec{A}_3 = EA_3$$

$$\phi_{net} = E\, 2\pi rL$$

The net charge enclosed by the gaussian surface is given by,

$q_{en} = \sigma \times 2\pi RL$

By applying Gauss's law, we get,

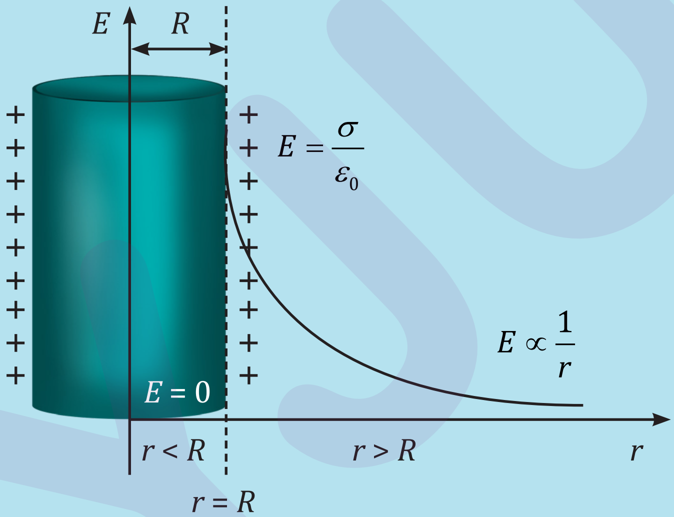
$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow E\oint dA = \frac{\sigma 2\pi RL}{\varepsilon_0}$$

$$\Rightarrow E\,2\pi rL = \frac{\sigma 2\pi RL}{\varepsilon_0}$$

$$\boxed{\Rightarrow E = \frac{\sigma R}{\varepsilon_0 r}}$$

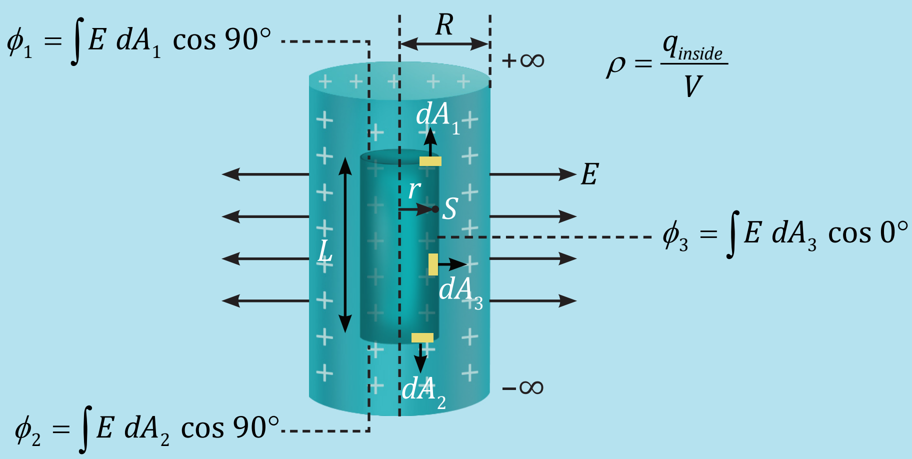
Electric field (E) vs distance (r) graph



Electric field due to the solid non-conducting cylinder

a. The electric field at a distance r inside the cylinder ($r < R$)

Consider an infinitely long, uniformly charged non-conducting solid cylinder of radius R, with volume charge density ρ. Consider a point S, which is present inside the cylinder. For this case, the gaussian surface will be a cylinder of radius r and length L. Now, the cylinder (gaussian surface) can be divided into three surfaces, two circular surfaces, and one curved surface as, A_1, A_2, and A_3, and their elements are marked as dA_1, dA_2, and dA_3, respectively, as shown in the figure.



For surface areas A_1 and A_2 , the angle between area vector and the electric field is 90°, so $\phi_1 = \phi_2 = 0$.

The net flux is given by,

$\phi_{net} = \phi_1 + \phi_2 + \phi_3$

$\phi_{net} = \phi_3 = \oint \vec{E}.d\vec{A}_3 = EA_3$

$\phi_{net} = E\ 2\pi rL$

The net charge enclosed by the gaussian surface is given by,

$q_{en} = \rho V = \rho\pi r^2 L$

By applying Gauss's law, we get,

$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_0}$$

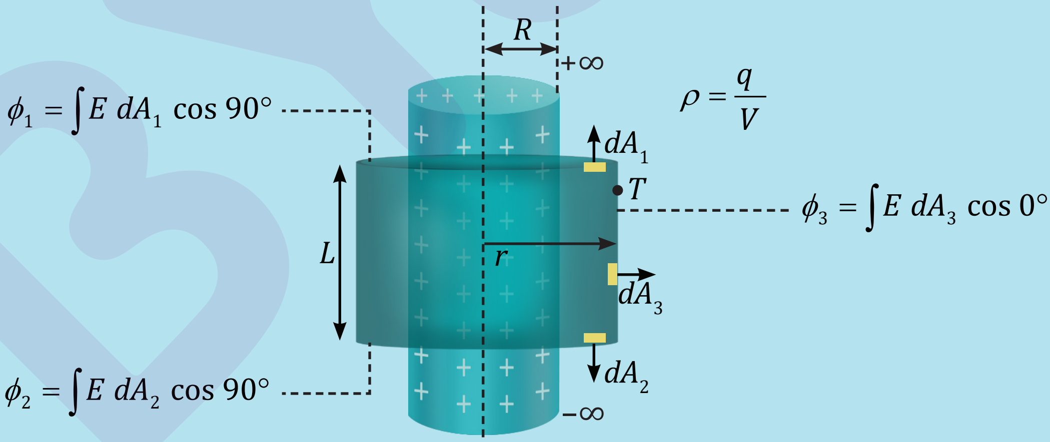
$$\Rightarrow E\oint dA = \frac{\rho\pi r^2 L}{\varepsilon_0}$$

$$\Rightarrow E\,2\pi rL = \frac{\rho\pi r^2 L}{\varepsilon_0}$$

$$\boxed{\Rightarrow E = \frac{\rho r}{2\varepsilon_0}}$$

b. The electric field at a distance *r* outside the cylinder ($r > R$)

Consider an infinitely long, uniformly charged solid cylinder of radius R, with volume charge density ρ. Consider a point T, which is present outside the cylinder. For this case, the gaussian surface will be a cylinder of radius r and length L. Now the cylinder (gaussian surface) can be divided into three surfaces, two circular surfaces, and one curved surface, A_1, A_2, and A_3, and their elements are marked as dA_1, dA_2, and dA_3, respectively, as shown in the figure.



For surface areas A_1 and A_2, the angle between area vector and the electric field is 90°, so $\phi_1 = \phi_2 = 0$.

The net flux is given by,

$\phi_{net} = \phi_1 + \phi_2 + \phi_3$

$\phi_{net} = \phi_3 = \oint \vec{E}.d\vec{A}_3 = EA_3$

$\phi_{net} = E\ 2\pi rL$

The net charge enclosed by the gaussian surface is given by,

$q_{en} = \rho V = \rho\pi R^2 L$

By applying Gauss's law, we get,

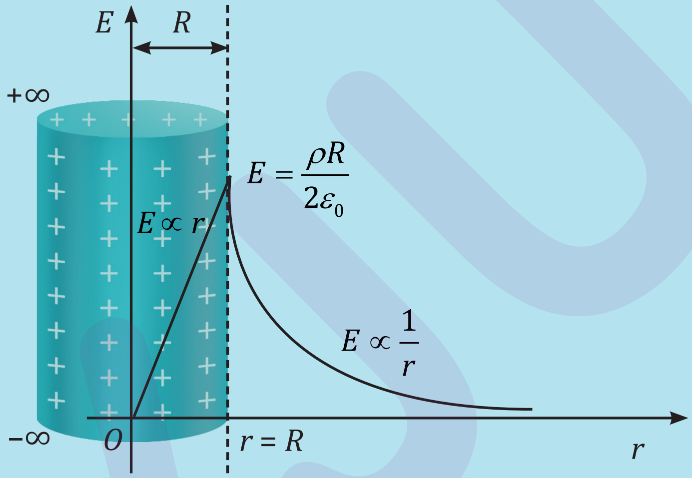
$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow E\oint dA = \frac{\rho\pi R^2 L}{\varepsilon_0}$$

$$\Rightarrow E\,2\pi rL = \frac{\rho\pi R^2 L}{\varepsilon_0}$$

$$\boxed{\Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r}}$$

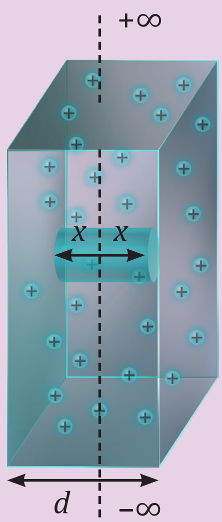
Electric field (E) vs distance (r) graph



Electric Field Due to an Infinitely Large, Uniformly Charged Non-Conducting Sheet

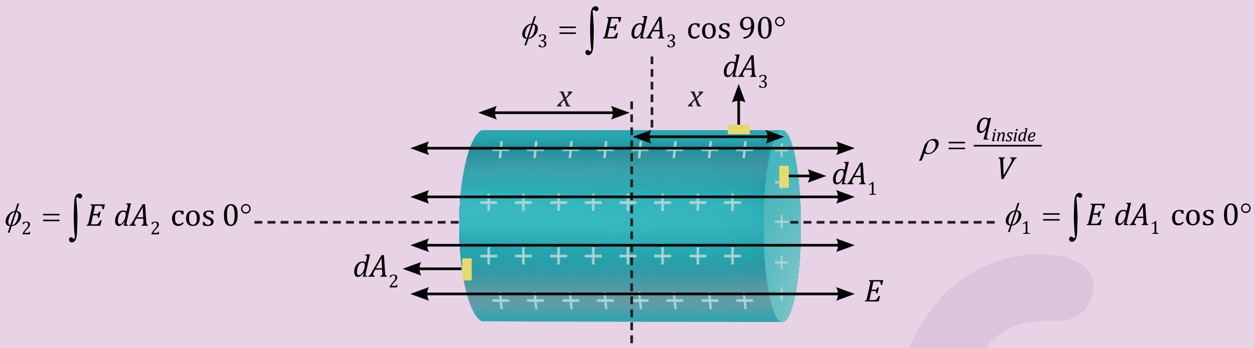
Consider a uniformly charged, non-conducting sheet of thickness d, with volume charge density ρ. To find the electric field, we have to assume a gaussian surface that is symmetrical to the charge distribution. For this case, the gaussian surface will be a cylinder whose end caps are parallel to the sheet and the axis is perpendicular to the axis of the sheet. The length of the cylinder is $2x$.

a. Electric field inside the sheet at point $x < \frac{d}{2}$



In this case, the field point lies inside the sheet at a distance of x from the axis of the sheet. For this case, the gaussian surface will be a cylinder of radius r and length $2x$. Now the cylinder (gaussian surface) can be divided into three surfaces, two circular surfaces, and one curved surface as, A_1, A_2, and A_3, and their elements are marked as dA_1, dA_2, and dA_3, respectively, as shown in the figure on the next page.

For surface area A_3, the angle between the area vector and the electric field is 90°, so $\phi_3 = 0$. Also, for surface area A_1 and A_2, the angle between the area vector and the electric field are 0°, and they are symmetrically located. Thus, the net flux passing through them will be equal, i.e., $\phi_1 = \phi_2$



The net flux is given by,

$\phi_{net} = \phi_1 + \phi_2 + \phi_3 = 2\phi_1$

$\phi_{net} = 2\oint \vec{E}.d\vec{A}_1$

$\phi_{net} = 2EA$ ($A_1 = A_2 = A$ and $\cos 0° = 1$)

The charge enclosed by the Gaussian surface is given by,

$q_{en} = \rho A(2x)$

Where, ρ is the volume charge density.

By applying Gauss's law, we get,

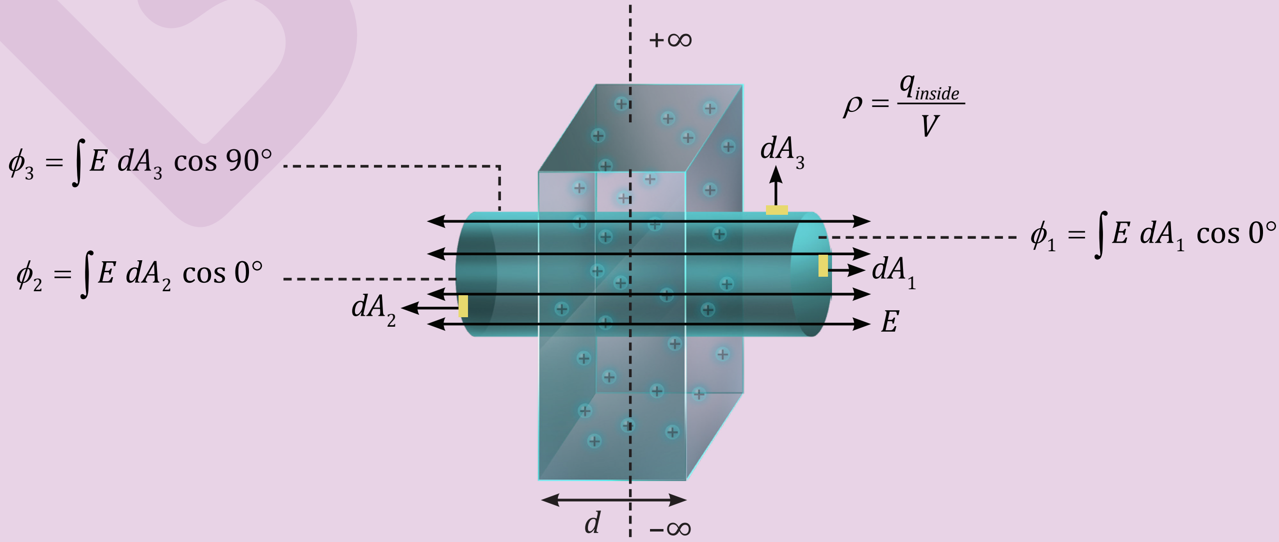
$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow 2EA = \frac{2\rho Ax}{\varepsilon_0}$$

$$\boxed{\Rightarrow E = \frac{\rho x}{\varepsilon_0}}$$

b. Electric field outside the sheet at point $x > \frac{d}{2}$

In this case, the point lies outside the sheet at a distance of x from the axis of the sheet. For this case, the gaussian surface will be a cylinder of radius r and length $2x$. Now, the cylinder (gaussian surface) can be divided into three surfaces, two circular surfaces, and one curved surface as, A_1, A_2, and A_3 and their elements are marked as dA_1, dA_2, and dA_3, respectively, as shown in the figure.



Similar to the previous case, for surface area A_3, the angle between the area vector and the electric field is 90°, so $\phi_3 = 0$. Also, for surface areas A_1 and A_2, the angle between the area vector and the electric field are 0° and they are symmetrically located. Thus, the net flux passing through them will be equal, i.e., $\phi_1 = \phi_2$.

The net flux is given by,

$\phi_{net} = \phi_1 + \phi_2 + \phi_3 = 2\phi_1$

$\phi_{net} = 2\oint \vec{E}.d\vec{A}_1$

$\phi_{net} = 2EA$ ($\because A_1 = A_2 = A$ and $\cos 0° = 1$)

The charge enclosed by the Gaussian surface is given by,

$q_{en} = \rho Ad$

Where, ρ is the volume charge density.

By applying Gauss's law, we get,

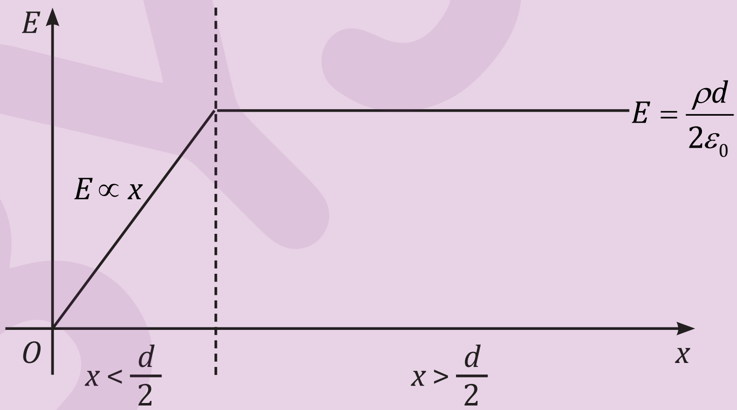
$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow 2EA = \frac{\rho Ad}{\varepsilon_0}$$

$$\boxed{\Rightarrow E = \frac{\rho d}{2\varepsilon_0}}$$

The electric field outside the sheet is constant.

Electric field (E) vs distance (x) graph

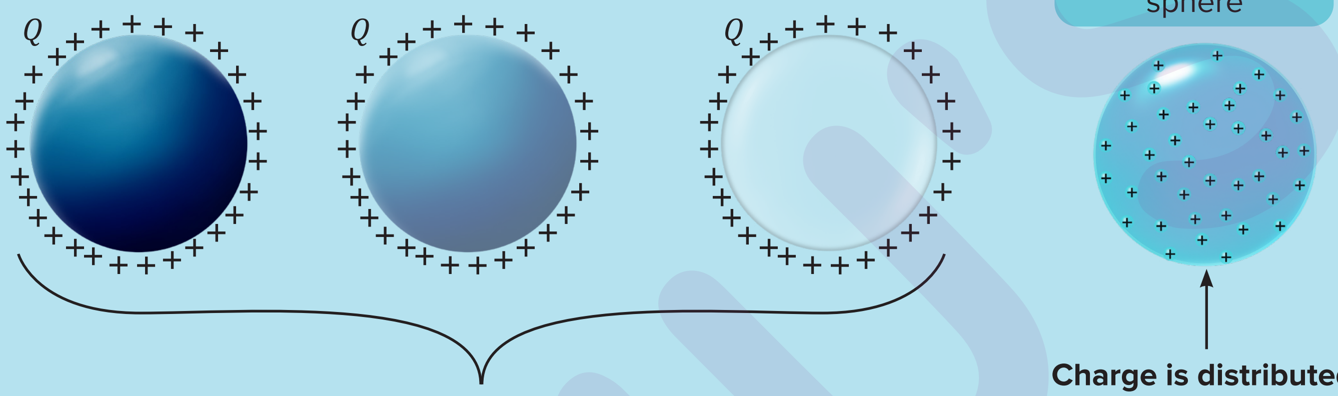


NEET

Application of Gauss's Law: Charged Spheres

Types of charged spheres

- Solid conducting Sphere
- Hollow conducting Sphere
- Thin non-conducting spherical shell
- Solid non-conducting Sphere / Dielectric sphere



Charge resides on the surface only

Charge is distributed over the volume

As in the case of cylinders, the same charge distribution happens in the case of spheres also. In the solid conducting, hollow conducting, and hollow/thin non-conducting spherical shell, the whole charge is distributed only on the surface so that we can treat them as a single case, i.e., due to the similar charge distribution, they have the same electric fields at equal distances.

Electric field due to solid conducting sphere/ hollow conducting sphere/ thin non-conducting spherical shells

Electric field due to a uniformly charged spherical shell

Let us consider a hollow sphere of radius R and charge density σ. Also, charge Q is distributed uniformly along the surface of the spherical shell.

a. Electric field inside the sphere at a distance r from the centre ($r < R$)

In this case, we can use a sphere of radius r as a Gaussian surface. The gaussian surface do not enclose any charge inside it, so the net charge enclosed is 0,

i.e., $q_{en} = 0$.

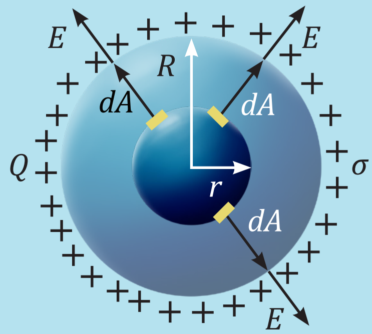
Therefore, by applying Gauss's law, we get,

$$\oint \vec{E}_{inside} \cdot d\vec{A} = \frac{q_{en}}{\varepsilon_0}$$

As the charge enclosed is 0,

$$q_{en} = 0$$

$$\therefore \vec{E}_{inside} = 0$$



b. Electric field outside the sphere at a distance r from the centre ($r > R$)

For this case as well, the Gaussian surface will be a sphere of radius r. At any given point on the sphere, the area vector and the electric field are in the same direction.

The net flux is given by,

$$\phi_{net} = \oint \vec{E}.d\vec{A}$$

$$\Rightarrow \phi_{net} = \oint E\,dA\cos\theta$$

$$\Rightarrow \phi_{net} = \oint E\,dA\cos 0^\circ$$

$$\Rightarrow \phi_{net} = E\oint dA$$

$$\Rightarrow \phi_{net} = E\,4\pi r^2$$

The charge enclosed by the Gaussian surface is given by,

$q_{en} = Q$

By applying Gauss's law, we get,

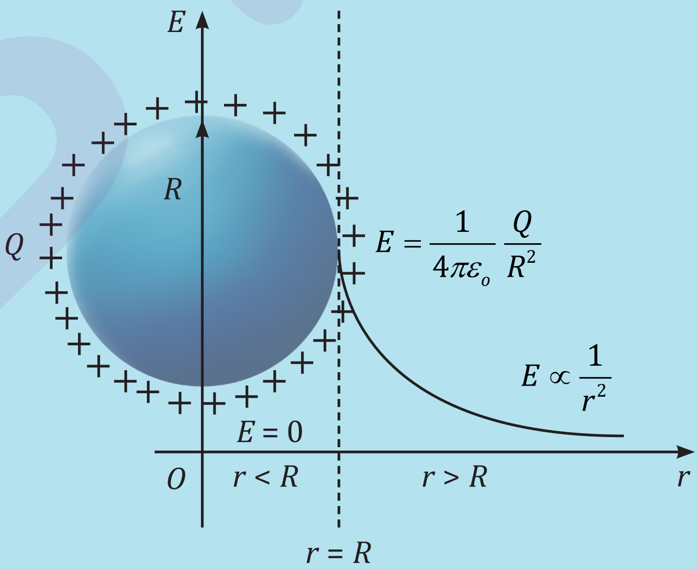
$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow E4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

Electric field at any point outside any uniform spherical symmetric charge distribution behaves as if all its charge is concentrated at the centre.

Electric field (E) vs distance (r) graph



BYJU'S Classes

PHYSICS

ELECTROSTATICS

ELECTRIC POTENTIAL

NOTES

What you already know

- Electric flux through the face of a cube
- Electric flux through the curved surface of a cylinder and a container
- Applications of Gauss's law

What you will learn

- Scalar and vector fields
- Conservative and non-conservative forces
- Electric potential
- Electric potential due to a point charge
- Electric potential for a system of charges

NEET

Application of Gauss's Law

Electric field due to a solid non-conducting uniformly charged sphere

Let us consider a uniformly charged solid non-conducting sphere of radius R with volume charge density ρ. Also, the total charge Q is distributed uniformly throughout the sphere.

1. Electric field inside the sphere at a distance r from the centre ($r < R$)

In this case, we can assume a sphere of radius r as a Gaussian surface. The angle between the area vector and the electric field is 0° throughout the periphery of the Gaussian surface, i.e., $\theta = 0°$. Thus, the magnitude of the electric flux at every point on the periphery of the Gaussian surface is the same.

The net flux through the Gaussian surface is given by,

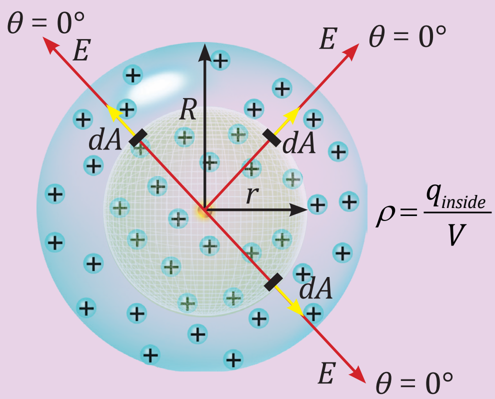
$$\phi_{net} = \oint \vec{E}.d\vec{A}$$

$$\Rightarrow \phi_{net} = \oint E\,dA\cos\theta$$

$$\Rightarrow \phi_{net} = \oint E\,dA\cos 0°$$

$$\Rightarrow \phi_{net} = E\oint dA$$

$$\Rightarrow \phi_{net} = E\left(4\pi r^2\right)$$



The net charge enclosed by the Gaussian surface is given by,

$$q_{en} = \rho V = \rho \times \frac{4}{3}\pi r^3$$

By applying Gauss's law, we get,

$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_o}$$

$$\Rightarrow E\left(4\pi r^2\right) = \frac{\rho \times \frac{4}{3}\pi r^3}{\varepsilon_o}$$

$$\Rightarrow E = \frac{\rho r}{3\varepsilon_o}$$

2. Electric field outside the sphere at distance *r* from the centre (*r* > *R*)

Similarly, in this case, we can assume a sphere of radius r as a Gaussian surface. The angle between the area vector and the electric field is 0° throughout the periphery of Gaussian surface, i.e., $\theta = 0°$. Thus, the magnitude of the electric flux at every point on the periphery of the Gaussian surface is the same.

The net flux through the Gaussian surface is given by,

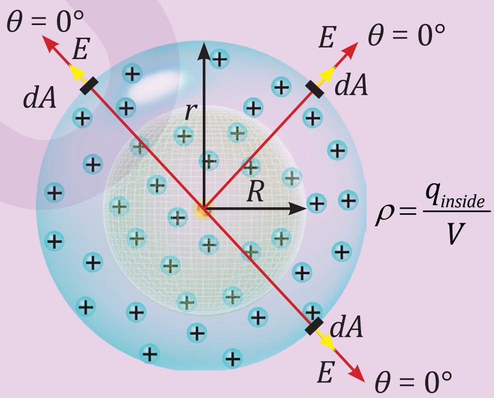
$$\phi_{net} = \oint \vec{E}.d\vec{A}$$

$$\Rightarrow \phi_{net} = \oint E\,dA \cos\theta$$

$$\Rightarrow \phi_{net} = \oint E\,dA \cos 0°$$

$$\Rightarrow \phi_{net} = E\oint dA$$

$$\Rightarrow \phi_{net} = E\left(4\pi r^2\right)$$



The net charge enclosed by the Gaussian surface is given by,

$q_{en} = Q$

By applying Gauss's law, we get,

$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_o}$$

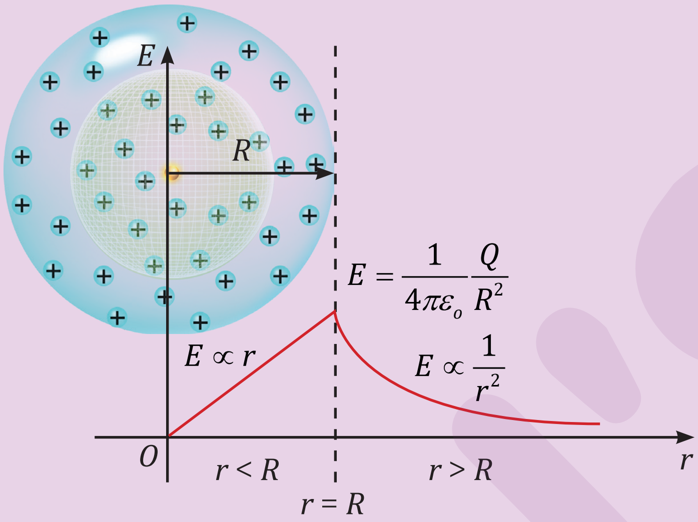
$$\Rightarrow E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_o}$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$\Rightarrow E = \frac{kQ}{r^2} \quad \left(\text{Where, } k = \frac{1}{4\pi\varepsilon_o}\right)$$

For the electric field at any point outside the uniform spherical symmetric charge distribution, it behaves as if all its charge is concentrated at the centre.

Electric field (E) vs distance (r) graph

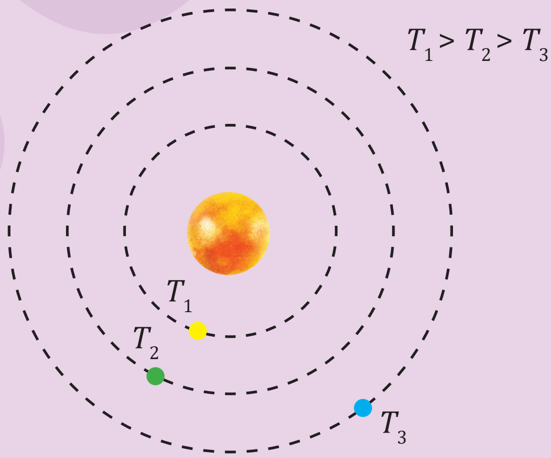


Field

A region in space where every point is characterized by a physical quantity is known as a field.

Scalar field

If the physical quantity associated with a field is a scalar quantity, then the field is known as the scalar field, i.e., a scalar field is a function that gives us a single value of some variable for every point in space.

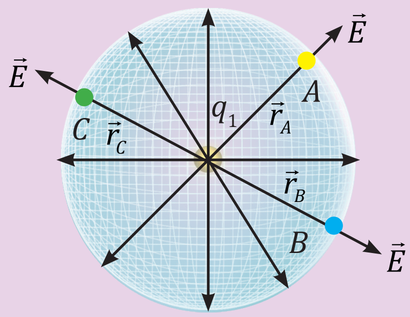


Example: Temperature field

Let us consider a heat source. The intensity of heat energy at different points around the heat source will be different as shown in the figure. Therefore, heat energy is a scalar quantity and the field associated with this is known as temperature field.

Vector field

If the physical quantity associated with a field is a vector quantity, then the field is known as the vector field.



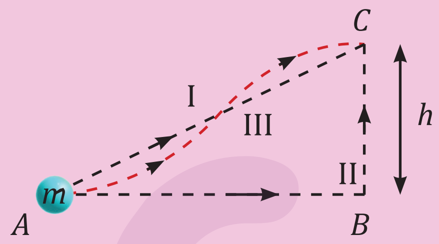
Example: Electrostatic field

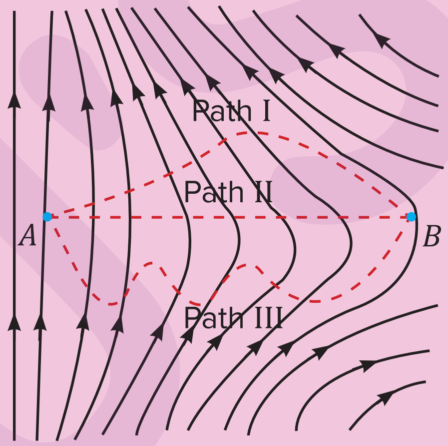
Consider a point charge $+q_1$ placed in space. The magnitude of the electric field at different points that are radially equidistant from the point charge is the same, but the direction of the electric field is different for each of them as shown in the figure.

Conservative Forces and Non-Conservative Forces

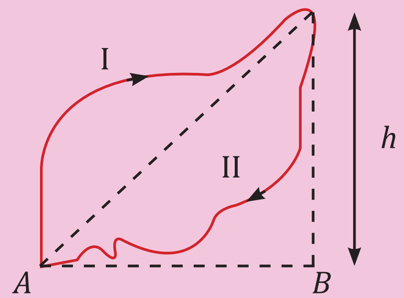
Conservative forces

1. The work done by a conservative force is independent of the path taken. It only depends on the initial and final positions.
   Examples:
   Gravitational force: To move a body of mass m to a height h, many paths can be taken, but the work done on each path will remain the same.
   $(W_g)_{\mathrm{I}} = (W_g)_{\mathrm{II}} = (W_g)_{\mathrm{III}} = -mgh$
   Electrostatic force: Electrostatic force is a conservative force as the work done to move a charge from A to B is independent of the path taken to move the charge.
   $(W_{el})_{\mathrm{I}} = (W_{el})_{\mathrm{II}} = (W_{el})_{\mathrm{III}}$
   Therefore, conservative forces are path independent.





2. A force is conservative if the work it does around any closed path is zero.
   Examples:
   Gravitational force: If a block of mass m is raised up to a height h by following path I and is brought back to the initial position following path II, then the net work done in the closed cycle is zero.
   $(W_g)_{\mathrm{I}} + (W_g)_{\mathrm{II}} = 0$



3. Potential energy can only be defined for conservative force fields.

Non-conservative forces

1. The work done by a non-conservative force depends on the path of the object. Non-conservative forces are path functions.
   Example: Friction force
2. Non-conservative forces are also known as dissipative forces because they dissipate mechanical energy into other forms.

BOARDS

Electric Potential (V)

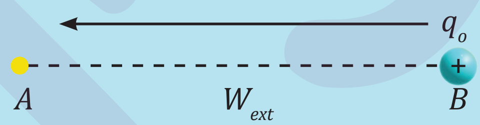
The electric field in a region of space is described by assigning a vector quantity ($\vec{E}$) at each point. Pictorially, it is represented by electric lines of force. The same electric field can also be described by assigning a scalar quantity (V) at each point known as electric potential.

Electric potential difference

The electric potential difference between two points is defined as **the work done by an external agent** in moving a unit positive charge from one point to another.

Suppose a positive charge $+q_o$ is brought from point B to point A. Thus, the potential difference between V_A and V_B is as follows:

$$V_A - V_B = \frac{W_{ext\ B \to A}}{q_o}$$

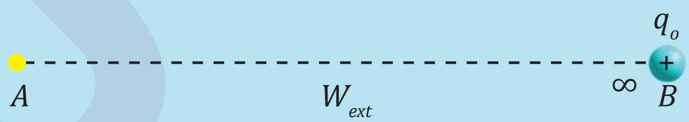


If the unit charge is, $+q_o = +1\ C$,

$$V_A - V_B = W_{ext\ B \to A}$$

If we assume that point B is at infinity and $V_\infty = 0$, then the **electric potential** at point A can be defined as the amount of work done by an external agent in moving a unit positive charge from infinity to point A.

$$V_A = \frac{W_{ext\ \infty \to A}}{q_o}$$



If the unit charge is, $+q_o = +1\ C$,

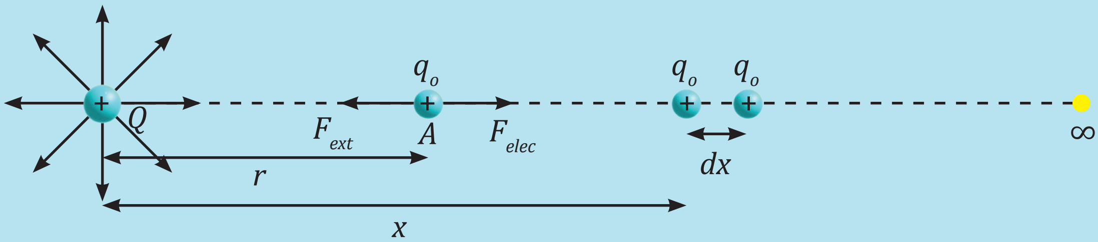
$$V_A = W_{ext\ \infty \to A}$$

The SI unit of electric potential is JC^{-1} or $Volt$.

1. The work done by the external agent is to be considered while calculating electric potential at a point.
2. The work done by the external agent is against the electric field.
3. Since electrostatic force is a conservative force, the work done by the electrostatic force is independent of its path.

Electric potential due to a point charge Q

Consider a point charge Q and a positive test charge q_o that is brought from infinity to point A by an external force. Let us assume that the test charge is brought very slowly without any change in velocity, such that, at any given instant, there is no acceleration. So, the net force acting on it at any point is zero.



Therefore, the electrostatic force is equal to the external force but in the opposite direction.

$$\left|\vec{F}_{ext}\right| = \left|\vec{F}_{elec}\right|$$

Since the electrostatic force is a conservative force and its magnitude is equal to the external force, we can consider the external force as the conservative force. The work done by the external force for moving the positive charge from A to infinity and infinity to A will be opposite to each other but equal in magnitude.

$$W_{ext_{\infty \to A}} = -W_{ext_{A \to \infty}}$$

While moving the positive charge from A to infinity, consider that the positive test charge is x distance away from charge Q at an instant. If the charge moves a small distance dx, then the work done by the external force is given by,

$$W_{ext_{A \to \infty}} = \int_r^\infty \vec{F}_{ext} . d\vec{x}$$

$$\Rightarrow W_{ext_{A \to \infty}} = \int_r^\infty \left|\vec{F}_{ext}\right| dx \cos 180°$$

$$\Rightarrow W_{ext_{A \to \infty}} = -\int_r^\infty \left|\vec{F}_{elec}\right| dx \qquad \left(\because \left|\vec{F}_{ext}\right| = \left|\vec{F}_{elec}\right| = \frac{kQq_o}{x^2}\right)$$

$$\Rightarrow W_{ext_{A \to \infty}} = -\int_r^\infty \frac{kQq_o}{x^2} dx$$

$$\Rightarrow W_{ext_{A \to \infty}} = -kQq_o \int_r^\infty x^{-2} dx$$

$$\Rightarrow W_{ext_{A \to \infty}} = -kQq_o \left[-\frac{1}{x}\right]_r^\infty \qquad \left(\text{Using} \int x^n \, dx = \frac{x^{n+1}}{n+1}\right)$$

$$\Rightarrow W_{ext_{A \to \infty}} = -kQq_o \left[-\frac{1}{\infty} - \left(-\frac{1}{r}\right)\right]$$

$$\Rightarrow W_{ext_{A \to \infty}} = -\frac{kQq_o}{r}$$

$$\Rightarrow W_{ext_{\infty \to A}} = -W_{ext_{A \to \infty}} = \frac{kQq_o}{r}$$

At point A, the electric potential is given by,

$$V_A = \frac{W_{ext_{\infty \to A}}}{q_o}$$

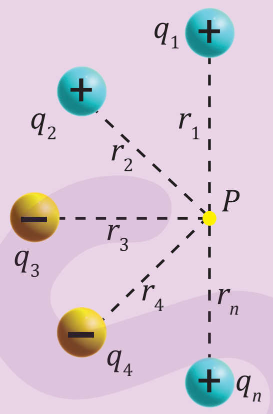
$$\Rightarrow V_A = \frac{kQq_o}{rq_o} = \frac{kQ}{r}$$

Since V is a scalar quantity, **the charge should be substituted along with its sign**. This means that a positive and a negative charge will have a positive and a negative potential, respectively, at all points.

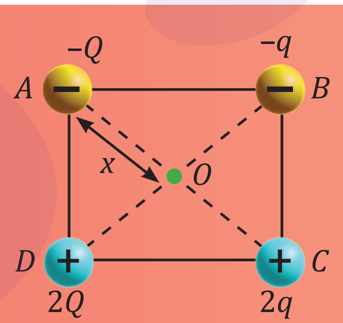
Potential Due to a System of Point Charges

If charges q_1, q_2, $-q_3$, $-q_4$, ..., q_n are placed in space at distance r_1, r_2, r_3, r_4, ..., r_n, respectively, then the value of the electric potential at point P is given by,

$$V_P = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{k(-q_3)}{r_3} + \frac{k(-q_4)}{r_4} + \ldots + \frac{kq_n}{r_n}$$



Four point charges, $-Q$, $-q$, $2q$, and $2Q$ are placed at each corner of a square. What is the relation between Q and q for which the potential at the centre of the square will be zero?



(A) $Q = -q$ (B) $Q = -\frac{1}{q}$ (C) $Q = q$ (D) $Q = \frac{1}{q}$

Solution

In a square, all the diagonals are of the same length. Therefore,

$OA = OB = OC = OD = x$

The value of the electric potential at the centre is given by,

$$V_O = \frac{k(-Q)}{x} + \frac{k(-q)}{x} + \frac{k(2Q)}{x} + \frac{k(2q)}{x}$$

$$\Rightarrow V_O = \frac{-kQ}{x} - \frac{kq}{x} + \frac{2kQ}{x} + \frac{2kq}{x}$$

$$\Rightarrow V_O = \frac{k}{x} Q + q$$

Since the potential at O is 0, we get,

$0 = Q + q$

$\Rightarrow Q = -q$

Thus, option (A) is the correct answer.

BYJU'S Classes

PHYSICS

ELECTROSTATICS

ELECTRIC POTENTIAL-2

NOTES

What you already know

- Applications of Gauss's law
- Scalar and vector field
- Conservative and non-conservative forces
- Electric potential
- Electric potential due to a point charge
- Electric potential for a system of charges

What you will learn

- Null potential points due to two point charges
- Electric potential due to extended charges, ring, and disc
- Electric potential energy for two-charge and three-charge systems

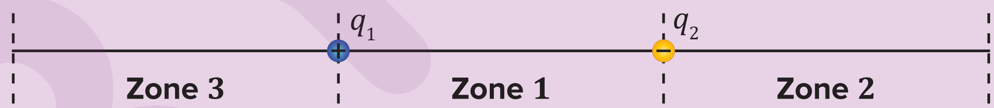
Null Potential Points Due to Two Point Charges

The point where the net potential is zero is known as a null potential point. In a **system of two like charges**, at all the points in space near the vicinity of the charges, the potential is either positive or negative. Hence, **null potential points are not possible.**

System of two unlike charges

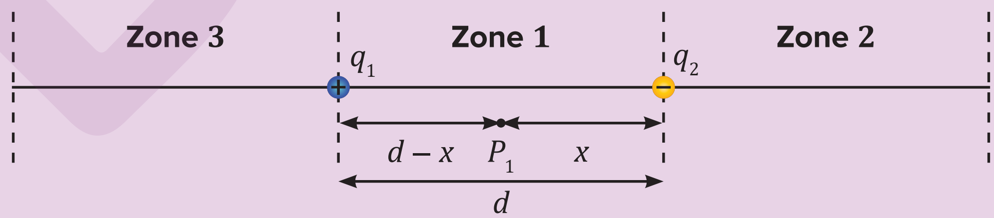
Let us consider two unlike charges, $+q_1$ and $-q_2$, where, $|q_1| > |q_2|$. Let the distance between the charges be d.

There are two points along the line joining the charges at which the potential will be zero. A null potential point will be possible near the smaller charge on either side of the line joining the two unlike charges. Let us divide the space around them into three zones, zone 1, zone 2, and zone 3, as shown in the figure. Thus, two null potential points will be possible, one in zone 1 and the other in zone 2.



Null point (P_1) in zone 1:

Let the null point be P_1, which is at a distance x from charge q_2.



The net potential at point P_1 is given by,

$$V_{P_1} = V_{+q_1} + V_{-q_2}$$

$$\Rightarrow V_{P_1} = \frac{kq_1}{(d-x)} + \frac{k(-q_2)}{x}$$

$$\Rightarrow V_{P_1} = \frac{kq_1}{(d-x)} - \frac{kq_2}{x}$$

For the net potential to be zero,

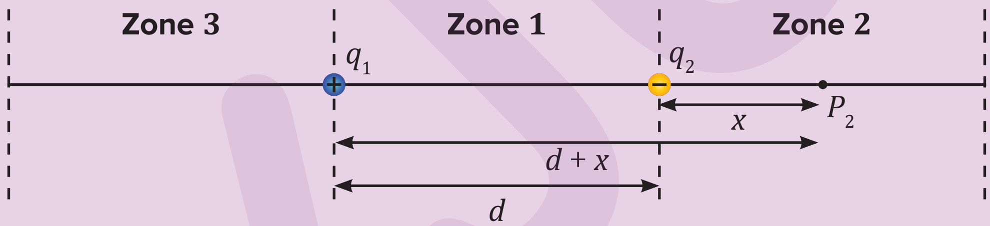
$$\Rightarrow V_{P_1} = \frac{kq_1}{(d-x)} - \frac{kq_2}{x} = 0$$

$$\Rightarrow q_1 x = q_2 (d-x)$$

$$\Rightarrow x = \frac{d}{\frac{q_1}{q_2} + 1}$$

Null point (P_2) in zone 2:

Let the null point be P_2, which is at a distance x from charge q_2.



The net potential at point P_2 is given by,

$$V_{P_2} = V_{+q_1} + V_{-q_2}$$

$$\Rightarrow V_{P_2} = \frac{kq_1}{(d+x)} + \frac{k(-q_2)}{x}$$

$$\Rightarrow V_{P_2} = \frac{kq_1}{(d+x)} - \frac{kq_2}{x}$$

For the net potential to be zero,

$$\Rightarrow V_{P_2} = \frac{kq_1}{(d+x)} - \frac{kq_2}{x} = 0$$

$$\Rightarrow q_1 x = q_2 (d+x)$$

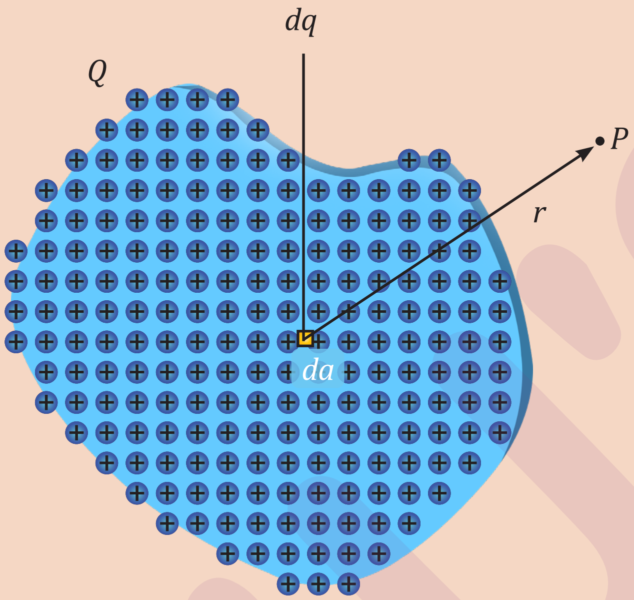
$$\Rightarrow x = \frac{d}{\frac{q_1}{q_2} - 1}$$

The general relation for null potential points along the line joining the two unlike charges is,

$$\boxed{x = \frac{d}{\frac{q_1}{q_2} \pm 1}}$$

Electric Potential Due to Extended Charges

Let us consider an extended body of area A and charge Q is distributed uniformly over the area. Let us consider a small element of the body having area da and charge dq.



The steps for obtaining the potential due to the extended charge systems are given as follows:

Step 1:

Find the small charge (charge of the element) according to the surface taken for study.

$$dq = \frac{Q}{A} da$$

Step 2:

Find the electric potential due to the small charge dq.

$$dV = \frac{k\,dq}{r}$$

Step 3:

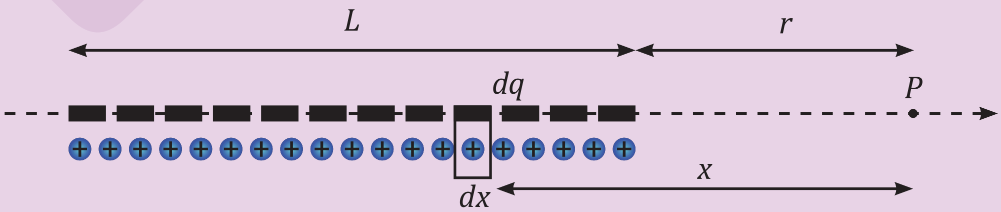
Substitute the value of dq in dV.

Step 4:

Integrate the potential dV with proper limits to obtain the net electric potential due to the extended charge system.

Electric Potential Due to a Uniformly Charged Rod at an Axial Point

Consider a uniformly charged rod of length L and charge Q. To find the electric potential at point P, which is at a distance r from one end of the rod, let us consider a small element dx having a small charge dq at a distance x from point P as shown in the figure.



The small charge on the elemental length can be written as follows:

$$dq = \frac{Q}{L} dx$$

The potential due to the small charge dq at point P is given by,

$$dV = \frac{k\,dq}{x} \qquad \cdots(i)$$

By substituting the value of dq in equation (i), we get,

$$dV = \frac{kQ\,dx}{Lx}$$

Integrating both the sides, we get,

$$\int dV = \frac{kQ}{L}\int \frac{dx}{x}$$

The limits of x will be from $x = r$ to $x = r + L$.

$$V = \frac{kQ}{L}\int_{r}^{r+L} \frac{dx}{x}$$

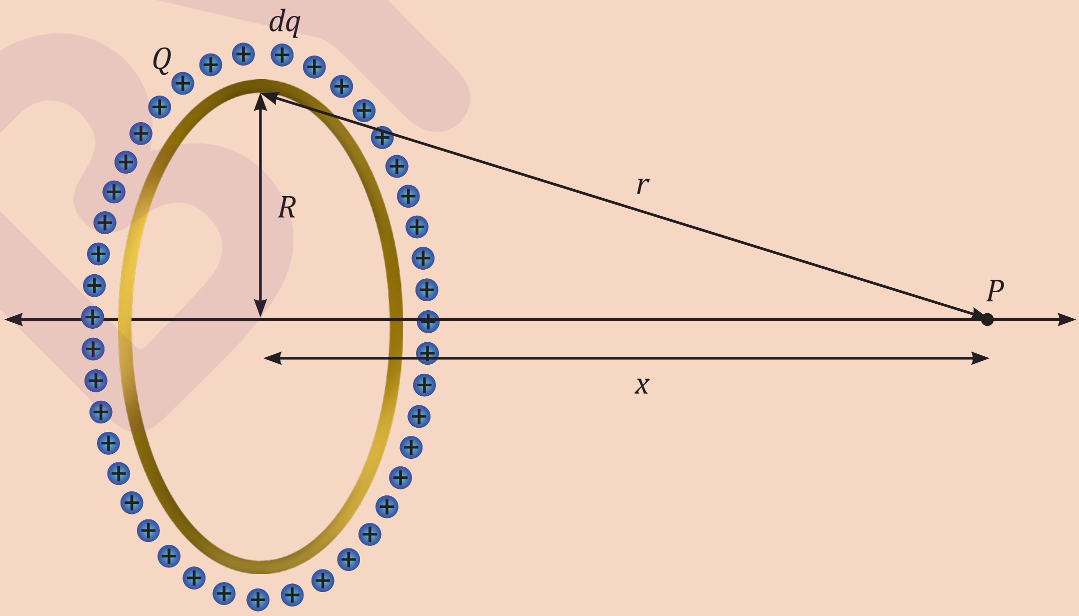
$$\Rightarrow V = \frac{kQ}{L}\left[\ln x\right]_{r}^{r+L}$$

$$\Rightarrow V = \frac{kQ}{L}\left[\ln(r+L) - \ln r\right]$$

$$\boxed{\Rightarrow V = \frac{kQ}{L}\ln\left(\frac{r+L}{r}\right)}$$

Electric Potential Due to a Uniformly Charged Ring

Consider a uniformly charged ring of radius R and charge Q. To find the electric potential along the axis of the ring at point P, which is at a distance x from the centre of the ring, let us consider a small element of the ring having length dx charge dq at a distance r from point P as shown in the figure.



The potential due to the small charge dq at point P is given by,

$$dV = \frac{k\,dq}{r}$$

$$\Rightarrow dV = \frac{k\,dq}{\sqrt{x^2 + R^2}} \qquad \left(\because r = \sqrt{x^2 + R^2}\right)$$

Here, $\sqrt{x^2 + R^2}$ is a constant at every element of the ring.

By integrating both the sides, we get,

$$\int dV = \frac{k}{\sqrt{x^2 + R^2}} \int dq$$

$$\boxed{\Rightarrow V = \frac{kQ}{\sqrt{x^2 + R^2}}}$$

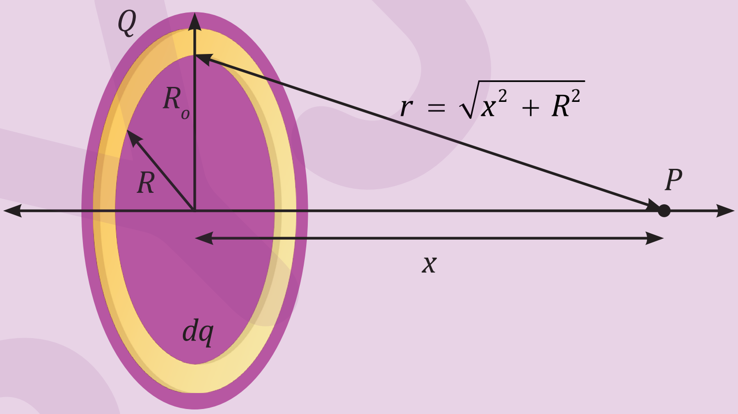
At centre, $x = 0$,

$$V_C = \frac{kQ}{R}$$

BOARDS

Electric Potential Due to a Uniformly Charged Disc

Consider a uniformly charged disc of radius R_o and charge Q. The disc is the combination of infinitesimally thin coaxial rings. To find the electric potential along the axis of the disc at point P, which is at a distance x from the centre of the disc, let us consider an element of the disc, which is nothing but an infinitesimally thin ring of radius R, having thickness dr and charge dq at a distance r from point P as shown in the figure.



The charge of each thin infinitesimal ring is given by,

$$dq = \frac{Q}{\pi R_o^2} 2\pi R\,dR$$

The potential at point P due to charge dq is given by,

$$dV = \frac{k\,dq}{r}$$

$$\Rightarrow dV = \frac{k\,dq}{\sqrt{x^2 + R^2}} \qquad \left(\because r = \sqrt{x^2 + R^2}\right) \quad ...(i)$$

By substituting the value of dq in equation (i), we get,

$$\Rightarrow dV = \frac{kQ2\pi R\,dR}{\pi R_o^2\sqrt{x^2+R^2}}$$

By integrating both the sides, we get,

$$\int dV = \frac{kQ}{R_o^2}\int\frac{2R\,dR}{\sqrt{x^2+R^2}}$$

The limits of R will be from $R = 0$ to $R = R_o$.

$$V = \frac{kQ}{R_o^2}\int_0^{R_o}\frac{2R\,dR}{\sqrt{x^2+R^2}} \qquad ....(ii)$$

Take $\left(x^2+R^2\right) = P \qquad ....(iii)$

By differentiating partially with respect to R, we get,

$2R\,dR = dP \qquad ....(iv)$

By substituting equations (iii) and (iv) in equation (ii), we get,

$$V = \frac{kQ}{R_o^2}\int_0^{R_o}\frac{dP}{P^{\frac{1}{2}}}$$

$$\Rightarrow V = \frac{kQ}{R_o^2}\left[2\sqrt{P}\right]_0^{R_o}$$

$$\Rightarrow V = \frac{kQ}{R_o^2}\left[2\sqrt{x^2+R^2}\right]_0^{R_o}$$

$$\Rightarrow V = \frac{kQ}{R_o^2}\left[2\sqrt{x^2+R_o^2}-2x\right]$$

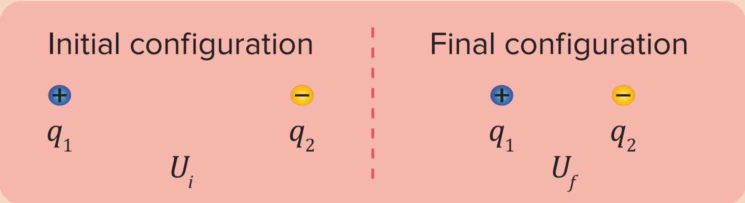
$$\boxed{\Rightarrow V = \frac{2kQ}{R_o^2}\left[\sqrt{x^2+R_o^2}-x\right]}$$

Electric Potential Energy (U)

Recall what we have learnt about potential energy till now. We cannot define absolute potential energy. However, what we can define is the change in potential energy.

The change in potential energy is the **negative of the work done by the conservative force,** as the system changes from the initial to the final configuration. Mathematically, the change in the potential energy is defined as, $\Delta U = U_f - U_i = (-W_{conservative\ force})_{i\rightarrow f}$

Let us consider that two charges $+q_1$ and $-q_2$ are initially separated by some distance, and are finally brought close to each other as shown in the figure.



Thus, the change in potential energy of the system is given by,

$\Delta U = U_f - U_i = (-W_{conservative\ force})_{i \to f}$

Now, suppose that those two charges are brought close to each other because of the work done (W_{ext}) by some external agent. According to the work-energy theorem,

$W_{ext} + W_{el} = \Delta(K.E.)$

Assuming that the charges are brought very slowly without changing their speeds, it can be said that, $\Delta(K.E.) = 0$.

Therefore,

$W_{ext} + W_{el} = 0$

$\Rightarrow W_{ext} = -W_{el}$

Thus, the change in potential energy of the system is given by,

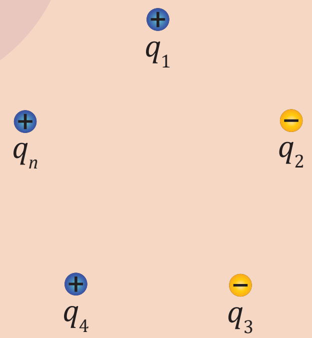
$\Delta U = W_{ext}$, if and only if $\Delta(K.E.) = 0$

It is very important to note that **potential energy can be defined only for conservative force fields**. Thus, the change in potential energy of a system in a conservative force field does not depend on the path through which the system changes from the initial to the final configuration. It is because **the change in potential energy is not a path function but a state function,** i.e., it depends only on the initial and final states of the system.

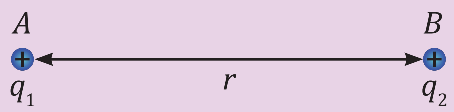
Definition

Electric potential energy is defined as the amount of work done in assembling a system of charges against the electric forces of the system by bringing individual charges from infinity to their respective positions in the system.

That is, initially, all these charges are at the infinite separation between them, and finally they are brought to the required configuration shown in the figure.



Electric Potential Energy of a Two-Charge System



Let us consider two like charges $+q_1$ and $+q_2$. Initially, they are infinitely separated and are finally brought to the configuration as shown in the figure, i.e., charges $+q_1$ and $+q_2$ are separated by a distance r. Let us assume that the electric potential energy at infinity is zero, i.e., $U_i = 0$

For bringing the first charge $+q_1$ from infinity to point A, there is no charge present near the vicinity. So, the work done against the electrostatic force is zero.

$W^{q_1}_{\infty \to A} = 0$

We know that due to a point charge, at all the points in space near the vicinity of the charges, the potential is either positive or negative. Thus, at point B, the potential due to $+q_1$ is given by,

$V_B = \dfrac{kq_1}{r}.$

For bringing the second charge q_2 from infinity to point B, which is at a distance r from point A, the work done is given by,

$$W^{q_2}_{\infty \to B} = q_2 V_B$$

$$\Rightarrow W^{q_2}_{\infty \to B} = \frac{kq_1q_2}{r}$$

We know that the change in electric potential energy is given by,

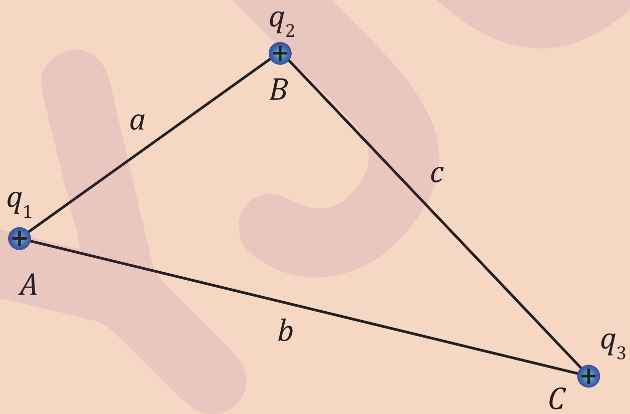
$$\Delta U = U_f - U_i = W^{q_1}_{\infty \to A} + W^{q_2}_{\infty \to B}$$

$$\Rightarrow U_f - 0 = 0 + \frac{kq_1q_2}{r}$$

$$\boxed{\Rightarrow U_f = \frac{kq_1q_2}{r}}$$

Electric Potential Energy of a Three-Charge System

Let us consider three like charges $+q_1$, $+q_2$, and $+q_3$. Initially, they are infinitely separated and are finally brought to the configuration as shown in the figure. Let us assume that the electric potential energy at infinity is zero, i.e., $U_i = 0$.



For bringing the first charge $+q_1$ from infinity to point A, there is no charge present near the vicinity. So, the work done against the electrostatic force is zero.

$$W^{q_1}_{\infty \to A} = 0$$

We know that due to a point charge at all the points in space near the vicinity of the charges, the potential is either positive or negative. Thus, at point B, the potential due to $+q_1$ is given by,

$$V_B = \frac{kq_1}{a}$$

For bringing the second charge $+q_2$ from infinity to point B, which is at a distance a from point A, the work done is given by,

$$W^{q_2}_{\infty \to B} = q_2 V_B$$

$$\Rightarrow W^{q_2}_{\infty \to B} = \frac{kq_1q_2}{a}$$

For bringing the third charge $+q_3$ from infinity to point C, which is at distance b and c from $+q_1$ and $+q_2$, respectively, the work done is given by,

$$W^{q_3}_{\infty \to C} = q_3 V_C$$

$$\Rightarrow W^{q_3}_{\infty \to C} = q_3\left(\frac{kq_1}{b} + \frac{kq_2}{c}\right) = \frac{kq_1q_3}{b} + \frac{kq_2q_3}{c}$$

The change in the electric potential energy is given by,

$$\Delta U = U_f - U_i = W^{q_1}_{\infty \to A} + W^{q_2}_{\infty \to B} + W^{q_3}_{\infty \to C}$$

$$\Rightarrow U_f - 0 = 0 + \frac{kq_1q_2}{a} + \frac{kq_1q_3}{b} + \frac{kq_2q_3}{c}$$

$$\boxed{\Rightarrow U_f = \frac{kq_1q_2}{a} + \frac{kq_1q_3}{b} + \frac{kq_2q_3}{c}}$$

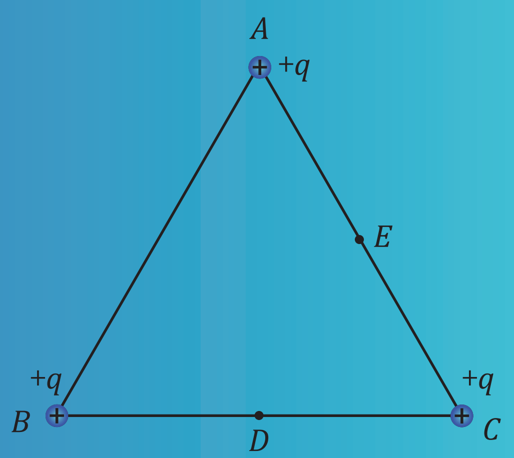
From the given relation, it is clear that the potential energy of a system is the sum of the potential energies of all the possible pairs of charges in the system (without repeating).

For a system having n charges, the number of pairs are given by,

$$N = \frac{n(n-1)}{2}$$

The work done in moving a charge between two points having the same potential is zero.
$W = q\Delta V = 0$ (since $\Delta V = 0$)

Three charges, each $+q$, are placed at the corners of an isosceles triangle ABC of sides BC and AC as $2a$. D and E are the midpoints of BC and CA. Find the work done in taking a charge Q from D to E.



(A) $\frac{3qQ}{\pi\varepsilon_o a}$ (B) $\frac{3qQ}{8\pi\varepsilon_o a}$ (C) $\frac{qQ}{4\pi\varepsilon_o a}$ (D) Zero

Solution

Due to the symmetric charge configuration,

Potential at D = Potential at E

$V_D = V_E$

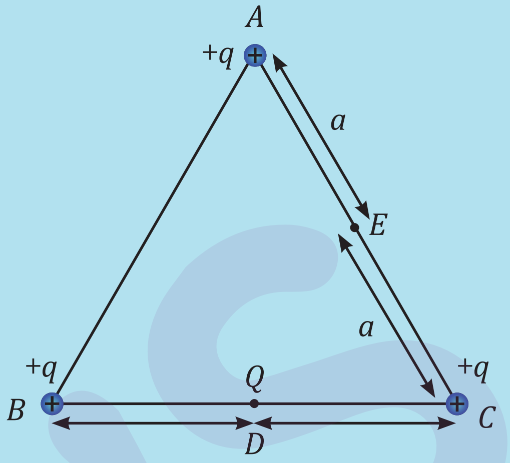
The change in the potential energy is given by,

$\Delta U = U_E - U_D = W_{Ext\,D \to E}$

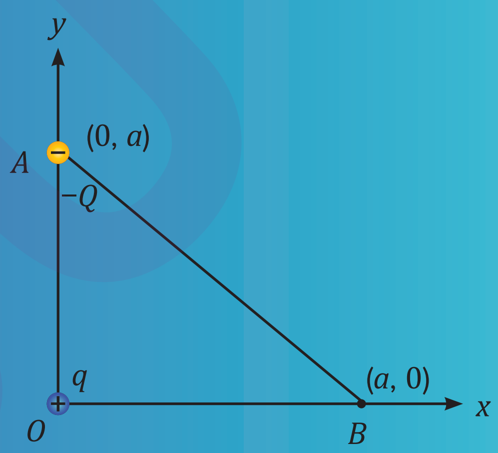
$\Rightarrow U_E - U_D = Q(V_E - V_D) = 0$

$\Rightarrow W_{Ext\,D \to E} = 0$

Thus, option (D) is the correct answer.



As per the diagram, a point charge $+q$ is placed at the origin O. Find the work done in taking another point charge $-Q$ from point A [coordinates (0, a)] to another point B [coordinates (a, 0)] along the straight path AB.



(A) Zero

(B) $\left(\frac{qQ}{4\pi\varepsilon_o}\frac{1}{a^2}\right)\sqrt{2}a$

(C) $\left(\frac{-qQ}{4\pi\varepsilon_o}\frac{1}{a^2}\right)\sqrt{2}a$

(D) $\left(\frac{qQ}{4\pi\varepsilon_o}\frac{1}{a^2}\right)\frac{a}{\sqrt{2}}$

Solution NEET

The potential at point A is given by, $V_A = \frac{kq}{a}$.

The potential at point B is given by, $V_B = \frac{kq}{a}$.

$\Rightarrow V_A = V_B$

The change in the potential energy is given by,

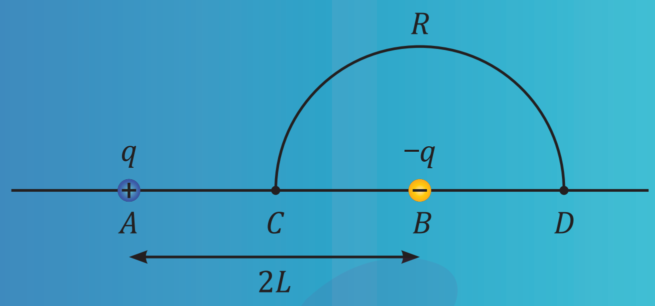
$\Delta U = U_B - U_A = W_{Ext\,A \to B}$

$\Rightarrow U_B - U_A = Q(V_B - V_A) = 0$

$\Rightarrow W_{Ext\,A \to B} = 0$

Thus, option (A) is the correct answer.

Charges $+q$ and $-q$ are placed at points A and B, respectively, which are at a distance $2L$ from each other. C is the midpoint between A and B. Find the work done in moving a charge $+Q$ along the semicircle CRD.



(A) $\dfrac{qQ}{2\pi\varepsilon_o L}$ (B) $\dfrac{qQ}{6\pi\varepsilon_o L}$ (C) $\dfrac{-qQ}{6\pi\varepsilon_o L}$ (D) $\dfrac{qQ}{4\pi\varepsilon_o L}$

Solution NEET

The potential at point C due to both the charges is given by,

$$V_C = \frac{kq}{L} - \frac{kq}{L} = 0$$

The potential at point D is given by,

$$V_D = \frac{kq}{3L} - \frac{kq}{L} = -2\frac{kq}{3L}$$

$$\Rightarrow V_D = -\frac{q}{6\pi\varepsilon_o L}$$

The net work done by the external force is given as follows:

$$\Delta U = U_D - U_C = W_{Ext\,C\to D}$$

$$\Rightarrow W_{Ext\,C\to D} = U_D - U_C = Q(V_D - V_C)$$

$$\Rightarrow W_{Ext\,C\to D} = Q\left(-\frac{q}{6\pi\varepsilon_o L} - 0\right)$$

$$\Rightarrow W_{Ext\,C\to D} = -\frac{qQ}{6\pi\varepsilon_o L}$$

Thus, option (C) is the correct answer.

BYJU'S Classes

PHYSICS

ELECTROSTATICS

ELECTRIC POTENTIAL - 3

NOTES

What you already know

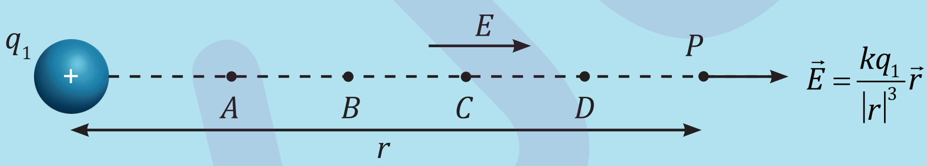
- Null potential points due to two point charges
- Electric potential due to extended charges, rings, discs
- Electric potential energy for two and three charge systems

What you will learn

- Relation between electric field and electric potential
- Calculation of electric potential from electric field and vice versa
- Electric potential due to a uniformly charged sphere and concentric shells

Relation between Electric Field and Electric Potential

Let us consider a point charge $+q_1$. Now, take a point P at a distance r from the charge.



At point P, the value of the electric field is given by,

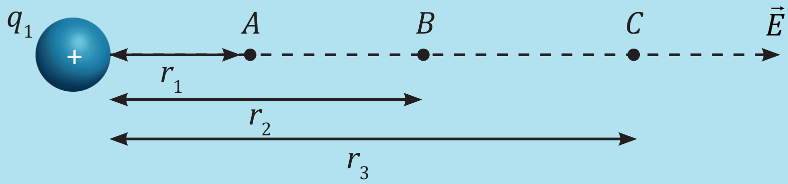
$$\vec{E}=\frac{kq_1}{|r|^3}\vec{r}$$

The direction of the electric field is away from the point charge.
Also, at point P, the value of the electric potential is given by,

$$V=\frac{kq_1}{r}$$

Since it is a scalar quantity, it does not have any direction. We can observe that the electric potential is inversely proportional to the distance from the charges.
Let us take three points A, B, and C, where A is near to charge $+q_1$ and C is far away from the charge along the line joining the charge and point P.

The electric potential at the three points is different as they are present at different distances from $+q_1$. Also, as the distance increases, the electric potential decreases.



Therefore,

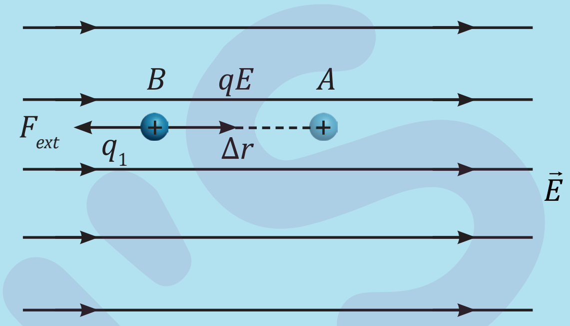
$$V_A > V_B > V_C$$

$$\frac{kq_1}{r_1} > \frac{kq_1}{r_2} > \frac{kq_1}{r_3}$$

Hence, we can conclude that the electric potential decreases along the direction of the electric field.

Case 1: When the electric field is uniform

Let us consider that a point charge $+q_1$ is placed in a uniform electric field. Thus, it experiences electrostatic force qE along the direction of the electric field. It is moved from point A to point B in the electric field very slowly without changing its speed by applying an external force equal in magnitude, which is opposite in direction as shown in the figure. While moving charge $+q_1$ from point A to point B for the entire path, the change in kinetic energy is zero ($\Delta KE = 0$).



The value of the external force is equal to the electrostatic force but is opposite in direction.

$$\vec{F}_{ext} = -q\vec{E}$$

The potential difference between points A and B is given by,

$$V_B - V_A = \frac{W_{ext\,A\to B}}{q_1} \quad ......(i)$$

The work done by the external force is given by,

$$W_{ext\,A\to B} = \vec{F}_{ext} \,.\, \Delta\vec{r}$$

$$\Rightarrow W_{ext\,A\to B} = -q_1\left(\vec{E}.\Delta\vec{r}\right)$$

$$\Rightarrow \frac{W_{ext\,A\to B}}{q_1} = -\left(\vec{E}.\Delta\vec{r}\right) \quad ......(ii)$$

From equations (i) and (ii), we get,

$$\Rightarrow V_B - V_A = \frac{W_{ext\,A\to B}}{q_1} = -\left(\vec{E}.\Delta\vec{r}\right)$$

$$\Rightarrow \Delta V = -\left(\vec{E}.\Delta\vec{r}\right)$$

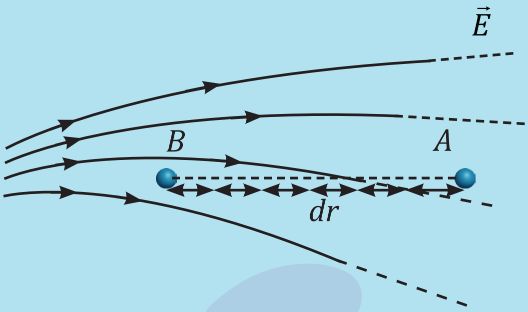
The negative sign shows that along the direction of electric field, the electric potential is decreasing. In other words, the electric field is directed from high potential to low potential.

Case 2: When the electric field is non-uniform

In case of a non-uniform electric field, we have to divide the whole path from point A to B into small lengths dr. For this small length dr, the electric field is almost uniform.

Therefore, the small potential difference is given by,

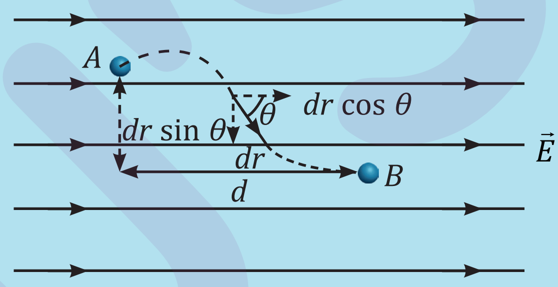
$$dV = -\vec{E}.d\vec{r}$$



The total potential difference is given by,

$$\int_A^B dV = -\int_A^B \vec{E}.d\vec{r}$$

Case 3: When the electric field is uniform but charges are moving in a random path



Consider that charge q_1 is placed in a uniform electric field. The charge is moved from A to B along an arbitrary path as shown in the figure. Let the separation between point A and point B be d.

The total path between A and B is broken into small lengths $d\vec{r}$. Resolve $d\vec{r}$ such that it is parallel and perpendicular to the electric field, i.e., $dr\cos\theta$ is along the electric field and $dr \sin\theta$ is perpendicular to the electric field.

Thus, the electric potential is given by,

$$V_B - V_A = -\int_A^B \vec{E}.d\vec{r}$$

$$\Rightarrow V_B - V_A = -\int_0^d E\, dr\cos\theta$$

$$\Rightarrow V_B - V_A = -E\int_0^d dr\cos\theta$$

If $dr\cos\theta$ is along $\vec{E}$, then the electric potential is given by,

$$\Rightarrow V_B - V_A = -Ed$$

If $dr\cos\theta$ is opposite to $\vec{E}$, then the electric potential is given by,

$$\Rightarrow V_B - V_A = Ed$$

Calculation of Potential From Electric Field

For calculating the value of potential, we have to take $\vec{E}$ and $d\vec{r}$ in a Cartesian coordinate system.

Let,

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

And, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

The potential difference is given by,

$$dV = -\vec{E}.d\vec{r}$$

$$\Rightarrow dV = -\left[\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right).\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)\right]$$

$$\Rightarrow dV = -E_x dx - E_y dy - E_z dz$$

Calculation of Electric Field From Potential

The potential is given in the following form:

$$dV = -E_x dx - E_y dy - E_z dz$$

On partially differentiating with respect to x, we get,

$$\frac{\partial V}{\partial x} = -E_x \ (\because dy = 0 \text{ and } dz = 0)$$

Similarly, on partially differentiating with respect to y and z, we get,

$$\frac{\partial V}{\partial y} = -E_y (\because dx = 0 \text{ and } dz = 0)$$

$$\frac{\partial V}{\partial z} = -E_z \ (\because dx = 0 \text{ and } dy = 0)$$

The electric field is given by,

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$\Rightarrow \vec{E} = \left(-\frac{\partial V}{\partial x}\right)\hat{i} + \left(-\frac{\partial V}{\partial y}\right)\hat{j} + \left(-\frac{\partial V}{\partial z}\right)\hat{k}$$

Therefore,

$$dV = |\vec{E}| = \sqrt{\left(-\frac{\partial V}{\partial x}\right)^2 + \left(-\frac{\partial V}{\partial y}\right)^2 + \left(-\frac{\partial V}{\partial z}\right)^2}$$

The electric field is the negative gradient of the electric potential.

The electric potential at a point (x, y, z) is given by, $V = -x^2y - xz^3 + 4$. At that point, what is the electric field?

(A) $\vec{E} = \hat{i}\,2xy + \hat{j}(x^2 + y^2) + \hat{k}(3xz - y^2)$

(B) $\vec{E} = \hat{i}\,z^3 + \hat{j}\,xyz + \hat{k}\,z^2$

(C) $\vec{E} = \hat{i}(2xy + z^3) + \hat{j}(xy^2) + \hat{k}(3z^2x)$

(D) $\vec{E} = \hat{i}(2xy + z^3) + \hat{j}(x^2) + \hat{k}(3xz^2)$

Solution NEET

Given,

$$V = -x^2y - xz^3 + 4$$

On partially differentiating with respect to x, y, and z, we get,

$$\frac{\partial V}{\partial x} = -2xy - z^3 + 0 = -2xy - z^3$$

$$\frac{\partial V}{\partial y} = -x^2 + 0 + 0 = -x^2$$

$$\frac{\partial V}{\partial z} = 0 - 3xz^2 + 0 = -3xz^2$$

The electric field is given by,

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$\Rightarrow \vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

By substituting the values of $\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial y}$, and $\frac{\partial V}{\partial z}$ in the given equation, we get,

$$\Rightarrow \vec{E} = -\left(\left(-2xy - z^3\right)\hat{i} + \left(-x^2\right)\hat{j} + \left(-3xz^2\right)\hat{k}\right)$$

$$\Rightarrow \vec{E} = \hat{i}\left(2xy + z^3\right) + \hat{j}\left(x^2\right) + \hat{k}\left(3xz^2\right)$$

Thus, option (D) is the correct answer.

In a region, the potential is represented by, $V(x, y, z) = 6x - 8xy - 8y + 6yz$, where, V is in volts and $x, y,$ and z are in meters. What is the electric force experienced by a charge of 2 coulombs situated at point (1, 1, 1)?

(A) $6\sqrt{5}\,N$ (B) $30\,N$ (C) $24\,N$ (D) $4\sqrt{35}\,N$

Solution NEET

Given,

$$V(x, y, z) = 6x - 8xy - 8y + 6yz$$

On partially differentiating with respect to x, y, and z, we get,

$$E_x = -\frac{\partial V}{\partial x} = -(6 - 8y)$$

$$\Rightarrow E_{x|(1,1,1)} = 2$$

$$E_y = -\frac{\partial V}{\partial y} = -(0 - 8x - 8 + 6z)$$

$$\Rightarrow E_{y|(1,1,1)} = 10$$

$$E_z = -\frac{\partial V}{\partial z} = -(6y)$$

$$\Rightarrow E_{z|(1,1,1)} = -6$$

The magnitude of the electric field is given by,

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

$$\Rightarrow |\vec{E}| = \sqrt{4 + 100 + 36}$$

$$\Rightarrow |\vec{E}| = 2\sqrt{35}\ NC^{-1}$$

The electric force is given by,

$$|\vec{F}| = q|\vec{E}| = 2 \times 2\sqrt{35}$$

$$\Rightarrow |\vec{F}| = 4\sqrt{35}\ N$$

Thus, option (D) is the correct answer.

Electric Potential Due to Uniformly Charged Spheres

In the last session, we discussed four kinds of charged spheres such as a solid conducting sphere; a thin, hollow conducting sphere; a solid non-conducting sphere; and a thin, hollow non-conducting sphere. Out of these four spheres, only in the solid non-conducting sphere, the charge is distributed throughout the sphere. However, for the other three types, the charge is distributed uniformly only over the surface so that we can treat them as a single case. Due to similar charge distribution, they will have the same electric fields and electric potentials at equal distances.

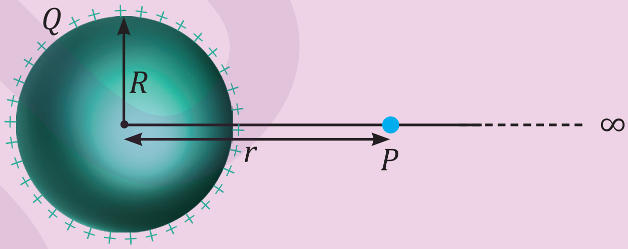
Potential due to solid conducting sphere; thin, hollow conducting sphere; thin, hollow non-conducting sphere

Here, we can take any of the three types of spheres, i.e., solid conducting, hollow conducting, or hollow non-conducting. For our study, we are taking a thin, hollow conducting sphere.

Case 1: Outside the sphere ($r > R$)

Let us consider a thin, hollow conducting sphere of radius R, having a charge Q uniformly distributed on it. Now, consider point P outside the sphere at a distance r from the sphere.

We know that an electric field at any point outside the uniform spherical symmetric charge distribution behaves as if all its charge is concentrated at its centre.



Therefore, the electric field at P is given by,

$$E_P = \frac{kQ}{r^2}$$

The electric potential difference is given by,

$$V_P - V_\infty = -\int_\infty^r \vec{E}.d\vec{r}$$

$$\Rightarrow V_P - 0 = -\int_\infty^r \frac{kQ}{r^2} dr$$

$$\Rightarrow V_P = \frac{kQ}{r}$$

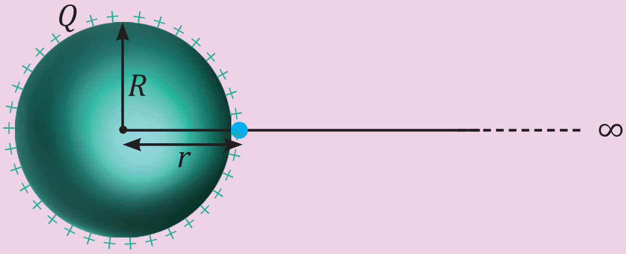
Case 2: On the surface of the sphere ($r = R$)

The potential difference at the surface of the sphere is given by,

$$V_P - V_\infty = -\int_\infty^R \vec{E}.d\vec{r}$$

$$\Rightarrow V_P - 0 = -\int_\infty^R \frac{kQ}{r^2} dr$$

$$\Rightarrow V_P = \frac{kQ}{R}$$



Case 3: Inside the sphere ($r < R$)

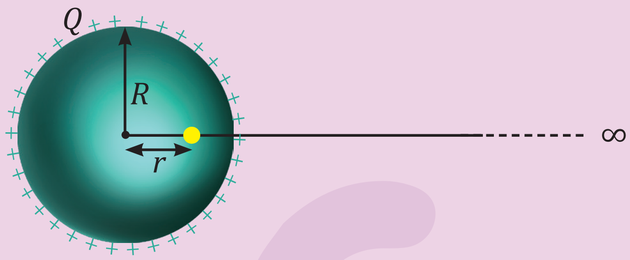
The electric field inside the sphere due to uniform distribution is given by,

$E_P = 0$

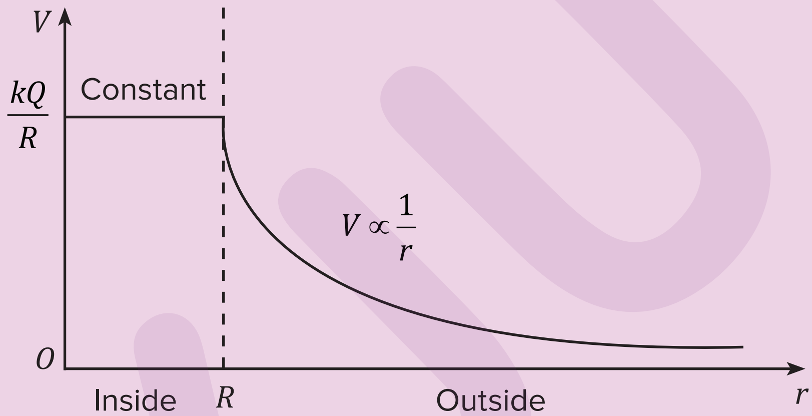
The electric field is zero inside as well as on the surface of the sphere, i.e., the potential is constant inside as well on the surface of the sphere.

Therefore, the electric potential difference at any point inside the sphere is given by,

$$V_P = \frac{kQ}{R}$$



Electric potential (V) vs distance (r) graph



Potential due to a solid non-conducting sphere

For a solid non-conducting sphere, the charge is distributed throughout the material so that the potential inside the conductor does not become zero.

Case 1: Outside the sphere ($r > R$)

Let us consider a solid non-conducting sphere of radius R, having a charge Q uniformly distributed throughout the material. Consider point P outside the sphere at a distance r from the sphere.

We know that the electric field at any point outside the uniform spherical symmetric charge distribution behaves as if all its charge is concentrated at its centre.

Therefore, the electric field at P is given by,

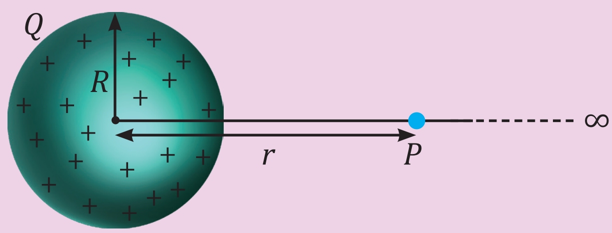
$$E_P = \frac{kQ}{r^2}$$

The electric potential difference is given by,

$$V_P - V_\infty = -\int_\infty^R \vec{E}.d\vec{r}$$

$$\Rightarrow V_P - 0 = -\int_\infty^R \frac{kQ}{r^2} dr$$

$$\Rightarrow V_P = \frac{kQ}{R}$$



Case 2: On the surface of the sphere (*r* = *R*)

The potential difference at the surface of the sphere is given by,

$$V_P - V_\infty = -\int_\infty^R \vec{E}.d\vec{r}$$

$$\Rightarrow V_P - 0 = -\int_\infty^R \frac{kQ}{r^2} dr$$

$$\Rightarrow V_P = \frac{kQ}{R}$$

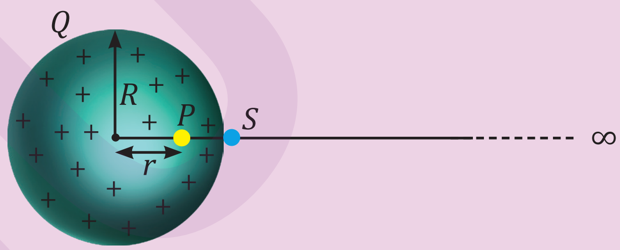
Case 3: Inside the sphere (*r* < *R*)

Since the charge is distributed throughout the sphere, i.e., inside the sphere as well, the electric field inside the sphere is non-zero.

The electric field inside the sphere is given by,

$$E_{inside} = \frac{kQr}{R^3}$$

The electric potential difference is given by,



$$\int_{V_s}^{V_P} dV = -\int_R^r E_{inside}\, dr$$

$$\Rightarrow V_P - V_s = -\int_R^r \frac{Qr}{4\pi\varepsilon_o R^3} dr$$

$$\Rightarrow V_P - V_s = -\frac{Q}{4\pi\varepsilon_o R^3} \int_R^r r\, dr$$

$$\Rightarrow V_P - V_s = -\frac{Q}{4\pi\varepsilon_o R^3} \left[\frac{r^2}{2}\right]_R^r$$

$$\Rightarrow V_P - V_s = -\frac{Q}{4\pi\varepsilon_o R^3} \left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$\Rightarrow V_P - \frac{Q}{4\pi\varepsilon_o R} = -\frac{Q}{4\pi\varepsilon_o R^3} \left[\frac{r^2}{2}\right] + \frac{1}{2}\frac{Q}{4\pi\varepsilon_o R} \qquad \left(\because V_s = \frac{Q}{4\pi\varepsilon_o R}\right)$$

$$\Rightarrow V_P = \frac{Q}{4\pi\varepsilon_o R} \left[\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right]$$

$$\Rightarrow V_P = \frac{1}{2}\frac{Q}{4\pi\varepsilon_o R} \left[3 - \frac{r^2}{R^2}\right]$$

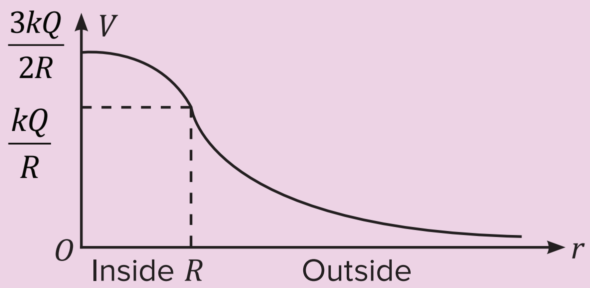
At centre, $r = 0$,

$$\therefore V_C = \frac{3}{2} V_S = \frac{3}{2}\frac{Q}{4\pi\varepsilon_o R}$$

Therefore, potential difference inside the sphere is given by,

$$V_P = \frac{kQ}{2R}\left[3 - \frac{r^2}{R^2}\right]$$

Electric potential (V) vs distance (r) graph



A conducting sphere of radius R is given charge Q. Find the electric potential and the electric field at the centre of the sphere, respectively.

(A) Zero and $\frac{Q}{4\pi\varepsilon_o R^2}$ (B) $\frac{Q}{4\pi\varepsilon_o R}$ and Zero (C) $\frac{Q}{4\pi\varepsilon_o R}$ and $\frac{Q}{4\pi\varepsilon_o R^2}$ (D) Both are zero

Solution NEET

For a conducting sphere, the charge is distributed uniformly only on the surface of the sphere. Due to symmetric and uniform charge distribution, the net electric field at any point inside the sphere is zero.
Therefore, the electric field at centre is given by,
$E = 0$
Since it is a solid conducting sphere, we know that a conductor is an equipotential surface. Thus, the electric potential is the same inside as well as on the surface of the conductor.
Therefore, the electric potential at the centre is given by,

$$V_P = \frac{kQ}{R} = \frac{Q}{4\pi\varepsilon_o R}$$

Thus, option (B) is the correct answer.

Two metallic spheres of radii 1 cm and 3 cm are given charge of -1×10^{-2} C and 5×10^{-2} C, respectively. If these spheres are connected by a conducting wire, then what is the final charge on the bigger sphere?

(A) 2×10^{-2} C (B) 3×10^{-2} C (C) 4×10^{-2} C (D) 1×10^{-2} C

Solution NEET

We know that the charge is conserved. The net charge in the system remains the same. If the two charged conductors are made to come in contact, the charges transfer from one conductor to another until they reach a common potential. Once their potential becomes equal, no more charge transfer happens.
$Q_{net} = Q_1 + Q_2 = (-1 \times 10^{-2}\ C) + (5 \times 10^{-2}\ C) = 4 \times 10^{-2}\ C$ .....(*i*)
Given, the spheres are charged and are connected by a conducting wire. Hence, they will attain a common potential. Thus, the potential of both spheres will become equal.

$V_1 = V_2$

$$\Rightarrow \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{R_1}{R_2} = \frac{1}{3}$$

$$\Rightarrow Q_1 = \frac{Q_2}{3}$$

By substituting the value of Q_1 in equation (i), we get,

$$\frac{Q_2}{3} + Q_2 = 4 \times 10^{-2}$$

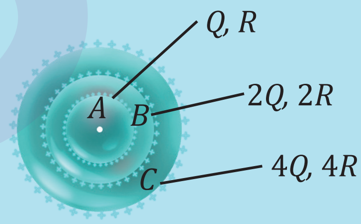
$$\Rightarrow 4Q_2 = 3 \times 4 \times 10^{-2}$$

$$\Rightarrow Q_2 = 3 \times 10^{-2}\ C$$

Thus, option (B) is the correct answer.

Potential Due to Concentric Shells

Consider three concentric hollow spheres, A, B, and C, with charges Q, $2Q$, and $4Q$, respectively. The electric potential at any point on the surface of any sphere will be the sum of all the potential that occurs due to all the shells.



Potential on shell B is due to its charge distribution on its surface itself and due to charge distribution on shell A and charge distribution on shell C.
The electric potential at any point on the surface of shell B is given by,

$$V_{B_{surface}} = V_A + V_B + V_C$$

$$\Rightarrow V_{B_{surface}} = \frac{kQ}{2R} + \frac{2kQ}{2R} + \frac{4kQ}{4R}$$

$$\Rightarrow V_{B_{surface}} = \frac{5kQ}{2R}$$

Potential on shell C is due to its charge distribution on its surface itself and due to charge distribution on shell A and charge distribution on shell B.

$$V_{C_{surface}} = V_A + V_B + V_C$$

$$\Rightarrow V_{C_{surface}} = \frac{kQ}{4R} + \frac{2kQ}{4R} + \frac{4kQ}{4R}$$

$$\Rightarrow V_{C_{surface}} = \frac{7kQ}{4R}$$

ELECTROSTATICS

POTENTIAL DUE TO DIPOLE AND EQUIPOTENTIAL SURFACES

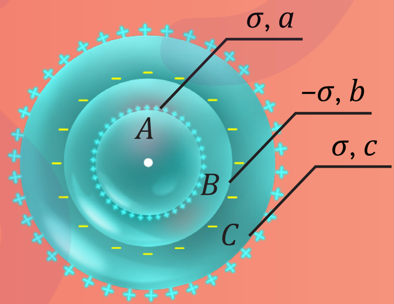
What you already know

- Null potential points due to two point charges
- Electric potential due to extended charges, ring, disc
- Electric potential energy for two charge and three charge system

What you will learn

- Potential due to dipole
- Potential due to dipole in a uniform electric field
- Equipotential surface and its properties

Three concentric spherical shells having radii a, b, and c ($a < b < c$) and have surface charge densities σ, $-\sigma$, and σ, respectively. If V_A, V_B, and V_C denote the potentials of the three shells, then, for $c = a + b$, we have,



(A) $V_C = V_B \neq V_A$ (B) $V_C \neq V_B \neq V_A$ (C) $V_C = V_B = V_A$ (D) $V_C = V_A \neq V_B$

Solution NEET

The charge on the shell a, b, and c is,

$$q_A = \sigma \times 4\pi a^2$$

$$q_B = \sigma \times 4\pi b^2$$

$$q_C = \sigma \times 4\pi c^2$$

The potential on the surface of shell A is due to all the charges in the system, and it is given by,

$$V_{A_{surface}} = V_A + V_B + V_C$$

$$\Rightarrow V_{A_{surface}} = \frac{kq_A}{a} - \frac{kq_B}{b} + \frac{kq_C}{c}$$

$$\Rightarrow V_{A_{surface}} = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_A}{a} - \frac{q_B}{b} + \frac{q_C}{c}\right)$$

By substituting the value of q_A, q_B, and q_C, we get,

$$\Rightarrow V_{A_{surface}} = \frac{1}{4\pi\varepsilon_o}\left(\frac{\sigma \times 4\pi a^2}{a} - \frac{\sigma \times 4\pi b^2}{b} + \frac{\sigma \times 4\pi c^2}{c}\right)$$

$$\Rightarrow V_{A_{surface}} = \frac{\sigma}{\varepsilon_o}(a - b + c)$$

$$\Rightarrow V_{A_{surface}} = \frac{\sigma}{\varepsilon_o}(a - b + a + b) \quad (\because c = a + b)$$

$$\Rightarrow V_{A_{surface}} = \frac{\sigma}{\varepsilon_o}(2a) \quad .....(i)$$

Similarly, the potential on the surface of shell B is given by,

$$V_{B_{surface}} = V_A + V_B + V_C$$

$$\Rightarrow V_{B_{surface}} = \frac{kq_A}{b} - \frac{kq_B}{b} + \frac{kq_C}{c}$$

$$\Rightarrow V_{B_{surface}} = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_A}{b} - \frac{q_B}{b} + \frac{q_C}{c}\right)$$

By substituting the value of q_A, q_B, and q_C, we get,

$$\Rightarrow V_{B_{surface}} = \frac{1}{4\pi\varepsilon_o}\left(\frac{\sigma \times 4\pi a^2}{b} - \frac{\sigma \times 4\pi b^2}{b} + \frac{\sigma \times 4\pi c^2}{c}\right)$$

$$\Rightarrow V_{B_{surface}} = \frac{\sigma}{\varepsilon_o}\left(\frac{a^2}{b} - b + c\right)$$

$$\Rightarrow V_{B_{surface}} = \frac{\sigma}{\varepsilon_o}\left(\frac{a^2}{b} - b + a + b\right) \quad (\because c = a + b)$$

$$\Rightarrow V_{B_{surface}} = \frac{\sigma}{\varepsilon_o}\left(\frac{a^2}{b} + a\right) \quad .....(ii)$$

The potential on the surface of shell C is given by,

$$V_{C_{surface}} = V_A + V_B + V_C$$

$$\Rightarrow V_{C_{surface}} = \frac{kq_A}{c} - \frac{kq_B}{c} + \frac{kq_C}{c}$$

$$\Rightarrow V_{C_{surface}} = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_A}{c} - \frac{q_B}{c} + \frac{q_C}{c}\right)$$

By substituting the value of q_A, q_B, and q_C, we get,

$$\Rightarrow V_{C_{surface}} = \frac{1}{4\pi\varepsilon_o}\left(\frac{\sigma \times 4\pi a^2}{c} - \frac{\sigma \times 4\pi b^2}{c} + \frac{\sigma \times 4\pi c^2}{c}\right)$$

$$\Rightarrow V_{C_{surface}} = \frac{\sigma}{\varepsilon_o}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$$

$$\Rightarrow V_{C_{surface}} = \frac{\sigma}{\varepsilon_o}\left(\frac{a^2 - b^2}{c} + c\right)$$

$$\Rightarrow V_{C_{surface}} = \frac{\sigma}{\varepsilon_o}\left(\frac{a^2 - b^2}{(a+b)} + (a+b)\right) \quad (\because c = a + b)$$

$$\Rightarrow V_{C_{surface}} = \frac{\sigma}{\varepsilon_o}(a - b + a + b)$$

$$\Rightarrow V_{C_{surface}} = \frac{\sigma}{\varepsilon_o}(2a) \quad .....(iii)$$

From equations (i), (ii), and (iii), we get,

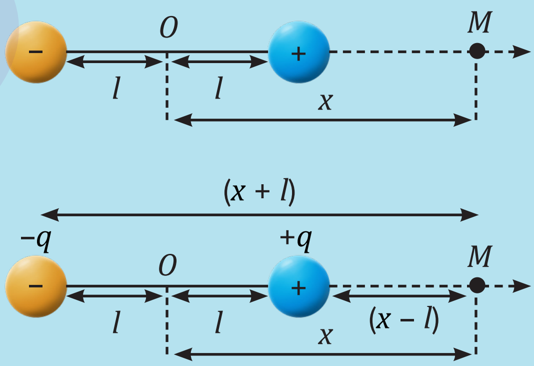
$V_C = V_A \neq V_B$

Thus, option (D) is the correct answer.

Potential Due to Dipole

1. On the axis of the dipole

Let us consider an electric dipole of length $2l$. On the axial line (axis) of the dipole, let us consider a point M at a distance x from the centre of the dipole, as shown in the figure.



The point M is at a distance of $x - l$ and $x + l$ from positive and negative charges, respectively.

The net potential at point M due to the dipole is given by,

$$(V_{net})_M = V_{+q} + V_{-q}$$

$$\Rightarrow (V_{net})_M = \frac{kq}{(x-l)} - \frac{kq}{(x+l)}$$

$$\Rightarrow (V_{net})_M = \frac{kq(2l)}{(x^2 - l^2)}$$

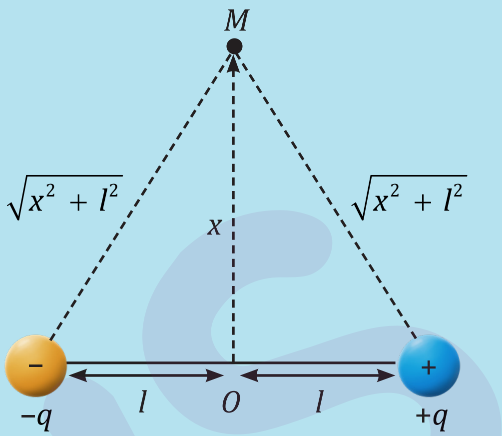
$$\Rightarrow \boxed{(V_{net})_M = \frac{kp}{(x^2 - l^2)}} \quad (\because p = q \times 2l)$$

If $l << x$,

$$\Rightarrow (V_{net})_M = \frac{kp}{(x^2)}$$

2. On the perpendicular bisector of the dipole or on the equatorial line

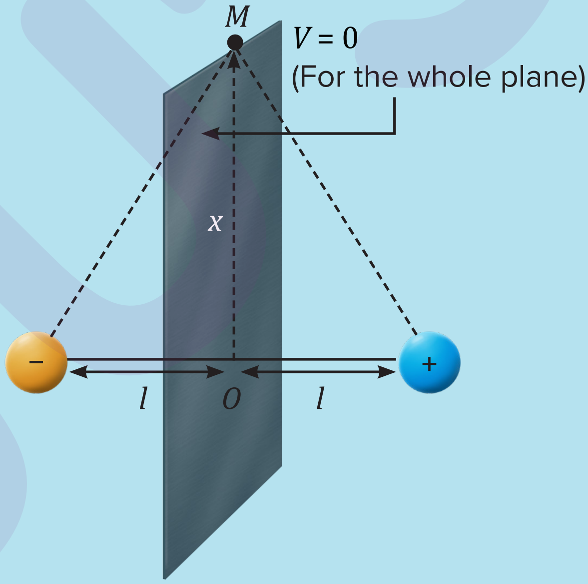
Let us consider an electric dipole of length $2l$. On the perpendicular bisector of the dipole, let us consider a point M at a distance x from the centre of the dipole, as shown in the figure.



The point M is at a distance of $\sqrt{x^2 + l^2}$ from both positive and negative charges, respectively.

$$(V_{net})_M = V_{+q} + V_{-q}$$

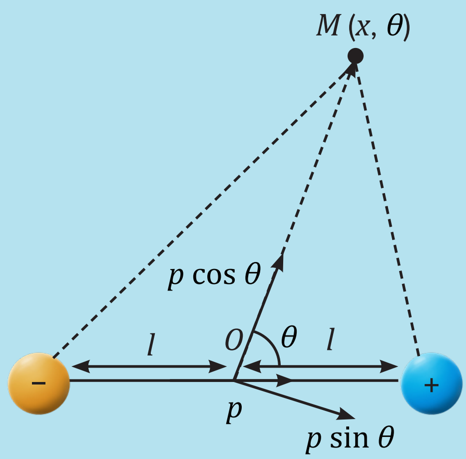
$$\Rightarrow (V_{net})_M = \frac{kq}{\sqrt{x^2 + l^2}} - \frac{kq}{\sqrt{x^2 + l^2}} = 0$$



Since every point on the perpendicular bisector of the dipole (equatorial line) is equidistant from both the positive and negative charges. The potential at any point on the perpendicular bisector of the dipole will be zero. If we consider a plane that is perpendicular to the dipole axis and passes through the equatorial line of the dipole as shown in the figure, the net potential due to the dipole on the whole plane is zero. This plane is known as "Equatorial plane".

3. The potential at any general point

Let us consider an electric dipole of length $2l$. Consider any general point M, neither on the axial line nor on the equatorial line of the dipole. Point M at a distance x from the centre of the dipole. The line joining the centre of the dipole (O) and point M makes an angle θ with the dipole moment vector $\vec{p}$ as shown in the figure. Since we know the electric potential on the axial point and on the equatorial point of a dipole, let us resolve the dipole moment p of the dipole into two components, one along the line joining the centre of the dipole and point M, and another one is perpendicular to the line joining the centre of the dipole. As a consequence, point M becomes the axial point of the dipole $p \cos\theta$ and the equatorial point of the dipole $p \sin\theta$.



So, the potential at point M due to $p\cos\theta$ is given by,

$$V_{p\cos\theta} = \frac{kp\cos\theta}{x^2}\ (\text{if } l << x)$$

For $p\sin\theta$, the point M is an equatorial point, and we know that the electric potential at an equatorial point is zero.

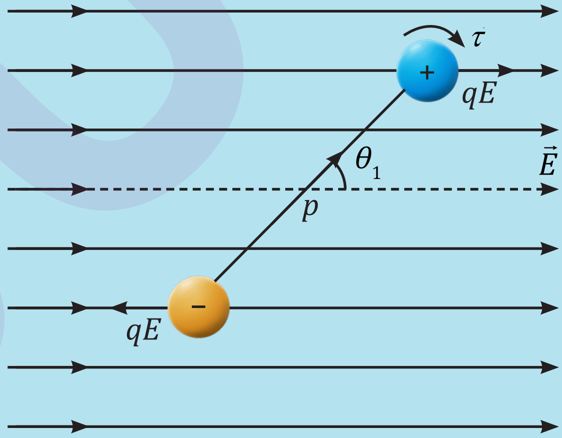
$$V_{p\sin\theta} = 0$$

So, the net electric potential at point M due to the dipole is given as follows:

$$(V_{net})_M = V_{p\sin\theta} + V_{p\cos\theta} = \frac{kp\cos\theta}{x^2}\ (\text{if } l << x)$$

Potential Energy Due to Dipole in a Uniform Electric Field

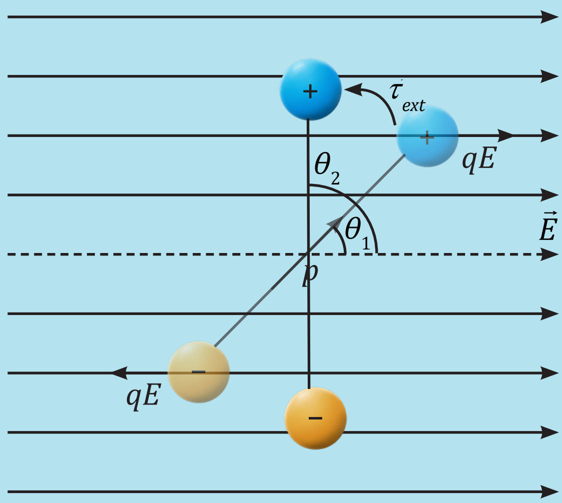
Consider an electric dipole placed in a uniform electric field at an angle θ_1 with the electric field. Due to the electric field, both positive and negative charge experience an electrostatic force equal in magnitude but opposite in direction. Therefore, the **net force on the dipole is zero.** But due to the two equal and opposite forces with different lines of action forms a couple. The couple generates a clockwise non-zero torque about the COM of the dipole, which rotates the dipole to align it in the direction of the electric field.



The torque acting on the dipole is given as follows:

$\vec{\tau} = \vec{p} \times \vec{E}$

Now, let us consider an external torque is given to the dipole to rotate the dipole from an angle θ_1 to angle θ_2, as shown in the figure. The external torque is given in such a way that there is no change in the kinetic energy of the system.



According to the work-energy theorem,

$W_{ext} + W_{el} = \Delta(K.E.)$

Since there is no change in kinetic energy of the dipole, $\Delta(K.E.) = 0$

$W_{ext} + W_{el} = 0$

$\Rightarrow W_{ext} = -W_{el}$

We know that, $\Delta U = -W_{el}$

Thus, $\Delta U = W_{ext}$, **if and only if $\Delta(K.E.) = 0$**

Therefore, we can say, $|W_{external}| = |W_{electrical}|$

The work done by the external torque is given by,

$$W_{ext} = \int_{\theta_1}^{\theta_2} \tau_{ext}\, d\theta$$

Also, $|\vec{\tau}_{ele}| = |\vec{\tau}_{ext}| = pE\sin\theta$

$$\Rightarrow W_{ext} = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$

$$\Rightarrow W_{ext} = pE\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$\Rightarrow W_{ext} = -pE[\cos\theta]_{\theta_1}^{\theta_2}$$

$$\Rightarrow W_{ext} = -pE[\cos\theta_2 - \cos\theta_1]$$

The change in potential energy is given by,

$$\Delta U = U_f - U_i = W_{ext}$$

$$\Rightarrow \Delta U = -pE[\cos\theta_2 - \cos\theta_1]$$

When $\theta = 90°$, we can assume that the potential energy is zero. That means we choose $\theta = 90°$ as the reference point.

$$\Delta U = U_{\theta_2} - U_{\theta_1} = -pE[\cos\theta_2 - \cos\theta_1]$$

Let $\theta_1 = 90°$ and $U_{\theta_1} = 0$

$$\Rightarrow U_{\theta_2} - 0 = -pE[\cos\theta_2 - \cos 90°]$$

$$\Rightarrow U_{\theta_2} = -pE[\cos\theta_2]$$

Therefore, for any general angle θ the potential energy is given by,

$$\Rightarrow U_\theta = -pE\cos\theta$$

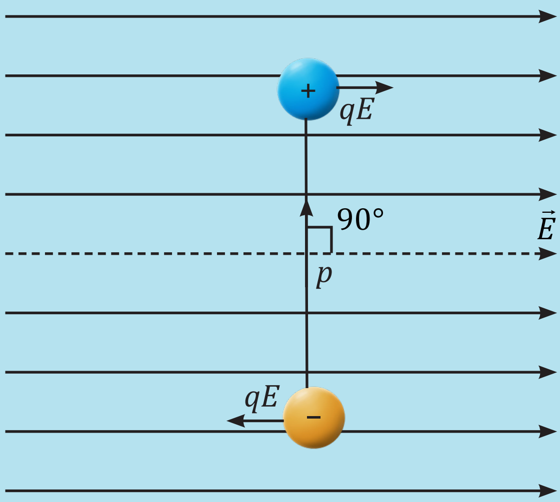
$$\Rightarrow U_\theta = -\vec{p}.\vec{E}$$

Some important cases

Case 1:

When dipole moment $\vec{p}$ makes an angle $\theta = 90°$ with the electric field, the net force acting on the dipole is given by,

$$F_{net} = 0$$



The net torque acting is given by,

$$\vec{\tau} = pE\sin 90° = pE$$

$\Rightarrow \vec{\tau}$ is maximum

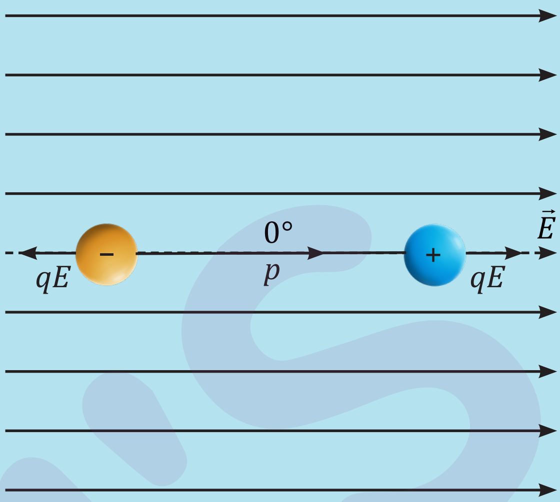
The potential energy is given by,

$$U_\theta = -pE\cos 90° = 0$$

Case 2:

When dipole moment $\vec{p}$ makes an angle $\theta = 0°$ with the electric field, the net force acting on the dipole is given by,

$F_{net} = 0$



The net torque acting is given by,

$\vec{\tau} = pE \sin 0° = 0$

The potential energy is given by,

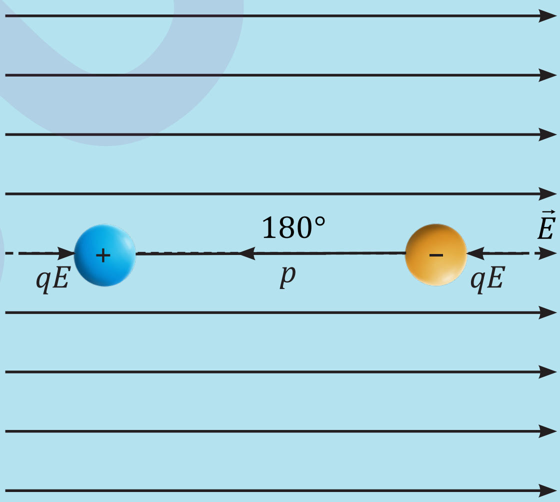
$U_\theta = -pE \cos 0° = -pE$

In this case, both the net torque and the net force acting on the dipole are zero, therefore, we can say that the **dipole is in equilibrium**. Also, **the potential energy at this position is minimum**, so the equilibrium will be a stable equilibrium.

Case 3:

When dipole moment $\vec{p}$ makes an angle $\theta = 180°$ with the electric field, the net force acting on the dipole is given by,

$F_{net} = 0$



The net torque acting is given by,

$\vec{\tau} = pE \sin 180° = 0$

The potential energy is given by,

$U_\theta = -pE \cos 180° = pE$

In this case also, both the net torque and the net force acting on the dipole are zero. Therefore, we can say that the dipole is in equilibrium. Also, **the potential energy at this position is maximum**, so the equilibrium will be an **unstable equilibrium**.

An electric dipole of the dipole moment $\vec{p}$ is lying along a uniform electric field $\vec{E}$. What is the work done in rotating the dipole by 90°?

(A) pE (B) $\sqrt{2}pE$ (C) $\frac{pE}{2}$ (D) $2pE$

Solution NEET

It is given that, initially the dipole of the moment $\vec{p}$ is lying along a uniform electric field $\vec{E}$ i.e., θ = 0° and it is rotated from 0° to 90°.

Therefore,

$\theta_1 = 0°$

and $\theta_2 = 90°$

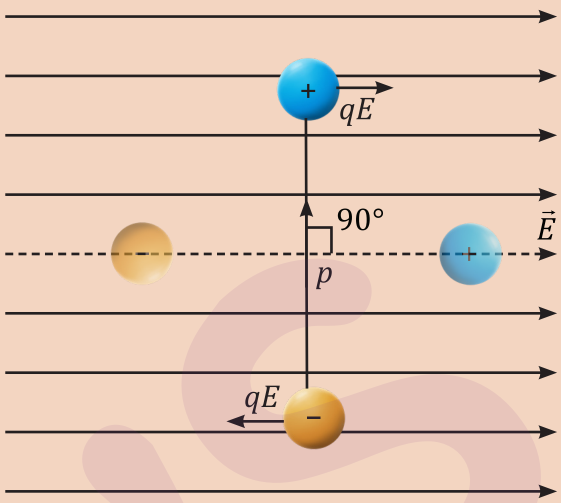
The work done in rotating the dipole is given as follows:

$\Rightarrow W_{ext} = -pE[\cos\theta_2 - \cos\theta_1]$

$\Rightarrow W_{ext} = -pE[\cos 90° - \cos 0°]$

$\Rightarrow W_{ext} = pE$

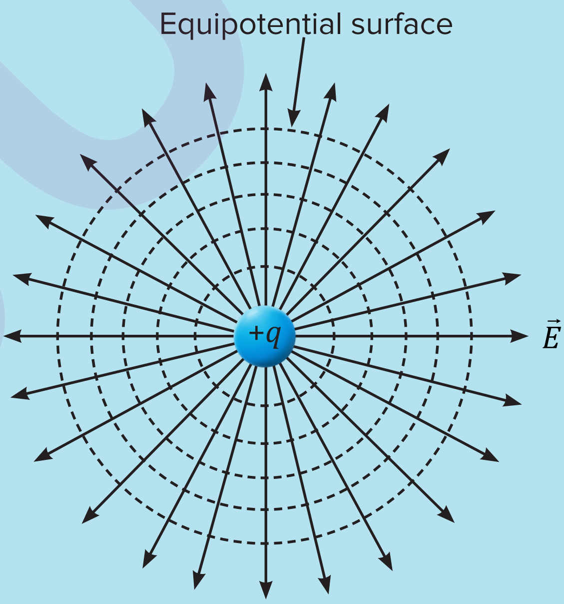
Thus, option (A) is the correct answer.



Equipotential Surfaces

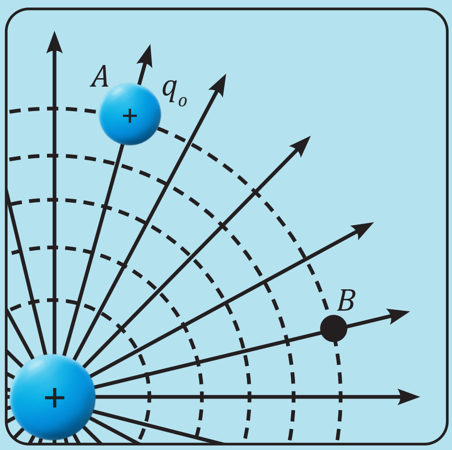
Consider a positive charge $+q$, due to which an electric field is generated in space. If we consider points that are radially equidistant from the charge $+q$, the electric potential at all those points will be the same, and if we connect those all points, it forms spherical shells. Similarly, there can be a number of shells in the electric field of a point charge. Thus, every point on each of the shells has equal potential i.e., these shells are considered as surfaces with the same potential that is also known as equipotential surfaces.

A surface on which the potential is the same at every point is known as an equipotential surface.



The properties of equipotential surfaces

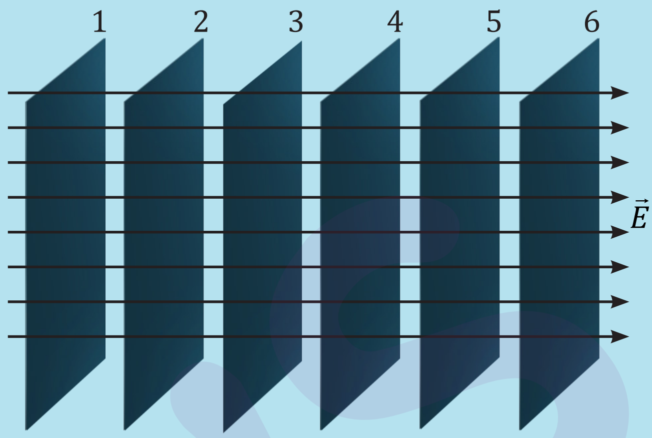
1. The work done in displacing a charge between any two points on an equipotential surface is zero.
   If a charge is moved from point A to B, then the work done is given as follows:
   The potential on every point on the equipotential surface is the same. Therefore,
   $W = q_o(V_B - V_B)$
   $W = 0$
2. The equipotential surfaces are always perpendicular to the electric field lines.
3. The two equipotential surfaces can never intersect each other.



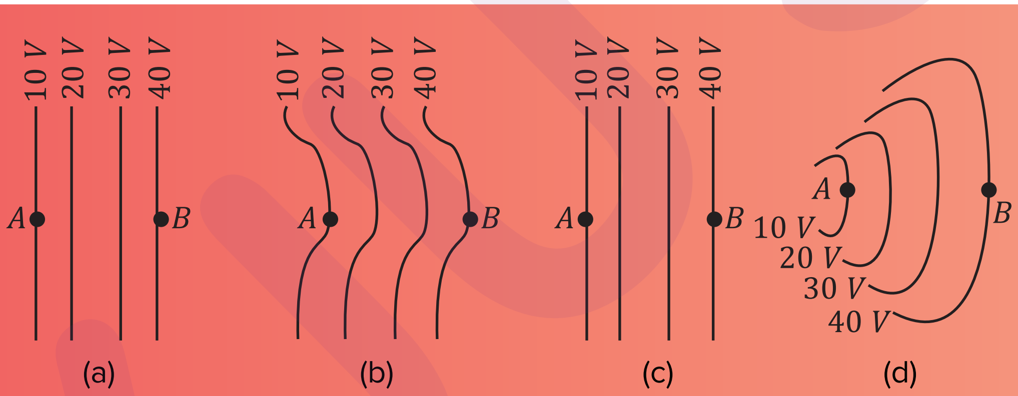
4. In the uniform electric field $\vec{E}$, the equipotential surfaces will be as shown in the figure.

We know that the electric potential decreases along the direction of the electric field or in other words, the direction of the electric field is from a higher potential to lower potential. Therefore, equipotential surfaces in the direction of the electric field will be in descending order of electric potential. Thus, the electric potential on surface 1 is greater than surface 2 i.e., $V_1 > V_2 > V_3 > V_4 > V_5 > V_6$. However, individually, the potential on each surface will be constant.

Plane equipotential surfaces



The given diagrams show equipotential region.



(A) The maximum work is required to move q in figure(c).

(B) In all four cases, the work done is the same.

(C) The minimum work is required to move q in figure (a).

(D) The maximum work done is required to move q in figure (b).

Solution

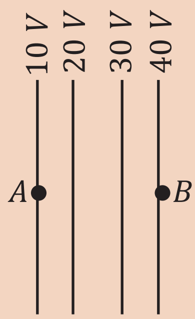
Work done for

(a) $W = q(\Delta V)$

$W = q(V_f - V_i)$

$W = q(40 - 10)$

$W = 30q$



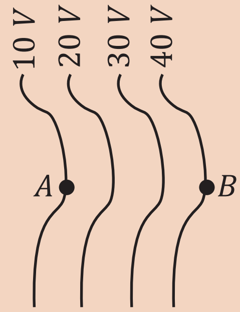
(a)

(b) $W = q(\Delta V)$

$W = q(V_f - V_i)$

$W = q(40 - 10)$

$W = 30q$



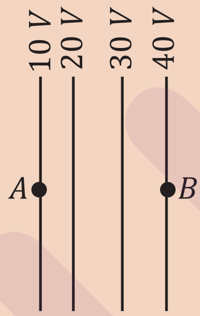
(b)

(c) $W = q(\Delta V)$

$W = q(V_f - V_i)$

$W = q(40 - 10)$

$W = 30q$



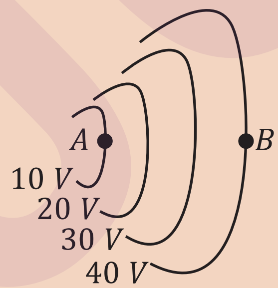
(c)

(d) $W = q(\Delta V)$

$W = q(V_f - V_i)$

$W = q(40 - 10)$

$W = 30q$



(d)

Thus, option (B) is the correct answer.