BYJU'S Study Planner for Board Term I
(CBSE Grade 12)

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Subject: Mathematics

Topic : Continuity and Differentiability

Class: Standard XII

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1. The function given by $f\left(x\right)=\dfrac{1}{\left|x\right|-1}-\dfrac{x^{2}}{2}$ is continuous in

   A. $\mathbb{R}-\{-1,1\}$

   B. $\mathbb{R}-\{1\}$

   C. $\mathbb{R}-\{-1\}$

   D. $\{-1,1\}$

$\dfrac{1}{\left|x\right|-1}$ is discontinuous where $\left|x\right|=1$

i.e., $x=-1,1$

while $\dfrac{x^{2}}{2}$ is continuous for all real x.

So, $f\left(x\right)=\dfrac{1}{\left|x\right|-1}-\dfrac{x^{2}}{2}$ is discontinuous at $x=-1,1$

Hence, $f(x)$ is continuous for all $x\in\mathbb{R}-\{-1,1\}$

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2. $f(x)=\operatorname{sgn}(x^3-x)$ is discontinuous at which of the following points

A. 0

B. 1

C. -1

D. All the above

We know that $\operatorname{sgn}(x)$ is discontinuous at $x=0$.
So, the function $f(x)=\operatorname{sgn}(x^3-x)$ will be discontinuous at all the points where
$x^3-x=0$
$\Rightarrow x\left(x^2-1\right)=0$
$\Rightarrow x(x+1)(x-1)=0$
$\Rightarrow x=\{0,1,-1\}$

3. If $f(x)=[[x]]-[x-1]$ then which of the following options is CORRECT, (where $[.]$ represents the greatest integer function.)

A. Discontinuous at $x=0$

B. Discontinuous at $x=1$

C. Discontinuous at $x=-1$

D. Continuous at every where

$f(x)=[[x]]-[x-1]=[x]-([x]-1)=1$
$\Rightarrow f(x)$ is continuous $\forall\, x\in\mathbb{R}$

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4. Let $f(x)=\dfrac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}}, x\neq 0$. If $f(x)$ is continuous at $x=0$, then the value of $f(0)$ is

A. $\frac{2}{3}$

B. 2

C. 4

D. 6

For continuity of $f(x)$ at $x=0$, we must have:

$$f(0)=\lim_{x\to 0} f(x)$$

$$=\lim_{x\to 0}\frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}}\left[\frac{0}{0}\text{form}\right]$$

$$=\lim_{x\to 0}\frac{\frac{1}{3}(27-2x)^{-2/3}\cdot(-2)}{-\frac{3}{5}(243+5x)^{-4/5}\cdot 5}$$

$$=\frac{2}{9}\lim_{x\to 0}\frac{(243+5x)^{4/5}}{(27-2x)^{2/3}}$$

$$=\frac{2}{9}\cdot\frac{(243)^{4/5}}{(27)^{2/3}}=\frac{2}{9}\cdot\frac{81}{9}=2$$

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5. The value of $f(0)$, so that the function $f(x)=\dfrac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}$ is continuous at $x=0$ is

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

For continuity of $f(x)$ at $x=0$, we must have,

$$f(0)=\lim_{x\to 0} f(x)=\lim_{x\to 0}\left(\frac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}\right)=\lim_{x\to 0}\frac{(1+x)^{1/2}-(1+x)^{1/3}}{x}$$

$$\left[\frac{0}{0}\text{form}\right]$$

$$=\lim_{x\to 0}\left\{\frac{1}{2(1+x)^{1/2}}-\frac{1}{3(1+x)^{2/3}}\right\}=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$$

6. The function given by $f(x)=\dfrac{3x+7}{x^2-5x+6}$ is continuous in

A. $(2,3]$

B. $\mathbb{R}-[2,3]$

C. $\mathbb{R}-\{2,3\}$

D. None of the above

$f(x)$ is not defined when denominator equals 0
So, for $f(x)$ to be continuous, $x^2-5x+6\neq 0$,
$\Rightarrow x\neq 2,3$
So, $f(x)$ is continuous for all $x\in\mathbb{R}-\{2,3\}$

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7. If $f(x)=\dfrac{1-\sin x}{\sin 2x}, x\neq\dfrac{\pi}{2}$ is continuous at $x=\dfrac{\pi}{2}$, then the value of $f\left(\dfrac{\pi}{2}\right)$ is

A. 0 ✔

B. $\dfrac{1}{2}$ ✘

C. 1 ✘

D. 2 ✘

For continuity of $f(x)$ at $x=\dfrac{\pi}{2}$, we must have

$$f\left(\frac{\pi}{2}\right)=\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\sin 2x}\left(\frac{0}{0}\text{form}\right)$$

Using L Hospital's Rule

$$=\lim_{x\to\frac{\pi}{2}}\frac{-\cos x}{2\cos 2x}=\frac{-\cos\frac{\pi}{2}}{2\cos\pi}=0$$

8. Let $f(x)=\begin{cases}(x-1)^{\frac{1}{2-x}}, & x>1, x\neq 2\\ k, & x=2\end{cases}$

The value of k for which f is continuous at $x=2$ is :

A. e^{-1}

B. e^{-2}

C. e

D. 1

$f(x)=\begin{cases}(x-1)^{\frac{1}{2-x}}, & x>1, x\neq 2\\ k, & x=2\end{cases}$

As $f(x)$ is continuous at $x=2$

$\therefore k=\lim\limits_{x\to 2}(x-1)^{\frac{1}{2-x}}$

$=\lim\limits_{x\to 2} e^{(x-1-1)\cdot\frac{1}{2-x}}$

$=\lim\limits_{x\to 2} e^{(-1)}$

$=e^{-1}$

9. For the function $f(x)=\dfrac{1-\sin x+\cos x}{1+\sin x+\cos x}$. The value of $f(\pi)$, so that $f(x)$ is continuous at $x=\pi$ is

- **A.** -1
- **B.** $-\dfrac{1}{2}$
- **C.** $\dfrac{1}{2}$
- **D.** 1

For continuity of $f(x)$ at $x=\pi$, we must have

$f(\pi)=\lim\limits_{x \to \pi} f(x)$

$$\lim_{x \to \pi} f(x)=\lim_{x \to \pi} \frac{1-\sin x+\cos x}{1+\sin x+\cos x} \quad \left[\frac{0}{0}\text{Form}\right]$$

Applying L Hospital's rule

$$=\lim_{x \to \pi} \frac{-\cos x-\sin x}{\cos x-\sin x}$$

$$=\frac{-\cos \pi-\sin \pi}{\cos \pi-\sin \pi}$$

$$=\frac{1-0}{-1-0}=-1$$

$\therefore f(\pi)=-1$

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10. If $f(x) = \begin{cases} \dfrac{(4^x - 1)^3}{\sin\left(\dfrac{x}{a}\right)\ln\left(1 + \dfrac{x^2}{3}\right)}, & x \neq 0 \\ 9(\ln 4)^3, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of a is

A. 0

B. 1

C. 2

D. 3 ✔

For $f(x)$ to be continuous at $x = 0$, we must have

$f(0) = \lim_{x \to 0} f(x) \quad \cdots (1)$

$$\Rightarrow \lim_{x \to 0} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{a}\right)\ln\left(1 + \frac{x^2}{3}\right)}$$

$$= \lim_{x \to 0} \frac{(4^x - 1)^3}{x^3} \cdot \frac{\left(\frac{x}{a}\right)}{\sin\left(\frac{x}{a}\right)} \cdot \frac{\frac{x^2}{3}}{\ln\left(1 + \frac{x^2}{3}\right)} \cdot 3a$$

$$= 3a \cdot \left(\lim_{x \to 0} \frac{4^x - 1}{x}\right)^3 \cdot \left(\frac{1}{\lim_{x \to 0} \frac{\sin(x/a)}{(x/a)}}\right) \cdot \left(\frac{1}{\lim_{x \to 0} \frac{\ln(1 + x^2/3)}{(x^2/3)}}\right)$$

$$= 3a \cdot (\ln 4)^3 \cdot \frac{1}{1} \cdot \frac{1}{1} = 3a(\ln 4)^3$$

Using (1), we get

$\Rightarrow 9(\ln 4)^3 = 3a(\ln 4)^3$

$\therefore a = 3$

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11. If $f(x) = \begin{cases} \dfrac{(1-\sin x)}{(\pi - 2x)^2}, & x \neq \dfrac{\pi}{2} \\ \lambda, & x = \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of λ is

- A. $\frac{1}{4}$
- B. $\frac{1}{2}$
- C. $\frac{1}{8}$
- D. 1

For $f(x)$ to be continuous at $x = \frac{\pi}{2}$, we must have

$$\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right) = \lambda$$

$$\Rightarrow \lambda = \lim_{x \to \frac{\pi}{2}} f(x) = \lim_{x \to \frac{\pi}{2}} \frac{(1-\sin x)}{(\pi - 2x)^2}$$

Using L Hospital's Rule

$$= \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-4(\pi - 2x)}$$

$$= \frac{1}{4} \cdot \lim_{x \to \frac{\pi}{2}} \frac{-\sin x}{-2} = \frac{1}{8}$$

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12.

Let $f(x) = \dfrac{\tan\left(\dfrac{\pi}{4} - x\right)}{\cot 2x}, x \neq \dfrac{\pi}{4}$. If $f(x)$ is continuous at $x = \dfrac{\pi}{4}$, then the value of $f\left(\dfrac{\pi}{4}\right)$ is

A. 1

B. $\dfrac{1}{2}$

C. 2

D. $\dfrac{1}{4}$

As $f(x)$ is continuous at $x = \dfrac{\pi}{4}$

$$\therefore f\left(\frac{\pi}{4}\right) = \lim_{x \to \frac{\pi}{4}} f(x)$$

$$\lim_{x \to \frac{\pi}{4}} f(x) = \lim_{x \to \frac{\pi}{4}} \frac{\tan\left(\frac{\pi}{4} - x\right)}{\cot 2x}$$

Using L Hospital's rule

$$= \lim_{x \to \frac{\pi}{4}} \frac{-\sec^2\left(\frac{\pi}{4} - x\right)}{-2\operatorname{cosec}^2 2x} = \frac{1}{2}$$

$$\therefore f\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

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13. The value of $f(0)$ such that the function $f(x)=\dfrac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$ is continuous at every point in its domain, is equal to

A. $\frac{1}{3}$

B. $-\frac{1}{3}$

C. $\frac{2}{3}$

D. 2

For $f(x)$ to be continuous at every point of its domain, it must be continuous at $x=0$ also,

$\therefore$ We must have,

$f(0)=\lim\limits_{x\to 0} f(x)$,

$\Rightarrow f(0)=\lim\limits_{x\to 0}\dfrac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$

Dividing numerator and denominator by x

$$=\lim_{x\to 0}\left\{\frac{2-\dfrac{\sin^{-1}x}{x}}{2+\dfrac{\tan^{-1}x}{x}}\right\}=\frac{1}{3}$$

$\therefore f(0)=\dfrac{1}{3}$

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14. The function $f(x) = x - |x - x^2|$ is

A. continuous at $x = 1$

B. discontinuous at $x = 0$

C. not defined at $x = 1$

D. not defined at $x = 0$

$f(x) = x - |x(1-x)|$

$$= \begin{cases} x + x(1-x) & x \geq 1 \\ x - x(1-x) & 0 \leq x < 1 \\ x + x(1-x) & x < 0 \end{cases}$$

So, doubtfull points are $x = 0, 1$
at $x = 1, f(x) = 1 = f(1^+)$ and
$\lim\limits_{x \to 1^-} f(x) = 1,$
at $x = 0, f(x) = 0 = f(0^+)$ and
$\lim\limits_{x \to 0^-} f(x) = 0$
Hence, $f(x)$ is continuous at $x \in \mathbb{R}$

15. The interval where the function $\log(1 + x)$ is continuous, is

A. $(0, \infty)$

B. $(-1, \infty)$

C. $(-\infty, -1)$

D. None of the above

$\because \log_a x$ is continuous where $x > 0$ and $a > 0, a \neq 1$, Therefore $\log(1 + x)$ is continuous when $1 + x > 0$
$\Rightarrow x > -1$
$\therefore$ Required interval $= (-1, \infty)$

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16. If $f(x)=\dfrac{\log_e(1+x^2\tan x)}{\sin x^3}, x\neq 0$ is continuous at $x=0$, then the value of $f(0)$ is

A. -1

B. 0

C. $\dfrac{1}{2}$

D. 1

For $f(x)$ to be continuous at $x=0$, we must have

$$f(0)=\lim_{x\to 0} f(x)$$

$$=\lim_{x\to 0}\frac{\log_e(1+x^2\tan x)}{\sin x^3}$$

$$=\lim_{x\to 0}\frac{\log_e(1+x^2\tan x)}{x^2\tan x}\cdot\frac{x^2\tan x}{x^3}\cdot\frac{x^3}{\sin x^3}$$

$$=\lim_{x\to 0}\left\{\frac{\log_e(1+x^2\tan x)}{x^2\tan x}\right\}\cdot\lim_{x\to 0}\left(\frac{\tan x}{x}\right)\cdot\frac{1}{\lim\limits_{x\to 0}\left(\dfrac{\sin x^3}{x^3}\right)}$$

$$=1\times 1\times\frac{1}{1}=1$$

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17. Let $f(x) = \begin{cases} \sqrt{1+x^2}, & x < \sqrt{3} \\ \sqrt{3}x - 1, & \sqrt{3} \le x < 4 \\ [x], & 4 \le x < 5 \\ |1-x|, & x \ge 5 \end{cases}$

where $[x]$ is the greatest integer less than or equal to x.
The number of point(s) of discontinuity of $f(x)$ in $\mathbb{R}$ is

- **A.** 3
- **B.** 0
- **C.** Infinite
- **D.** 1 ✔

Checking discontinuity at the points where $f(x)$ changes it's branches i.e.
$\sqrt{3}, 4, 5$
$f(\sqrt{3}^-) = 2$
$f(\sqrt{3}^+) = f(\sqrt{3}) = 2$
$f(4^-) = 4\sqrt{3} - 1$
$f(4^+) = f(4) = 4$
$f(5^-) = 4$
$f(5^+) = f(5) = 4$
So, $f(x)$ is discontinuous only at $x = 4$.

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18. The value of a so that the function $f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x > 0 \end{cases}$

is continuous at $x = 0$ is

A. 2

B. 4

C. 6

D. 8 ✔

$f(0)^{-} = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \dfrac{1-\cos 4x}{x^2}$

$= \lim\limits_{x \to 0^-} \dfrac{2\sin^2 2x}{x^2} = 8$

$f(0)^{+} = \lim\limits_{x \to 0^+} f(x)$

$= \lim\limits_{x \to 0^+} \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$

$= \lim\limits_{x \to 0^+} \dfrac{\sqrt{x} \cdot \left(\sqrt{16+\sqrt{x}}+4\right)}{16+\sqrt{x}-16}$

$= \lim\limits_{x \to 0^+} \left(\sqrt{16+\sqrt{x}}+4\right) = 8$

For $f(x)$ to be continuous,

$f(0^-) = f(0^+) = f(0)$

$\Rightarrow a = 8$

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19. Let $f(x)=5-|x-2|$ and $g(x)=|x+1|, x\in\mathbb{R}$. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β, then $\lim\limits_{x\to-\alpha\beta}\dfrac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to:

- A. $\frac{1}{2}$ ✓
- B. $\frac{-1}{2}$ ✗
- C. $\frac{3}{2}$ ✗
- D. $\frac{-3}{2}$ ✗

$f(x)=5-|x-2|$

$f(x)$ attains maximum value when

$|x-2|=0\Rightarrow x=2=\alpha$

$g(x)=|x+1|$

$g(x)$ attains minimum value when $x=-1=\beta$

$$\lim_{x\to-\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$$

$$=\lim_{x\to2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}=\frac{1}{2}$$

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20. If the function $f(x) = \begin{cases} a|\pi - x| + 1, & x \le 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$

is continuous at $x = 5$, then the value of $a - b$ is :

A. $\dfrac{2}{\pi - 5}$

B. $\dfrac{2}{\pi + 5}$

C. $\dfrac{2}{5 - \pi}$ ✔

D. $\dfrac{-2}{\pi + 5}$

$f(x) = \begin{cases} a|\pi - x| + 1, & x \le 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$

is continuous at $x = 5$

$\therefore \lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = f(5)$

$\Rightarrow a|\pi - 5| + 1 = b|5 - \pi| + 3$

$\Rightarrow a(5 - \pi) + 1 = b(5 - \pi) + 3$

$\Rightarrow (a - b)(5 - \pi) = 2$

$\therefore a - b = \dfrac{2}{5 - \pi}$