Date: 15/11/2021

Subject: Mathematics

Topic : Continuity and Differentiability

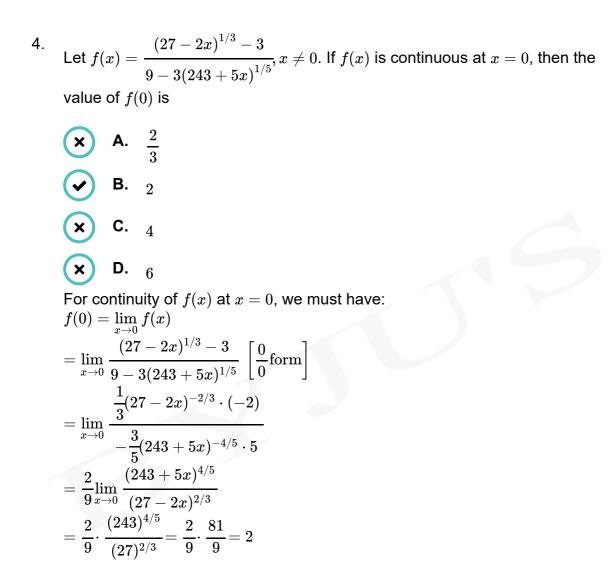
Class: Standard XII

1. The function given by
$$f(x) = \frac{1}{|x| - 1} - \frac{x^2}{2}$$
 is continuous in
A. $\mathbb{R} - \{-1, 1\}$
B. $\mathbb{R} - \{1\}$
C. $\mathbb{R} - \{1\}$
C. $\mathbb{R} - \{-1\}$
D. $\{-1, 1\}$
 $\frac{1}{|x| - 1}$ is discontinuous where $|x| = 1$
i.e., $x = -1, 1$
while $\frac{x^2}{2}$ is continuous for all real x .
So, $f(x) = \frac{1}{|x| - 1} - \frac{x^2}{2}$ is discontinuous at $x = -1, 1$
Hence, $f(x)$ is continuous for all $x \in \mathbb{R} - \{-1, 1\}$





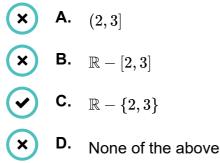
- 2. $f(x) = sgn(x^3 x)$ is discontinuous at which of the following points
 - **X** A. $_0$ **X** B. $_1$ **X** C. $_{-1}$ **V** D. All the above We know that sgn(x) is discontinuous at x = 0. So, the function $f(x) = sgn(x^3 - x)$ will be discontinuous at all the points where $x^3 - x = 0$ $\Rightarrow x (x^2 - 1) = 0$ $\Rightarrow x (x + 1)(x - 1) = 0$ $\Rightarrow x = \{0, 1, -1\}$
- 3. If f(x) = [[x]] [x 1] then which of the following options is CORRECT, (where [.] represents the greatest integer function.)
 - **X A.** Discontinuous at x = 0
 - **B.** Discontinuous at x = 1
 - **× C.** Discontinuous at x = -1
 - **D.** Continuous at every where
 - $f(x) = [[x]] [x-1] = [x] ([x]-1) = 1 \ \Rightarrow f(x) ext{ is continuous } orall x \in \mathbb{R}$



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The value of f(0), so that the function $f(x) = \frac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}$ is continuous 5. at x = 0 is $\frac{1}{6}$ Α. $\frac{1}{3}$ В. $\frac{1}{2}$ × С. $\frac{2}{3}$ X D. For continuity of f(x) at x = 0, we must have, $f(0) = \lim_{x o 0} f(x) = \lim_{x o 0} \left(rac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}
ight) = \lim_{x o 0} rac{(1+x)^{1/2} - (1+x)^{1/3}}{x}$ $\left|\frac{0}{0}$ form $= \lim_{x o 0} \left\{ rac{1}{2(1+x)^{1/2}} - rac{1}{3(1+x)^{2/3}}
ight\} = rac{1}{2} - rac{1}{3} = rac{1}{6}$

6. The function given by $f(x) = rac{3x+7}{x^2-5x+6}$ is continuous in



f(x) is not defined when denominator equals 0 So, for f(x) to be continuous, $x^2 - 5x + 6 \neq 0$, $\Rightarrow x \neq 2, 3$ So, f(x) is continuous for all $x \in \mathbb{R} - \{2, 3\}$

7. If $f(x) = \frac{1 - \sin x}{\sin 2x}, x \neq \frac{\pi}{2}$ is continuous at $x = \frac{\pi}{2}$, then the value of $f\left(\frac{\pi}{2}\right)$ is A. 0 A. 0 A. 0 X B. $\frac{1}{2}$ X C. 1 X D. 2 For continuity of f(x) at $x = \frac{\pi}{2}$, we must have $f\left(\frac{\pi}{2}\right) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\sin 2x} \left(\frac{0}{0} \text{ form}\right)$ Using L Hospital's Rule $= \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{2\cos 2x} = \frac{-\cos \frac{\pi}{2}}{2\cos \pi} = 0$



8.
Let
$$f(x) = \begin{cases} (x-1)^{\frac{1}{2}-x}, & x > 1, x \neq 2\\ k, & x = 2 \end{cases}$$

The value of k for which f is continuous at $x = 2$ is :

A. e^{-1}
B. e^{-2}
C. e
D. 1
 $f(x) = \begin{cases} (x-1)^{\frac{1}{2}-x}, & x > 1, x \neq 2\\ k, & x = 2 \end{cases}$
As $f(x)$ is continuous at $x = 2$
 $\therefore k = \lim_{x \to 2} (x-1)^{\frac{1}{2}-x}$
 $= \lim_{x \to 2} e^{(x-1-1)\cdot \frac{1}{2}-x}$
 $= \lim_{x \to 2} e^{(-1)}$
 $= e^{-1}$



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9. For the function $f(x) = \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$. The value of $f(\pi)$, so that f(x) is continuous at $x = \pi$ is

$$\begin{array}{c|ccc} \checkmark & \mathsf{A.} & -1 \\ \hline \bigstar & \mathsf{B.} & -\frac{1}{2} \\ \hline \bigstar & \mathsf{C.} & \frac{1}{2} \\ \hline \bigstar & \mathsf{C.} & \frac{1}{2} \\ \hline \bigstar & \mathsf{D.} & 1 \\ \end{array}$$
For continuity of $f(x)$ at $x = \pi$, we must have $f(\pi) = \lim_{x \to \pi} f(x)$

$$\lim_{x \to \pi} f(x) = \lim_{x \to \pi} \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x} \quad \left[\frac{0}{0} \text{Form} \right]$$
Applying L Hospital's rule
$$= \lim_{x \to \pi} \frac{-\cos x - \sin x}{\cos x - \sin x}$$

$$= \frac{-\cos \pi - \sin \pi}{\cos \pi - \sin \pi}$$

$$= \frac{1 - 0}{-1 - 0} = -1$$

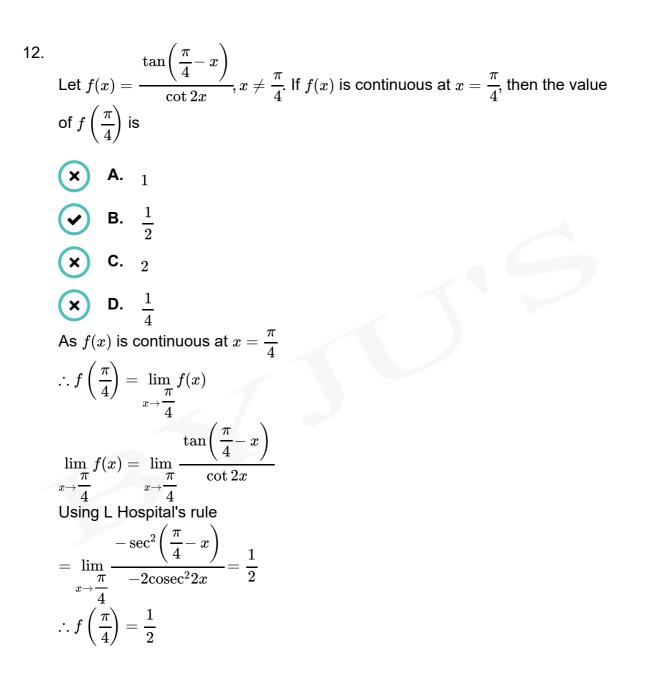
$$\therefore f(\pi) = -1$$

10.
If
$$f(x) = \begin{cases} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{a}\right)\ln\left(1 + \frac{x^2}{3}\right)}, & x \neq 0 \\ 9(\ln 4)^3, & x = 0 \end{cases}$$
 is continuous at $x = 0$, then the 9(\ln 4)^3, $x = 0$
Value of a is
 \checkmark A. 0
 \checkmark B. 1
 \checkmark C. 2
 \checkmark D. 3
For $f(x)$ to be continuous at $x = 0$, we must have $f(0) = \lim_{x \to 0} f(x) \cdots (1)$
 $\Rightarrow \lim_{x \to 0} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{a}\right)\ln\left(1 + \frac{x^2}{3}\right)}$
 $= \lim_{x \to 0} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{a}\right)\ln\left(1 + \frac{x^2}{3}\right)} \cdot \frac{\frac{x^2}{3}}{\ln\left(1 + \frac{x^2}{3}\right)} \cdot 3a$
 $= 3a \cdot \left(\lim_{x \to 0} \frac{4^x - 1}{x}\right)^3 \cdot \left(\frac{1}{\lim_{x \to 0} \frac{\sin(x/a)}{(x/a)}}\right) \cdot \left(\frac{1}{\lim_{x \to 0} \frac{\ln(1 + x^2/3)}{(x^2/3)}}\right)$
 $= 3a \cdot (\ln 4)^3 \cdot \frac{1}{1} \cdot \frac{1}{1} = 3a(\ln 4)^3$
Using (1), we get
 $\Rightarrow 9(\ln 4)^3 = 3a(\ln 4)^3$

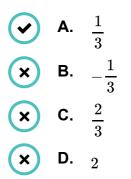
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11. If $f(x) = \begin{cases} \frac{(1-\sin x)}{(\pi-2x)^2}, & x \neq \frac{\pi}{2} \\ \lambda, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of λ is $\lambda, & x = \frac{\pi}{2}$ $\lambda, & x = \frac{\pi}{2}$, we must have $\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right) = \lambda$ $x \to \frac{\pi}{2}$ $\lambda = \lim_{x \to \frac{\pi}{2}} f(x) = \lim_{x \to \frac{\pi}{2}} \frac{(1-\sin x)}{(\pi-2x)^2}$ Using L Hospital's Rule $= \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-4(\pi-2x)}$ $= \frac{1}{4} \cdot \lim_{x \to \frac{\pi}{2}} \frac{-\sin x}{-2} = \frac{1}{8}$





^{13.} The value of f(0) such that the function $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at every point in its domain, is equal to



For f(x) to be continuous at every point of its domain, it must be continuous at x = 0 also,

$$\therefore$$
 We must have, $f(0) = \lim_{x o 0} f(x),$

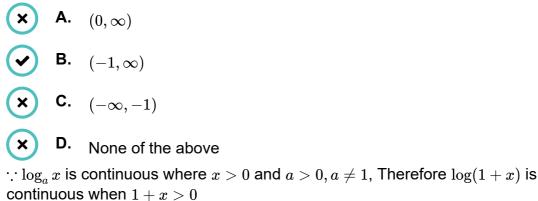
 $\Rightarrow f(0) = \lim_{x o 0} rac{2x - \sin^{-1}x}{2x + an^{-1}x}$

Dividing numerator and denominator by \boldsymbol{x}

$$=\lim_{x\to 0} \left\{ \frac{2-\frac{\sin^{-1}x}{x}}{2+\frac{\tan^{-1}x}{x}} \right\} = \frac{1}{3}$$
$$\therefore f(0) = \frac{1}{3}$$

14. The function $f(x) = x - |x - x^2|$ is A. continuous at x = 1B. discontinuous at x = 0C. not defined at x = 1

15. The interval where the function log(1 + x) is continuous, is



continuous when $\Rightarrow x > -1$

 \therefore Required interval $= (-1,\infty)$

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16. If $f(x) = \frac{\log_e(1+x^2 \tan x)}{\sin x^3}$, $x \neq 0$ is continuous at x = 0, then the value of f(0) is (**x**) **A**. -1(**x**) **B**. 0(**x**) **C**. $\frac{1}{2}$ (**v**) **D**. 1For f(x) to be continuous at x = 0, we must have $f(0) = \lim_{x \to 0} f(x)$ $= \lim_{x \to 0} \frac{\log_e(1+x^2 \tan x)}{\sin x^3}$ $= \lim_{x \to 0} \frac{\log_e(1+x^2 \tan x)}{x^2 \tan x} \cdot \frac{x^2 \tan x}{x^3} \cdot \frac{x^3}{\sin x^3}$ $= \lim_{x \to 0} \left\{ \frac{\log_e(1+x^2 \tan x)}{x^2 \tan x} \right\} \cdot \lim_{x \to 0} \left(\frac{\tan x}{x} \right) \cdot \frac{1}{\lim_{x \to 0} \left(\frac{\sin x^3}{x^3} \right)}$ $= 1 \times 1 \times \frac{1}{1} = 1$



$$\mathsf{Let}\; f(x) = egin{cases} \sqrt{1+x^2}, & x < \sqrt{3} \ \sqrt{3}x - 1, & \sqrt{3} \leq x < 4 \ [x], & 4 \leq x < 5 \ |1-x|, & x \geq 5 \end{cases}$$

where [x] is the greatest integer less than or equal to x. The number of point(s) of discontinuity of f(x) in \mathbb{R} is

X A. 3 **X** B. 0 **X** C. Infinite **D**. 1 Checking discontinuity at the points where f(x) changes it's branches i.e. $\sqrt{3}, 4, 5$ $f(\sqrt{3}^{-}) = 2$

$$f(\sqrt{3}^{+}) = f(\sqrt{3}) = 2$$

 $f(\sqrt{3}^{+}) = f(\sqrt{3}) = 2$
 $f(4^{-}) = 4\sqrt{3} - 1$
 $f(4^{+}) = f(4) = 4$
 $f(5^{-}) = 4$
 $f(5^{+}) = f(5) = 4$
So, $f(x)$ is discontinuous only at $x = 4$.

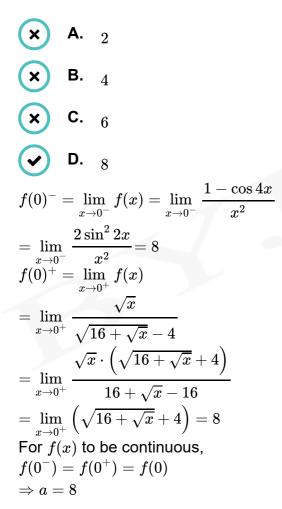


18.

The value of *a* so that the function f(x) =

$$\left\{egin{array}{ccc} rac{1-\cos 4x}{x^2}, & x < 0\ a, & x = 0\ rac{\sqrt{x}}{\sqrt{x}}, & x > 0 \end{array}
ight.$$

is continuous at x = 0 is



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19. Let f(x) = 5 - |x - 2| and $g(x) = |x + 1|, x \in \mathbb{R}$. If f(x) attains maximum value at α and g(x) attains minimum value at β , then $\lim_{x \to -\alpha\beta} \frac{(x - 1)(x^2 - 5x + 6)}{x^2 - 6x + 8}$ is equal to:

$$\begin{array}{|c|c|} \checkmark & \mathsf{A.} & \frac{1}{2} \\ \hline \bigstar & \mathsf{B.} & \frac{-1}{2} \\ \hline \bigstar & \mathsf{B.} & \frac{-1}{2} \\ \hline \bigstar & \mathsf{C.} & \frac{3}{2} \\ \hline \bigstar & \mathsf{D.} & \frac{-3}{2} \\ f(x) = 5 - |x-2| \\ f(x) \text{ attains maximum value when } \\ |x-2| = 0 \Rightarrow x = 2 = \alpha \\ g(x) = |x+1| \\ g(x) \text{ attains minimum value when } x = -1 = \beta \\ \lim_{x \to -\alpha\beta} \frac{(x-1)(x^2 - 5x + 6)}{x^2 - 6x + 8} \\ = \lim_{x \to 2} \frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)} = \frac{1}{2} \\ \end{array}$$

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20. If the function $f(x) = \begin{cases} a|\pi - x| + 1, & x \leq 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$ is continuous at x = 5, then the value of a - b is :

$$\begin{array}{c|cccc} \bigstar & \mathsf{A.} & \frac{2}{\pi-5} \\ \hline \bigstar & \mathsf{B.} & \frac{2}{\pi+5} \\ \hline \checkmark & \mathsf{C.} & \frac{2}{5-\pi} \\ \hline \bigstar & \mathsf{D.} & \frac{-2}{\pi+5} \\ f(x) = \begin{cases} a|\pi-x|+1, & x \le 5 \\ b|x-\pi|+3, & x > 5 \\ b|x-\pi|+3, & x > 5 \\ \hline & b|x-\pi|+3, & b|x-\pi|+3, & b|x-\pi|+3, & b|x-\pi|+3, \\ \hline & b|x-\pi|+3, & b|x-\pi|+3, & b|x-\pi|+3, & b|x-\pi|+3, \\ \hline & b|x-\pi|+3, & b|x-\pi|+3, & b|x-\pi|+3, & b|x-\pi|+3, & b|x-\pi|+3, \\ \hline & b|x-\pi|+3, & b|x-\pi|$$

