

BYJU'S Study Planner for Board Term I (CBSE Grade 12)

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Subject: Mathematics

Topic : Inverse Trigonometric Functions

Class: Standard XII

1. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is

- A. $\frac{\pi}{3}$
- B. $-\frac{\pi}{3}$
- C. $\frac{\pi}{6}$
- D. $-\frac{\pi}{6}$

Let the principal value be y

$$y = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right), y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin y = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin y = \sin\left(-\frac{\pi}{3}\right)$$

$$\Rightarrow y = -\frac{\pi}{3}$$

2. The domain of the function $f(x) = \sin^{-1}(5x)$ is

- A. $\left[-\frac{\pi}{5}, \frac{\pi}{5}\right]$
- B. $\left[-\frac{\pi}{10}, \frac{\pi}{10}\right]$
- C. \mathbb{R}
- D. $\left[-\frac{1}{5}, \frac{1}{5}\right]$

Given,

$$f(x) = \sin^{-1}(5x)$$

$$\because -1 \leq 5x \leq 1$$

$$\therefore -\frac{1}{5} \leq x \leq \frac{1}{5}$$

Hence, domain is $\left[-\frac{1}{5}, \frac{1}{5}\right]$

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3. The value of $\cos^{-1}\left(\cos \frac{5\pi}{3}\right) + \sin^{-1}\left(\cos \frac{5\pi}{3}\right)$ is

- A. $\frac{\pi}{2}$
- B. $\frac{5\pi}{3}$
- C. $\frac{10\pi}{3}$
- D. 0

$$\cos^{-1}\left[\cos \frac{5\pi}{3}\right] + \sin^{-1}\left[\frac{\cos 5\pi}{3}\right] = \frac{\pi}{2} \left(\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right)$$

4. The value of $\sin^{-1}\left(\sin \frac{4\pi}{3}\right) + \cos^{-1}\left(\cos \frac{4\pi}{3}\right)$ is

- A. $\frac{8\pi}{3}$
- B. $\frac{4\pi}{3}$
- C. $\frac{2\pi}{3}$
- D. $\frac{\pi}{3}$

$$\begin{aligned} & \sin^{-1}\left(\sin \frac{4\pi}{3}\right) + \cos^{-1}\left(\cos \frac{4\pi}{3}\right) \\ &= \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) \\ &= -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \pi - \cos^{-1}\left(\frac{1}{2}\right) \\ &= -\frac{\pi}{3} + \pi - \frac{\pi}{3} = \frac{\pi}{3} \end{aligned}$$

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5. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to

- A. $\frac{\pi}{2}$
- B. $\frac{\pi}{4}$
- C. π
- D. $\frac{3\pi}{4}$

$$\text{Given : } \sin^{-1} x + \sin^{-1} y = \frac{\pi}{2} \dots (1)$$

We know that,

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x$$

\therefore Equation (1) becomes

$$\frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \cos^{-1} y = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$$

6. $\cos \left[\cos^{-1} \left(\frac{-1}{7} \right) + \sin^{-1} \left(\frac{-1}{7} \right) \right] =$

- A. $\frac{-1}{3}$
- B. 0
- C. $\frac{1}{3}$
- D. $\frac{4}{9}$

$$\cos \left\{ \cos^{-1} \left(\frac{-1}{7} \right) + \sin^{-1} \left(\frac{-1}{7} \right) \right\} = \cos \frac{\pi}{2} = 0$$

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7. The value of $\tan^{-1}\left(\tan\left(-\frac{3\pi}{4}\right)\right) + \cot^{-1}\left(\cot\left(-\frac{3\pi}{4}\right)\right)$ is

A. $\frac{\pi}{2}$

B. π

C. $-\frac{3\pi}{2}$

D. $\frac{3\pi}{2}$

$$\begin{aligned} & \tan^{-1}\left(\tan\left(-\frac{3\pi}{4}\right)\right) + \cot^{-1}\left(\cot\left(-\frac{3\pi}{4}\right)\right) \\ &= \tan^{-1}\left(-\tan\left(\frac{3\pi}{4}\right)\right) + \cot^{-1}\left(-\cot\left(\frac{3\pi}{4}\right)\right) \\ &= \tan^{-1}\left(\tan\left(\frac{\pi}{4}\right)\right) + \cot^{-1}\left(\cot\left(\frac{\pi}{4}\right)\right) \\ &= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} \end{aligned}$$

8. The value of $\tan^{-1} \cot \frac{12\pi}{7}$ is

A. $\frac{12\pi}{7}$

B. $\frac{5\pi}{7}$

C. $\frac{3\pi}{7}$

D. $-\frac{3\pi}{14}$

$$\begin{aligned} \tan^{-1} \cot \frac{12\pi}{7} &= \frac{\pi}{2} - \cot^{-1} \cot \frac{12\pi}{7} \\ &= \frac{\pi}{2} - \cot^{-1} \cot\left(\pi + \frac{5\pi}{7}\right) = \frac{\pi}{2} - \frac{5\pi}{7} = -\frac{3\pi}{14} \end{aligned}$$

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9. In the interval $x \in [0, 1]$ the value of $\cos^{-1} \sqrt{1-x} + \sin^{-1} \sqrt{1-x}$ is

- A. π
- B. $\frac{\pi}{2}$
- C. 1
- D. 0

$\sqrt{1-x}$ is defined for $x \in [0, 1]$

Here, the argument of \cos^{-1} and \sin^{-1} is same.

$$\cos^{-1} \sqrt{1-x} + \sin^{-1} \sqrt{1-x} = \frac{\pi}{2}$$

10. A solution of the equation $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$ is

- A. $x = 1$
- B. $x = -1$
- C. $x = 0$
- D. $x = \pi$

$$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1}(1+x) = \frac{\pi}{2} - \tan^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x) = \cot^{-1}(1-x)$$

$$\Rightarrow \tan^{-1}(1+x) = \tan^{-1}\left(\frac{1}{1-x}\right)$$

$$\Rightarrow 1+x = \frac{1}{1-x} \Rightarrow 1-x^2 = 1$$

$$\Rightarrow x = 0$$

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11. The value of $\cos^{-1} \left[-\sin \left(\frac{7\pi}{6} \right) \right]$ is

A. $\frac{5\pi}{3}$

B. $\frac{7\pi}{6}$

C. $\frac{\pi}{3}$

D. $-\frac{7\pi}{6}$

$$\cos^{-1} \left[-\sin \left(\frac{7\pi}{6} \right) \right]$$

$$= \cos^{-1} \left[\cos \left(\frac{\pi}{2} + \frac{7\pi}{6} \right) \right]$$

$$= \cos^{-1} \left[\cos \left(\frac{10\pi}{6} \right) \right]$$

$$= \cos^{-1} \left[\cos \left(\frac{5\pi}{3} \right) \right]$$

$$= \cos^{-1} \left[\cos \left(2\pi - \frac{\pi}{3} \right) \right]$$

$$= \cos^{-1} \left[\cos \frac{\pi}{3} \right]$$

$$= \frac{\pi}{3} \quad (\because \cos^{-1}(\cos x) = x, \quad x \in [0, \pi])$$

Alternate solution

$$\cos^{-1} \left[-\sin \left(\frac{7\pi}{6} \right) \right]$$

$$= \cos^{-1} \left[-\sin \left(\pi + \frac{\pi}{6} \right) \right]$$

$$= \cos^{-1} \left[\sin \left(\frac{\pi}{6} \right) \right]$$

$$= \cos^{-1} \left(\frac{1}{2} \right)$$

$$= \frac{\pi}{3}, \quad \because \cos^{-1}(x) \in [0, \pi]$$

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12. The value of $\sin^{-1} \sin \frac{36\pi}{7} + \cos^{-1} \sin \frac{39\pi}{7}$ is

A. $\frac{4\pi}{7}$

B. $\frac{\pi}{7}$

C. $\frac{11\pi}{14}$

D. $\frac{3\pi}{14}$

$$\sin^{-1} \sin \frac{36\pi}{7}$$

$$= \sin^{-1} \sin \left(5\pi + \frac{\pi}{7} \right)$$

$$= -\frac{\pi}{7}$$

$$\cos^{-1} \sin \frac{39\pi}{7}$$

$$= \frac{\pi}{2} - \sin^{-1} \sin \left(6\pi - \frac{3\pi}{7} \right)$$

$$= \frac{\pi}{2} - \left(-\frac{3\pi}{7} \right)$$

$$= \frac{\pi}{2} + \left(\frac{3\pi}{7} \right) = \frac{13\pi}{14}$$

\therefore The value of the above expression is $-\frac{\pi}{7} + \frac{13\pi}{14} = \frac{11\pi}{14}$

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13. The value of $\cos\left[2\cos^{-1}\frac{1}{5} + \sin^{-1}\frac{1}{5}\right]$ is

- A. $\frac{2\sqrt{6}}{5}$
- B. $-\frac{2\sqrt{6}}{5}$
- C. $\frac{1}{5}$
- D. $-\frac{1}{5}$

$$\cos\left(\cos^{-1}\frac{1}{5} + \sin^{-1}\frac{1}{5} + \cos^{-1}\frac{1}{5}\right)$$

$$= \cos\left(\frac{\pi}{2} + \cos^{-1}\frac{1}{5}\right)$$

$$= -\sin\left(\cos^{-1}\frac{1}{5}\right)$$

$$= -\sin\left(\sin^{-1}\sqrt{\frac{24}{25}}\right)$$

$$= -\frac{2\sqrt{6}}{5}$$

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14. If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then x equals

- A. -1
- B. 1
- C. 0
- D. None of these

$$\begin{aligned}
 & (\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8} \\
 \Rightarrow & (\tan^{-1} x + \cot^{-1} x)^2 - 2 \tan^{-1} x \left(\frac{\pi}{2} - \tan^{-1} x \right) = \frac{5\pi^2}{8} \\
 \Rightarrow & \frac{\pi^2}{4} - 2 \times \frac{\pi}{2} \tan^{-1} x + 2(\tan^{-1} x)^2 = \frac{5\pi^2}{8} \\
 \Rightarrow & 2(\tan^{-1} x)^2 - \pi \tan^{-1} x - \frac{3\pi^2}{8} = 0 \\
 \Rightarrow & \tan^{-1} x = -\frac{\pi}{4}, \frac{3\pi}{4} \\
 \Rightarrow & \tan^{-1} x = -\frac{\pi}{4} \\
 \Rightarrow & x = -1
 \end{aligned}$$

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15. If α and β ($\alpha > \beta$) are the roots of the equation $x^2 - \sqrt{2}x + \sqrt{3 - 2\sqrt{2}} = 0$, then the value of $(\cos^{-1} \alpha + \tan^{-1} \alpha + \tan^{-1} \beta)$ is equal to

- A. $\frac{3\pi}{8}$
- B. $\frac{5\pi}{8}$
- C. $\frac{7\pi}{8}$
- D. $\frac{\pi}{3}$

$$\begin{aligned}x^2 - \sqrt{2}x + \sqrt{3 - 2\sqrt{2}} &= 0 \\ \Rightarrow x^2 - \sqrt{2}x + \sqrt{(\sqrt{2} - 1)^2} &= 0 \\ \Rightarrow x^2 - 1 - \sqrt{2}(x - 1) &= 0 \\ \Rightarrow (x - 1)(x + 1 - \sqrt{2}) &= 0 \\ \Rightarrow x = 1, \sqrt{2} - 1 &\\ \therefore \alpha = 1 \text{ and } \beta = \sqrt{2} - 1 &\end{aligned}$$

Hence, $\cos^{-1} \alpha + \tan^{-1} \alpha + \tan^{-1} \beta = 0 + \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$

16. The value of $\sin^{-1} \left(\frac{3}{5} \right) + \tan^{-1} \left(\frac{1}{7} \right)$ is

- A. $\frac{\pi}{4}$
- B. $\frac{\pi}{2}$
- C. $\cos^{-1} \left(\frac{4}{5} \right)$
- D. $\frac{3\pi}{4}$

$$\begin{aligned}\sin^{-1} \frac{3}{5} + \tan^{-1} \frac{1}{7} &= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} \\ &= \tan^{-1} \left(\frac{\frac{3}{4} + \frac{1}{7}}{1 - \left(\frac{3}{4} \times \frac{1}{7} \right)} \right) \quad \left[\because \frac{3}{4} \times \frac{1}{7} < 1 \right] \\ &= \tan^{-1} \left(\frac{25}{25} \right) \\ &= \tan^{-1} 1 = \frac{\pi}{4}\end{aligned}$$

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17. The value of $\sum_{x=0}^4 \sin^{-1}(\sin x)$ is equal to

- A. $3\pi - 8$
- B. $3\pi - 7$
- C. $3\pi - 9$
- D. $3\pi - 6$

Given:

$$\sum_{x=0}^4 \sin^{-1}(\sin x) = \sin^{-1}(\sin 0) + \sin^{-1}(\sin 1) + \sin^{-1}(\sin 2) + \sin^{-1}(\sin 3) + \sin^{-1}(\sin 4)$$

We know, $\sin^{-1}(\sin x) = \begin{cases} x, & -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x \leq \frac{3\pi}{2} \end{cases}$

$$\begin{aligned} \therefore \sin^{-1}(\sin 0) + \sin^{-1}(\sin 1) + \sin^{-1}(\sin 2) + \sin^{-1}(\sin 3) + \sin^{-1}(\sin 4) \\ = (0) + (1) + (\pi - 2) + (\pi - 3) + (\pi - 4) \\ = 3\pi - 8 \end{aligned}$$

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18. If $f(x) = x^{11} + x^9 - x^7 + x^3 + 1$ and $f(\sin^{-1}(\sin 8)) = \alpha$, where α is constant, then $f(\tan^{-1}(\tan 8))$ is equal to

- A. α
- B. $\alpha - 2$
- C. $\alpha + 2$
- D. $2 - \alpha$

We observe $8 \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

But $3\pi - 8 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore f(\sin^{-1}(\sin 8)) = f(\sin^{-1}(\sin(3\pi - 8))) = f(3\pi - 8)$$

$$f(3\pi - 8) = \alpha$$

$$\Rightarrow (3\pi - 8)^{11} + (3\pi - 8)^9 - (3\pi - 8)^7 + (3\pi - 8)^3 + 1 = \alpha \quad \dots (1)$$

$$\text{Now, } f(\tan^{-1}(\tan 8))$$

$$= f(\tan^{-1}(\tan(8 - 3\pi)))$$

$$= f(8 - 3\pi)$$

$$= (8 - 3\pi)^{11} + (8 - 3\pi)^9 - (8 - 3\pi)^7 + (8 - 3\pi)^3 + 1$$

$$= 2 - ((3\pi - 8)^{11} + (3\pi - 8)^9 - (3\pi - 8)^7 + (3\pi - 8)^3 + 1)$$

$$\text{From (1), } f(\tan^{-1}(\tan 8)) = 2 - \alpha$$

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19. Consider

$$f(x) = \sin^{-1}(\sec(\tan^{-1} x)) + \cos^{-1}(\cosec(\cot^{-1} x))$$

Statement–1 : Domain of $f(x)$ is a singleton set.

Reason

Statement–2 : Range of the function $f(x)$ is a singleton set.

- A. Statement–1 is true, Statement–2 is true and Statement–2 is correct explanation for Statement–1.
- B. Statement–1 is true, Statement–2 is true and Statement–2 is NOT the correct explanation for Statement–1.
- C. Statement–1 is true, Statement–2 is false.
- D. Statement–1 is false, Statement–2 is true.

$$f(x) = \sin^{-1}(\sec(\tan^{-1}(x))) + \cos^{-1}(\cosec(\cot^{-1} x))$$

$$\Rightarrow f(x) = \sin^{-1}\left(\sqrt{1 + \tan^2(\tan^{-1}(x))}\right) + \cos^{-1}\left(\sqrt{1 + \cot^2 \cot^{-1}(x)}\right)$$

$$\Rightarrow f(x) = \sin^{-1}\left(\sqrt{x^2 + 1}\right) + \cos^{-1}\left(\sqrt{x^2 + 1}\right)$$

Hence,

$$-1 \leq \sqrt{x^2 + 1} \leq 1$$

$$\Rightarrow x^2 + 1 \leq 1$$

$$\Rightarrow x^2 \leq 0$$

$$\therefore x = 0$$

Hence, domain is a singleton set and

$$f(x) = \frac{\pi}{2}$$

Therefore, the range is also a singleton set.

Both statements are correct but second statement is not the correct explanation.

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20. Assertion (*A*) : $\cos^{-1} x$ and $\tan^{-1} x$ are positive for all positive real values of x in their domain.

Reason (*R*) : The domain of $f(x) = \cos^{-1} x + \tan^{-1} x$ is $[-1, 1]$.

- A. Both *A* and *R* are true and *R* is the correct explanation of *A*.
- B. Both *A* and *R* are true but *R* is not correct explanation of *A*.
- C. *A* is true but *R* is false.
- D. *A* is false but *R* is true.

Assertion:

For $\cos^{-1} x$, range is $[0, \pi]$ and domain is $[-1, 1]$.
 $\therefore \cos^{-1} x > 0$ for all x in its domain

For $\tan^{-1} x$, range is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and domain is \mathbb{R} .

$$\forall x > 0 \Rightarrow \tan^{-1} x > 0$$

Reason:

The domain of $\cos^{-1} x$ is $[-1, 1]$

The domain of $\tan^{-1} x$ is \mathbb{R}

So, the domain of $\cos^{-1} x + \tan^{-1} x$ is $\mathbb{R} \cap [-1, 1]$ i.e., $[-1, 1]$

So, it's also true but not the correct explanation of *A* as *R* doesn't give any information about range.