BYJU'S Study Planner for Board Term I
(CBSE Grade 12)

Date: 09/11/2021

Subject: Mathematics

Topic : Relations and Functions

Class: Standard XII

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1. Let A be a non-empty set such that $A \times A$ has 9 elements and $(-1,0),(0,1)$ are elements of $A \times A$, then $A =$

   A. $\{-1,0,2\}$

   B. $\{-1,0,1\}$

   C. $\{-2,0,1\}$

   D. Cannot be determined

Given that $(-1,0) \in A \times A$ and
$(0,1) \in A \times A$
$\Rightarrow -1,0,1$ are elements of set A
and given that $n(A \times A)=9 \Rightarrow n(A)=3$
$\therefore A=\{-1,0,1\}$

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2. Let $\mathbb{N}$ denote the set of all natural numbers. Define two binary relations on $\mathbb{N}$ as
$R_1 = \{(x, y) \in \mathbb{N} \times \mathbb{N} : 2x + y = 10\}$ and
$R_2 = \{(x, y) \in \mathbb{N} \times \mathbb{N} : x + 2y = 10\}$. Then

- **A.** Both R_1 and R_2 are transitive relations.
- **B.** Range of R_2 is $\{1, 2, 3, 4\}$.
- **C.** Range of R_1 is $\{2, 4, 8\}$.
- **D.** Both R_1 and R_2 are symmetric relations.

$R_1 = \{(1, 8), (2, 6), (3, 4), (4, 2)\}$
$R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$
$\text{Range of } R_2 = \{1, 2, 3, 4\}$

$(1, 8) \in R_1$ but $(8, 1) \notin R_1$, so R_1 is not symmetric.
$(4, 2) \in R_1$ & $(2, 6) \in R_1$ but $(4, 6) \notin R_1$, So R_1 is not transitive.

Similarly, R_2 is neither symmetric nor transitive.

3. If the function $f : B \to [-5, \infty)$ defined by $f(x) = x^2 - 4x + 5$ is one-one function, then B is

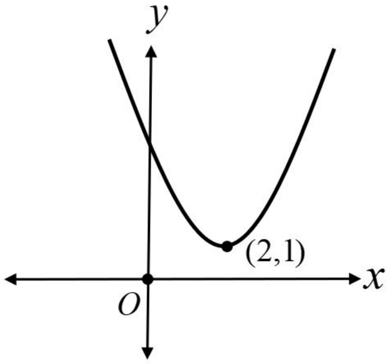
A. $[2, \infty)$

B. $[0, \infty)$

C. $[-5, \infty)$

D. $[-1, \infty)$

$f(x) = x^2 - 4x + 5$
$= (x - 2)^2 + 1$



Clearly, we can conclude from above graph and given options $f(x)$ will be one-one function for $x \in [2, \infty)$

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4. Consider the set $A=\{1,2,3\}$ and the relation on A as $R=\{(1,2),(1,3)\}$, then R is

   - **A.** a reflexive relation
   - **B.** a symmetric relation
   - **C.** a transitive relation
   - **D.** None of the above

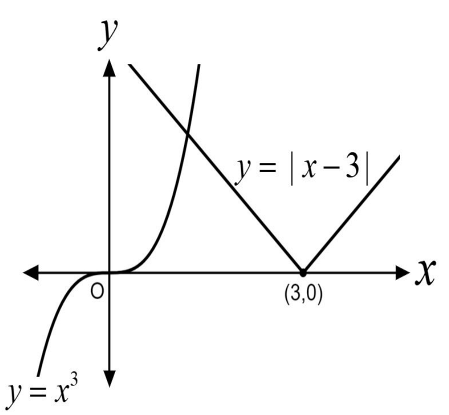
   R is not reflexive as $(1,1),(2,2),(3,3)$ does not belong to R
   R is not symmetric as$(1,2)\in R$ but $(2,1)\notin R$

   R is transitive as $(1,2),(1,3)\in R$
   but there is no ordered pair starting with 2 or 3 in relation
   i.e., the antecedent statement in transitive relation is false. So, given relation is transitive.

5. The number of solution(s) of the equation $|x-3|=x^3$ is

   - **A.** 1
   - **B.** 0
   - **C.** 2
   - **D.** 3

   The graph of $y=|x-3|$ and $y=x^3$



   Clearly, there is only one intersection point.
   Hence, the number of solution is 1.

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6. Let $f(x)=\dfrac{x^2-1}{x}, g(x)=\dfrac{x+2}{x-3}$ then domain of $\dfrac{f(x)}{g(x)}$ is

A. $\mathbb{R}-\{0,-2\}$

B. $\mathbb{R}-\{-2,0,3\}$

C. $\mathbb{R}$

D. $\mathbb{R}-\{0,-3\}$

Let $h(x)=\dfrac{f(x)}{g(x)}$

$$\therefore h(x)=\frac{\frac{x^2-1}{x}}{\frac{x+2}{x-3}}=\frac{(x^2-1)(x-3)}{x(x+2)}$$

But for $x=3, g(x)$ not defined
Hence $h(x)$ is not defined for $x=0,-2,3$
Hence required domain is
$\mathbb{R}-\{-2,0,3\}$

7. The function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=|(x-1)(x-2)|$ is

A. One-one function

B. Many-one function

C. Constant function

D. None of these

We can clearly see that $f(x)=0$ for $x=1$ and $x=2$
$\therefore f(x)$ is many-one function.

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8. Let $A = \{1, 2, 3\}$ and R, S be two relations on A given by
$R = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}, S = \{(1,1),(2,2),(3,3),(2,3),(3,2)\}$
then $R \cup S$ is

A. Reflexive, symmetric and transitive relation

B. reflexive and transitive relation only

C. not a transitive relation

D. Reflexive relation but not Symmetric relation

$R \cup S = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}$

$\therefore$ It is reflexive, symmetric but not transitive because $(1,2),(2,3) \in R \cup S$ but $(1,3) \notin R \cup S$.

9. Let $f(x) = (1+b^2)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is

A. $[0, 1]$

B. $\left[0, \frac{1}{2}\right]$

C. $\left[\frac{1}{2}, 1\right]$

D. $(0, 1]$

$f(x) = (1+b^2)x^2 + 2bx + 1$

Coefficient of x^2 is $(1+b^2) > 0$

Minimum value of $f(x)$ is $\frac{-D}{4a}$

$\therefore m(b) = -\frac{\{4b^2 - 4(1+b^2)\}}{4(1+b^2)}$

$\Rightarrow m(b) = \frac{1}{1+b^2}$

$\because 1 \le 1 + b^2 < \infty \ \forall\, b \in \mathbb{R}$

$\Rightarrow 0 < \frac{1}{1+b^2} \le 1$

$\therefore$ Range of $m(b)$ is $(0, 1]$

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10. The domain of $f(x)=\sqrt{\dfrac{4-x^2}{[x]+2}}$ is

(where [.] represents the greatest integer function)

- ✗ **A.** $(-\infty, 1)$
- ✓ **B.** $(-\infty,-2)\cup[-1,2]$
- ✗ **C.** $(-\infty,-1)\cup[2,\infty)$
- ✗ **D.** $(-\infty,1)\cup[2,\infty)$

Given : $f(x)=\sqrt{\dfrac{4-x^2}{[x]+2}}$

For $f(x)$ to be defined, we get

$\dfrac{4-x^2}{[x]+2}\geq 0 \text{ and } [x]+2\neq 0$

$\Rightarrow [x]\neq -2$

$\therefore x\notin[-2,-1)$

Case $1: 4-x^2\geq 0,\quad [x]+2>0$

$\Rightarrow x\in[-2,2], \qquad x\in[-1,\infty)$

$\therefore x\in[-1,2]\cdots(1)$

Case $2: 4-x^2<0,\quad [x]+2<0$

$\Rightarrow x\in(-\infty,-2)\cup(2,\infty),\ x\in(-\infty,-2)$

$\therefore x\in(-\infty,-2)\cdots(2)$

So, from (1) and (2), we get

$x\in(-\infty,-2)\cup[-1,2]$

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11. Let the function $f:\mathbb{R}-\{-b\}\to\mathbb{R}-\{1\}$ be defined by $f(x)=\dfrac{x+a}{x+b}, a\neq b,$ then

A. f is one–one but not onto function

B. f is onto but not one–one function

C. f is bijective function ✔

D. f is neither one–one nor onto function

For one-one :
Let $x, y \in \mathbb{R}-\{-b\}$ such that $f(x)=f(y)$
$\Rightarrow \dfrac{x+a}{x+b}=\dfrac{y+a}{y+b}$
$\Rightarrow xy+ay+bx+ab=xy+xa+yb+ab$
$\Rightarrow x(b-a)=y(b-a)$
$\Rightarrow x=y \ (\because a\neq b)$
$\therefore f$ is one-one function.

For onto :
Let $y\in\mathbb{R}$ such that $f(x)=y$
$\Rightarrow \dfrac{x+a}{x+b}=y$
$\Rightarrow x+a=xy+yb$
$\Rightarrow x=\dfrac{a-by}{y-1}$
$\Rightarrow y\in\mathbb{R}-\{1\}$
$\therefore f$ is onto function.

Hence, f is one-one and onto function, it is bijective function.

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12. If relation R is defined as aRb if "a is the father of b". Then R is

A. reflexive

B. symmetric

C. transitive

D. none of these ✔

Given
$R = \{(a, b); a$ is the father of $b\}$
$(i)\ \because (a, a) \notin R$ [as a can not be father of a]
So, R is not reflexive.

(ii) Now if aRb is true, then a is the father of $b\}$
$\Rightarrow (b, a) \notin R$
[as b cannot be father of a]
So, R is not symmetric.

(iii) Let $(a, b) \in R$ and $(b, c) \in R$
Hence a is father of b and b is father of c
$\Rightarrow (a, c) \notin R$
[as a is not the father of c]
So, R is not transitive.

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13. Let $\mathbb{N}$ denote the set of natural numbers and R be a relation on $\mathbb{N} \times \mathbb{N}$ defined by
$(a, b)R(c, d) \Longleftrightarrow ad(b+c) = bc(a+d)$. Then on $\mathbb{N} \times \mathbb{N}$, R is

- ✔ **A.** An equivalence relation
- ✖ **B.** Reflexive and symmetric relation only
- ✖ **C.** Transitive relation only
- ✖ **D.** Symmetric and transitive relation only

Given $(a, b)R(c, d) \Leftrightarrow ad(b+c) = bc(a+d)$ where $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$
$ad(b+c) = bc(a+d)$
$\Rightarrow \dfrac{b+c}{bc} = \dfrac{a+d}{ad}$
$\Rightarrow \dfrac{1}{b} + \dfrac{1}{c} = \dfrac{1}{a} + \dfrac{1}{d}$
$\therefore (a, b)R(c, d) \Leftrightarrow \dfrac{1}{b} + \dfrac{1}{c} = \dfrac{1}{a} + \dfrac{1}{d}$

As $\dfrac{1}{b} + \dfrac{1}{a} = \dfrac{1}{a} + \dfrac{1}{b}\ \forall\ (a, b) \in \mathbb{N} \times \mathbb{N}$
So, $(a, b)R(a, b)$
$\therefore R$ is reflexive relation.

If $(a, b)R(c, d) \Rightarrow \dfrac{1}{b} + \dfrac{1}{c} = \dfrac{1}{a} + \dfrac{1}{d}$
$\Rightarrow \dfrac{1}{d} + \dfrac{1}{a} = \dfrac{1}{c} + \dfrac{1}{b}$
$\Rightarrow (c, d)R(a, b)$
$\therefore R$ is symmetric relation.

Let $(a, b), (c, d), (e, f) \in \mathbb{N} \times \mathbb{N}$
Let $(a, b)R(c, d)$ and $(c, d)R(e, f)$ $(a, b)R(c, d) \Rightarrow ad(b+c) = bc(a+d)$
$\Rightarrow \dfrac{1}{b} + \dfrac{1}{c} = \dfrac{1}{a} + \dfrac{1}{d} \cdots (1)$
$(c, d)R(e, f) \Rightarrow cf(d+e) = de(c+f)$
$\Rightarrow \dfrac{1}{d} + \dfrac{1}{e} = \dfrac{1}{f} + \dfrac{1}{c} \cdots (2)$
$(1) + (2) \Rightarrow \left(\dfrac{1}{b} + \dfrac{1}{c} + \dfrac{1}{d} + \dfrac{1}{e}\right) = \left(\dfrac{1}{a} + \dfrac{1}{d} + \dfrac{1}{f} + \dfrac{1}{c}\right)$
$\dfrac{1}{b} + \dfrac{1}{e} = \dfrac{1}{a} + \dfrac{1}{f}$
$\Rightarrow (a, b)R(e, f)$
$\therefore R$ is a Transitive relation.
Since R is Reflexive, Symmetric and Transitive relation.
$\therefore R$ is an Equivalence relation.

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14. Let a relation f defined on $(0, \infty)$ as $f(x) = \left|1 - \frac{1}{x}\right|$. Then which among the following is true

A. $f(-1) = 2$

B. f is many-one function

C. f is one-one function

D. Relation f is not a function

$f(-1) = 2$ is not true as -1 is not an element in domain of relation.
$f(x) = \left|1 - \frac{1}{x}\right|$
For $x = 1$
$f(x) = \left|1 - \frac{1}{1}\right| = 0$
But, the co-domain of relation is $(0, \infty)$ which is not including 0.
So, relation f is not a function.

15. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \dfrac{(e^x - e^{-x})}{2}$.
The inverse of the given function is:

A. $f^{-1}(x) = \log_e(x + \sqrt{x^2 + 1})$

B. $f^{-1}(x) = \log_e(x - \sqrt{x^2 + 1})$

C. $f^{-1}(x) = \log_e(x + \sqrt{x^2 - 1})$

D. $f^{-1}(x) = \log_e(x - \sqrt{x^2 - 1})$

Let $y = f(x) = (e^x - e^{-x})/2$
$\Rightarrow 2y = e^x - e^{-x}$
$\Rightarrow e^{2x} - 2ye^x - 1 = 0$
$\Rightarrow e^x = \dfrac{2y \pm \sqrt{4y^2 + 4}}{2} = y \pm \sqrt{y^2 + 1}$
$\Rightarrow e^x = y + \sqrt{y^2 + 1}$
$(\because y - \sqrt{y^2 + 1} < 0$ for all y but e^x is always positive$)$
$\Rightarrow x = \log_e(y + \sqrt{y^2 + 1})$
$\therefore f^{-1}(x) = \log_e(x + \sqrt{x^2 + 1})$

16. Which among the following relations on $\mathbb{Z}$ is an equivalence relation

- **A.** $xRy \Leftrightarrow |x| = |y|$ ✔
- **B.** $xRy \Leftrightarrow x \geq y$ ✘
- **C.** $xRy \Leftrightarrow x > y$ ✘
- **D.** $xRy \Leftrightarrow x < y$ ✘

Let's consider $xRy \Leftrightarrow |x| = |y|$
Now, $|x| = |x|$
$\Rightarrow xRx$
Hence, R is reflexive.
Now, let $xRy \Rightarrow |x| = |y|$
or $|y| = |x|$ as equality commutative
$\Rightarrow yRx$
Hence, R is symmetric.
Checking for transitive,
Let (x, y) and (y, z) satisfies R
Now, xRy, yRz
$\Rightarrow |x| = |y|, |y| = |z|$
$\Rightarrow |x| = |z|$
$\Rightarrow xRz$
$\therefore (x, z)$ satisfies R
Hence, R is transitive.
Hence, R is an equivalence relation.

R is not symmetric relation as $3 \geq 2$ does not implies $2 \geq 3$. Hence R is not an equivalence relation.

R is not symmetric relation as $3 > 2$ does not implies $2 > 3$. Hence R is not an equivalence relation.

R is not symmetric relation as $1 < 3$ is true but $3 < 1$ is not true. Hence R is not an equivalence relation.

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17. Let S be the set of all triangles and R^+ be the set of all positive real numbers. If the relation f defined from S to R^+ such that $f(\triangle) =$ area of triangle, $\triangle \in S$, then which of the following is true about relation f

- ✗ **A.** f is not a function
- ✓ **B.** relation f is a many-one function
- ✗ **C.** If $\triangle_1 \cong \triangle_2$, then $f(\triangle_1) \neq f(\triangle_2)$
- ✗ **D.** None of the above

Relation f is a function as a triangle can't have two different values of it's area.
f is not one-one as many triangles exist for same area.

18. If a relation R is defined on set of real numbers as $xRy \iff x - y + \sqrt{2}$ is an irrational number, then the relation R is

- ✓ **A.** a reflexive relation
- ✗ **B.** a symmetric relation
- ✗ **C.** a transitive relation
- ✗ **D.** both reflexive and transitive relation

$xRy \iff x - y + \sqrt{2}$ is irrational,
$x, y \in \mathbb{R}$

(i) R is reflexive relation as $x - x + \sqrt{2} = \sqrt{2}$ is irrational.

(ii) R is not symmetric relation as $(\sqrt{2}, 1) \in R$ but $(1, \sqrt{2}) \notin R$

(iii) R is not transitive relation as $(\sqrt{2}, 1) \in R$ and $(1, 2\sqrt{2}) \in R$ but $(\sqrt{2}, 2\sqrt{2}) \notin R$

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19. Consider the functions $f(x) = \begin{cases} x+1, & x \leq 1 \\ 2x+1, & 1 < x \leq 2 \end{cases}$

$g(x) = \begin{cases} x^2, & -1 \leq x < 2 \\ x+2, & 2 \leq x \leq 3 \end{cases}$

Domain of $f(g(x))$ is

- ✗ **A.** $[0, \sqrt{2}]$
- ✗ **B.** $[-1, 2]$
- ✓ **C.** $[-1, \sqrt{2}]$
- ✗ **D.** $[\sqrt{2}, \sqrt{2}]$

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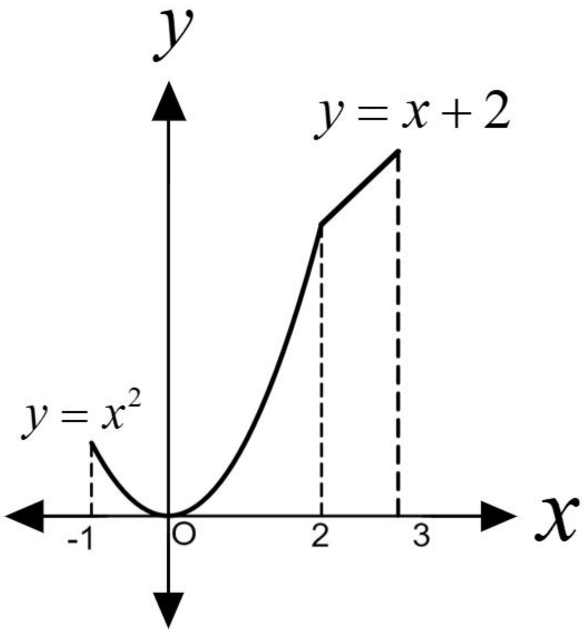
$$f\left(g\left(x\right)\right)=\begin{cases} g\left(x\right)+1 & g\left(x\right)\le 1 \\ 2g\left(x\right)+1 & 1<g\left(x\right)\le 2 \end{cases}$$



Now from graph of $g(x)$,
$g(x)\le 1$
$\Rightarrow x\in[-1,1]$
and $g(x)\in(1,2]\Rightarrow x\in(1,\sqrt{2}]$

$$\therefore f\left(g\left(x\right)\right)=\begin{cases} g(x)+1, & x\in[-1,1] \\ 2g(x)+1, & x\in(1,\sqrt{2}\,] \end{cases}$$

$$=\begin{cases} x^2+1, & x\in[-1,1] \\ 2x^2+1, & x\in(1,\sqrt{2}\,] \end{cases}$$

So, domain of $f(g(x))=[-1,\sqrt{2}]$

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20. Consider the functions $f(x) = \begin{cases} x+1, & x \leq 1 \\ 2x+1, & 1 < x \leq 2 \end{cases}$

$g(x) = \begin{cases} x^2, & -1 \leq x < 2 \\ x+2, & 2 \leq x \leq 3 \end{cases}$

Range of the function $f(g(x))$ is

A. $[1, 5]$

B. $[2, 3]$

C. $[1, 2] \cup (3, 5]$

D. $[1, 5] - \{3\}$

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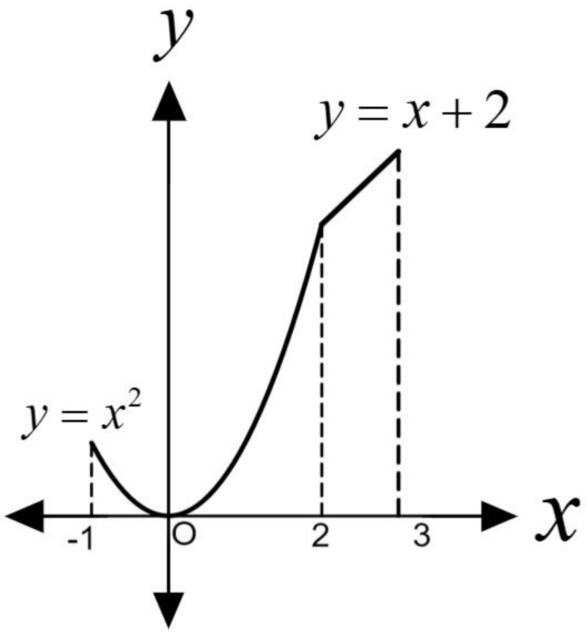
$$f(g(x)) = \begin{cases} g(x)+1 & g(x) \le 1 \\ 2g(x)+1 & 1 < g(x) \le 2 \end{cases}$$



Now from graph of $g(x)$,
$g(x) \le 1$
$\Rightarrow x \in [-1, 1]$
and $g(x) \in (1, 2] \Rightarrow x \in (1, \sqrt{2}]$

$$\therefore f(g(x)) = \begin{cases} g(x)+1, & x \in [-1, 1] \\ 2g(x)+1, & x \in (1, \sqrt{2}] \end{cases}$$

$$= \begin{cases} x^2+1, & x \in [-1, 1] \\ 2x^2+1, & x \in (1, \sqrt{2}] \end{cases}$$

For $-1 \le x \le 1$
$x^2 \in [0, 1] \Rightarrow x^2 + 1 \in [1, 2]$
For $1 < x \le \sqrt{2}$
$2x^2 + 1 \in (3, 5]$
$\therefore$ range is $[1, 2] \cup (3, 5]$