

BYJU'S Study Planner for Board Term I (CBSE Grade 12)

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Subject: Mathematics

Topic : Linear Programming

Class: Standard XII

1. For an LPP,

maximize $z = ax + by$ where $a, b \in R$ subject to the constraints

$$a_1x + b_1y \leq 0$$

$$a_2x + b_2y \leq 0$$

$$x, y \geq 0,$$

consider the following statements:

(I) The solution depends on the optimizing function.

(II) The solution depends on the constraints.

Which of the following statement(s) is/are correct?



A. Both (I) and (II)



B. (I) only



C. (II) only



D. The solution depends upon some more aspects.

For an arbitrary LPP, the solution depends upon the values at the corner points of the feasible region (if there is any). So, the solution depends only on the constraints.

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2. The corner points of the feasible region determined by a system of linear constraints are $(0, 10)$, $(5, 5)$, $(15, 15)$, $(0, 20)$. Let $z = px + qy$ where $p, q > 0$. If the maximum of z occurs at both the points $(15, 15)$ and $(0, 20)$, then which of the following is true:

- ☒ A. $p = q$
- ☒ B. $p = 2q$
- ☒ C. $p = 3q$
- ☒ D. $q = 3p$

Let z_0 be the maximum value of z in the feasible region. Since maximum occurs at both $(15, 15)$ and $(0, 20)$, the value z_0 is attained at both $(15, 15)$ and $(0, 20)$.

$$\Rightarrow z_0 = p(15) + q(15) \cdots (i)$$

$$\text{and } z_0 = p(0) + q(20) \cdots (ii)$$

From (i) and (ii), we get:

$$15p + 15q = 20q$$

$$\Rightarrow 15p = 5q$$

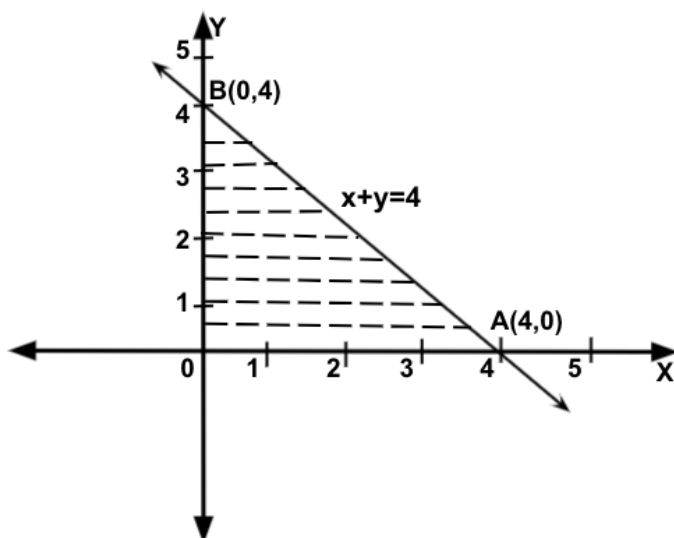
$$\Rightarrow 3p = q$$

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3. If $Z = 3x + 4y$, subject to the constraints: $x + y \leq 4$, $x \geq 0$, $y \geq 0$, then Z_{\max} is equal to

- ☐ A. 12
- ☒ B. 16
- ☐ C. 20
- ☐ D. 24

The feasible region determined by the constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$ is as follows.



The corner points of the feasible region are $O(0, 0)$, $A(4, 0)$ and $B(0, 4)$. The values of Z at these points are as follows.

Corner points	$Z = 3x + 4y$
$O(0, 0)$	0
$A(4, 0)$	12
$B(0, 4)$	16 \rightarrow maximum

Hence, the maximum value of Z is 16 at the point $B(0, 4)$.

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4. A toy company manufactures two types of dolls, A and B . Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A . Further, the production level of dolls of type A can exceed three times the production of dolls of other types by at most 600 units. If the company makes a profit of $Rs.12$ and $Rs.16$ per doll respectively on dolls A and B , how many of each should be produced weekly in order to maximize the profit

- ☐ A. $(A, B) \equiv (400, 800)$
- ☒ B. $(A, B) \equiv (800, 400)$
- ☐ C. $(A, B) \equiv (600, 600)$
- ☐ D. $(A, B) \equiv (0, 1200)$

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Let's assume that the number of dolls of type A is X and number of dolls of type B be Y

Since combined production level should not exceed 1200 dolls

$$\therefore X + Y \leq 1200 \dots\dots (i)$$

Since production levels of dolls of type A exceeds 3 times the production of type B by at most 600 units

$$\therefore X - 3Y \leq 600 \dots (ii)$$

Also, the demands of dolls of type B is at most half of that for dolls of type A

$$\therefore Y \leq \frac{X}{2} \Rightarrow 2Y - X \leq 0 \dots (iii)$$

Since the count of an object can't be negative.

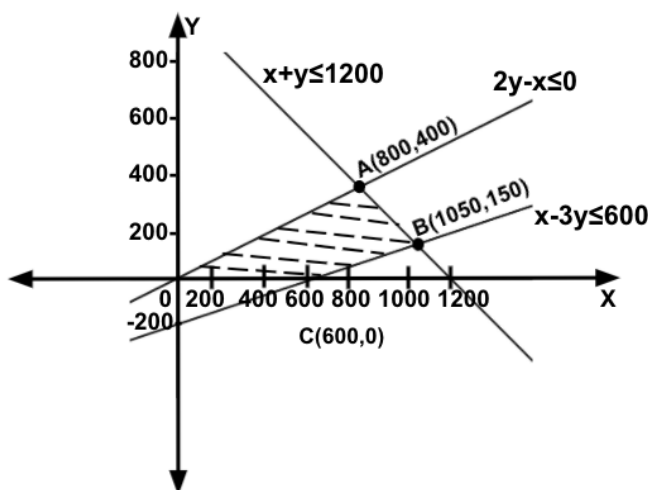
$$\text{So, } X \geq 0, Y \geq 0 \dots (iv)$$

Now, profit on type A dolls = Rs 12 and profit on type B dolls = Rs 16

So, total profit $Z = 12X + 16Y$

We have to maximize the total profit Z of the manufacturers.

After plotting all the constraints given by equation (i), (ii), (iii) and (iv)

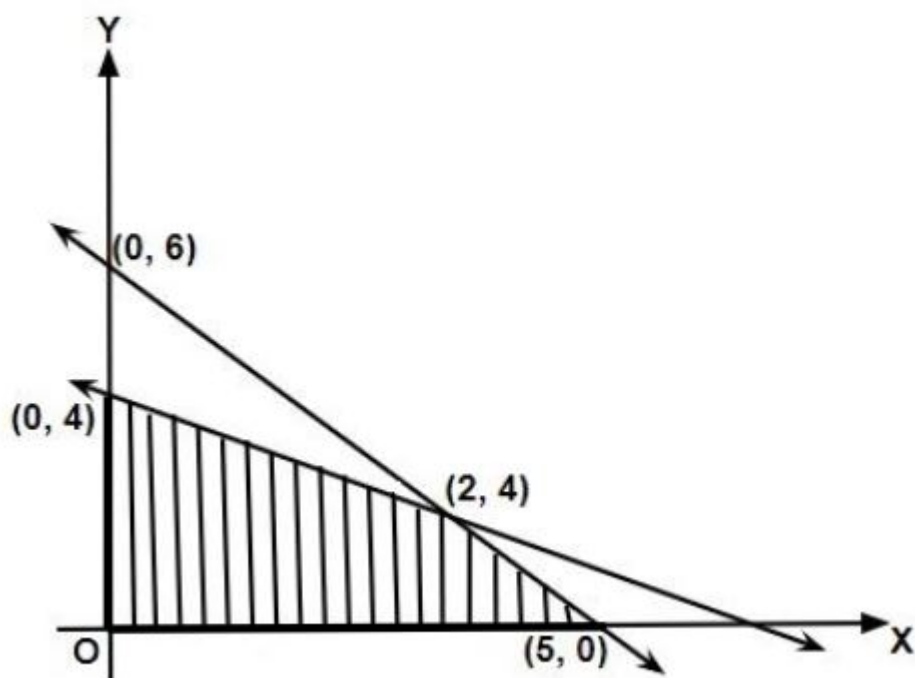


Corner points	$Z = 12X + 16Y$
$A(800, 400)$	16000 (maximum)
$B(1050, 150)$	15000
$C(600, 0)$	7200
$O(0, 0)$	0

So, in order to maximize the profit, the company should produce 800 type A dolls and 400 type B dolls

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5. The feasible region of a LPP is shown in the figure. If $Z = 5x + 2y$, then the maximum value of Z occurs at



- ☐ A. $(0, 0)$
- ☒ B. $(5, 0)$
- ☐ C. $(2, 4)$
- ☐ D. $(0, 4)$

Extreme Points	$Z = 5x + 2y$
$(0, 0)$	$5 \times 0 + 2 \times 0 = 0$
$(5, 0)$	$5 \times 5 + 2 \times 0 = 25$
$(2, 4)$	$5 \times 2 + 2 \times 4 = 18$
$(0, 4)$	$5 \times 0 + 2 \times 4 = 8$

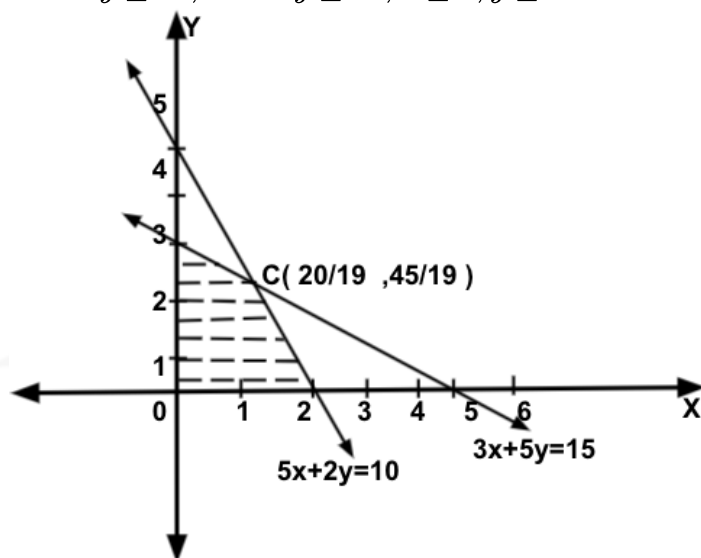
Maximum value of $Z = 25$ which occurs at $(5, 0)$.

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6. If $Z = 5x + 3y$, subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$, then Z_{\max} is equal to

- ☐ A. 10
- ☐ B. 9
- ☒ C. $\frac{235}{19}$
- ☐ D. $\frac{255}{19}$

The feasible region determined by the system of constraints, $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$ are as shown



The corner points of the

feasible region are $O(0,0)$, $A(2,0)$, $B(0,3)$ and $C\left(\frac{20}{19}, \frac{45}{19}\right)$.

The values of Z at these corner points are as follows.

corner point	$Z = 5x + 3y$
$O(0,0)$	0
$A(2,0)$	10
$B(0,3)$	9
$C\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{235}{19}$ maximum

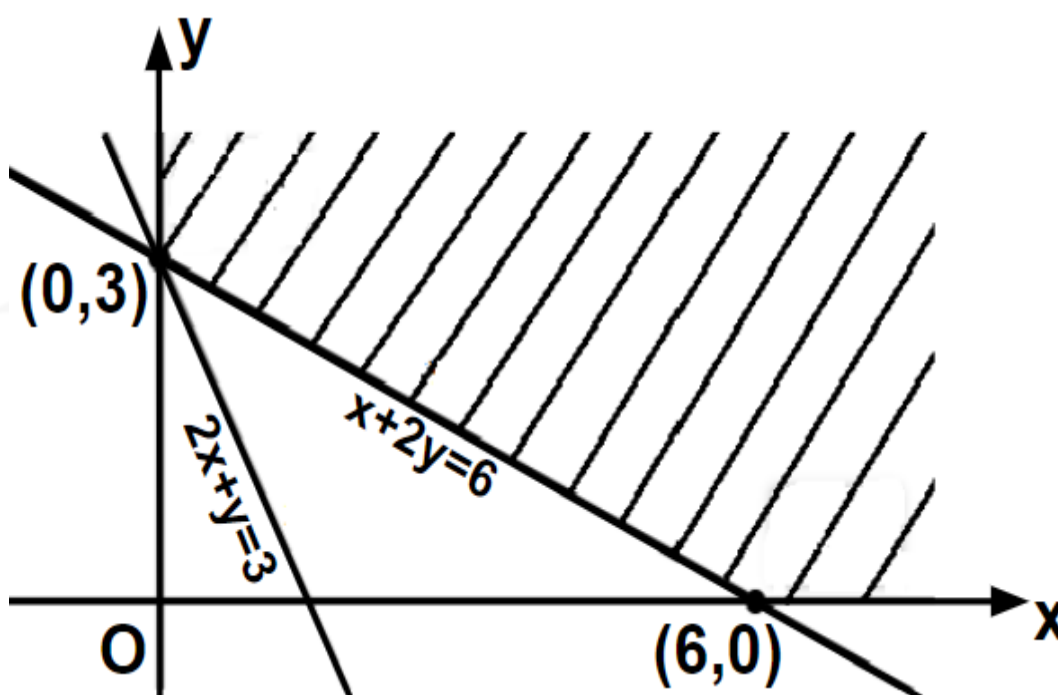
\therefore Maximum value of Z is $\frac{235}{19}$ at the point $\left(\frac{20}{19}, \frac{45}{19}\right)$

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7. If $Z = x + 2y$, subject to the constraints: $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$, then the minimum of Z occurs at

- ☒ A. only one point
- ☒ B. two points
- ☒ C. more than two points
- ☒ D. minimum of Z is not possible

The feasible region under the given constraints is



The coordinates of corner points and values of Z are

Corner points	$Z = x + 2y$
(0, 3)	6
(6, 0)	6

From the above table minimum value of $Z = 6$

Now, compare the constraints and objective function.

We observe that the slope of the constraint ($x + 2y \geq 6$) and objective function is same, so Z will be minimum at all the points which lie on the constraint line ($x + 2y = 6$) between the points (0, 3) and (6, 0).

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8. The point which lies in the half plane $3x - 2y - 2 \geq 0$ is

☒ A. $(-3, -1)$

☒ B. $(2, 3)$

☒ C. $(3, 2)$

☒ D. $(1, 3)$

Only point $(3, 2)$ satisfies the condition $3x - 2y - 2 \geq 0$ i.e.,
 $3 \times 3 - 2 \times 2 - 2 = 3 \geq 0$.

Therefore, option (c) is correct.

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9. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class and 80 for economy class, then the number of tickets of each class must be sold in order to maximise the profit for the airline is [where $n(E)$ = number of executive class tickets and $n(E')$ = number of economy class tickets]

- ☒ A. $n(E) = 120, n(E') = 80$
- ☐ B. $n(E) = 80, n(E') = 120$
- ☐ C. $n(E) = 140, n(E') = 60$
- ☐ D. $n(E) = 60, n(E') = 140$

Suppose x is the number of executive class tickets and y is the number of economy class tickets .

Then, total profit (in Rs) = $1000x + 600y$

Let $Z = 1000x + 600y$

We now have the following mathematical model for the given problem.

Maximise $Z = 1000x + 600y \dots (i)$

subject to the constraints:

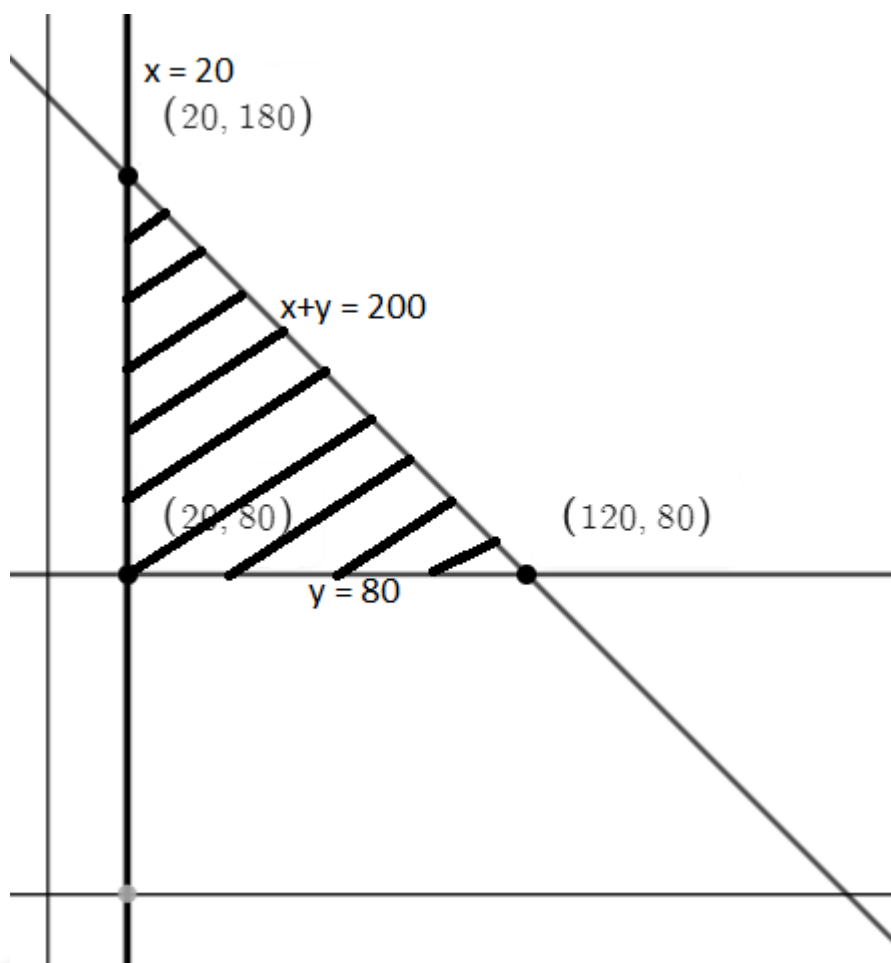
$x + y \leq 200$ (Total seat constraint) $\dots (ii)$

$x \geq 20$ (executive class seat constraint) $\dots (iii)$

$y \geq 80$ (economy class seat constraint) $\dots (iv)$

The feasible region (shaded) determined by the linear inequalities (ii) to (iv) is shown in the Figure

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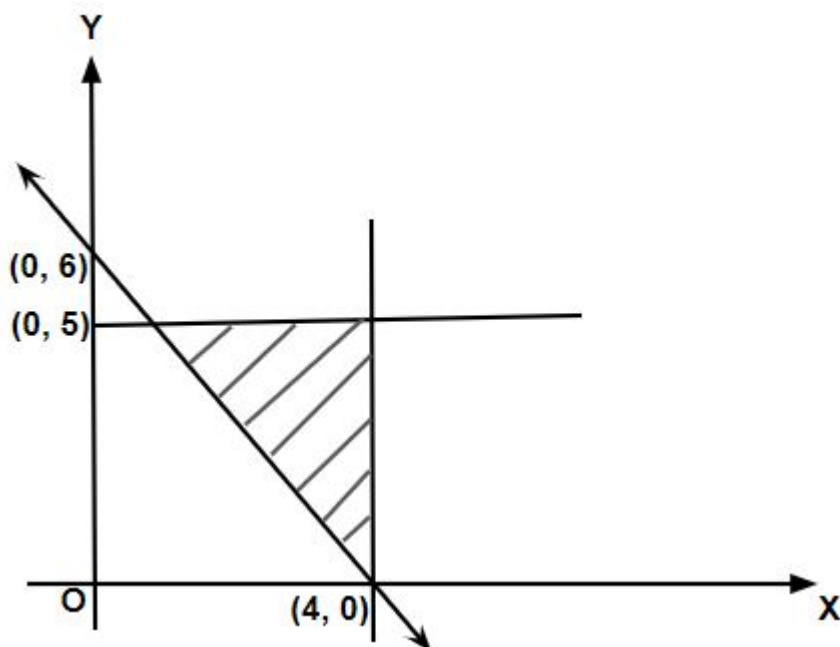
Let us evaluate the objective function Z at each corner point as shown below

Corner points	$Z = 1000x + 600y$
$(20, 180)$	128000
$(20, 80)$	68000
$(120, 80)$	168000

We find that maximum value of Z is 168000 at $(120, 80)$. Hence, 120 tickets of executive class and 80 tickets of economy class must be sold to realise the maximum profit and the maximum profit will be Rs 168000.

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10. The shaded region in the figure is the solution set of the inequations



- ☒ A. $6x + 4y \geq 24, x \leq 4, y \leq 5, x, y \geq 0$
- ☐ B. $6x + 4y \leq 24, x \geq 4, y \leq 5, x, y \geq 0$
- ☐ C. $6x + 4y \leq 24, x \geq 4, y \geq 5, x, y \geq 0$
- ☐ D. $6x + 4y \geq 24, x \geq 4, y \leq 5, x, y \geq 0$

The line joining (4, 0) and (6, 0) is $6x + 4y = 24$

The shaded region is bounded by $6x + 4y \geq 24, x \leq 4, y \leq 5, x, y \geq 0$

Therefore, option (a) is correct.

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11. Sum of maximum and minimum value of the objective function $z = 10x + 7y$, subjected to the constraints $0 \leq x \leq 60$, $0 \leq y \leq 45$, $5x + 6y \leq 420$ is

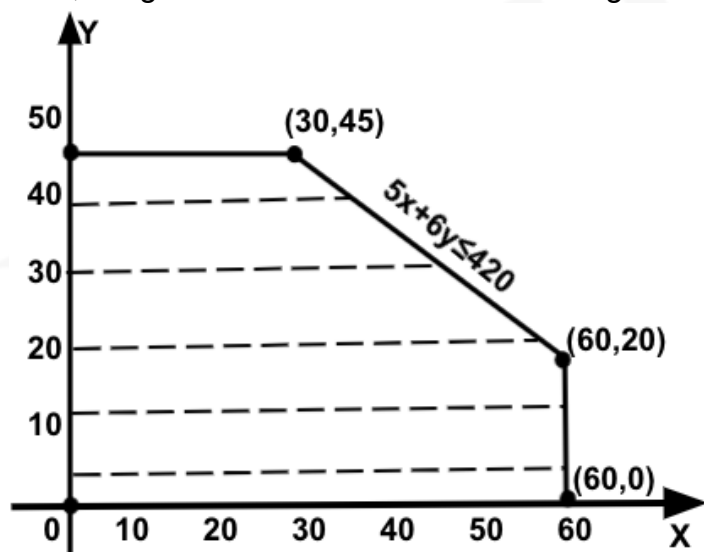
- ☒ A. 740
☐ B. 1025
☐ C. 1055
☐ D. 1100

Given, $z = 10x + 7y$ and constraints are

$$0 \leq x \leq 60, 0 \leq y \leq 45, 5x + 6y \leq 420$$

$$\Rightarrow 0 \leq x \leq 60, 0 \leq y \leq 45 \text{ and } \frac{x}{84} + \frac{y}{70} \leq 1$$

Now, we get common area as marked in figure



Corner points	$z = 10x + 7y$
(0, 45)	315
(60, 20)	740 (Maximum)
(60, 0)	600
(30, 45)	615
(0, 0)	0 (Minimum)

$$\text{Thus, } z_{\max} + z_{\min} = 740 + 0 = 740$$

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12. Minimum value of objective function $z = 2.5x + y$ subject to the constraints:
 $x \geq 0, y \geq 0, 3x + 5y \leq 15, 5x + 2y \leq 10$ is

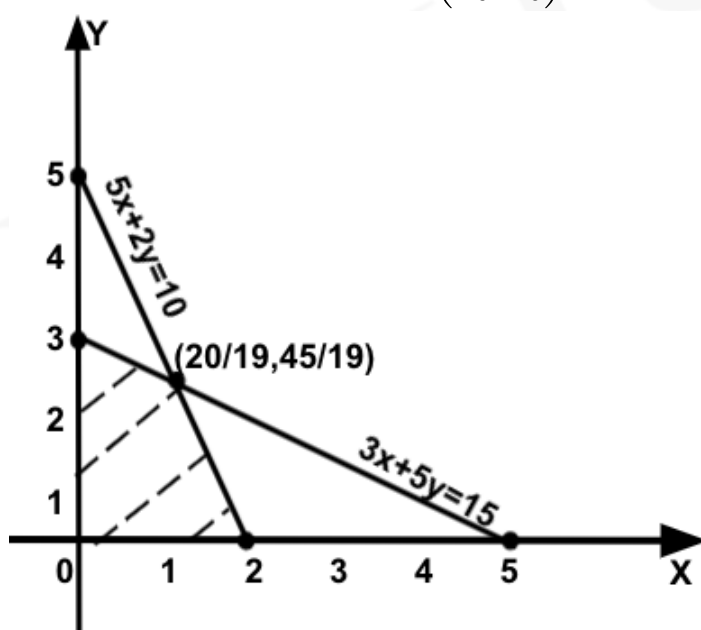
- ☒ A. 20
- ☒ B. 5
- ☒ C. 0
- ☒ D. Does not exist

Given, $z = 2.5x + y$ subject to the constraints:

$$x \geq 0, y \geq 0, 3x + 5y \leq 15, 5x + 2y \leq 10$$

$$\Rightarrow \frac{x}{5} + \frac{y}{3} \leq 1, \frac{x}{2} + \frac{y}{5} \leq 1$$

We get intersection points as $\left(\frac{20}{19}, \frac{45}{19}\right)$



Now, values of objective

function

Corner points	$z = 2.5x + y$
$(2, 0)$	5
$(0, 3)$	3
$\left(\frac{20}{19}, \frac{45}{19}\right)$	5
$(0, 0)$	0 (Minimum)

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13. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled men and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is Rs.15 and on an item of model B is Rs.10, then the total number of items of each model should be made per day in order to maximize daily profit.
[where, $n(A)$ = number of items of artical A and $n(B)$ = number of items of artical A]

- ☐ A. $n(A) = 10, n(B) = 15$
- ☐ B. $n(A) = 15, n(B) = 10$
- ☒ C. $n(A) = 10, n(B) = 20$
- ☐ D. $n(A) = 20, n(B) = 10$

Given that total available hours for skilled men = $8 \times 5 = 40$

and total available hours for semi-skilled men = $8 \times 10 = 80$

Let x be the number of items produced of model A and y be the number of items produced of model B .

Let Z be the maximizing function.

Then $Z = 15x + 10y$

subject to the constraints

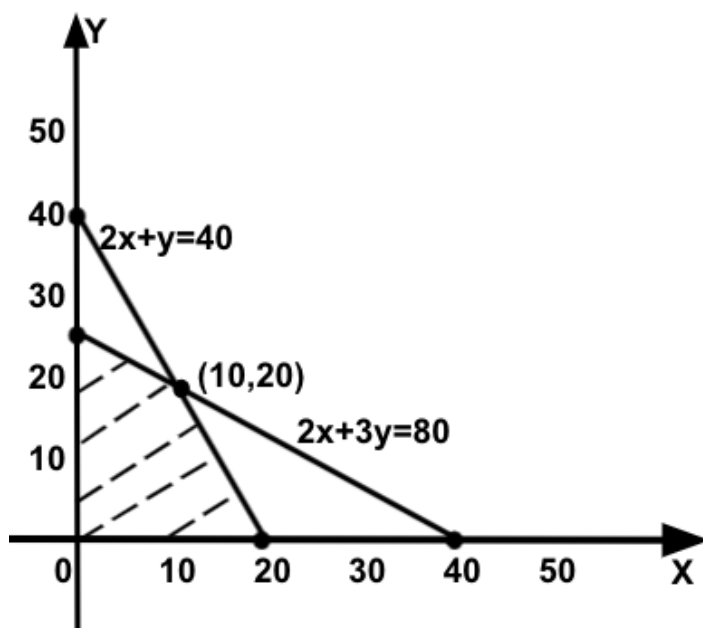
$2x + y \leq 40$ (skilled men work time constraint)

$2x + 3y \leq 80$ (semi-skilled men work time constraint)

$x \geq 0, y \geq 0$ (non-negative constraints)

Plotting the graphs form the above constraints, we have

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From the above graph, we get

Corner points	$Z = 15x + 10y$
(0, 26.667)	266.67
(10, 20)	350
(0, 0)	0
(20, 0)	300

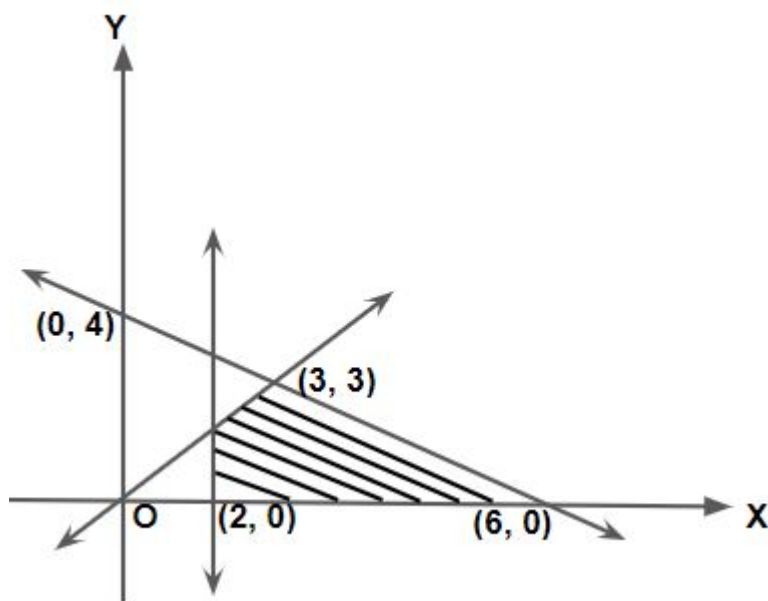
As we can see that the maximum value of Z occurs at (10, 20).

So, manufacturer should produce 10 items of model A and 20 items of model B in order to maximize the profit.

The maximum profit is Rs.350

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14. The shaded region in the figure is the solution set of the inequations.



- ☒ A. $4x + 6y \leq 24, x \leq 2, y \leq x, x, y \geq 0$
- ☒ B. $4x + 6y \leq 24, x \geq 2, y \geq x, x, y \geq 0$
- ☒ C. $4x + 6y \geq 24, x \leq 2, y \geq x, x, y \geq 0$
- ☒ D. $4x + 6y \leq 24, x \geq 2, y \leq x, x, y \geq 0$

The line joining $(6, 0)$ and $(0, 4)$ is $\frac{x}{6} + \frac{y}{4} = 1$ i.e, $4x + 6y = 24$

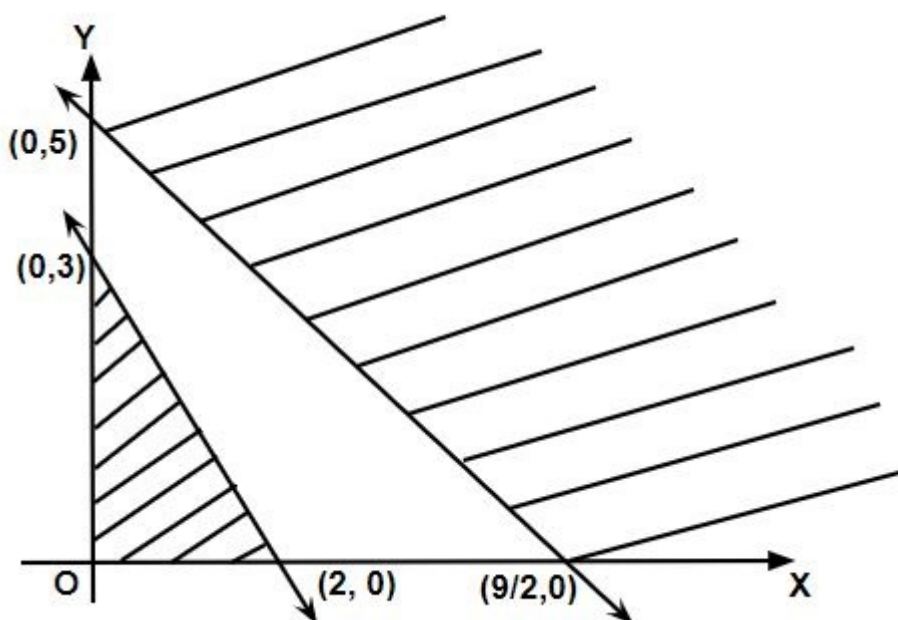
The line joining $(0, 0)$ and $(3, 3)$ is $y = x$

On observation, we can conclude that option (d) is the correct set of inequations which represents the shaded region.

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15. For the LPP; maximize $z = 5x - 2y$ subject to the constraints
 $3x + 2y \leq 6$, $10x + 9y \geq 45$, $x, y \geq 0$

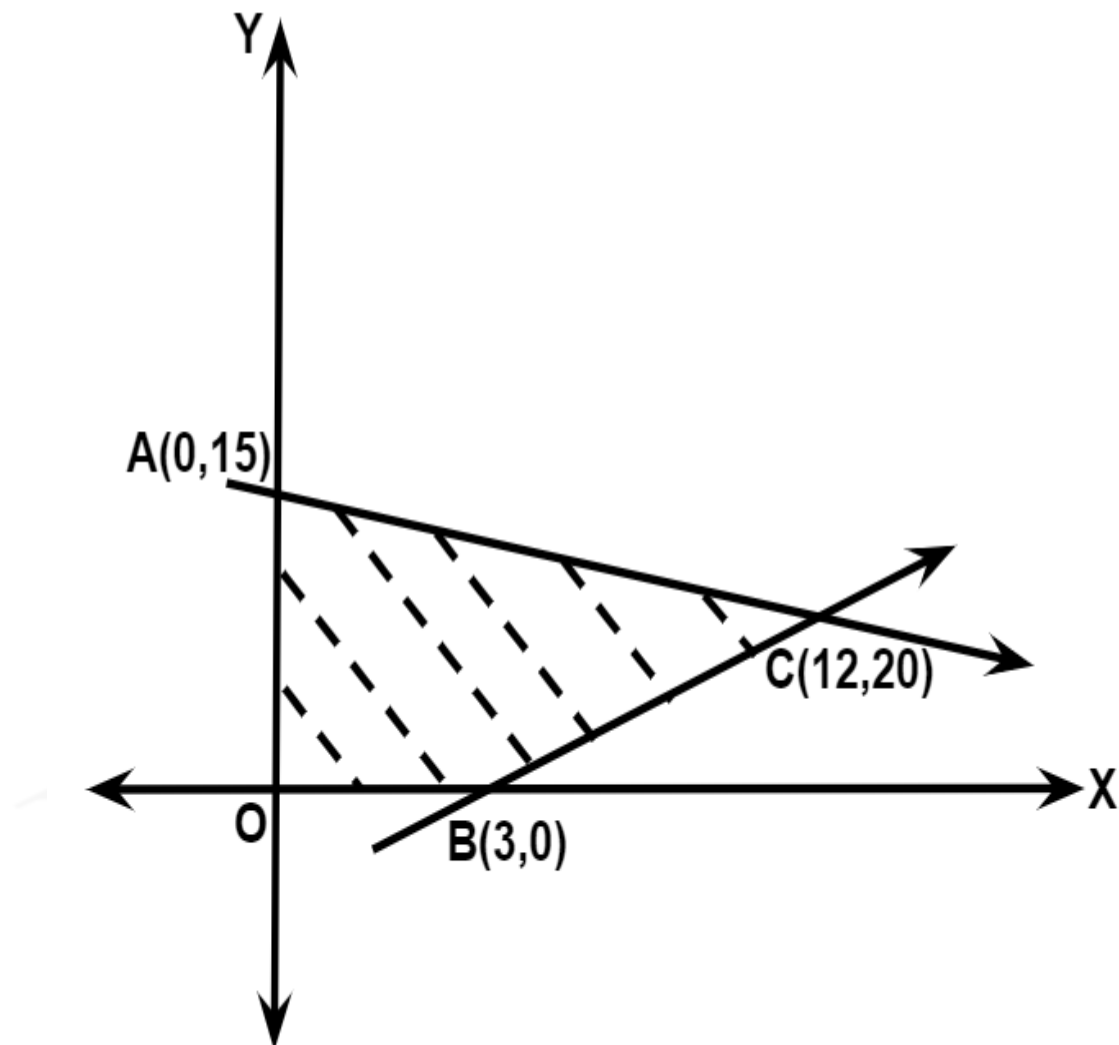
- ☒ A. $z_{\min} = -15$
- ☒ B. $z_{\min} = 0$
- ☒ C. $z_{\min} = -6$
- ☒ D. The LPP has no feasible solution.



From the graph, we can conclude that the given LPP has no feasible solution.

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16. The feasible region of a LPP is shown in the figure. If $Z = 3x - y$, then the maximum value of Z occurs at



- ☒ A. (0, 0)
- ☒ B. (3, 0)
- ☒ C. (12, 20)
- ☒ D. (0, 15)

The table of values at corner points for subjective function $Z = 3x - y$ is given below

Corner points : (x, y)	value : $Z = 3x - y$
$O(0, 0)$	0
$A(0, 15)$	-15
$B(3, 0)$	9
$C(12, 20)$	16

So, Z_{max} occurs at $C(12, 20)$.

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17. A fruit grower has 150 acres of land available to raise two crops, A and B . It takes one day to trim an acre of crop A and two days to trim an acre of crop B and there are 240 days per year available for trimming. It takes 0.3 day to pick an acre of crop A and 0.1 day to pick an acre of crop B and there are 30 days per year available for picking. The number of acres of each fruit that should be planted to maximize profit, assuming that the profit is 140 per acre for crop A and 235 per acre for crop B , is

- ☒ A. 29820
- ☒ B. 35250
- ☒ C. 29550
- ☒ D. 25640

let x = number of acres of crop A .

let y = number of acres of crop B .

the equation we want to maximize is the profit equation.

the profit on crop A is 140 per acre so the total profit on crop A would be:

$$140x$$

the profit on crop B is 235 per acre so the total profit on crop B would be:

$$235y$$

So, objective function is : $z = 140x + 235y$

it takes 1 day to trim an acre of crop A and 2 days to trim an acre of crop B with a total of 240 days available for trimming.

$$1 \times x + 2 \times y \leq 240$$

Also, 0.3 day to pick an acre of crop A and 0.1 day to pick an acre of crop B with a total of 30 days available for picking.

$$3x + y \leq 300$$

total number of acres has to be less than or equal to 150.

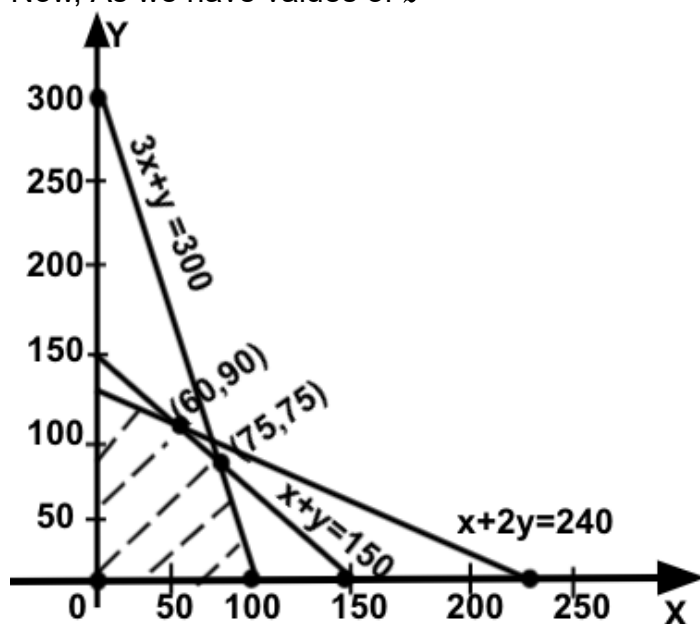
$$x + y \leq 150$$

So, constraints are:

$$x + y \leq 150, 3x + y \leq 300, x + 2y \leq 240, x, y \geq 0$$

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Now, As we have values of z



Corner Points	$z = 140x + 235y$
(60, 90)	29550(maximum)
(75, 75)	28125
(0, 120)	28200
(100, 0)	14000

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18. The corner points of the feasible region for optimization problem determined by a system of linear constraints are $(6, 5)$, $(9, 7)$, $(15, 0)$, $(0, 10)$. Let $z = px + qy$ and the maximum of z occurs at both the points $(6, 5)$ and $(9, 7)$, then which of the following is true:

- ☒ A. $3p + 2q = 0$ and $p > 0, q > 0$
- ☒ B. $3p + 2q = 0$ and $p > 0, q < 0$
- ☒ C. $3p + 2q = 0$ and $p < 0, q < 0$
- ☒ D. $3p + 2q = 0$ and $p < 0, q > 0$

Let z_0 be the maximum value of z in the feasible region. Since maximum occurs at both $(6, 5)$ and $(9, 7)$, the value z_0 is attained at both $(6, 5)$ and $(9, 7)$.

$$\Rightarrow z_0 = p(6) + q(5) \dots (i)$$

$$\text{and } z_0 = p(9) + q(7) \dots (ii)$$

From (i) and (ii), we get:

$$6p + 5q = 9p + 7q$$

$$\Rightarrow 3p + 2q = 0 \dots (iii)$$

and the values at other corner points $(15, 0)$ and $(0, 10)$ should be less than the z_0

$$\Rightarrow 6p + 5q > 15p$$

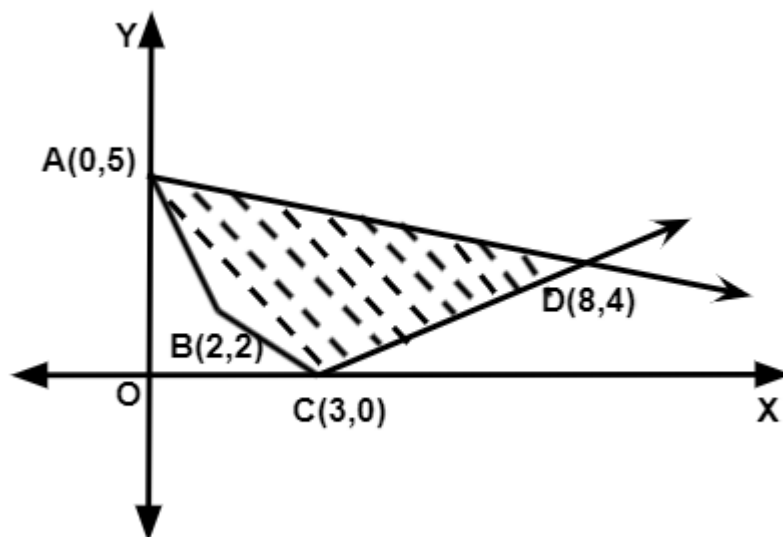
$$\Rightarrow p < 0 \dots (A) \text{ [from (iii)]}$$

$$\text{and } 6p + 5q > 10q$$

$$\Rightarrow q < 0 \dots (B) \text{ [from (iii)]}$$

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19. The feasible region of an LPP is shown in the figure. If $Z = 8x + 3y$, then the minimum value of Z occurs at



- ☒ A. (2, 2)
- ☒ B. (3, 0)
- ☒ C. (0, 5)
- ☒ D. (8, 4)

The table of values at corner points for objective function $Z = 8x + 3y$ is given below :

Corner points : (x, y)	Value : $Z = 8x + 3y$
$A(0, 5)$	15
$B(2, 2)$	22
$C(3, 0)$	24
$D(8, 4)$	76

So, $Z_{\min} = 15$ occurs at $A(0, 5)$.

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20. A vanilla cake requires 200g of flour and 25g of fat, and a strawberry cake requires 100g of flour and 50g of fat. From 5kg of flour and 1kg of fat, the maximum number of cakes that can be made is

- ☐ A. 45
- ☒ B. 30
- ☐ C. 25
- ☐ D. 20

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Let x number of vanilla cakes and y number of strawberry cakes be made.

Then objective function is

Maximize $Z = x + y$

subject to constraints : maximum 5kg flour is available.

$$\Rightarrow \frac{200}{1000}x + \frac{100}{1000}y \leq 5$$

$$\Rightarrow 2x + y \leq 50 \quad \dots (i)$$

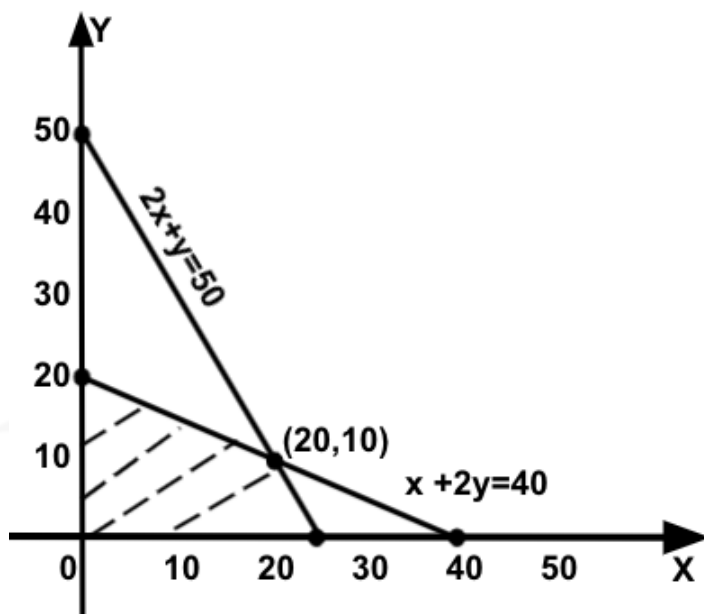
and maximum 1kg of fat is available.

$$\Rightarrow \frac{25}{1000}x + \frac{50}{1000}y \leq 1$$

$$\Rightarrow x + 2y \leq 40 \quad \dots (ii)$$

$$\text{and } x, y \geq 0 \quad \dots (iii)$$

Now, plotting the graph of given constraints to get feasible region :



Now, tabulating the value of $Z = x + y$ at corner points :

Corner point : (x, y)	Value : $Z = x + y$
$(0, 0)$	0
$(0, 20)$	20
$(25, 0)$	25
$(20, 10)$	30 (maximum)

So, $Z_{\max} = 30$