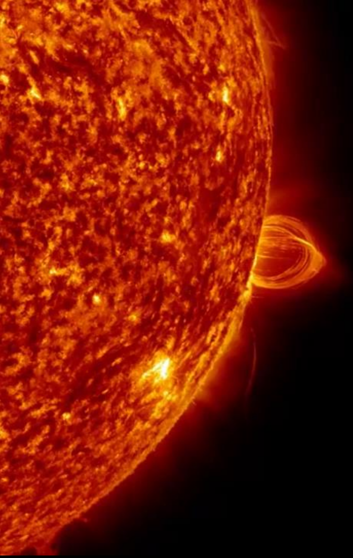
Welcome to

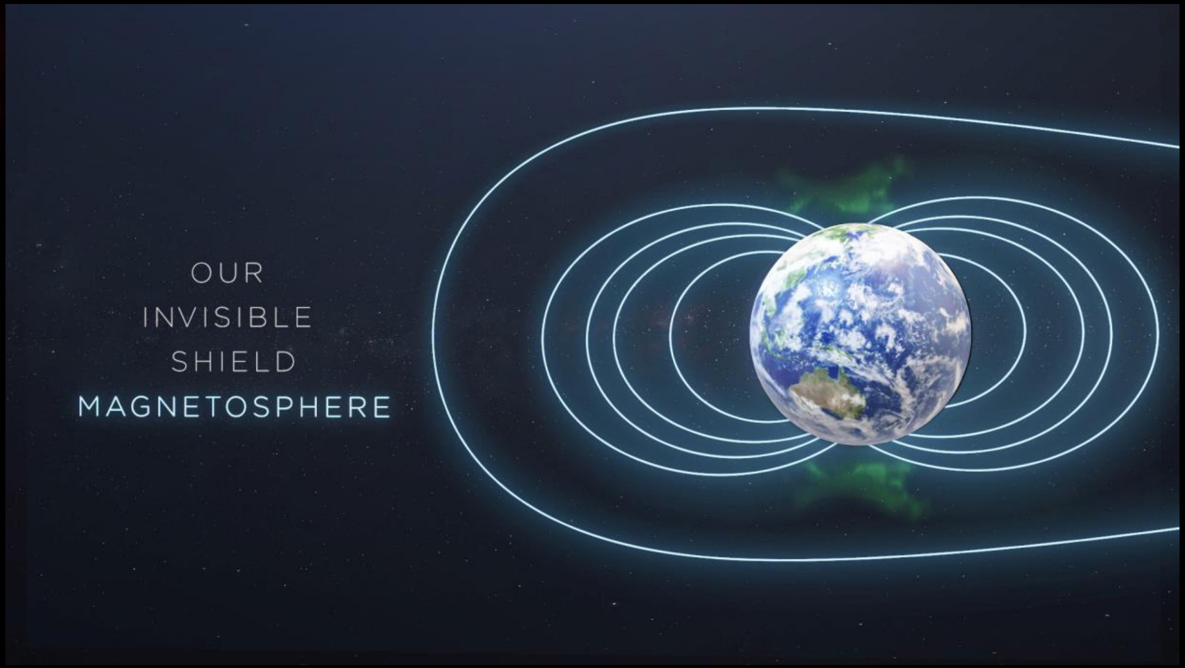
BYJU'S Classes

MAGNETISM

Science behind Northern lights



Source : NASA

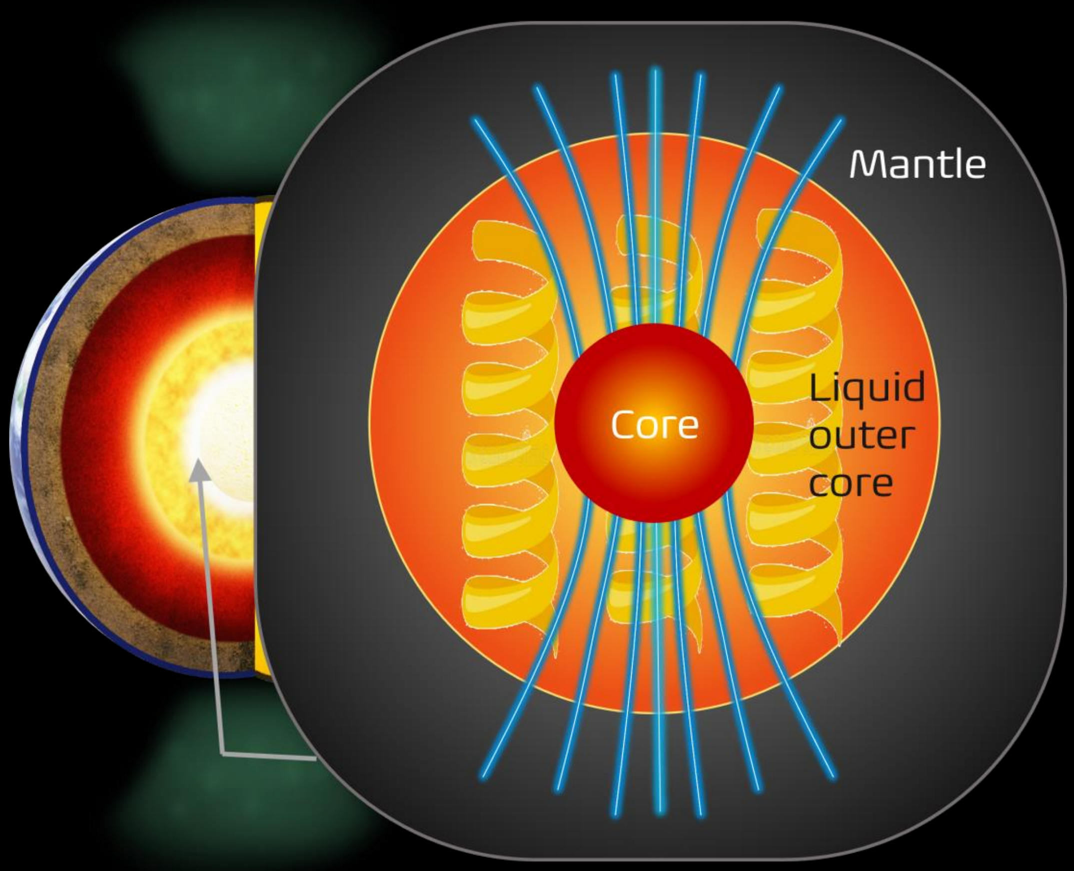


Nuclear reaction in the Sun causes the solar flare/storm containing high energy charge particles to approach towards Earth. Now, because of the shape of Earth’s magnetic field, the charges particles gets accumulated onto the poles. At the poles, the electrons from atmospheric gases get excited by gaining the energy from these charged particles and because of this process of gaining and losing energy, they emit light of different colours.

Our Earth’s magnetic field act as shield to protect us from the wrath of the solar storm. This magnetic field around the Earth is also known as the magnetosphere.

Magnetic field of Earth

- Earth is a natural source of magnetic field.
- The most prominent cause of this magnetic field is the molten liquid deep inside the Earth.
- The motion of ionized particle i.e., charge particle in the molten core of Earth constitutes the convection currents.
- In case of the Earth, these currents behave like a coil and produce magnetic field in surrounding.
- This type of formation of current inside the Earth can be assumed as a bar magnet. Thus, we can imagine that a bar magnet is placed inside the Earth.



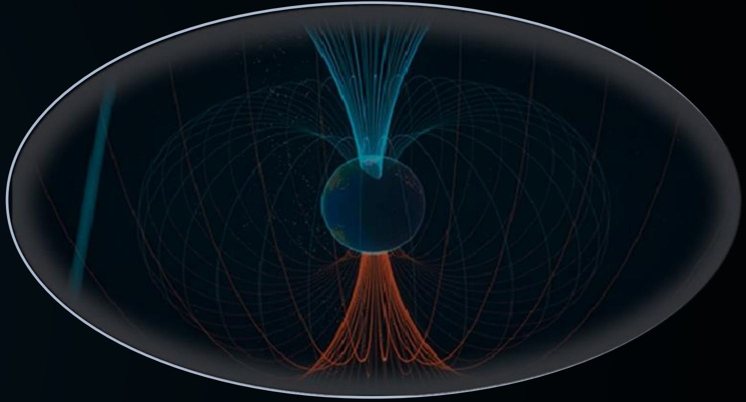
ELECTROMAGNETISM {Study of charges}.

ELECTRO

charge is at rest

In case of electrostatics the charges are at rest/static.

MAGNETISM



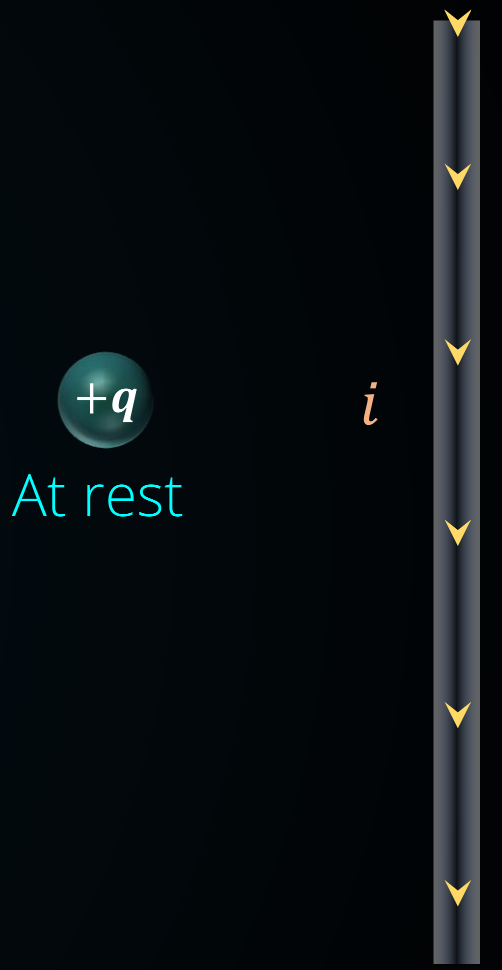
Charges are Moving.

In case of magnetism the charges are at moving.

❖ Electromagnetism is based on the principle that moving charges can produce electric as well as magnetic field.

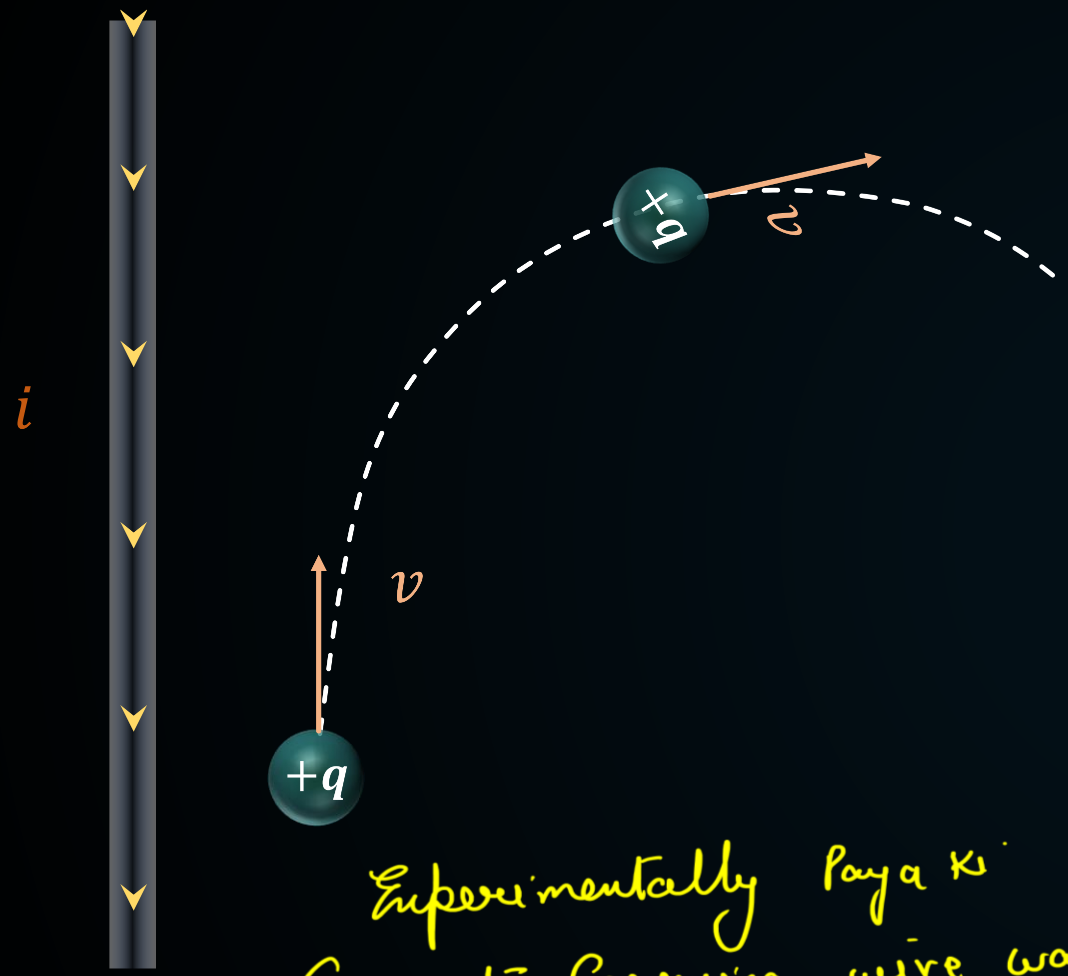
Charge at rest

- Current carrying wire is neutral and thus, does not produce any electric field. Hence, it should not produce any force on a charge particle.
- Practically, when a charge particle is placed at rest in the vicinity of the current carrying wire, it is seen that the charge particle remains at rest. Thus, force on the charge particle due to the wire, $F_{q,\,wire} = 0$



Theory & Practical matches.

Moving Charge



Experimentally Paya ki Current Carrying wire waisi hi fields generate karti hain jaisi Magnets karti hain.

- When the positive charge is given some initial velocity, it is seen that the charge is deflected. Therefore, there must be a field which exerts force on the moving charge only and the field is not electric field.
- It is also noticeable that the force acting on the charge is not along the direction of velocity of the charge.
- Practically, it is seen that the force which is acted on the moving charge particle by the current carrying wire is dependent on:
  - Nature of the charge particle ($+ve$ or $-ve$)
  - Velocity of the charge particle
  - Angle of projection of the charge particle
- Experimentally, it is concluded that the field is similar to the field that a magnet generates.

Oersted's Experiment

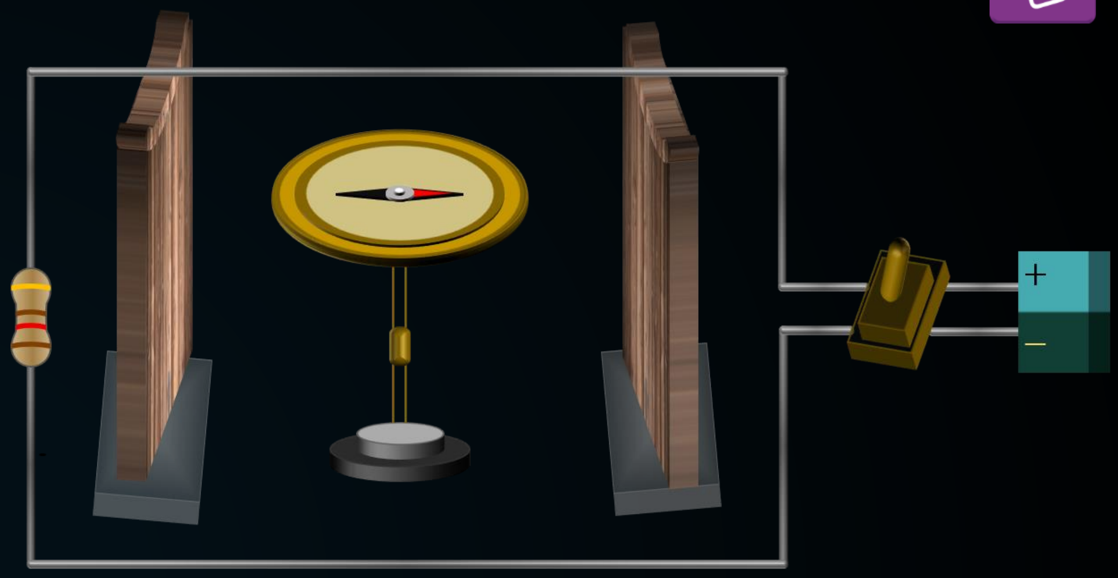
Hans Christian Oersted performed an important experiment which showed that there was a connection between electricity and magnetism. In this experiment he placed a magnetic compass in a current carrying loop. When current was switched on through the loop, it deflects compass needle. Since we all know that a magnet attracts iron, Oersted explained the observation of the experiment as if the current had produced a magnetic field strong enough to cause the compass needle to turn.

Hans Christian Oersted
(Denmark,1777-1851)

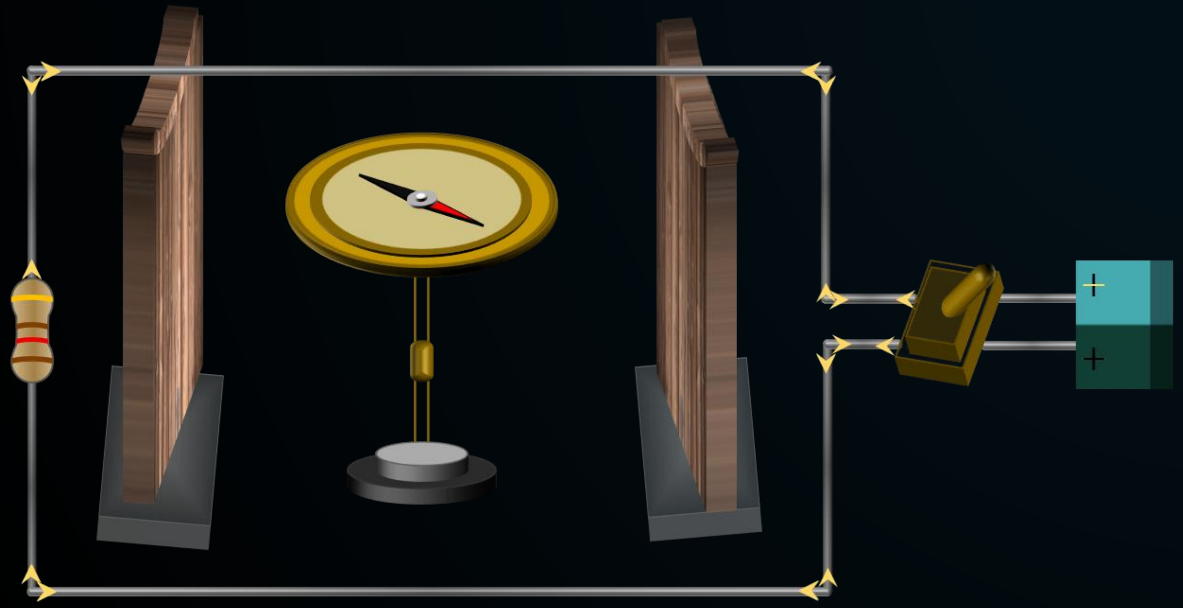
Oersted’s Experiment

When Switch Is OFF

- When the switch is OFF, the needle of the magnetic compass directs towards the magnetic North and magnetic South pole of the earth.



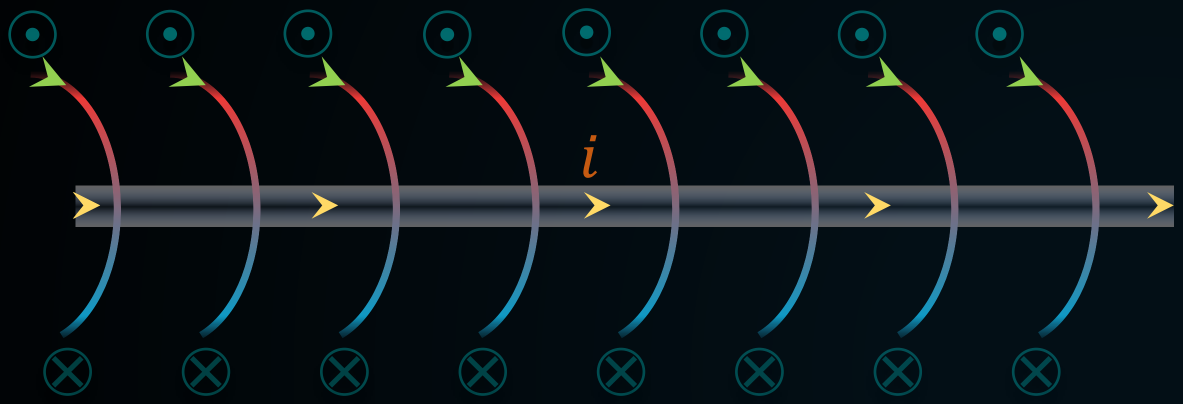
When Switch Is ON



- When the switch is on, we can see the deflection in the needle of magnetic compass.

Magnetic Field

- A space around a magnet or a current carrying conductor up to which a moving charge or another magnet or another conductor can experience a force.



$\odot$ Bahar
$\otimes$ Ander

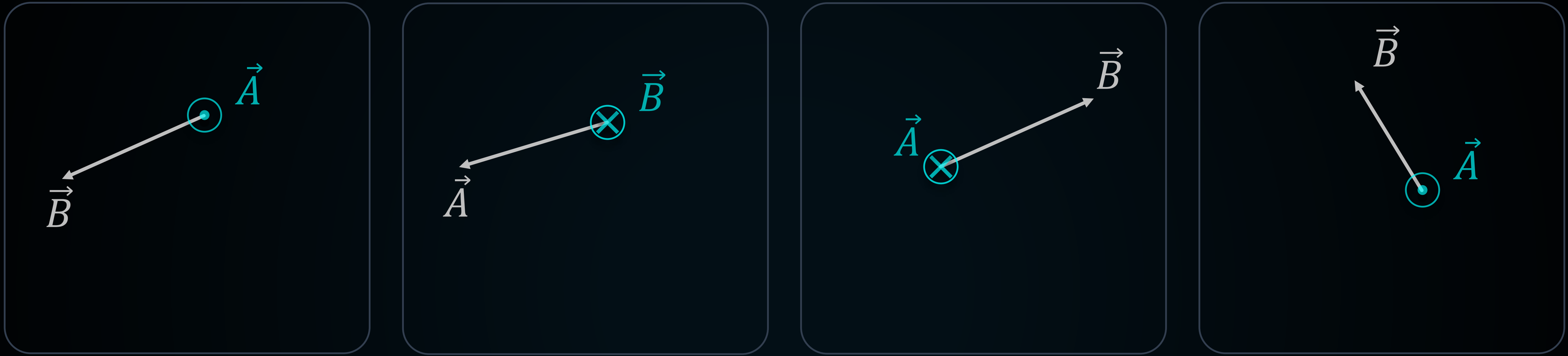
- A vector quantity.
- SI Unit: $Tesla = \frac{Weber}{m^2}$
- CGS Unit: $Gauss$
- $1\ Gauss = 10^{-4}\ Tesla$

Kahani :- 1st Aspect Calculation of M.f due to Current Carrying wire. → Biot Savart Law / Ampere's Circuital Law.

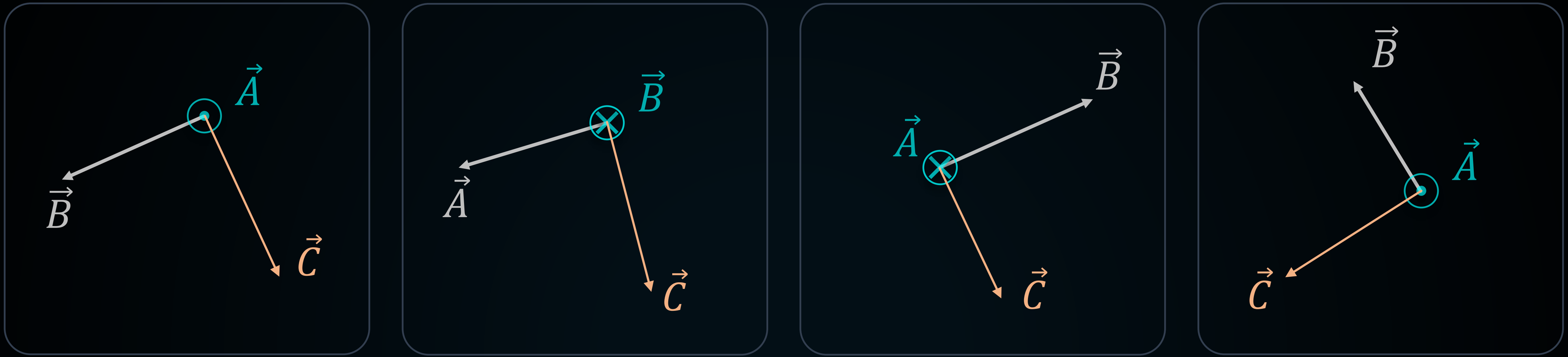
2nd Aspect :- Calculation of force on Moving charge due to M.f.

Question Solution

If $\vec{A} \times \vec{B} = \vec{C}$, then find the direction of $\vec{C}$ for the following cases.



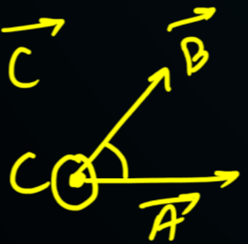
Question | **Solution**



$\odot$ Bahar
$\otimes$ Ander

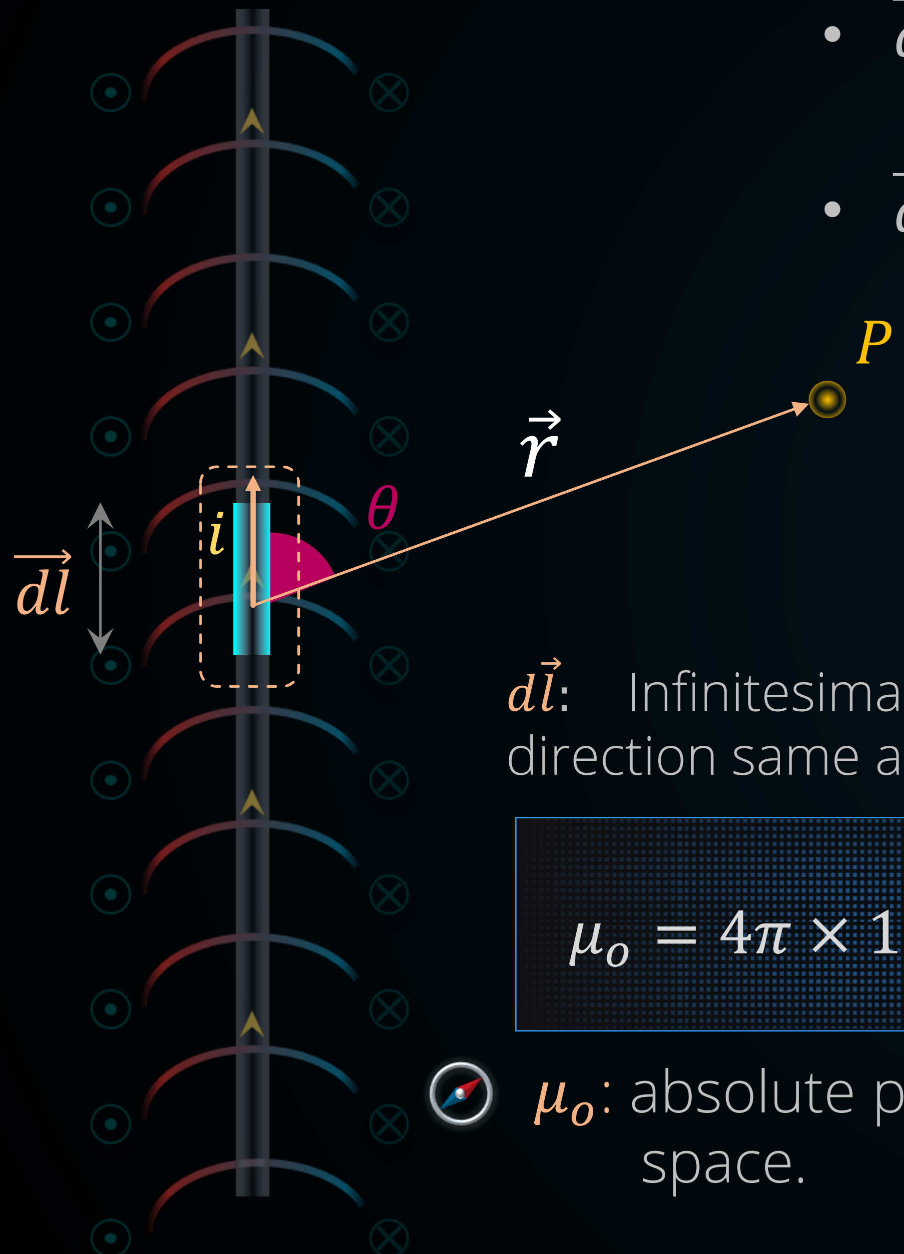
If you keep fingers of your right hand in the direction of $\vec{A}$ and curl them along $\vec{B}$ then the direction of thumb will be the direction of $\vec{C}$.

$\vec{A} \times \vec{B} = \vec{C}$



If you keep fingers of your left hand as shown in the figure and your index finger along $\vec{A}$, middle figure along $\vec{B}$, then the thumb will point towards $\vec{C}$.

Biot - Savart's Law



- $\overrightarrow{dB} \propto i$
- $\overrightarrow{dB} \propto \dfrac{1}{|\vec{r}|^2}$
- $\overrightarrow{dB} \propto \sin\theta$
- $\overrightarrow{dB} \propto \vec{dl}$

$$\overrightarrow{dB} = \frac{\mu_o}{4\pi}\frac{(i\vec{dl} \times \vec{r})}{|\vec{r}|^3}$$

{Yaad hona chahiye}.

Tesla

$\otimes \overrightarrow{dB} = \dfrac{\mu_o i (\vec{dl} \times \vec{r})}{4\pi |\vec{r}|^3}$

$\vec{dl}$: Infinitesimal wire segment having direction same as the current.

$\vec{dl}$ → direction udhar hogi jidhar current jayega

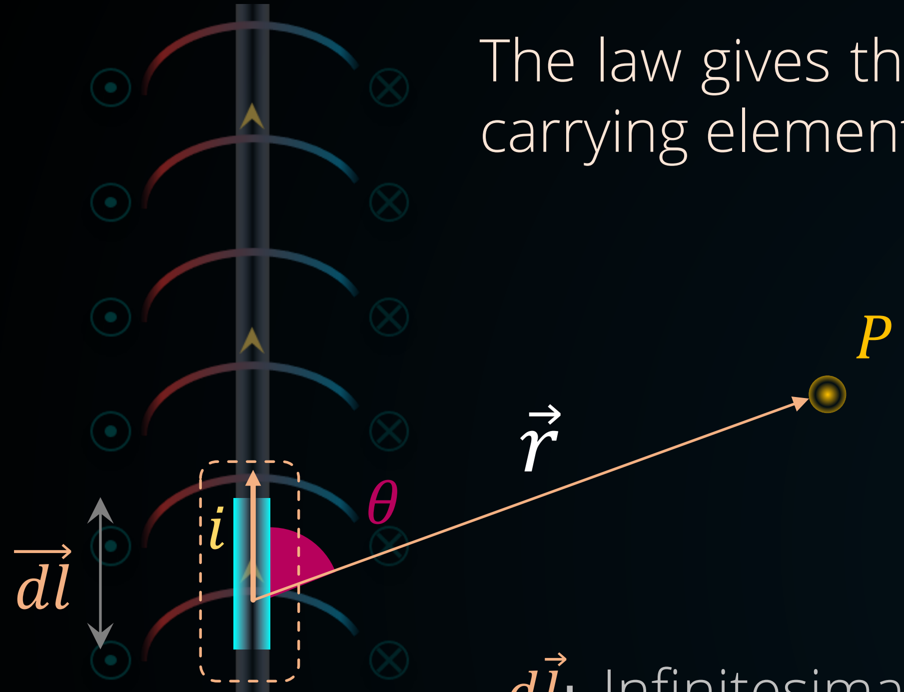
$\vec{r}$ jiska head wahan hoga jahan M.F nikalni hogi.

$$\mu_o = 4\pi \times 10^{-7} \frac{T\,m}{A}$$

μ_o: absolute permeability in free space.

Biot - Savart's Law

BOARDS NEET



The law gives the magnetic field generated at a point in vicinity of current carrying element.

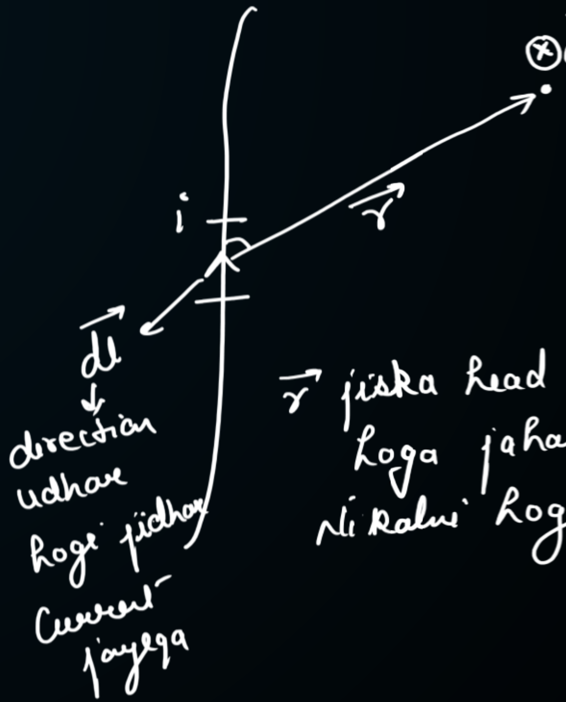
- $\vec{dB} \propto i$
- $\vec{dB} \propto \frac{1}{|\vec{r}|^2}$
- $\vec{dB} \propto \sin\theta$
- $\vec{dB} \propto \vec{dl}$

$$\vec{dB} = \frac{\mu_o}{4\pi}\frac{(i\vec{dl}\times\vec{r})}{|\vec{r}|^3}$$

{Yaad hona chahiye}.

Tesla

$$\otimes dB = \frac{\mu_0 i(\vec{dl}\times\vec{r})}{4\pi\,|\vec{r}|^3}.$$



$\vec{r}$ jiska head wahan hoga jahan M.F nikalni hogi.

$\vec{dl}$: Infinitesimal wire segment having direction same as the current.

$$\mu_o = 4\pi\times10^{-7}\ \frac{T\,m}{A}$$

μ_o: absolute permeability in free space.

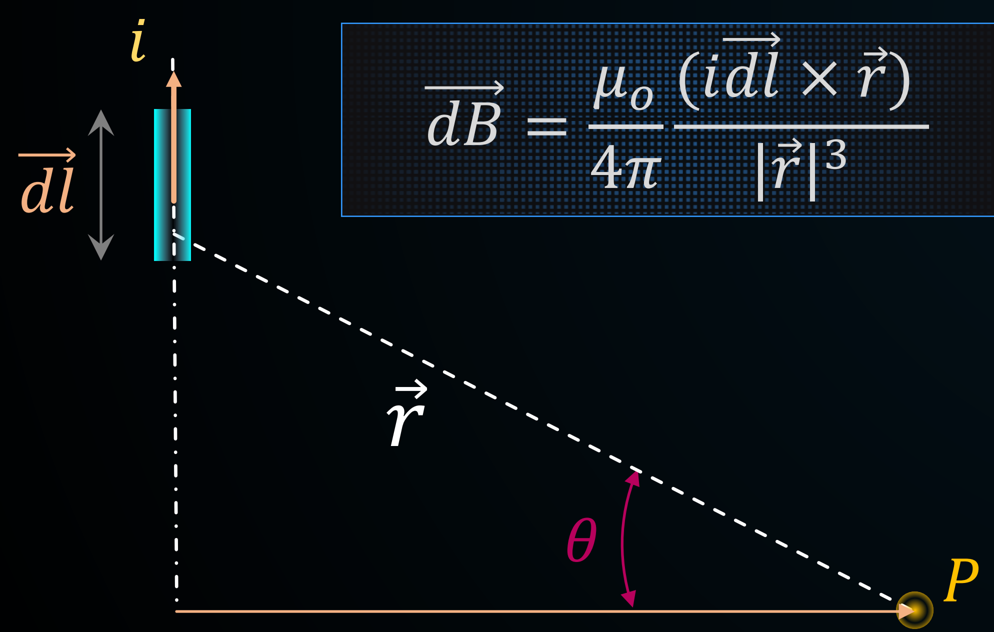
Magnetic Field Due to Straight Current Carrying Conductor

Suppose we want to find the magnetic field at point P which is asymmetrically placed w.r.t the finite wire.

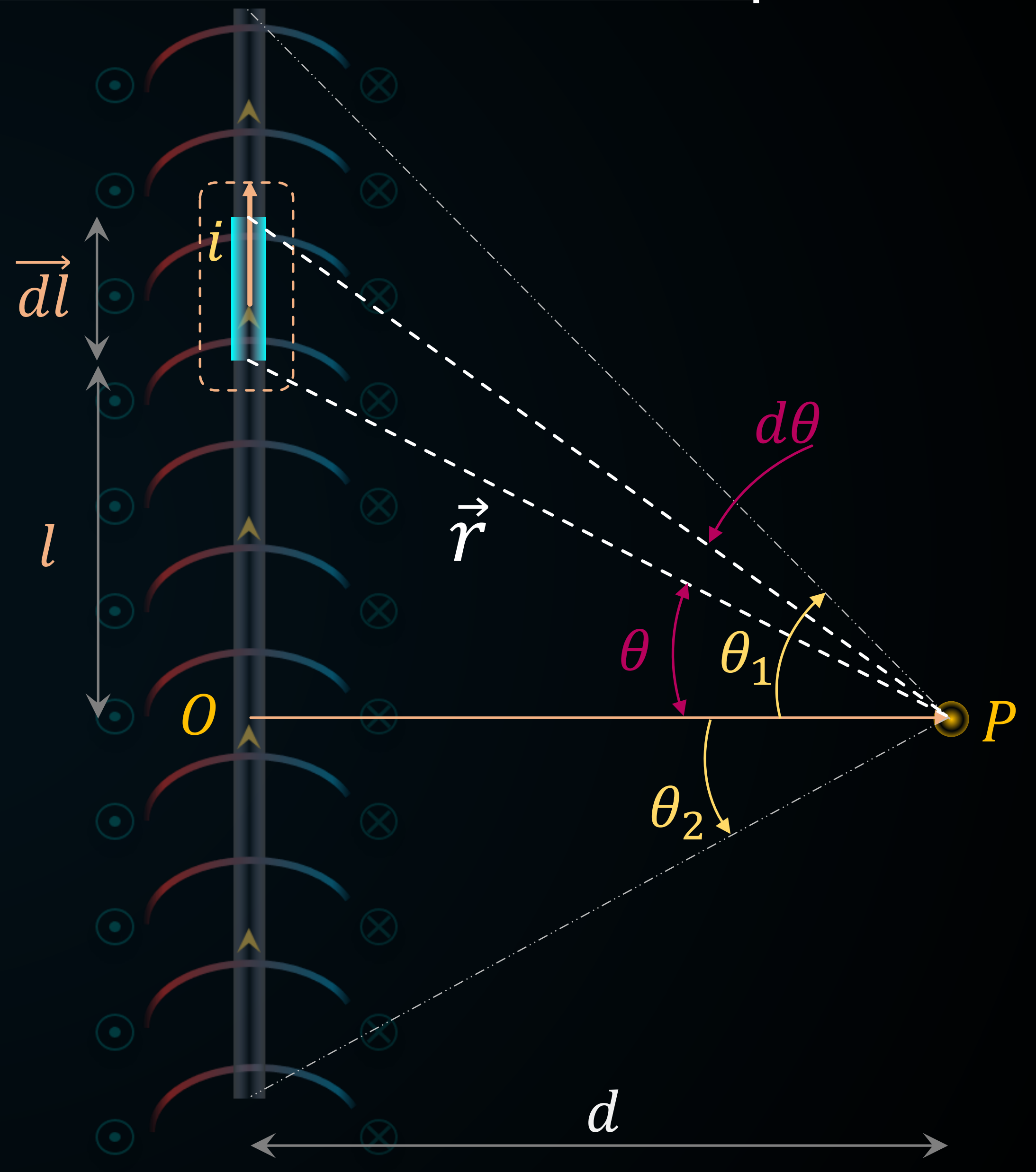
Consider an element dl at distance l from the foot of the perpendicular O drawn from P.

Let the element dl subtends an angle $d\theta$ at point P and the position vector of the element from point P is $\vec{r}$.

Magnetic field at point P due to the element dl is given by,

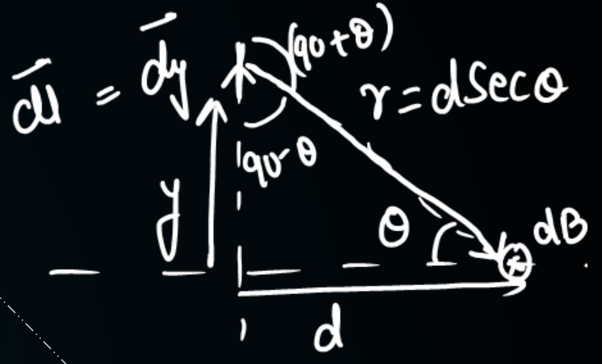


$$\overrightarrow{dB} = \frac{\mu_o}{4\pi}\frac{(i\overrightarrow{dl} \times \vec{r})}{|\vec{r}|^3}$$



Magnetic Field Due to Straight Current Carrying Conductor

From figure, $\vec{dl} = \vec{dy}$, $r = d\sec\theta$ | $\tan\theta = y/d \Rightarrow y = d\tan\theta$, $dy = d\sec^2\theta\, d\theta$



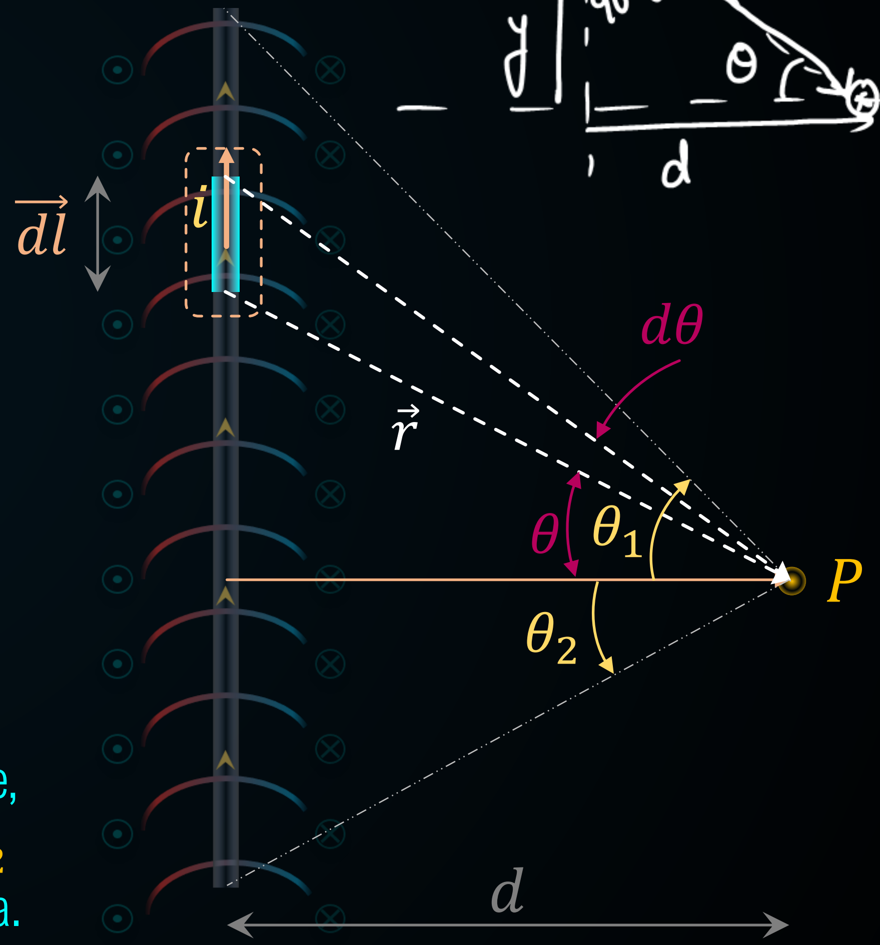
$$\Rightarrow d\vec{B} = \frac{\mu_0 i\,(\vec{dl}\times\vec{r})}{4\pi|\vec{r}|^3} = \frac{\mu_0 i\, dy\, r\cdot\sin(90+\theta)}{4\pi r^3}$$

$$dB = \frac{\mu_0 i\, dy\cos\theta}{4\pi d^2\sec^2\theta} = \frac{\mu_0 i\, d\sec^2\theta\cos\theta\, d\theta}{4\pi d^2\sec^2\theta} = \frac{\mu_0 i}{4\pi d}\cos\theta\, d\theta$$

$$B_{net} = \frac{\mu_0 i}{4\pi d}\int_{-\theta_2}^{\theta_1}\cos\theta\, d\theta = \boxed{\frac{\mu_0 i}{4\pi d}\left[\sin\theta_1 + \sin\theta_2\right]}$$

$$B = \frac{\mu_o\, i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

While finding B due to finite wire, only the magnitude of θ_1 and θ_2 should be inserted in the formula.



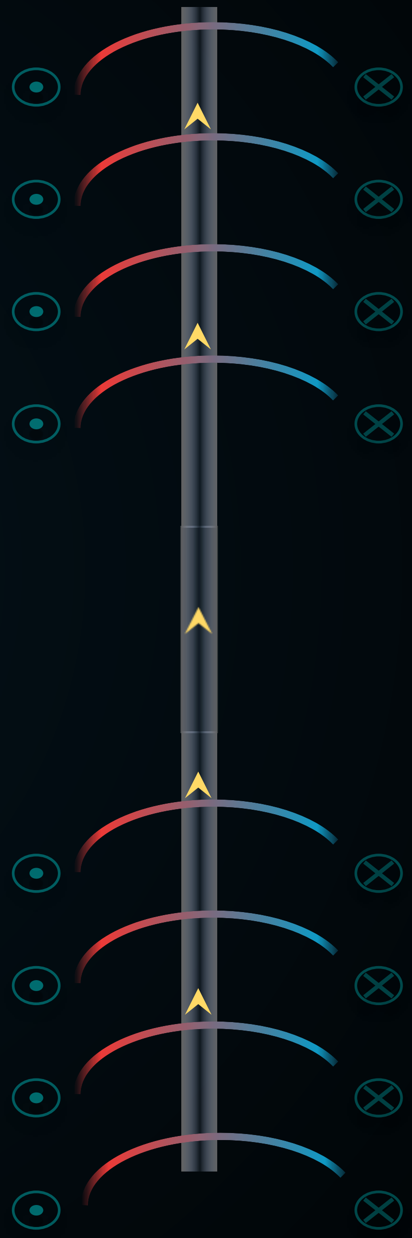
Right Hand Thumb Rule

- Point your thumb in the direction of the current flow and curl your fingers.

  Curled fingers give the direction of magnetic field.

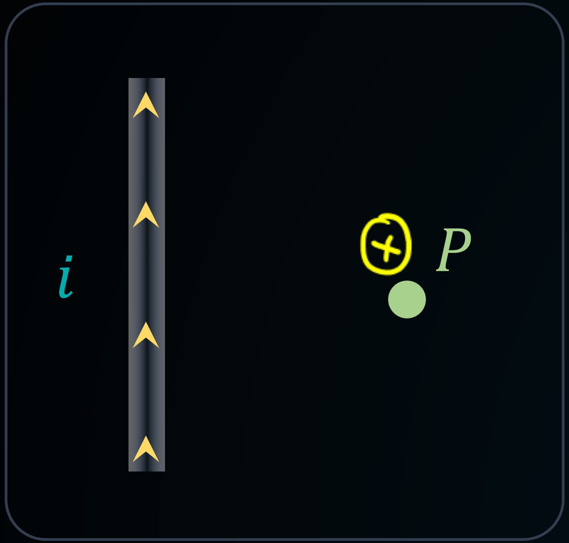
OR

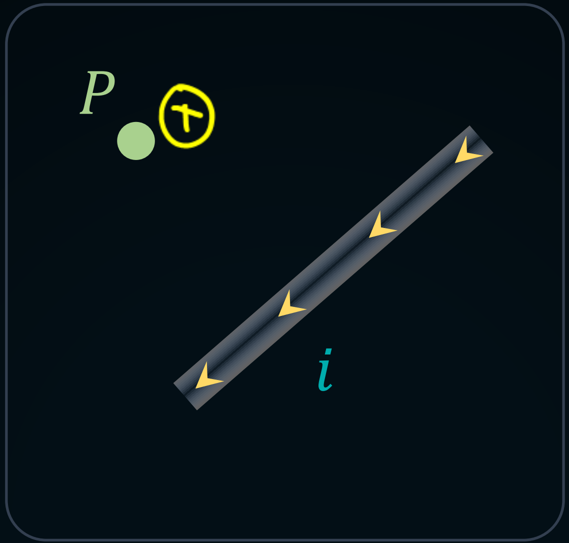
$$\overrightarrow{dB} = \frac{\mu_o}{4\pi}\frac{(i\overrightarrow{dl} \times \vec{r})}{|\vec{r}|^3}$$

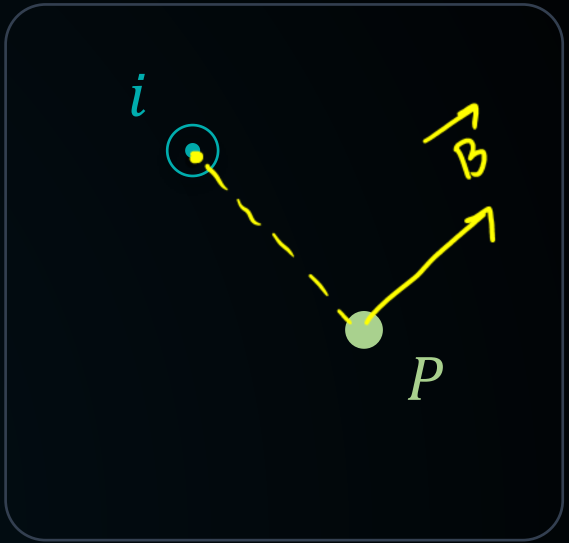


Direction of Magnetic Field

Examples

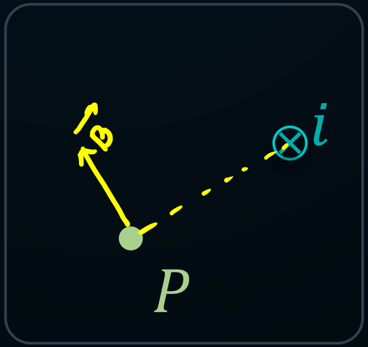


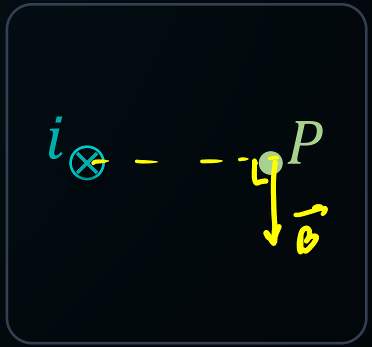


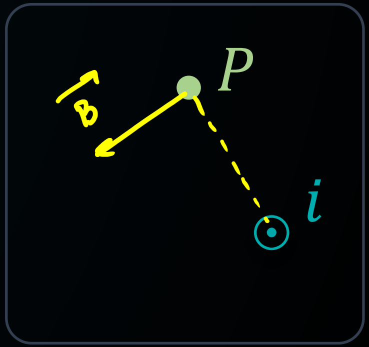


Right hand palm rule

Point your thumb in the direction of the current flow and stretch your fingers towards the point where you want to find the magnetic field. Then, you palm will give you the direction of magnetic field.







Question

A straight wire carries a current i as shown. Calculate the magnetic field due to the wire at point P located at a distance d away from the wire.

Solution

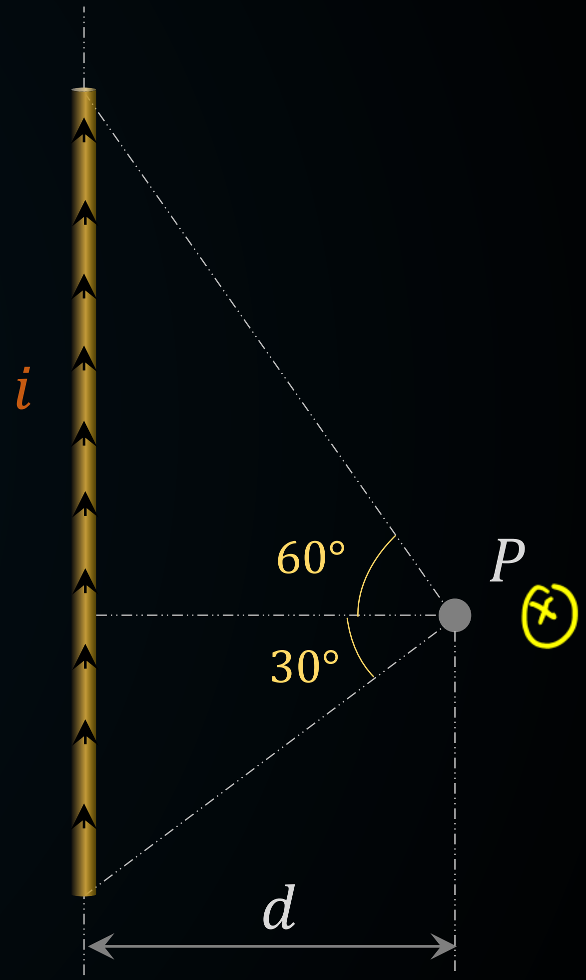
Given, $d = d$ $\quad \theta_1 = 60^\circ$

$\theta_2 = 30^\circ$

We have, $B = \dfrac{\mu_o\, i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

$\Rightarrow$ $\boxed{B = \dfrac{\mu_0 i}{4\pi d}\left[\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\right]}$

$$B = \frac{\mu_o i\ (\sqrt{3} + 1)}{8\pi d} \otimes$$



Question

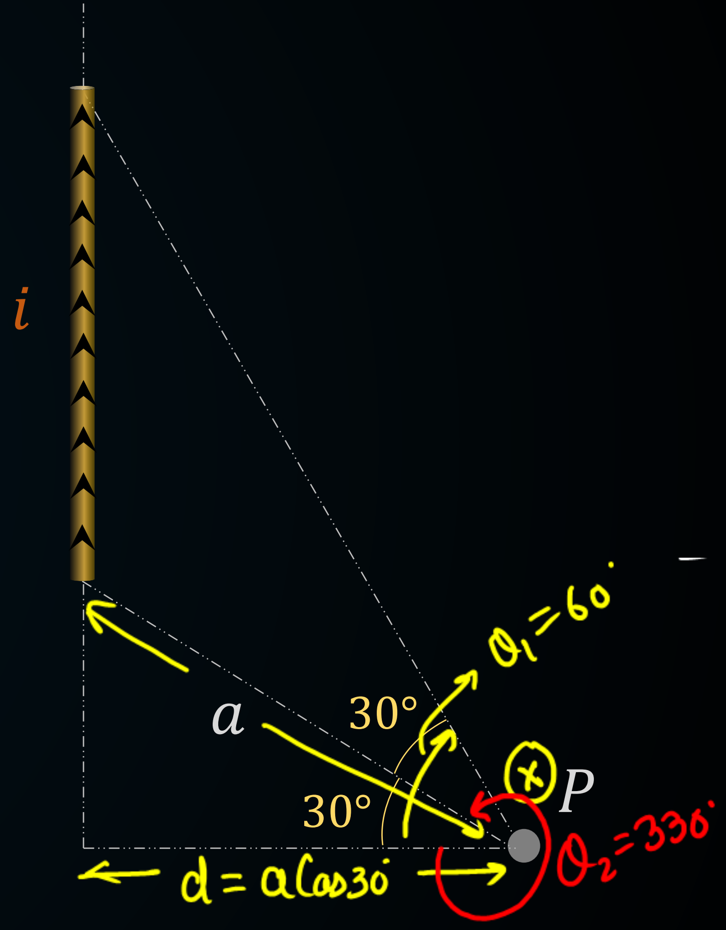
A straight wire carries a current i as shown. Calculate the magnetic field due to the wire at point P.

Solution

$d = \perp$ distance of wire from Point.

$\theta_1 \& \theta_2$ should be taken from $\perp$.

$\theta_1 \& \theta_2$ should be taken in opposite Sense.



Given,

$\theta_1 = 60°$ $\quad \theta_2 = 330° = -30°$ $\quad d = a\cos 30° = \frac{\sqrt{3}a}{2}$

Question Solution

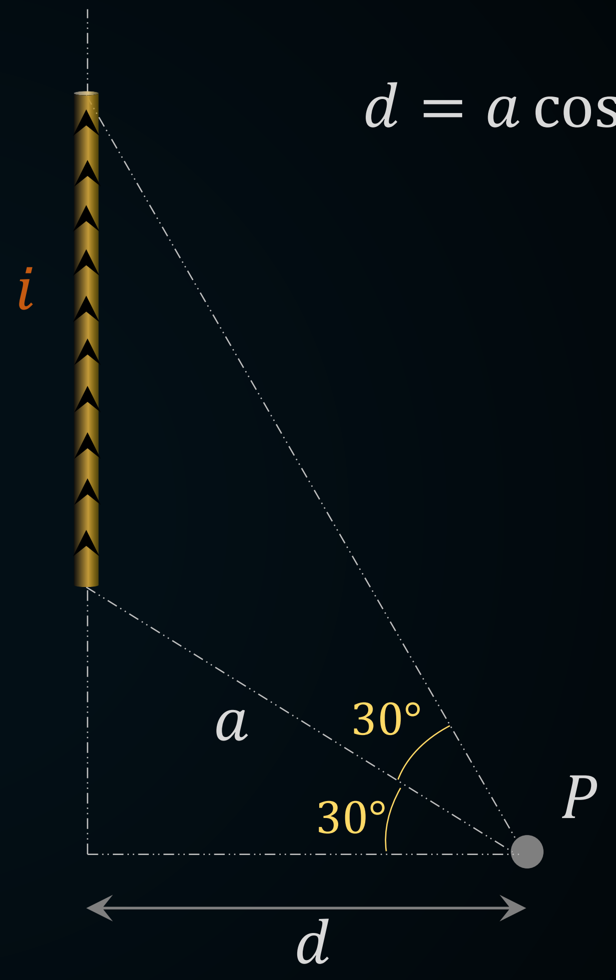
$$B = \frac{\mu_o\, i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

$\Rightarrow$ $B = \frac{\mu_o\, i}{4\pi d}(\sin 60° + \sin(-30°))$

$\Rightarrow$ $B = \frac{\mu_o i}{\cancel{4}\pi \frac{\sqrt{3}a}{\cancel{2}}}\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$

$= \boxed{\frac{\mu_o i}{4\sqrt{3}\pi a}[\sqrt{3}-1]}$ $\otimes$

$$B = \frac{\mu_o i\,(\sqrt{3}-1)}{4\sqrt{3}\pi a} \quad \otimes$$



$d = a\cos 30°$

Magnetic Field Due to Straight Current Carrying Conductor

Infinite Wire

Magnetic field due to a finite wire at a distance d is:

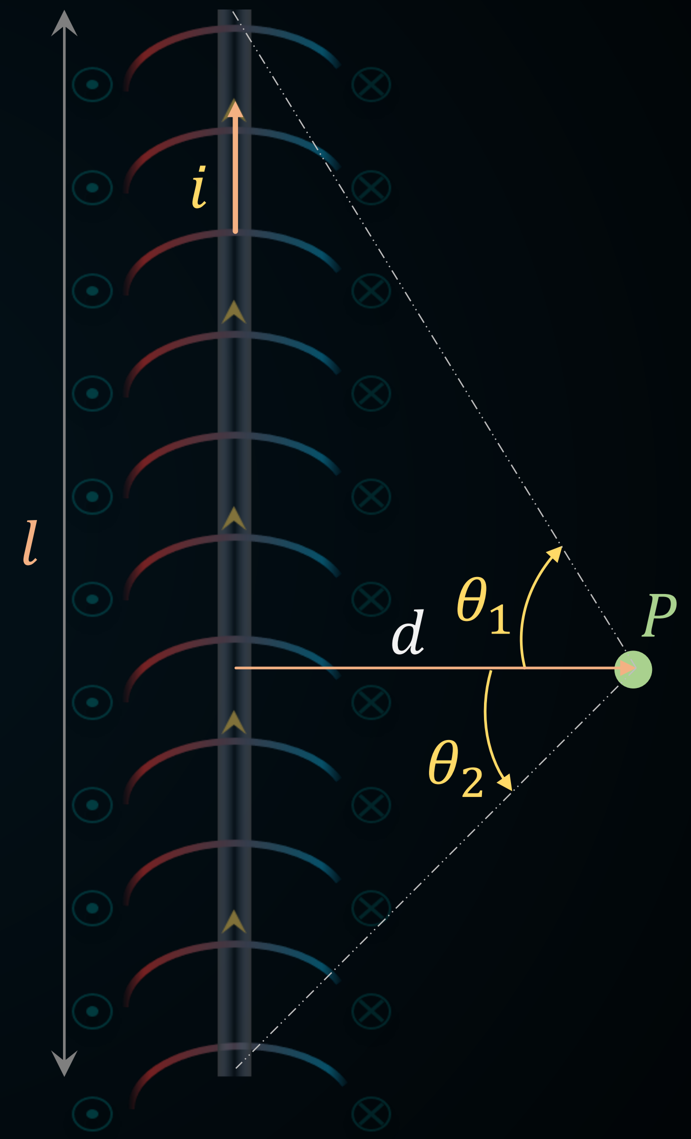
$$B = \frac{\mu_o\, i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

For infinite wire: $\theta_1 = \theta_2 = 90°\quad (\because l \gg d)$

Therefore, the magnetic field at point P due to the infinite wire will be given by:

$$B = \frac{\mu_0 i}{4\pi d}(1+1)$$

$$B = \frac{\mu_o\, i}{2\pi d}$$



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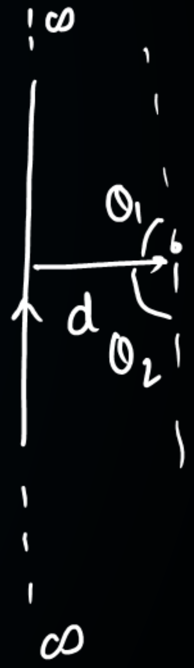


Semi – Infinite Wire

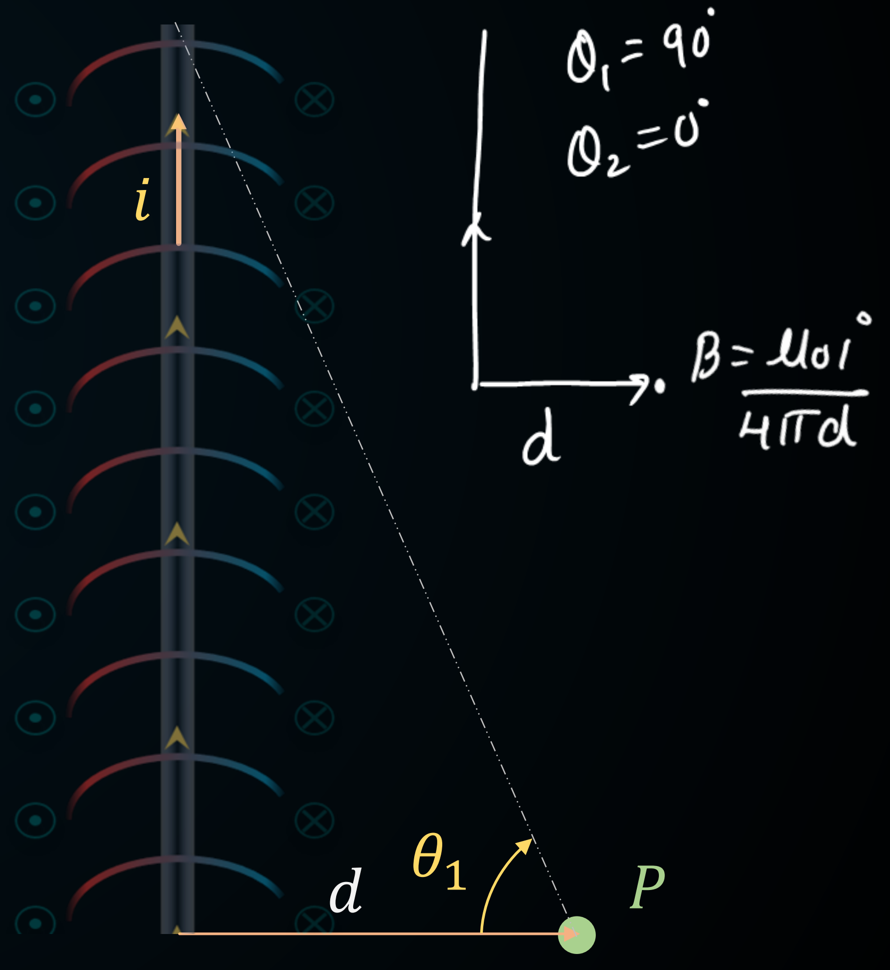
Magnetic field due to a finite wire at a distance d is:

$$B = \frac{\mu_o\, i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

For semi-infinite wire: $\theta_1 = 90°$ $\theta_2 = 0°$

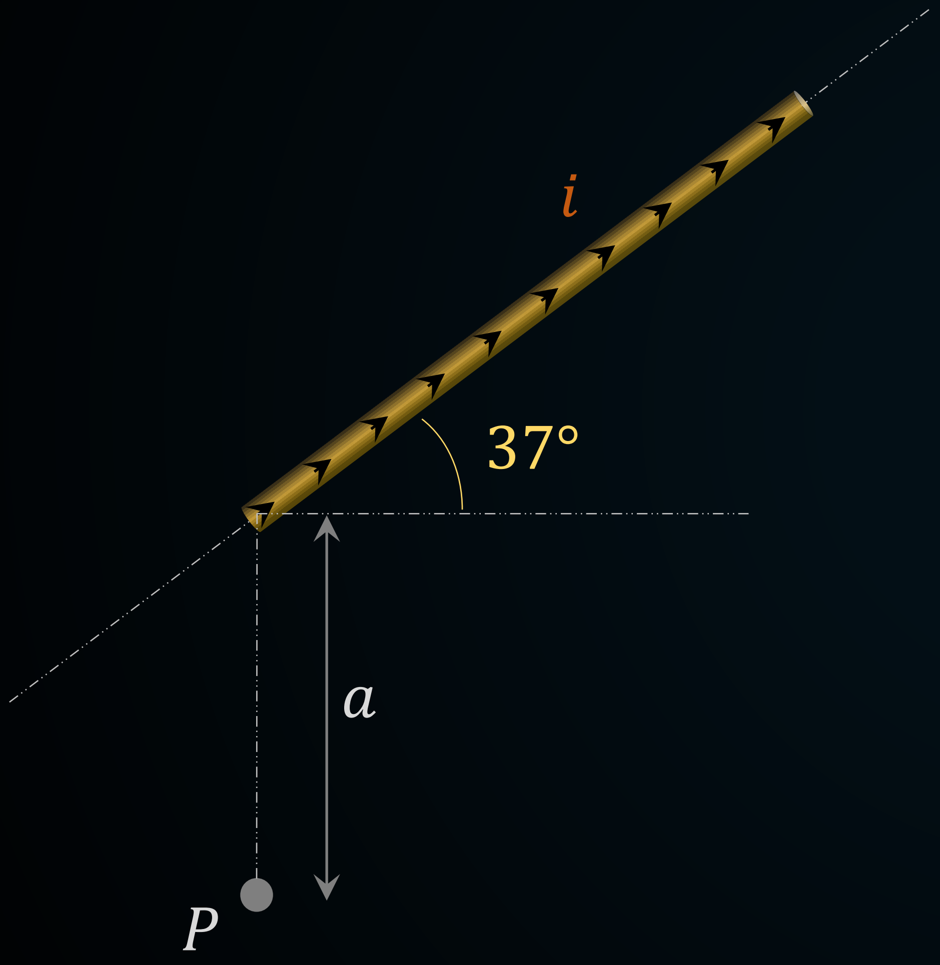
Therefore, the magnetic field at point P due to the semi-infinite wire will be given by:

$$B = \frac{\mu_o\, i}{4\pi d}$$



Question Solution

An infinitely long wire carries a current i as shown. Find the magnetic field at point P.



Question Solution

Perpendicular distance of wire from the point P: $d = a\cos 37° = \frac{4a}{5}$

From figure, $\theta_1 = 90°$ $\theta_2 = -37°$

We have, $B = \frac{\mu_o i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

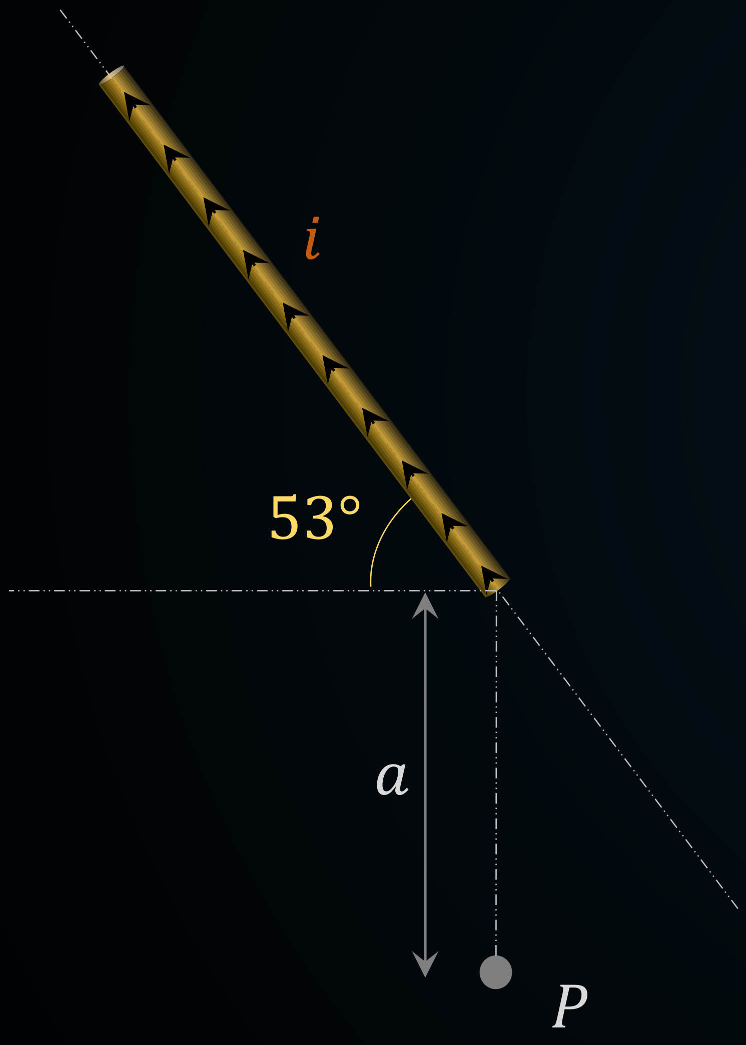
$$B = \frac{\mu_0 i}{4\pi \frac{4a}{5}}\left[1 - 3/5\right]$$

$$= \frac{\mu_0 i \cdot 2}{16\pi a} = \boxed{\frac{\mu_0 i}{8\pi a}}$$

$$B = \frac{\mu_o i}{8\pi a} \otimes$$

Question Solution

An infinitely long wire carries a current i as shown. Find the magnetic field at point P.



Question | Solution

Perpendicular distance of wire from the point P: $d = \frac{39}{5}$

From figure, $\theta_1 = 90$

$\theta_2 = -53^\circ$

We have, $B = \frac{\mu_o i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

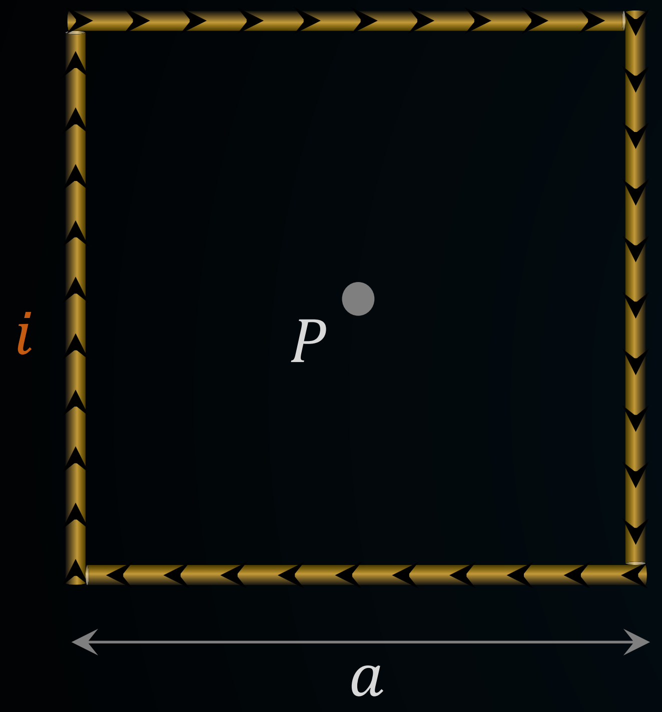
$$B = \frac{\mu_0 i}{4\pi \cdot \frac{39}{\cancel{5}}}\left(1 - \frac{4}{\cancel{5}}\right)$$

$$= \frac{\mu_0 i}{12\pi a} \odot$$

$$\boxed{B = \frac{\mu_o i}{12\pi a} \otimes}$$

Question Solution

A square loop of side a carries current i . Find the magnetic field due to the loop at its centre P.



Question Solution

Consider one side of a square loop. We can use formula of finite wire for this side. The total field is the sum of field due to the four wires.

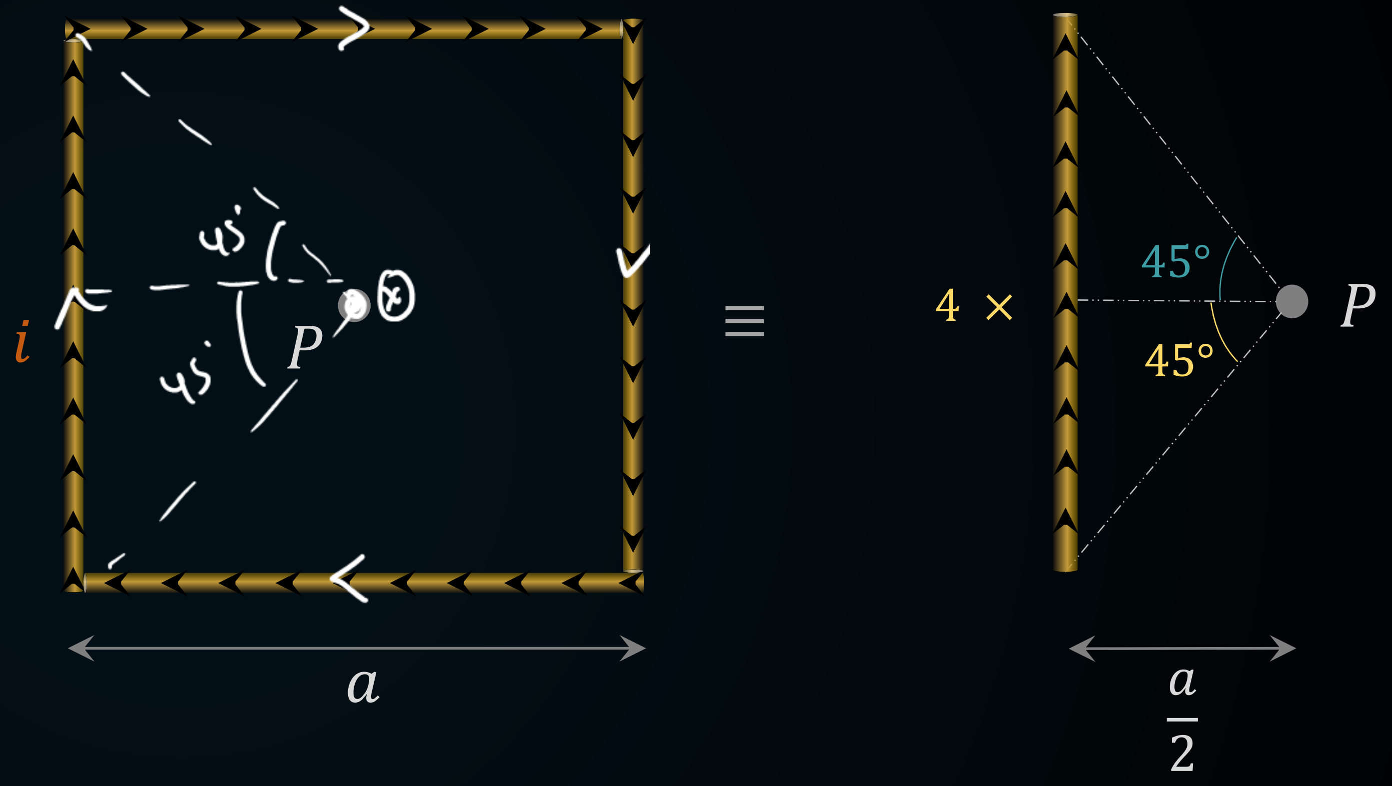
Given, $d = a/2 \quad \theta_1 = 45^\circ \quad \theta_2 = 45^\circ$

Net magnetic field at point P:

$B_{net} = 4 \times B$ due to one wire

$= 4 \times \frac{\mu_0 i}{4\pi \cdot a/2}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$

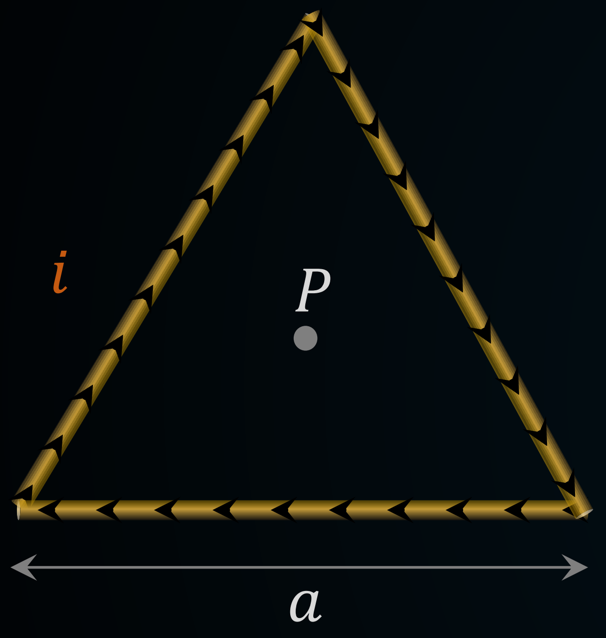
$= \frac{2\mu_0 i}{\pi a}\sqrt{2} \cdot \otimes$



$$B = \frac{2\sqrt{2}\mu_o i}{\pi a} \otimes$$

Question | Solution

An equilateral triangular loop of side a carries current i . Find the magnetic field due to the loop at its centroid P.



Question Solution

Consider one side of a triangular loop. We can use formula of finite wire for this side. The total field is the sum of field due to the three wires.

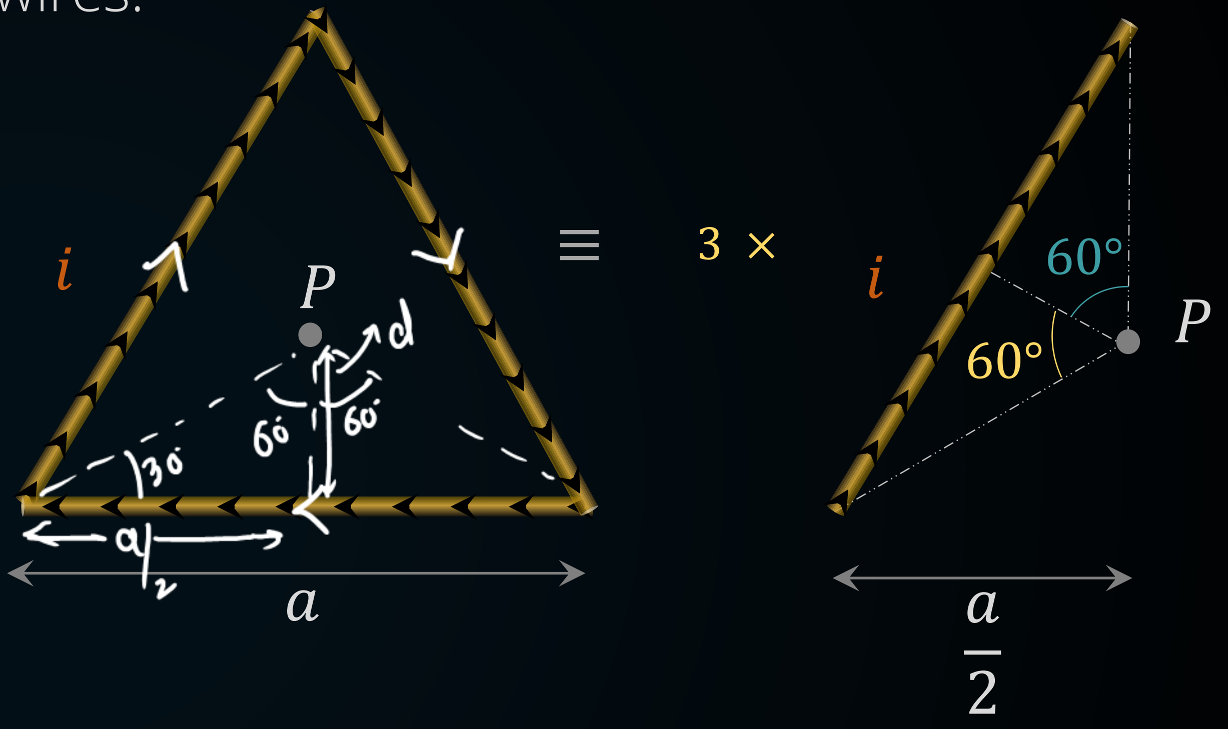
Given, $d = \frac{a}{2\sqrt{3}}$ $\theta_1 = 60^\circ = \theta_2$

Net magnetic field at point P:

$B_{net} = 3 \times$ B due to one wire

$$= 3 \times \frac{\mu_0 i}{4\pi \cdot \frac{a}{\cancel{2}\sqrt{3}}}\left[\frac{\sqrt{3}}{\cancel{2}} + \frac{\sqrt{3}}{\cancel{2}}\right]$$

$$= \frac{3\,\mu_0 i\,\sqrt{3}\cdot\cancel{2}\sqrt{3}}{\cancel{4}\pi a} = \boxed{\frac{9\mu_0 i}{2\pi a}} \otimes$$



$$\tan 30^\circ = \frac{d}{a/2} = \frac{2d}{a} = \frac{1}{\sqrt{3}}$$

$$d = \boxed{\frac{a}{2\sqrt{3}}}$$

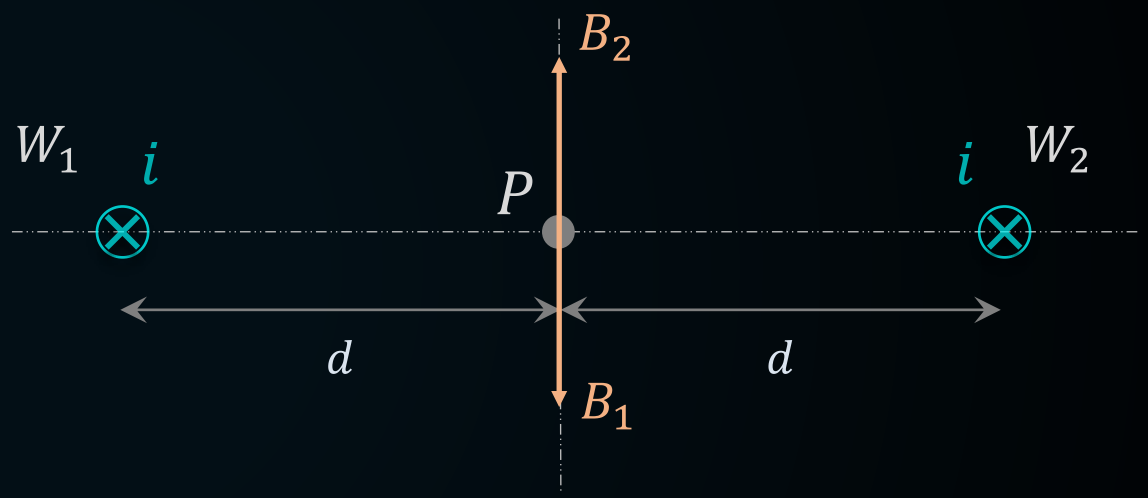
$$B_{net} = \frac{9\mu_o i}{2\pi a} \otimes$$

Question

Two infinitely long wires W_1 and W_2 carry the same current i inside the plane as shown. Calculate the magnetic field at point P.

Solution

As the wires are placed symmetrically with respect to point P, the magnitude of magnetic field due to wires is same. As the wires are on opposite direction, direction of field due to then at P will also be opposite.

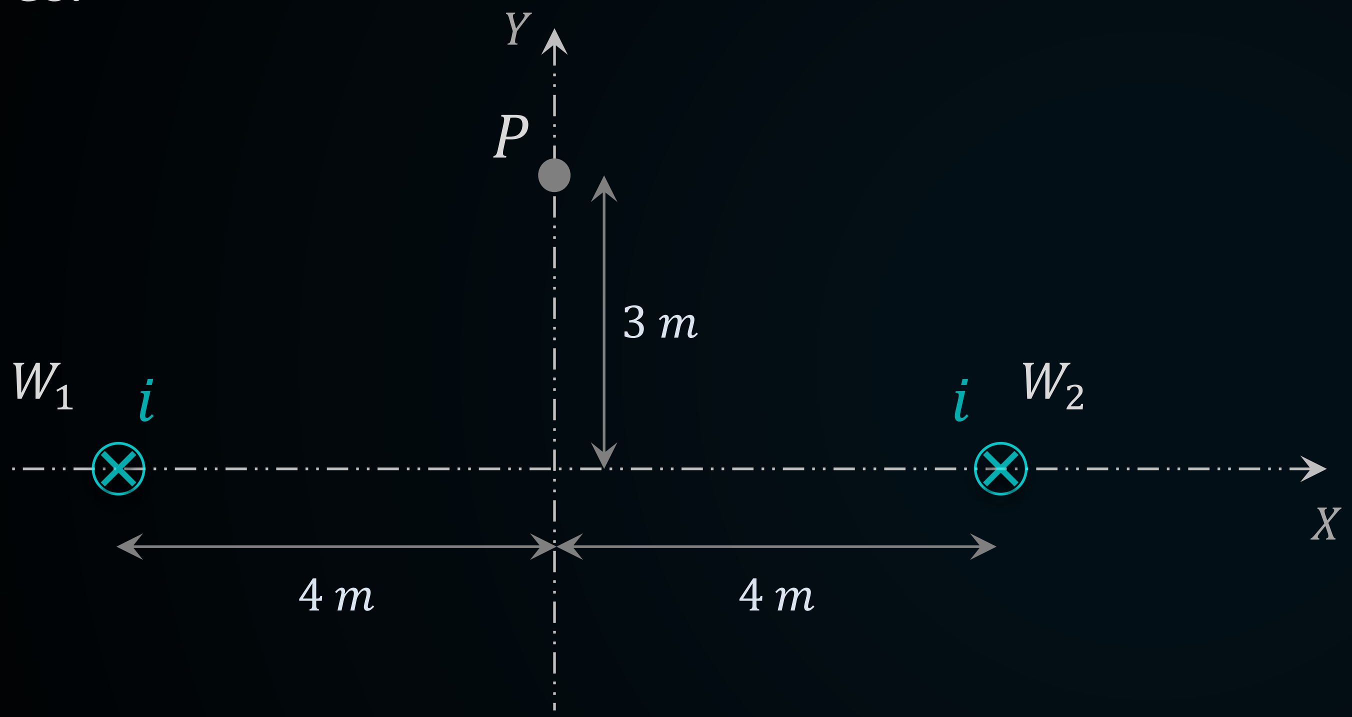


Thus, field due to wires get cancelled and the net field at point P becomes zero.

$$B_{net} = Zero$$

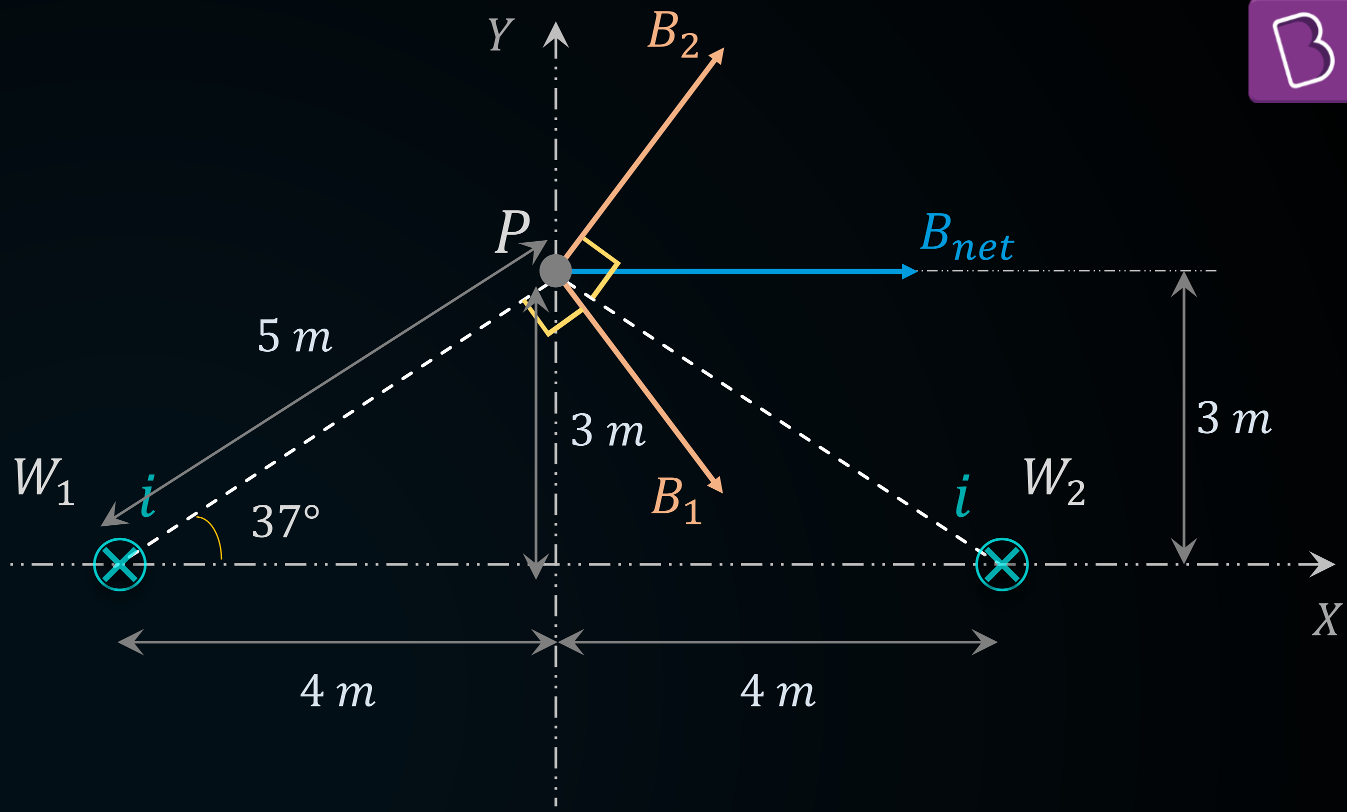
Question Solution

Two infinitely long wires W_1 and W_2 carry the same current i inside the plane as shown. Calculate the magnetic field at point P at a distance $3\ m$ away from the line joining the two wires.



Question Solution

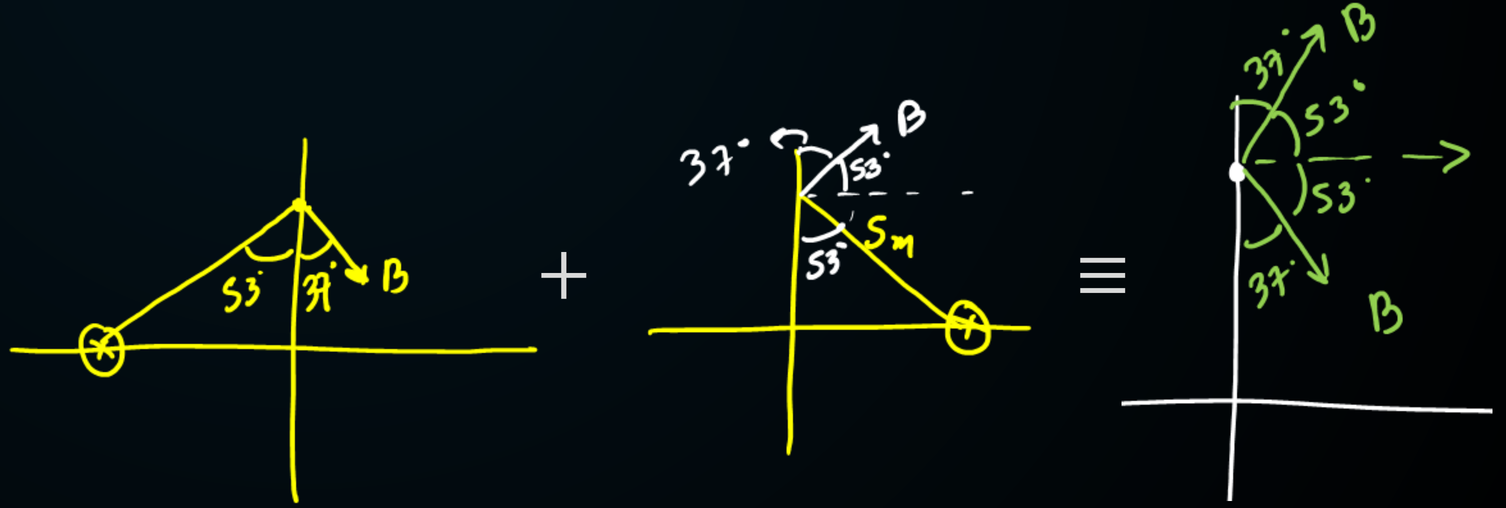
Direction of magnetic field due to individual wires is shown in the figure. As the wires are placed symmetrically, vertical components of the field are equal and opposite. Thus, vertical component gets cancelled. Net magnetic field is,



$B_{net} = 2B\cos 53^\circ$

$= \frac{2 \times \mu_0 i}{10\pi} \times \frac{3}{5}$

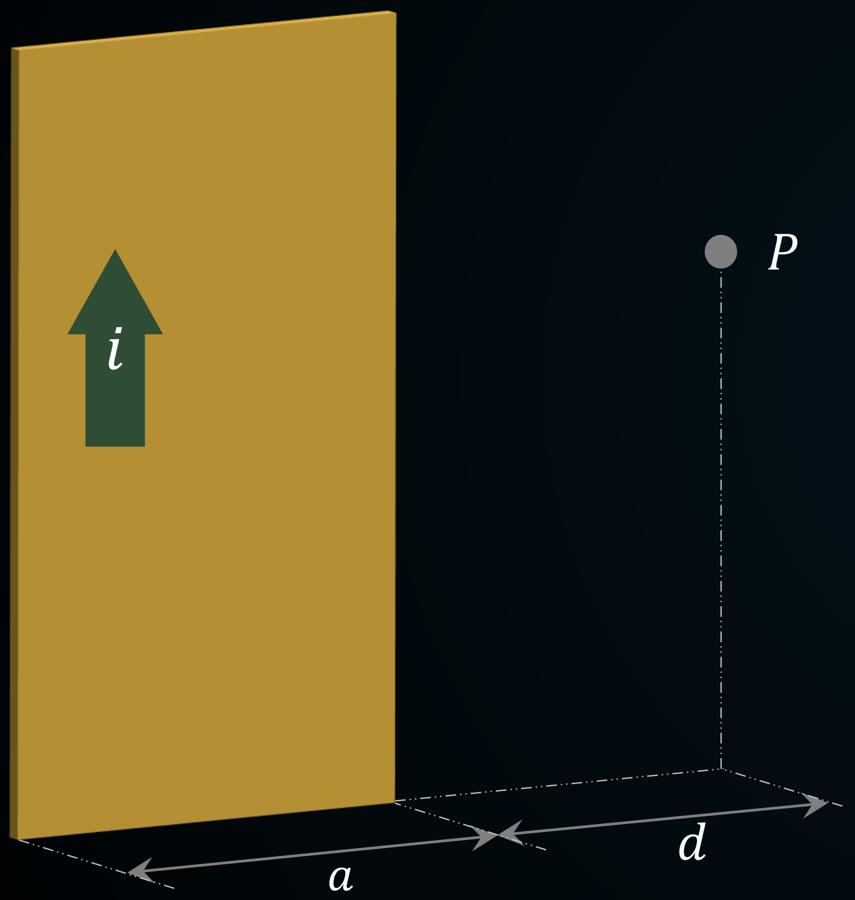
$= \boxed{\frac{3\mu_0 i}{25\pi} \rightarrow}$



$$B_{net} = \frac{3\mu_0 i}{25\pi} \text{ along } X - \text{axis}$$

Question Solution

An infinitely long plate of width a carries a current i as shown. Find the magnetic field due to the plate at a point P, located at a distance d away from one of the ends of the plate.



Question Solution

Consider a small element of the plate having width dx. This element can be assumed as a wire. Current per unit width of the plate is $\frac{i}{a}$.

Current in the small element $= \frac{i}{a} \times dx$

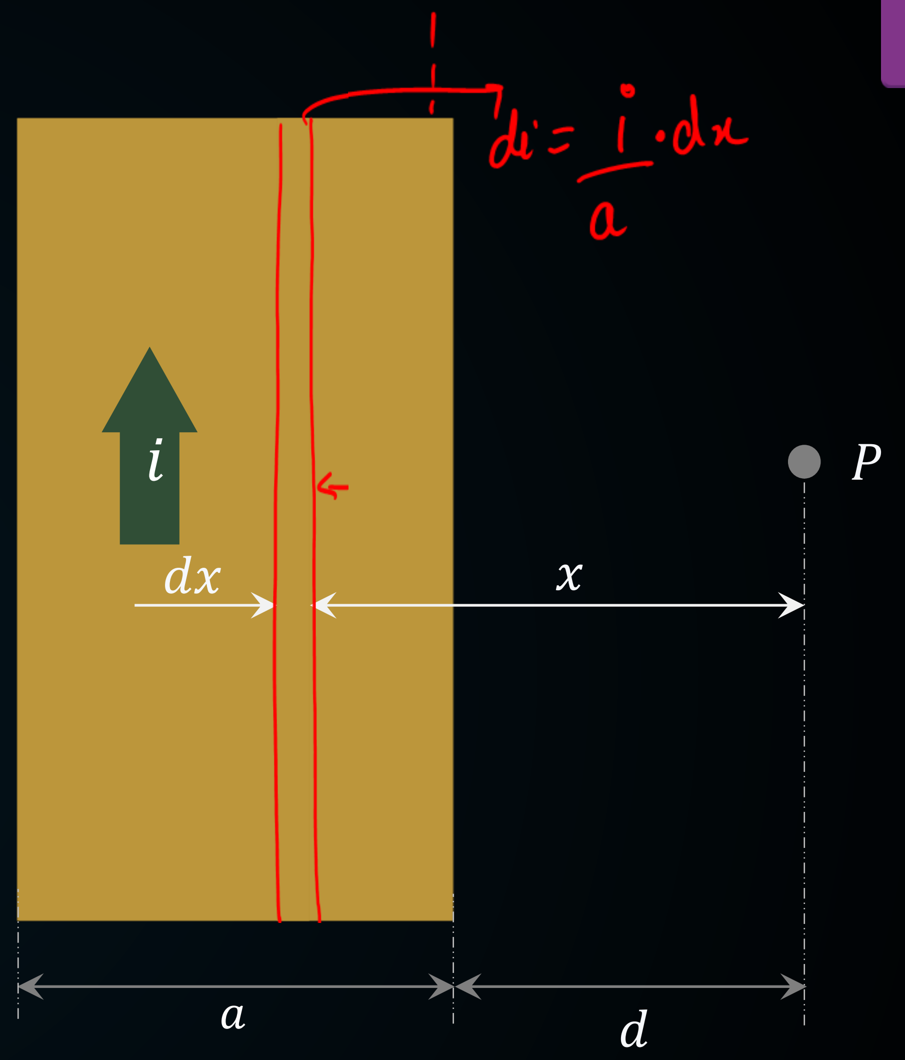
Magnetic field due to small element at point P:

$$dB = \frac{\mu_0\, di}{2\pi x} = \frac{\mu_0 i}{2\pi a} \cdot \frac{dx}{x}$$

Net magnetic field at P due to the plate is given by

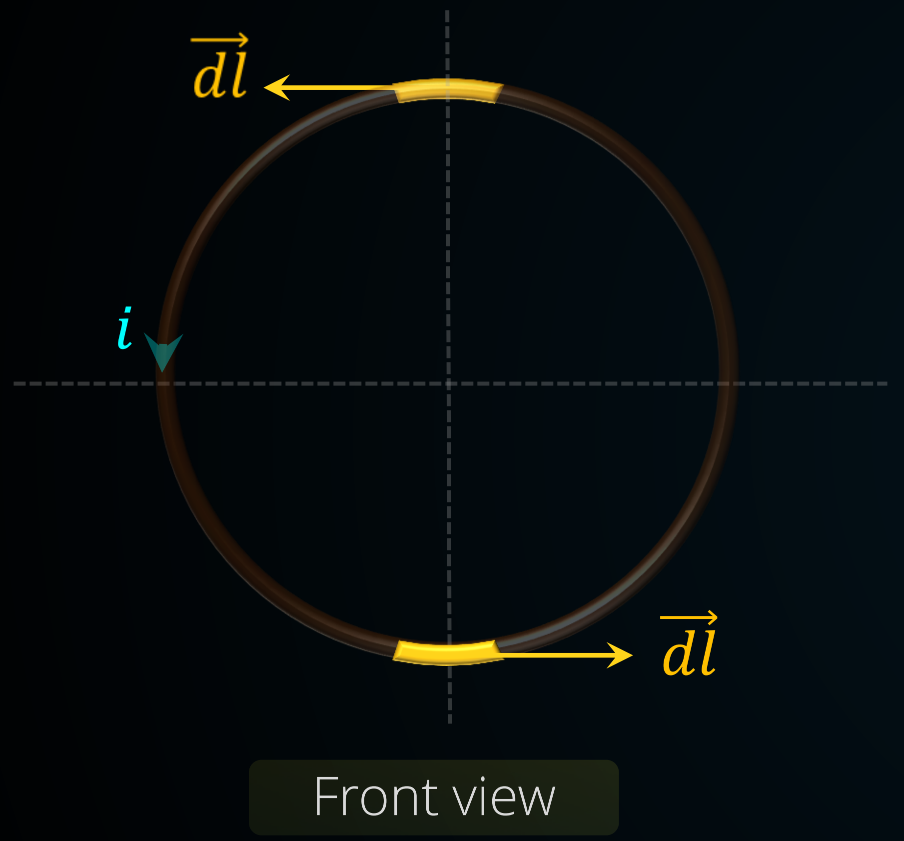
$$B_{net} = \int dB = \frac{\mu_0 i}{2\pi a} \int_d^{a+d} \frac{dx}{x} = \frac{\mu_0 i}{2\pi a} \ln\left[\frac{a+d}{d}\right]$$

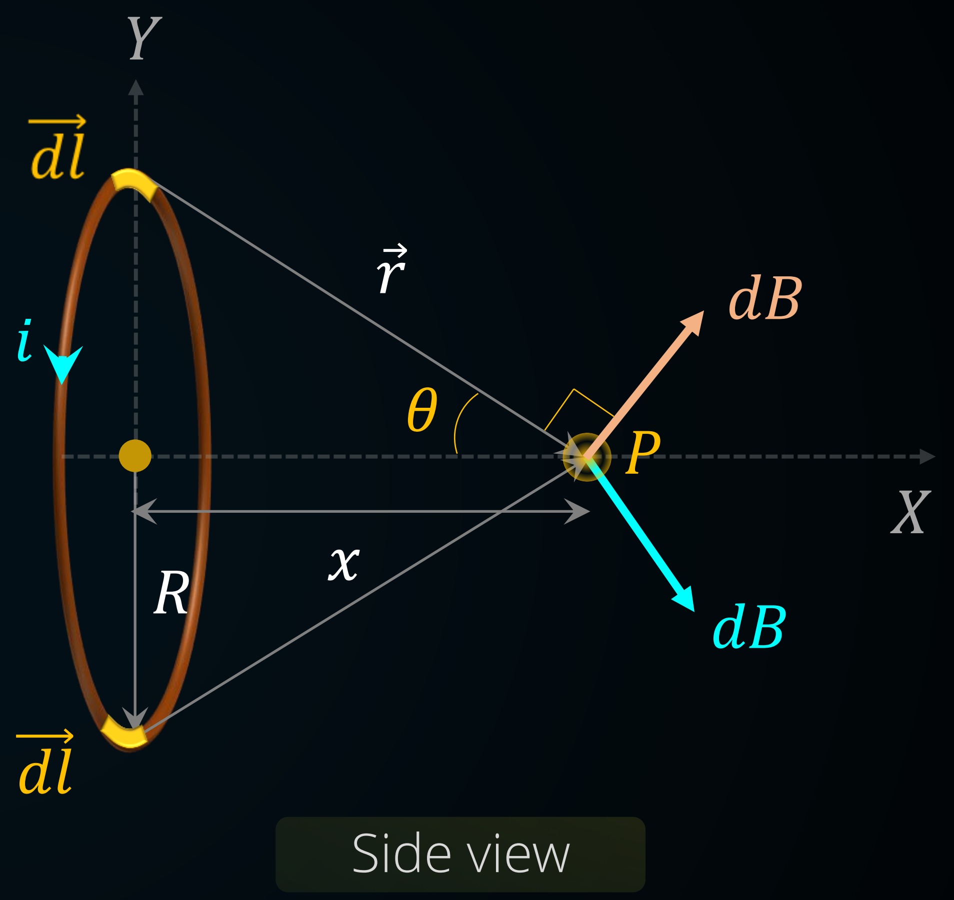
$$\boxed{B = \frac{\mu_0 i}{a(2\pi)} \ln\left(1 + \frac{a}{d}\right) \otimes}$$



MAGNETIC FIELD ON THE AXIS OF A CURRENT CARRYING RING

Magnetic field at point P :

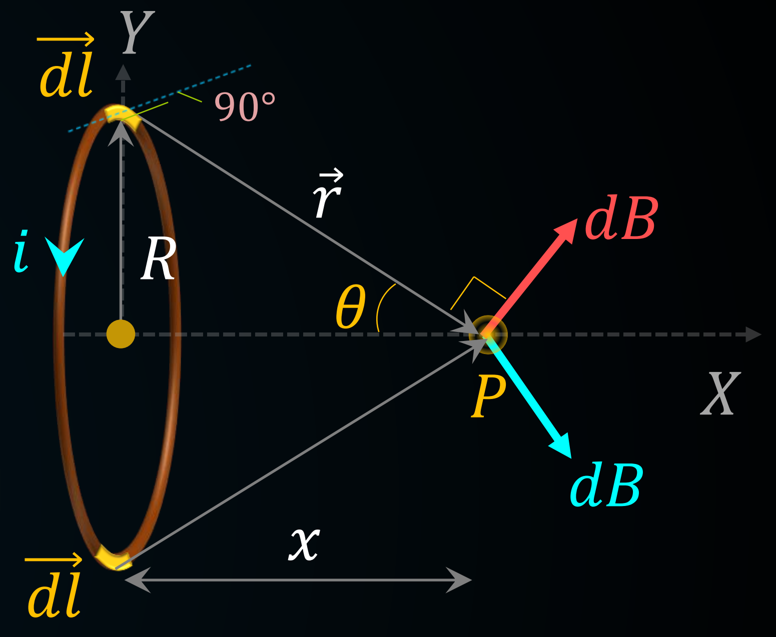




Magnetic field at point P : BOARDS

- Consider a current carrying element dl at the top-most point of the coil. The current in the wire is in the anti-clockwise direction from the viewpoint of P. Using 'Biot-savart law' and 'Right hand thumb rule', we can say that direction of the magnetic field at point P on the axis of the coil due to this element will be in slanted upward direction, as shown by the pink arrow in the adjacent figure.
- Similarly, choose a mirror current carrying element dl at the bottom most point of the coil. Using 'Biot-savart law' and 'Right hand thumb rule', we can say that direction of the magnetic field at point P on the axis of the coil due to this element will be in slanted downward direction, as shown by the blue arrow in the adjacent figure.
- The magnetic field vectors due to the element will be,

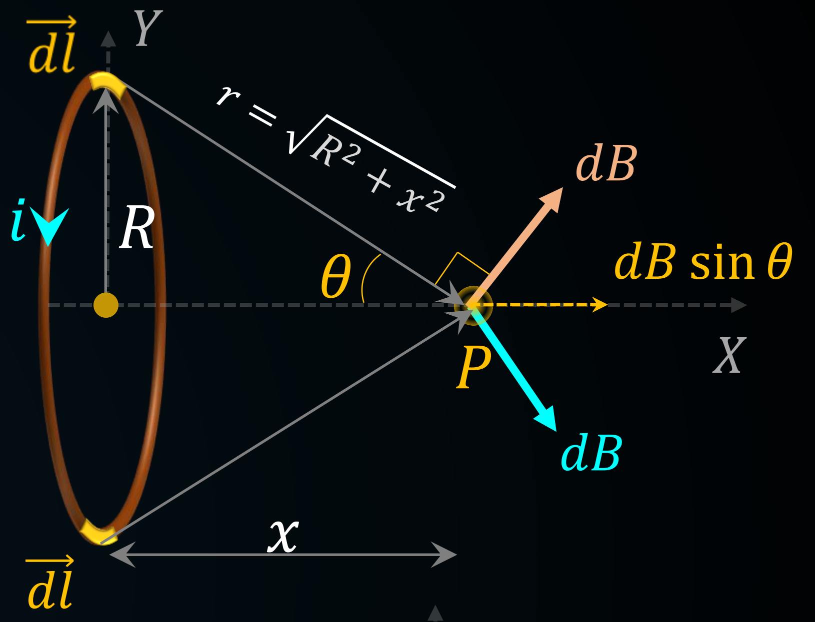
$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{i(\overrightarrow{dl} \times \vec{r})}{|\vec{r}|^3} \Rightarrow dB = \frac{\mu_0 i\, dl\, r \sin 90^\circ}{4\pi r^3} = \frac{\mu_0 i\, dl}{4\pi r^2}$$



MAGNETIC FIELD ON THE AXIS OF A CURRENT CARRYING RING

Magnetic field at point P :

- If we divide the magnetic field vectors component-wise, the vertical components get cancelled and the horizontal components get added. Therefore, the net magnetic field at point P will be along the axis of the ring and directed away from the centre of the coil (anti-clockwise current) and the magnitude of the total magnetic field due to the whole coil at point P will be,



$B_{net} = \int dB \sin\theta$ ← along Axis of Ring.

$B_{net} = \int \frac{\mu_0 i \cdot R\, dl}{4\pi (R^2+x^2)^{3/2}}$

will add

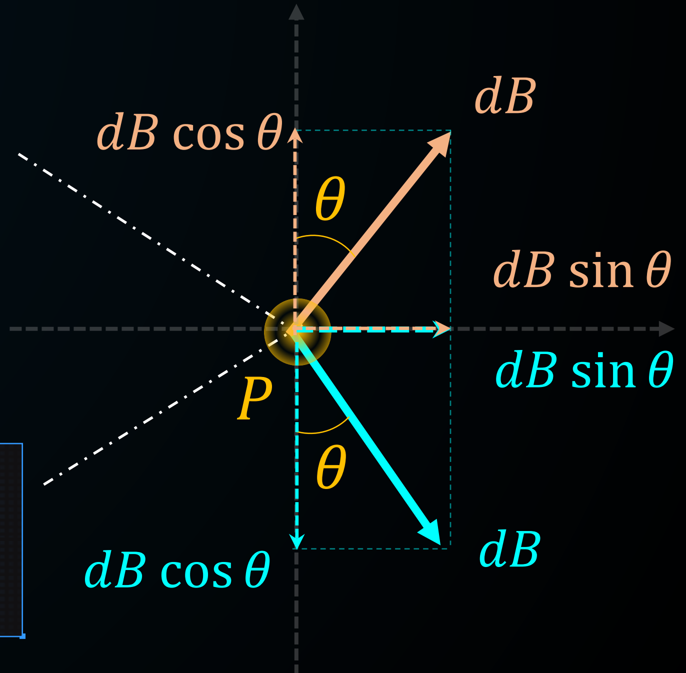
$= \frac{\mu_0 i R}{4\pi (R^2+x^2)^{3/2}} \int_0^{2\pi R} dl.$

$$\left(\because \sin\theta = \frac{R}{r} = \frac{R}{\sqrt{R^2 + r^2}}\right)$$

$$B_Y = 0$$

$B_{net} = \boxed{\frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}}$ → along the Axis - Yaad Karlo

$$B_{net} = B_X = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$$

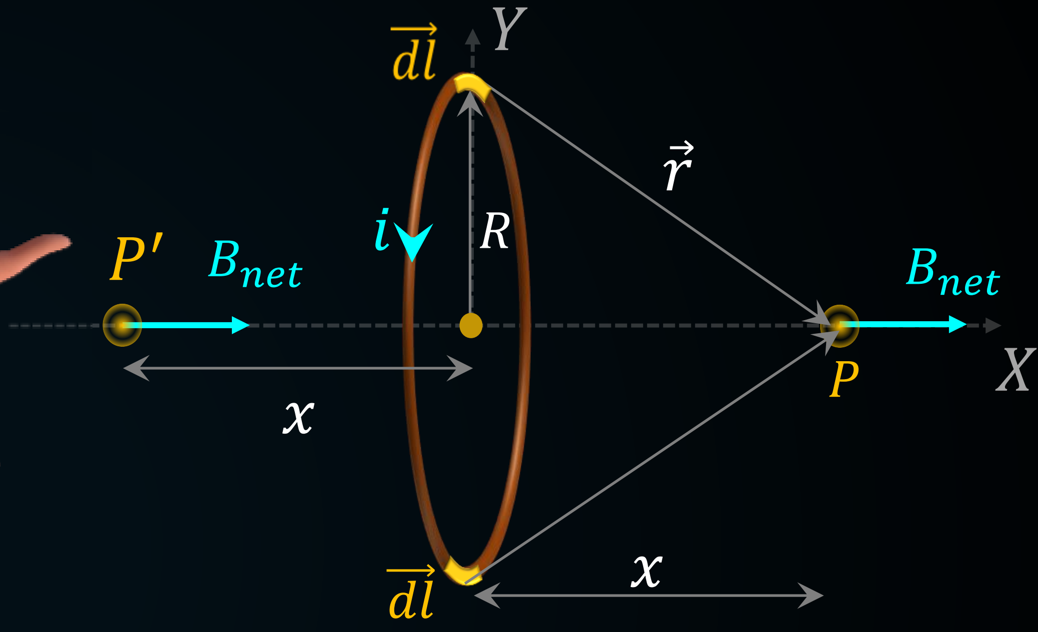


MAGNETIC FIELD ON THE AXIS OF A CURRENT CARRYING RING

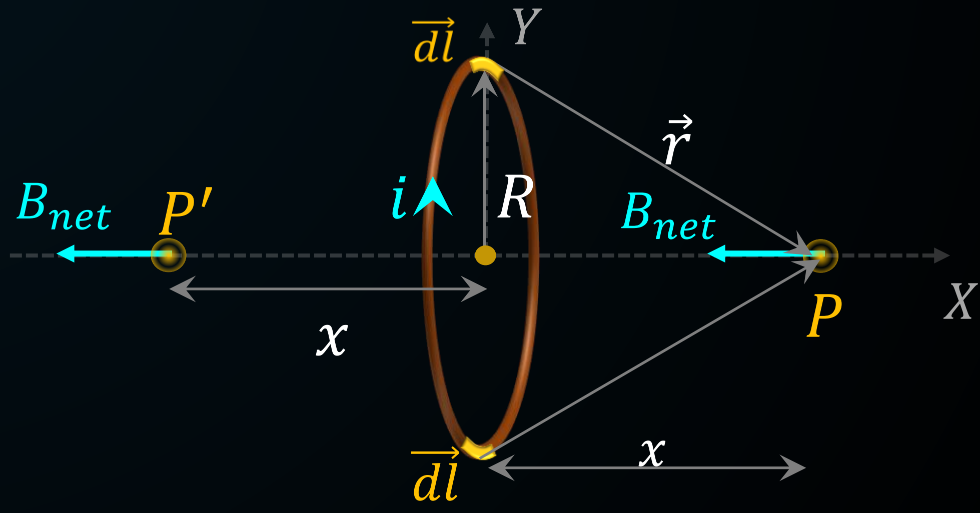
Direction of magnetic field

Right hand thumb rule

- Curl fingers of the right hand in the direction of the current.
- Thumb will give the direction of the magnetic field.



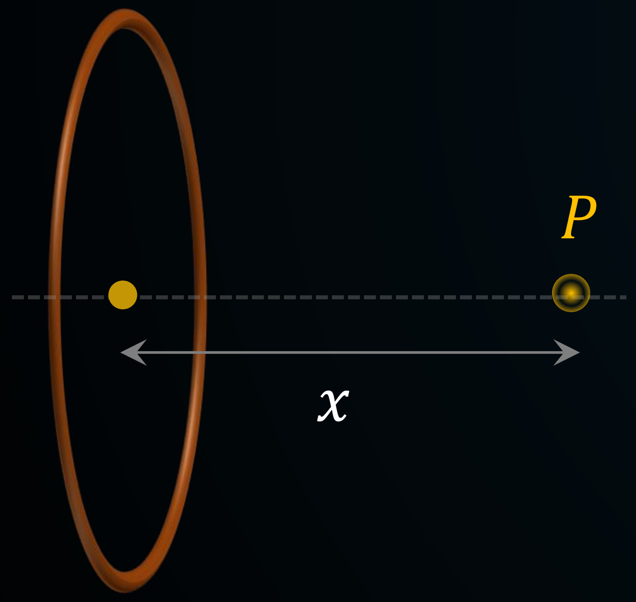
- The direction of the magnetic field remains same on both side of the ring until the direction of the current changes.
- The direction of the magnetic field changes as the direction of the current flowing through the ring changes.



MAGNETIC FIELD ON THE AXIS OF A CURRENT CARRYING RING

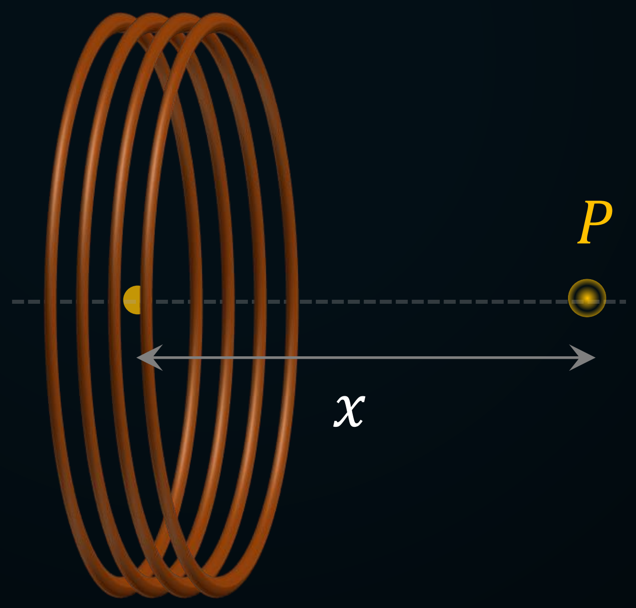
For a single loop

$$B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$$



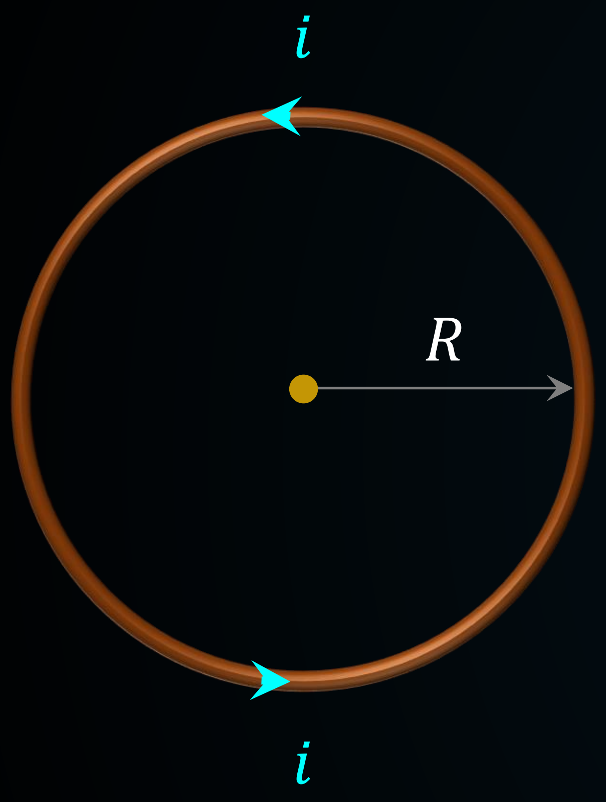
For n loops

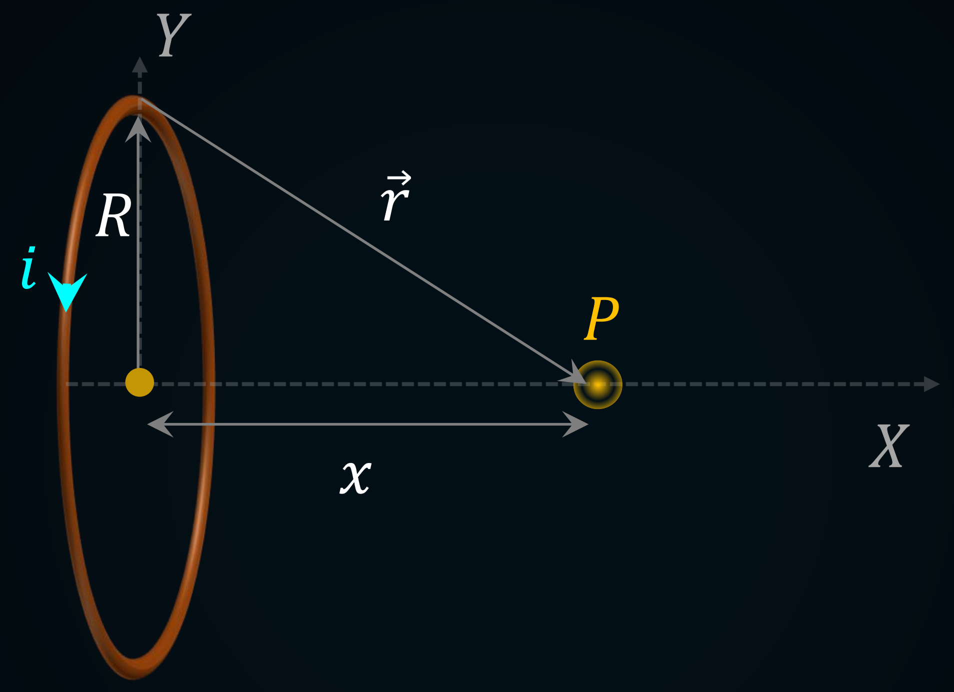
$$B = \frac{n\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$$



MAGNETIC FIELD ON THE AXIS OF A CURRENT CARRYING RING

Magnetic field at the center of the current carrying ring





$$B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$$

At centre, $x = 0$

$$B = \frac{\mu_0 i R^2}{2(R^2)^{3/2}} = \boxed{\frac{\mu_0 i}{2R}}$$

$$B_O = \frac{\mu_0}{2}\frac{i}{R}$$

MAGNETIC FIELD ON THE AXIS OF A CURRENT CARRYING RING

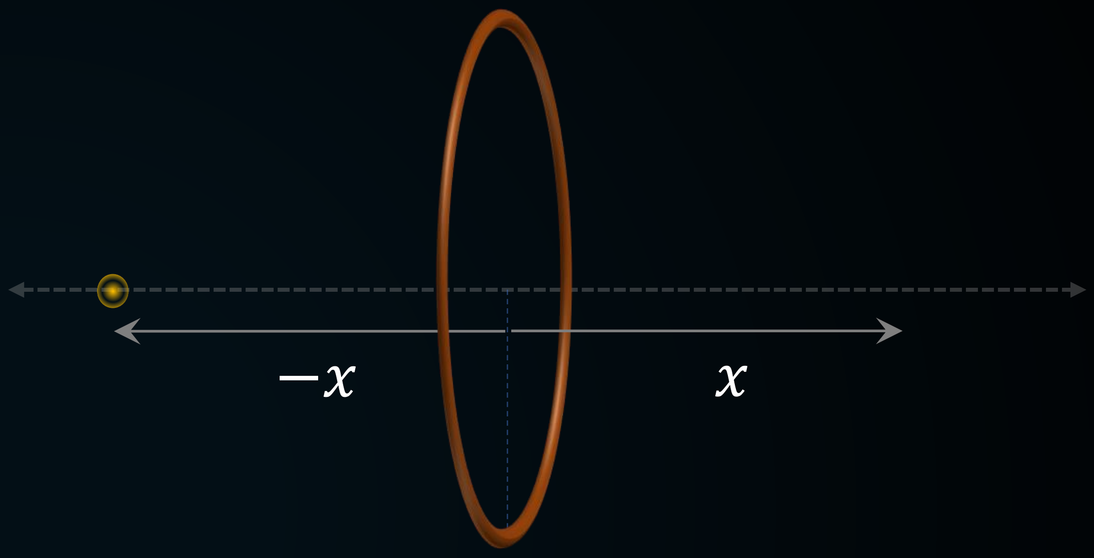
B vs x plot

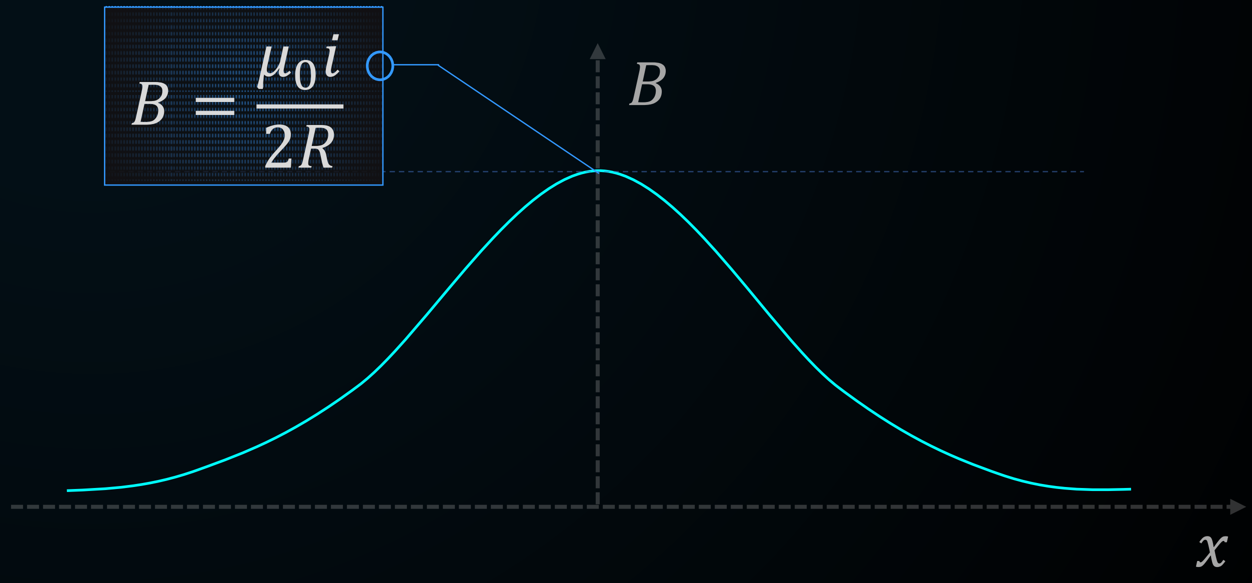
$$B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$$

- As x increases, the magnetic field decreases.
- At the centre of the ring, the magnitude of the magnetic field is maximum.

$$B = \frac{\mu_0 i}{2R} = B_{max}$$

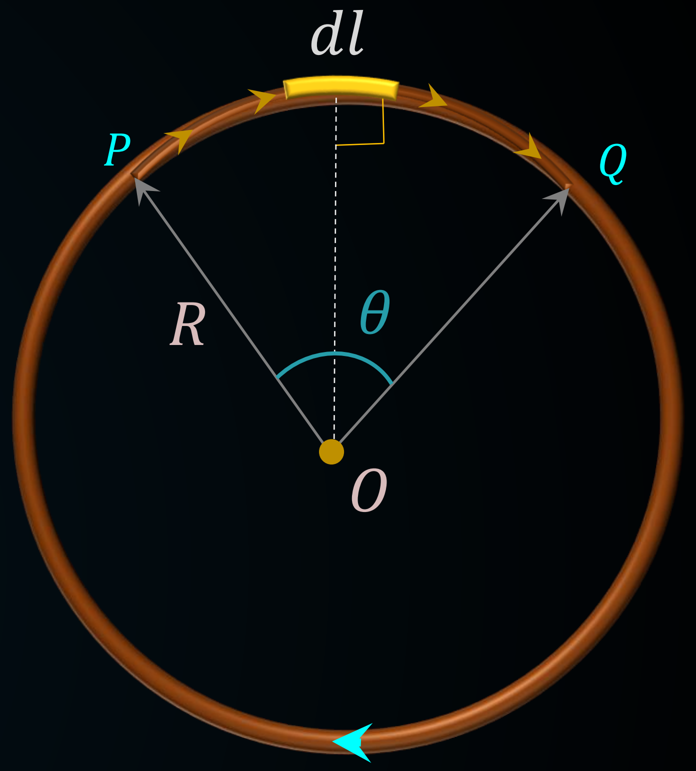
- At $x \to \infty$, the magnitude of the magnetic field due to the ring is zero.





MAGNETIC FIELD AT THE CENTRE OF A CIRCULAR ARC

- Suppose we want to find the magnetic field at the centre of the circular arc PQ having radius R. Let the angle subtended by the arc at O be θ.
- Consider a small length element dl. The angle between $d\vec{l}$ and $\vec{R}$ is 90° at each point on the arc.
- Therefore, the magnetic field at O due to the element dl is given by,



$$dB = \frac{\mu_0 i\, dl\, R \sin 90}{4\pi R^{3}}$$

$$dB = \frac{\mu_0 i}{4\pi R^2}\, dl$$

$$B_{net} = \frac{\mu_0 i}{4\pi R^2}\int dl$$

$$= \frac{\mu_0 i}{4\pi R^{2}} \cdot \cancel{R}\theta = \boxed{\frac{\mu_0 i\, \theta}{4\pi R}}$$

is in radians.

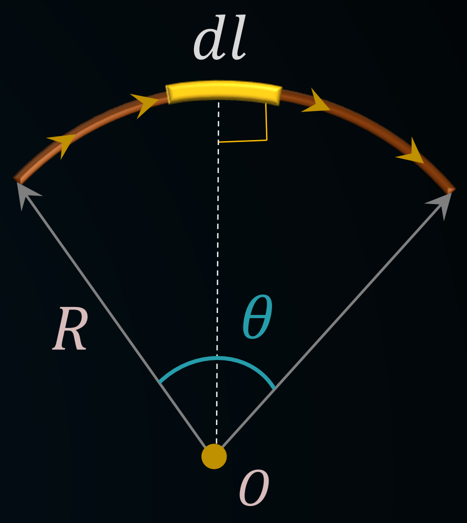
$$\int dl = R\theta$$

$$B_O = \frac{\mu_0}{4\pi}\frac{i\theta}{R}$$

- According to the direction of current, the magnetic field at O is directed into the plane of the ring i.e., "⊗".

MAGNETIC FIELD AT THE CENTRE OF A CIRCULAR ARC

General Formula: $\boxed{B_O = \frac{\mu_0}{4\pi}\frac{i\theta}{R}}$



CASE 1 : $\theta = 2\pi$ $(Circular\ loop)$

$$B_O = \frac{\mu_0}{4\pi}\frac{i(2\pi)}{R} \Rightarrow \boxed{B_O = \frac{\mu_0}{2}\frac{i}{R}}$$

CASE 2 : $\theta = \pi$ $(semi-circular\ loop)$

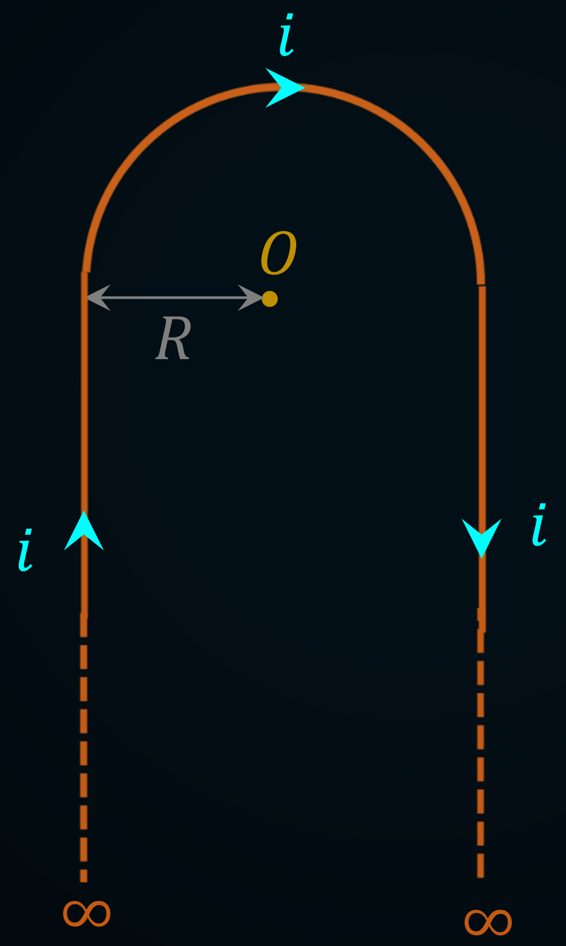
$$B_O = \frac{\mu_0}{4\pi}\frac{i(\pi)}{R} \Rightarrow \boxed{B_O = \frac{\mu_0}{4}\frac{i}{R}}$$

CASE 3 : $\theta = \pi/2$ $(quarter\ loop)$

$$B_O = \frac{\mu_0}{4\pi}\frac{i(\pi/2)}{R} \Rightarrow \boxed{B_O = \frac{\mu_0}{8}\frac{i}{R}}$$

Question Solution BOARDS

Consider the following infinite wire and find the magnetic field at point $'O'$.



Question Solution

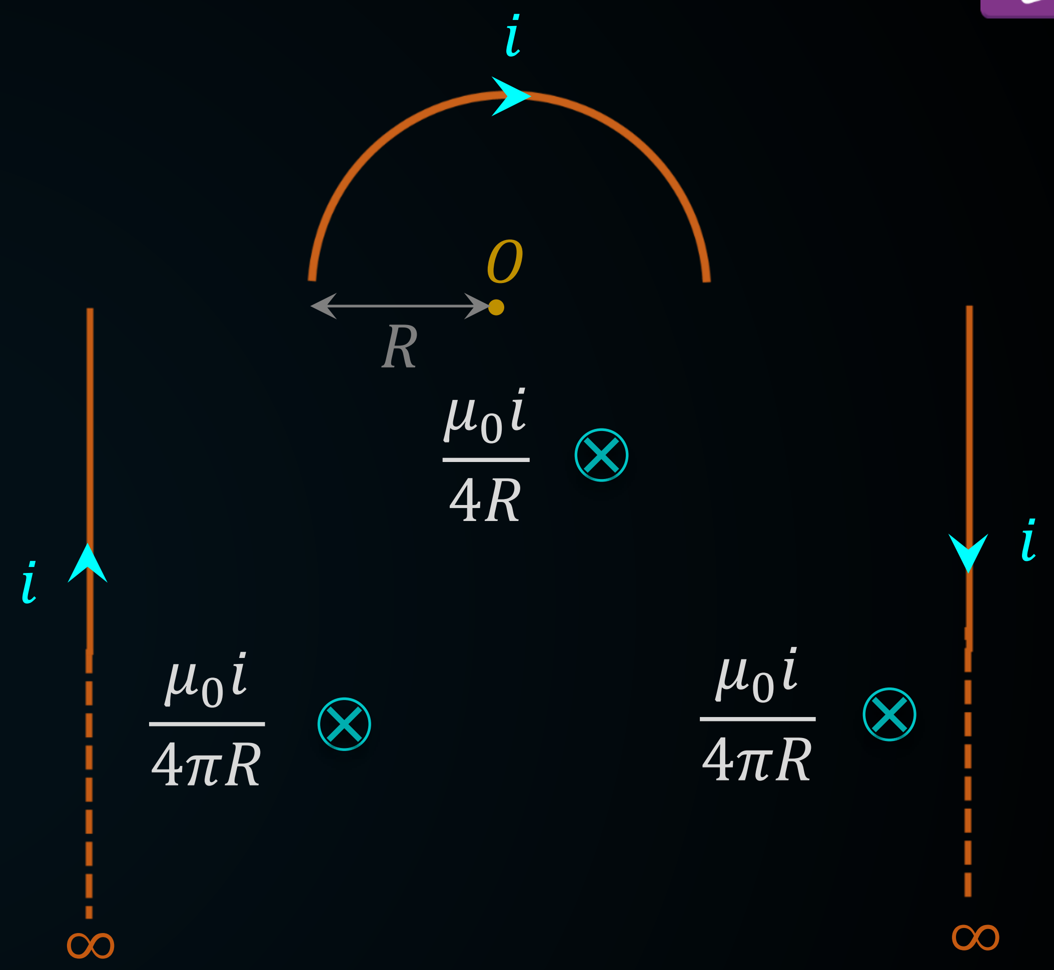
The configuration given in the figure can be splitted as shown in the figure. Here, we have two semi-infinite wire and one semi circular loop.

The magnetic field due to a semi-infinite wire is, $B_{SIW} = \frac{\mu_0 i}{4\pi R}$

The magnetic field due to a semicircular wire at its centre is,
$B_{SCW} = \frac{\mu_0 i}{4R}$

At point, the field due to all semi-infinite wire and semicircular wire is directed into the plane of the paper. Thus, the net magnetic field at point O will be,

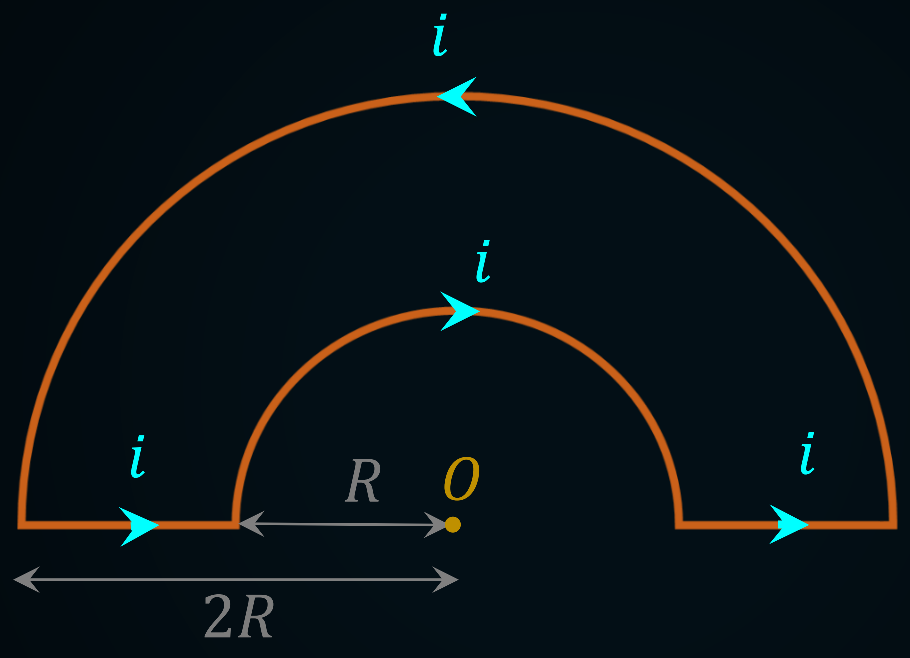
$$B = \frac{\mu_0 i}{4\pi R} \otimes + \frac{\mu_0 i}{4R} \otimes + \frac{\mu_0 i}{4\pi R} \otimes \Rightarrow B = \left(\frac{\mu_0 i}{2\pi R} + \frac{\mu_0 i}{4R}\right) \otimes$$



$$\boxed{B = \left(\frac{\mu_0 i}{2\pi R} + \frac{\mu_0 i}{4R}\right) \otimes}$$

Question Solution

Consider the following wire and find the magnetic field at point $'O'$.



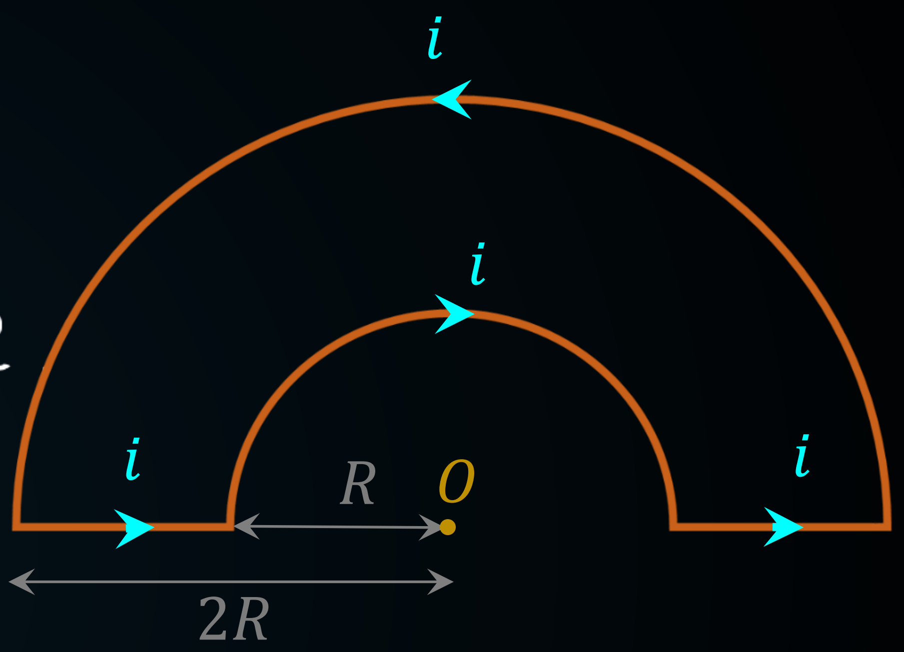
Question Solution

The configuration consists of: (1) A semicircular ring of radius $2R$, (2) A semicircular ring of radius R and these two rings are connected by conducting wires.

Since point O is lying on the line of the connecting wires, the magnetic field due to them is zero.

$B=0$

$d\vec{l} \times \vec{r} = 0$



The magnetic field due to the semicircular wire of radius R at its centre is, $B_1 = \frac{\mu_0 i}{4R} \otimes$

The magnetic field due to the semicircular wire of radius $2R$ at its centre is, $B_2 = \frac{\mu_0 i}{8R} \odot$

Since $B_1 > B_2$, the direction of net magnetic field at O will be "$\otimes$" and since the direction of B_1 and B_2 are opposite of each other, the magnitude of net magnetic field at O will be,

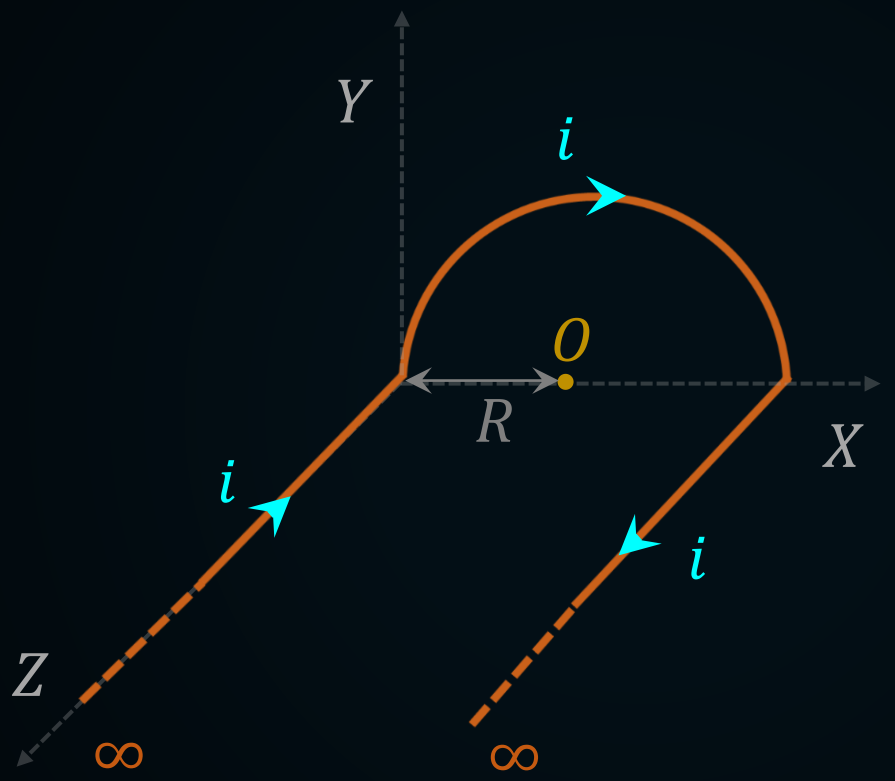
$$B = \frac{\mu_0 i}{4R} - \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}$$

$$\boxed{B = \frac{\mu_0 i}{8R} \otimes}$$

Question

Solution

Consider the following infinite wire and find the magnetic field at point $'O'$.



Question Solution

The configuration given in the figure consists of two semi-infinite wire and one semi circular loop. The semicircular loop is in XY plane.

The magnetic field at point O due to a semi-infinite wire is,

$\vec{B}_{SIW} = \frac{\mu_0 i}{4\pi R}(-\hat{j})$

The magnetic field due to a semicircular wire at its centre is,

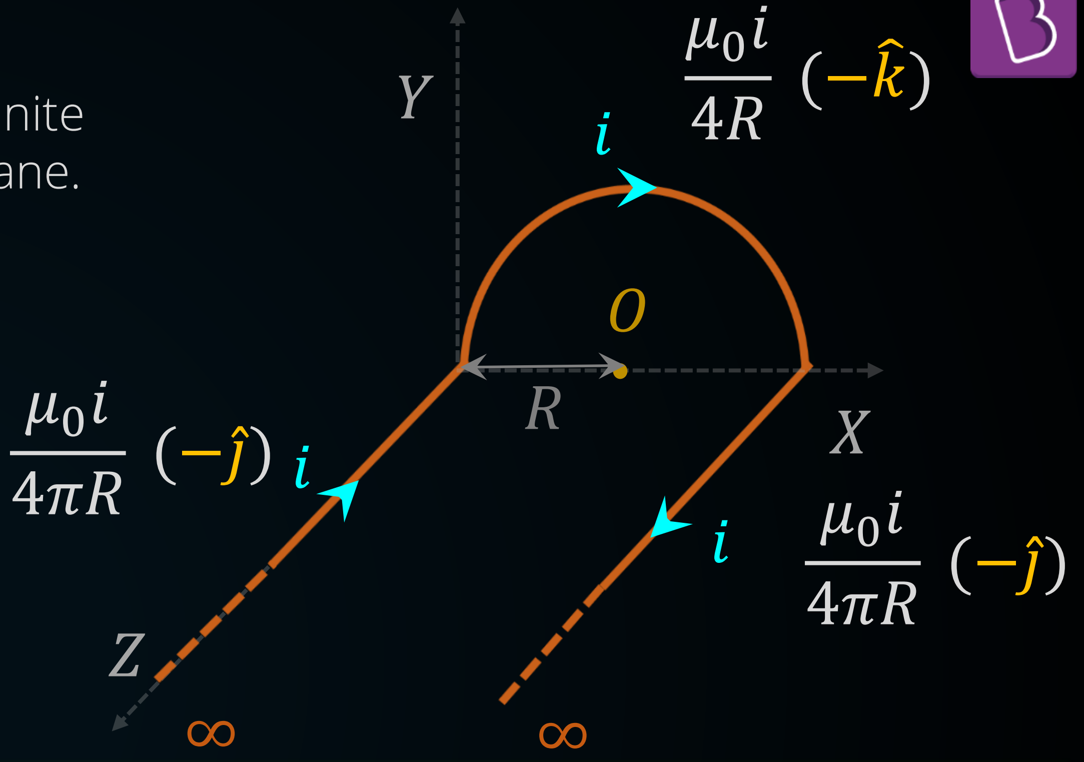
$\vec{B}_{SCW} = \frac{\mu_0 i}{4R}(-\hat{k})$

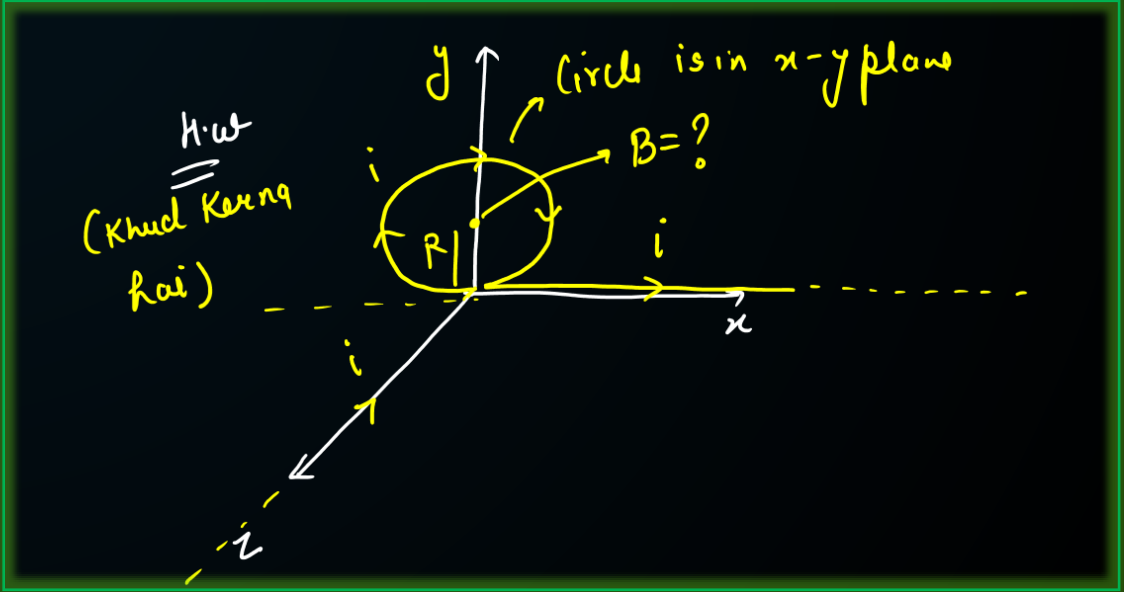
The net magnetic field at point O will be,

$$B = \frac{2 \times \mu_0 i}{4\pi R}(-\hat{j}) + \frac{\mu_0 i}{4R}(-\hat{k})$$

$$B = \frac{\mu_0 i}{2\pi R}(-\hat{j}) + \frac{\mu_0 i}{4R}(-\hat{k})$$

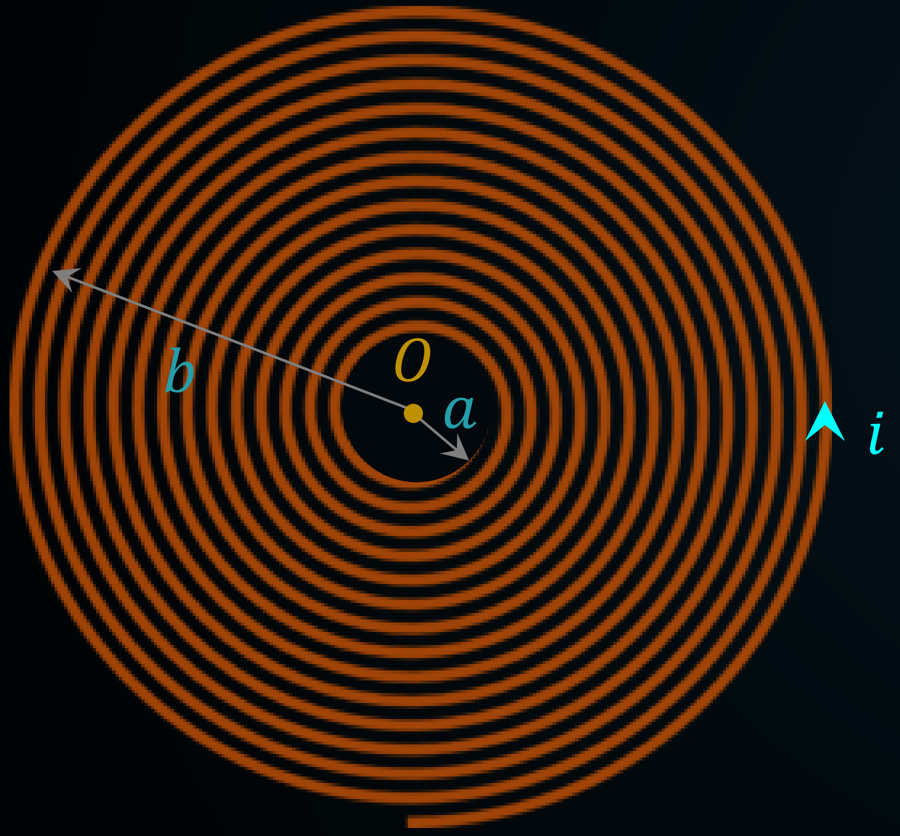
$$\boxed{B = \frac{\mu_0 i}{2\pi R}(-\hat{j}) + \frac{\mu_0 i}{4R}(-\hat{k})}$$





Question Solution

A long, insulated wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. A steady current i flows through the wire. Find the magnetic field at the center of the spiral.



Question Solution NEET

Consider an elementary ring of thickness dx and radius x. Therefore, total number of turns in thickness dx is $\frac{N}{(b-a)}dx$.

We know that the magnetic field at the centre of a current carrying ring of radius R and having N number of turns is, $B = \frac{\mu_0 N i}{2R}$

Therefore, the magnetic field at the centre of the elementary ring is,

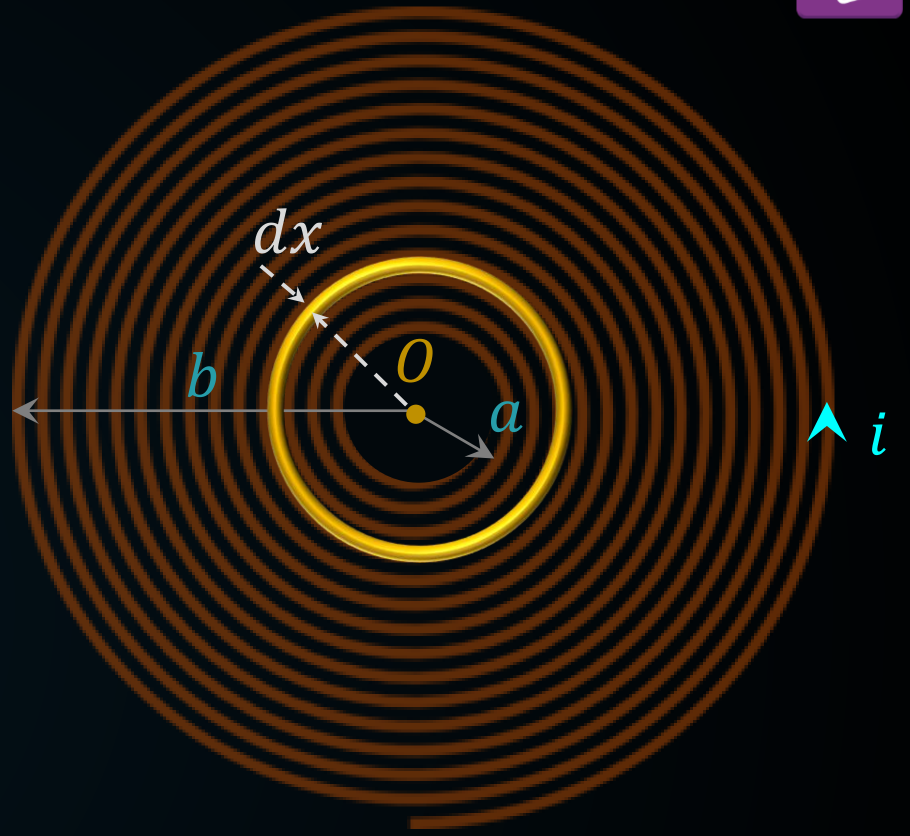
$$dB = \frac{\mu_0 i}{2x} \times \frac{N\,dx}{(b-a)}$$

Therefore, the net magnetic field due to the whole wire at the centre is given by,

$$B_{net} = \frac{\mu_0 N i}{2(b-a)}\int_a^b \frac{dx}{x} = \boxed{\frac{\mu_0 N i}{2(b-a)} \ln\left(b/a\right)}$$

The direction of current in the spiral wire suggests that the magnetic field will be coming out from the plane of the wire.

$$B = \frac{\mu_0 N i}{2(b-a)} \ln\left(\frac{b}{a}\right) \odot$$



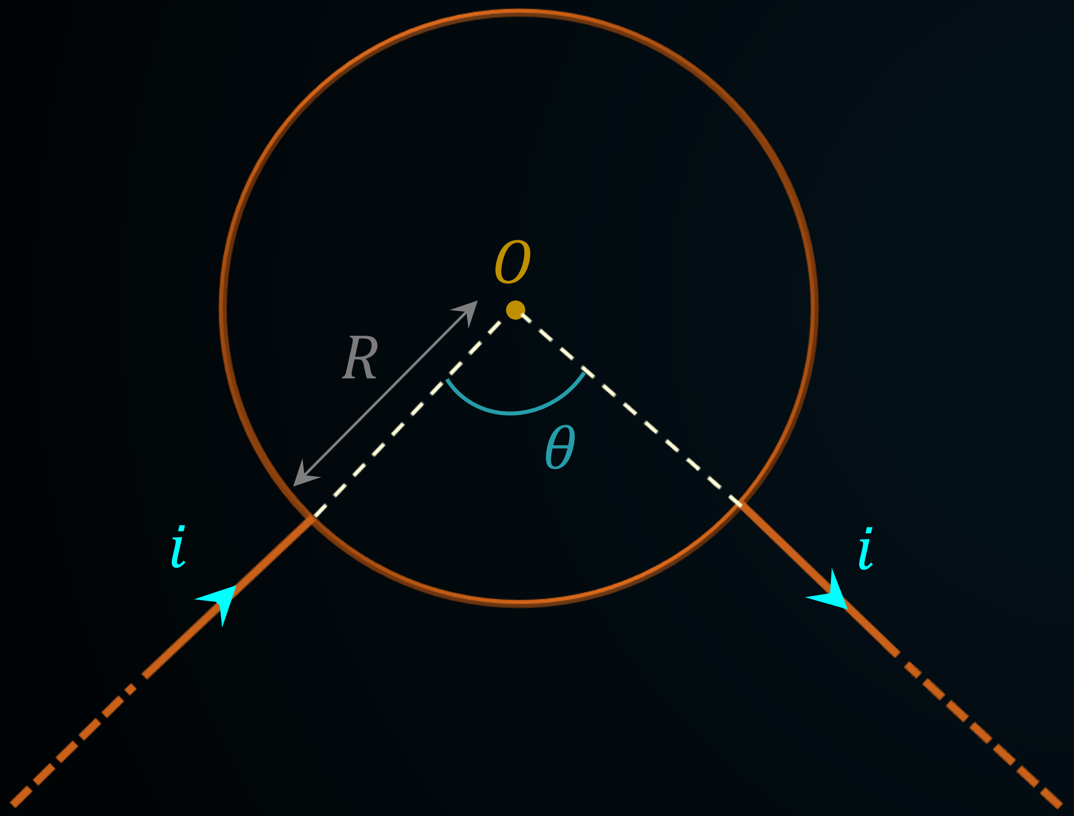
$b-a$ width me $\longrightarrow$ N turns.

1 width $\longrightarrow$ $\frac{N}{b-a}$ turns.

dx width $\longrightarrow$ $\frac{N}{(b-a)} \cdot dx$ turns.

Question Solution

A uniform circular ring and two infinite wires are connected as shown. Find the magnetic field at the center of the uniform ring.



Question | **Solution** | NEET

Since point O is lying on the line of the of the straight wire, the magnetic field at O due to the straight wire will be zero.

Let the resistance per unit length of the wire is $\lambda\ \Omega/m$.

The resistance in the lower part of the circle is, $R_1 = \lambda R\theta$

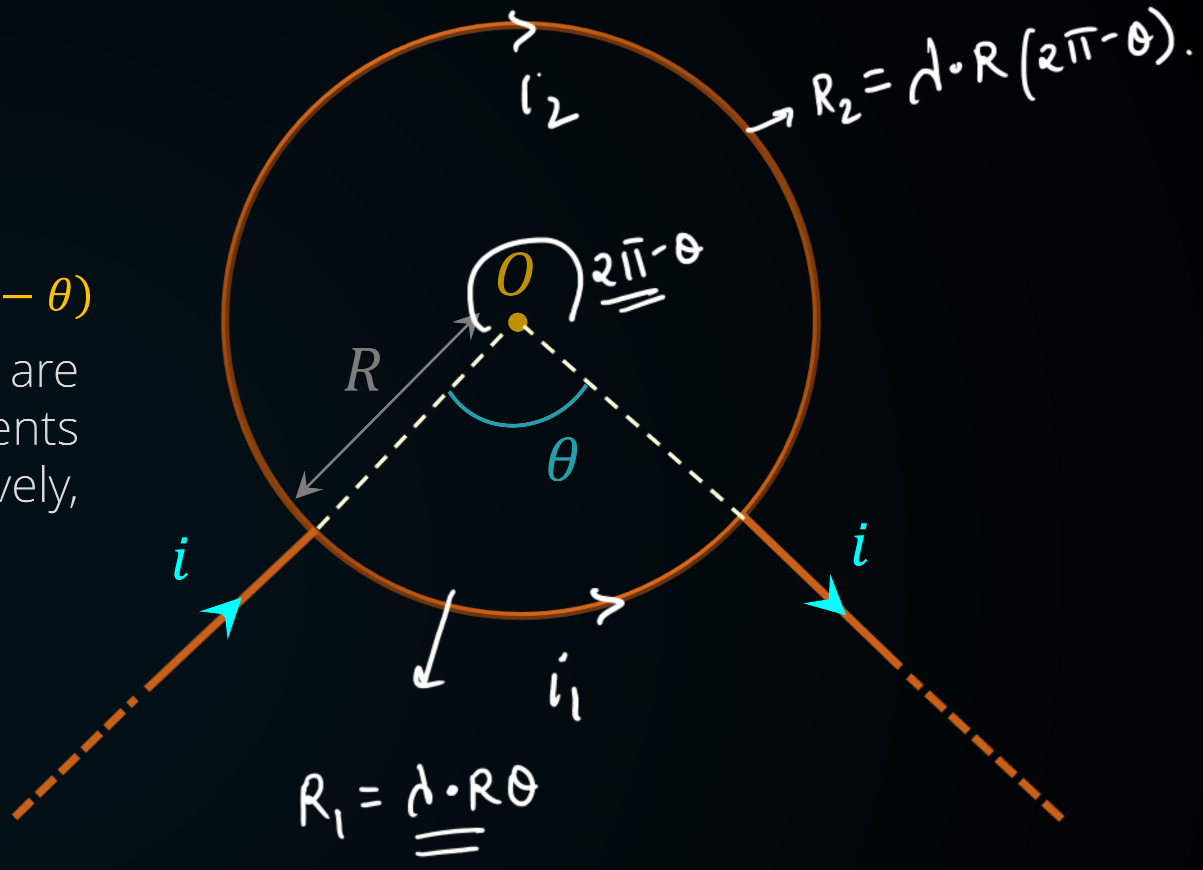
The resistance in the upper part of the circle is, $R_2 = \lambda R(2\pi - \theta)$

From the configuration shown in the figure, the resistances are parallel to each other. Thus, if i_1 and i_2 are the currents through the lower part and upper part of the ring, respectively, then,

$$i_1 : i_2 = \frac{1}{R_1} : \frac{1}{R_2} = \frac{1}{\lambda R\theta} : \frac{1}{\lambda R(2\pi - \theta)}$$

$$i_1 : i_2 = (2\pi - \theta) : (\theta)$$

$$i_1 = \frac{2\pi - \theta}{2\pi} \cdot i \qquad i_2 = \frac{\theta}{2\pi} \cdot i$$



Question | Solution

The magnetic field at O due to the lower part of the ring is,

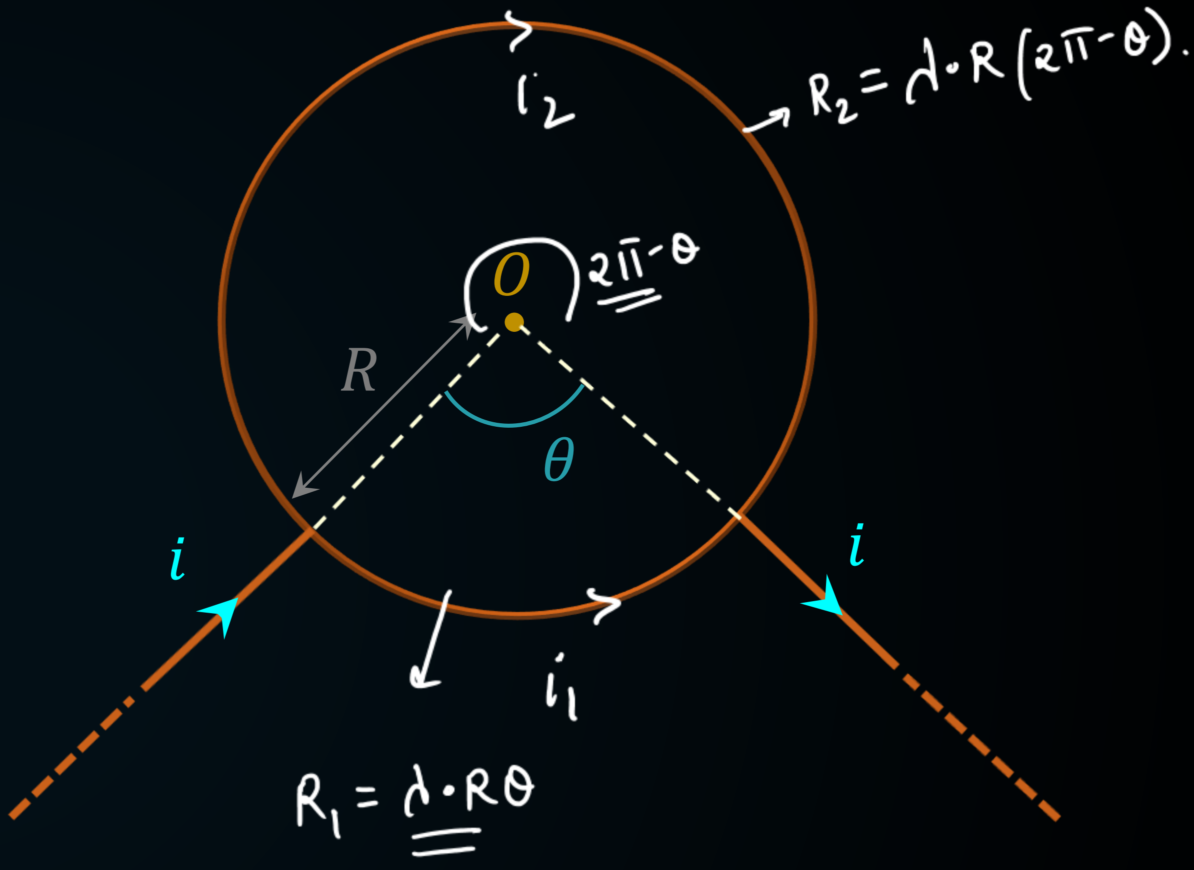
$$B_{i_1} = \frac{\mu_0}{4\pi}\frac{i_1\theta}{R} \odot$$

$$B_{i_1} = \frac{\mu_0 (2\pi-\theta)\, i^{\circ}\, \theta}{8\pi^2 \cdot R} \odot$$

The magnetic field at O due to the upper part of the ring is,

$$B_{i_2} = \frac{\mu_0 i_2}{4\pi R} \cdot (2\pi-\theta) \otimes$$

$$= \frac{\mu_0\, \theta \cdot (2\pi-\theta)\, i^{\circ}}{8\pi^2 R} \otimes$$



Therefore, the net magnetic field at point O is given by,

$$\boxed{B_{net} = B_0 = 0}$$ independent of θ

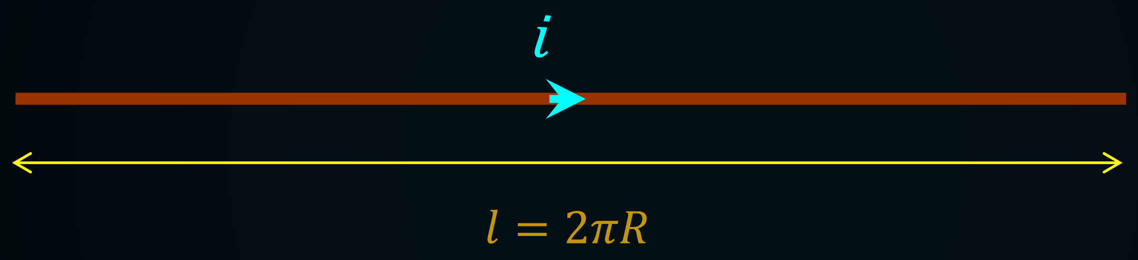
$$B_{net} = 0$$

Question Solution

Suppose a current carrying straight wire of length $l = 2\pi R$ is converted into a n-sided polygon. Find

(a) The magnetic field at the centre of the polygon.

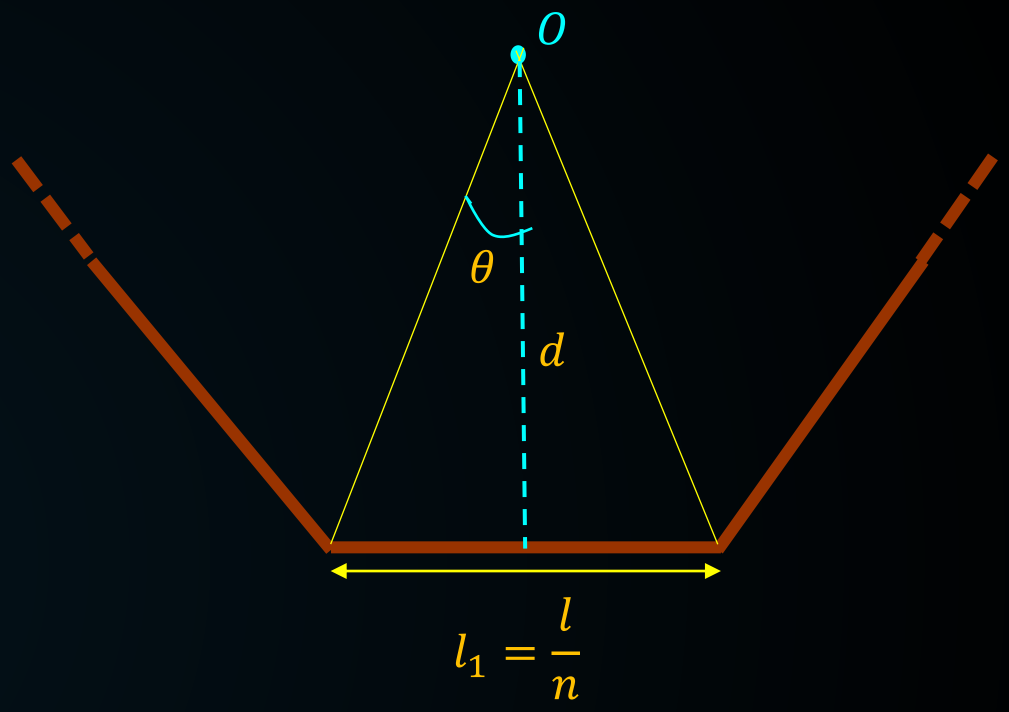
(b) The magnetic field at the centre of the polygon if $n \to \infty$.



The magnetic field at O due to n-sided polygon is, $B_{net} = n \times B_1$

Where B_1 is the magnetic field due to one side of the polygon

For one side of the polygon



Length, $l_1 = \frac{l}{n} = \frac{2\pi R}{n}$

Total angle subtended at $O = \frac{2\pi}{n}$. Therefore, half-angle is, $\theta = \frac{\pi}{n}$

Therefore, $\tan\theta = \frac{\left({l_1}/{2}\right)}{d} = \frac{\pi R}{nd} \Rightarrow d = \frac{\pi R}{n}\cot\theta \Rightarrow d = \frac{\pi R}{n}\cot\frac{\pi}{n}$

The net magnetic field at point O due to one side of the polygon is,

$$B_1 = \frac{\mu_0 i}{4\pi d}[2\sin\theta]$$

Therefore, the net magnetic field at point O due to whole polygon is,

$$B_{net} = \frac{\mu_0 n i}{4\pi\left[\frac{\pi R}{n}\cot\frac{\pi}{n}\right]}\left[2\sin\frac{\pi}{n}\right] \Rightarrow B_{net} = \frac{\mu_0 i}{2R}\left[\frac{\tan\frac{\pi}{n}}{\frac{\pi}{n}}\right]\left[\frac{\sin\frac{\pi}{n}}{\frac{\pi}{n}}\right]$$

Question Solution

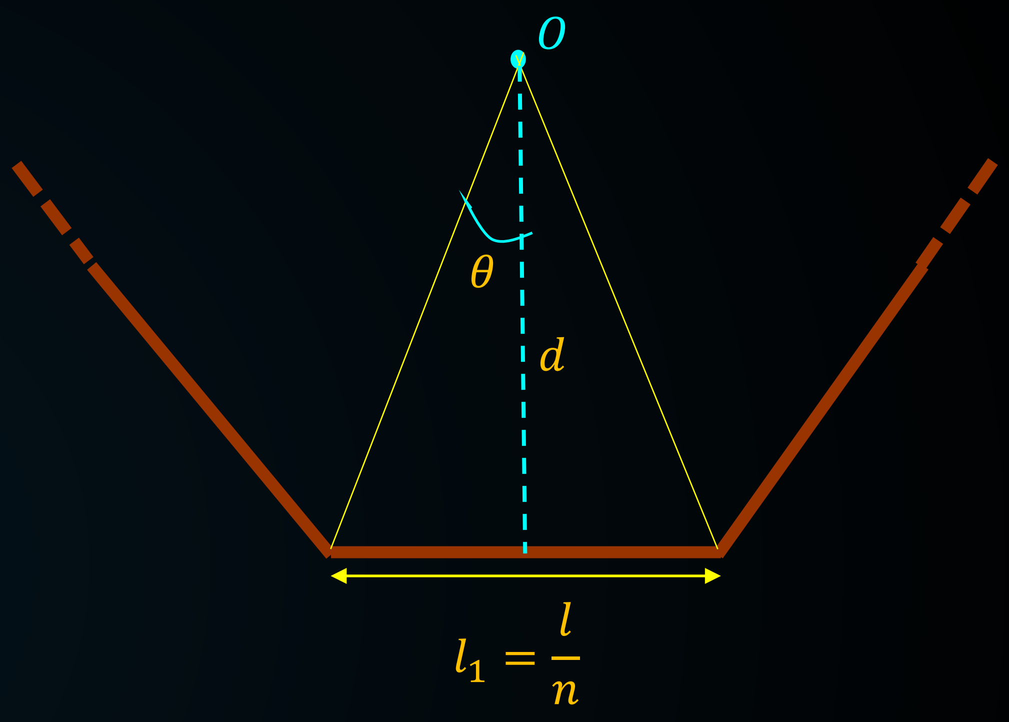
We know that: $\lim_{x\to 0}\left[\frac{\tan x}{x}\right]=1$ and $\lim_{x\to 0}\left[\frac{\sin x}{x}\right]=1$

Let $x=\frac{\pi}{n}$. Thus, for "$n\to\infty$", "$x\to 0$"

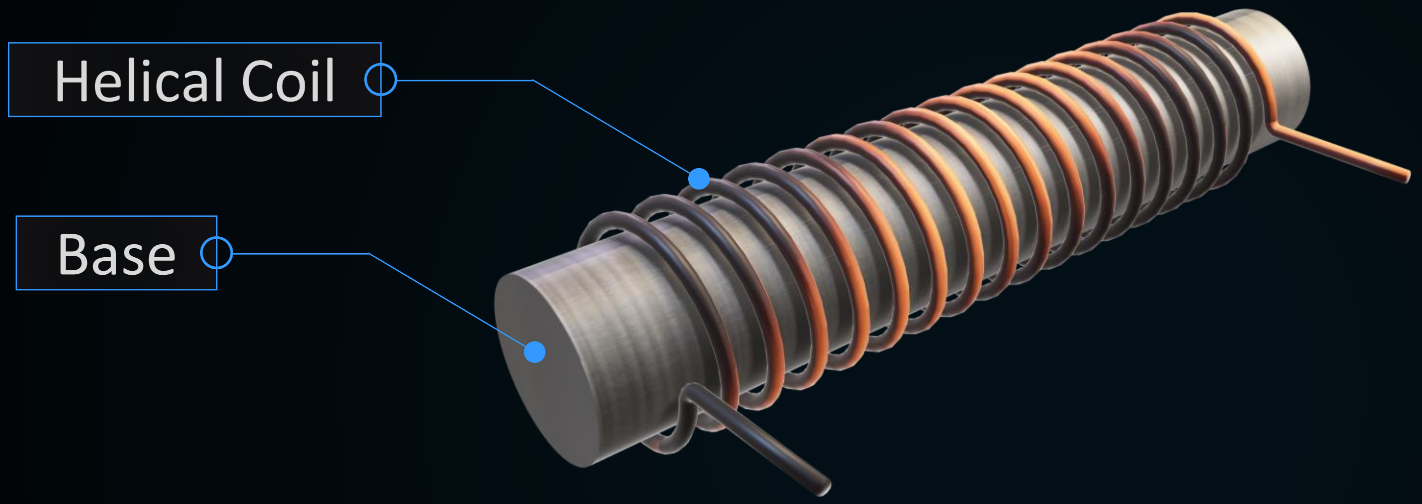
Therefore, for "$n\to\infty$", the expression of magnetic field becomes:

$$B_{net}=\frac{\mu_0 i}{2R}$$

Hence, the n-sided polygon becomes a circle for "$n\to\infty$".



SOLENOID

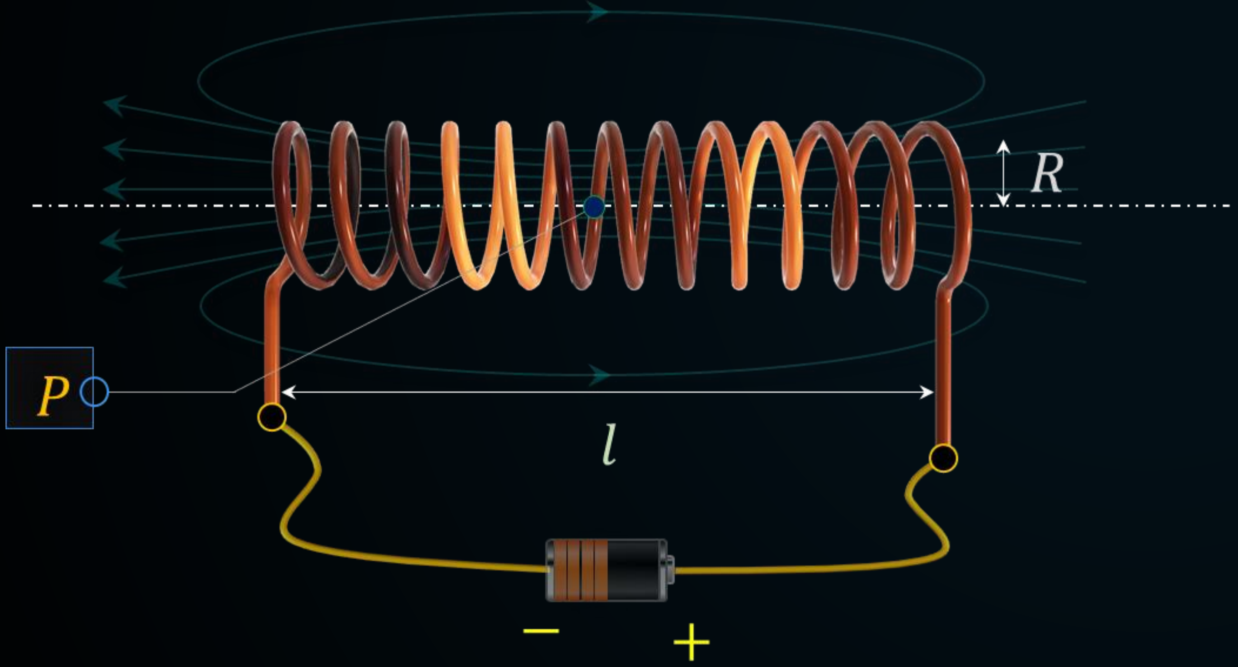


- A solenoid is a type of electromagnet.
- It is used to generate a controlled magnetic field through a coil wound into a tightly packed helix.

SOLENOID

Magnetic field at point P present on the axis of the solenoid

- We can assume a spring or helix as a configuration of solenoid provided the pitch of the helix is infinitesimal.
- The solenoid can also be assumed as composition of infinite number of rings provided the distance between each ring is very small.



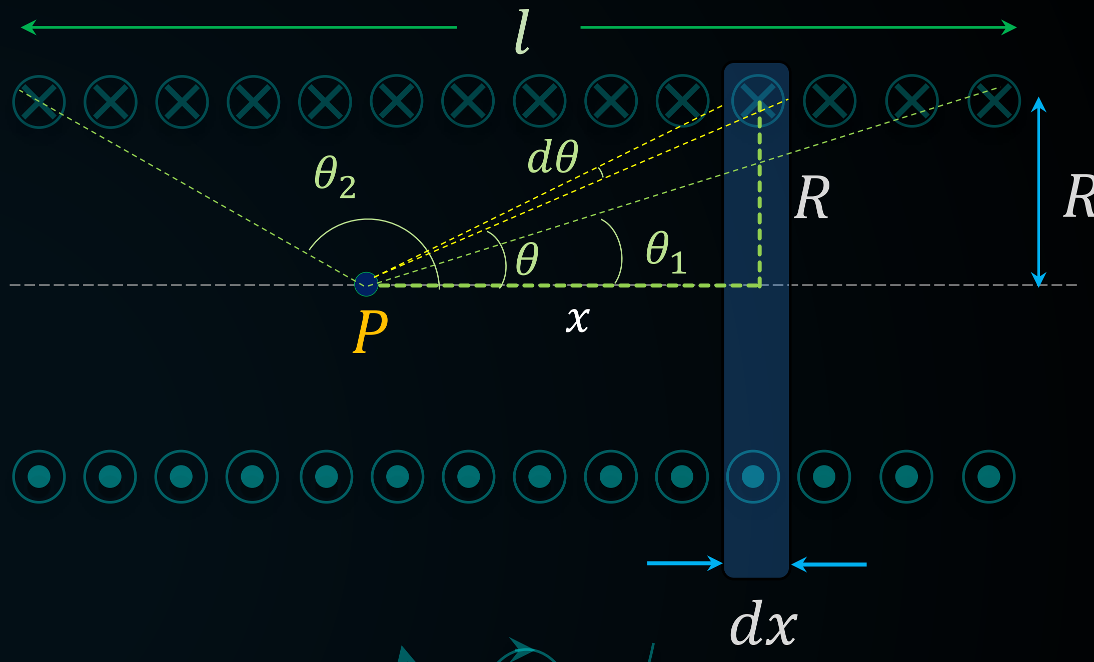
$$n = \frac{N}{l}$$

$n = no.\,of\ turns\ per\ unit\ length$

$N = total\ turns$

- The magnetic field inside the solenoid is nearly uniform and the field outside the solenoid is negligible.

Magnetic field at point P present on the axis of the solenoid



Consider a ring element of the solenoid of thickness dx at distance x from the point P.
Let the angles made by extreme points of solenoid with axis at point P be θ_1 and θ_2 and the angle made by ring element be $d\theta$.

Number of turns of wire in ring element,

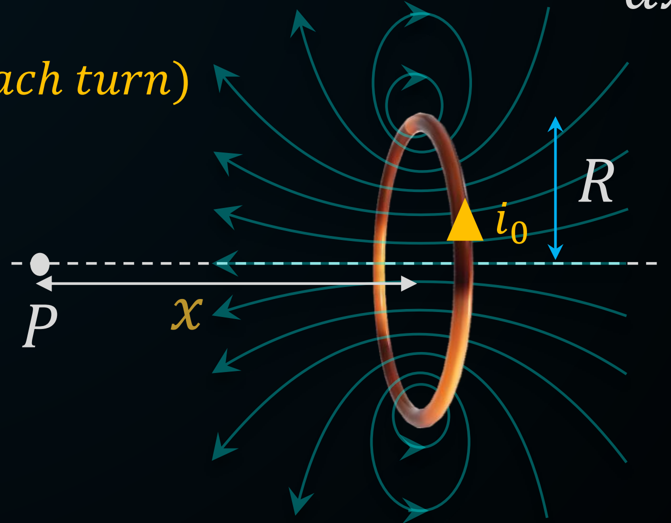
$$n_{ring} = \frac{N}{l} \times dx = ndx$$

$$\boxed{n = \frac{N}{l}}$$

Therefore, total current in the ring element is,

$i_0 = (Number\ of\ turns\ in\ the\ element) \times (Current\ through\ each\ turn)$

$i_0 = ndx \times i$ $\qquad$ $i =$ Current flowing through solenoid



We know the magnetic field due to ring at an axial point, so for ring element ;

$$dB = \frac{\mu_0 i_0 R^2}{2(R^2 + x^2)^{\frac{3}{2}}} = \frac{\mu_0 ni(dx)R^2}{2(R^2 + x^2)^{\frac{3}{2}}}$$

SOLENOID

Magnetic field at point P present on the axis of the solenoid

$$dB = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}} . n dx \ldots (1)$$

From figure,

$$\tan\theta = \frac{R}{x} \quad \Rightarrow x = R\cot\theta$$

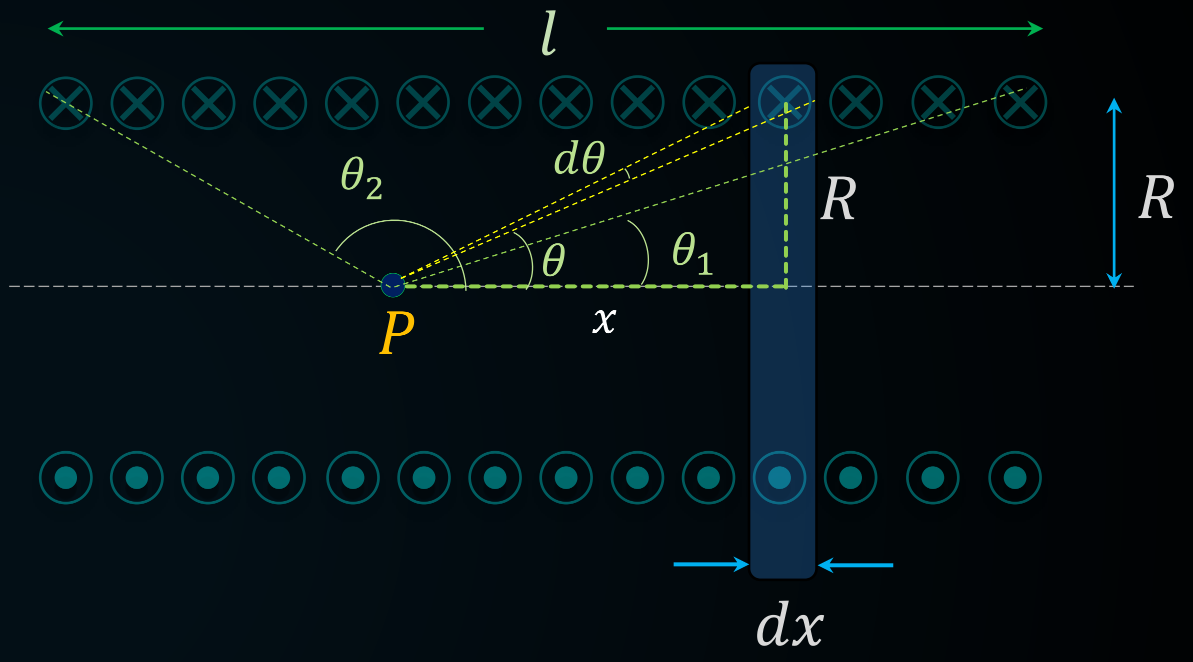
Differentiating, $dx = R(-\,cosec^2\theta\, d\theta)$

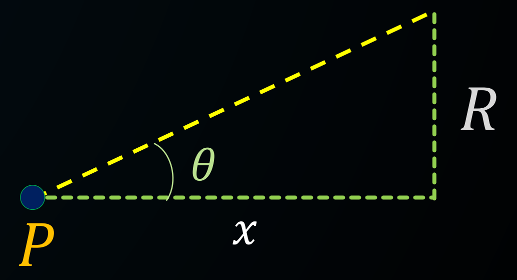
Substituting the value of x and dx in equation (1),

$$dB = \frac{\mu_0 i R^2}{2(R^2 + (R\cot\theta)^2)^{\frac{3}{2}}} . n(-R\, cosec^2\theta\, d\theta)$$

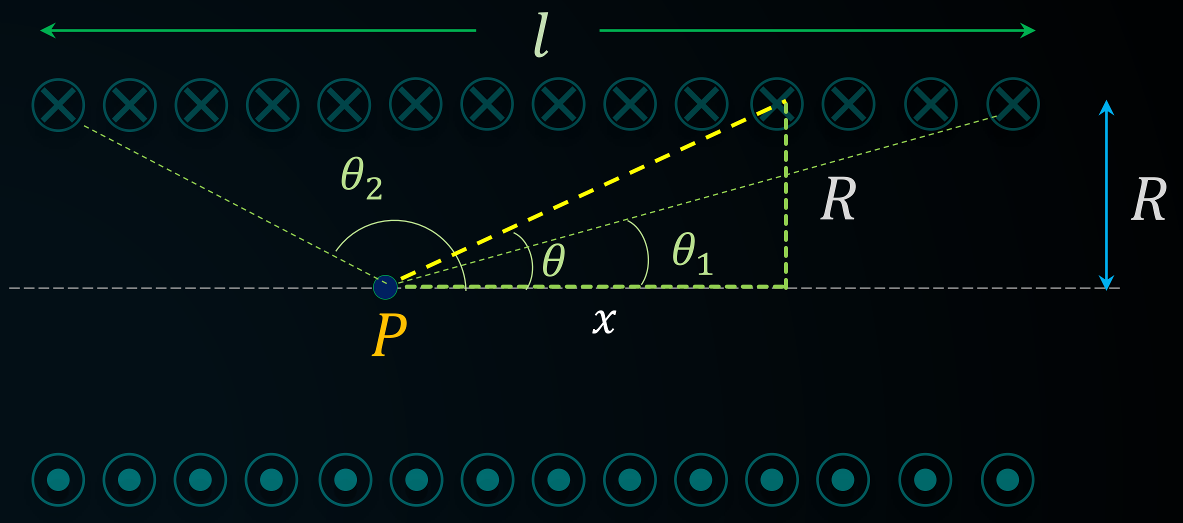
$$\Rightarrow dB = \frac{\mu_0 i R^2}{2R^3(cosec^2\theta)^{\frac{3}{2}}} . n(-R\, cosec^2\theta\, d\theta) \quad \because (1 + \cot^2\theta = cosec^2\theta)$$

$$\Rightarrow dB = -\frac{\mu_0 n i \sin\theta\, d\theta}{2}$$





Magnetic field at point P present on the axis of the solenoid



$$dB = -\frac{\mu_0 ni \sin\theta \, d\theta}{2}$$

Integrating for net magnetic field:

$$B_{net} = \int dB = -\frac{\mu_0 ni}{2} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$B_{net} = -\frac{\mu_0 ni}{2} [-\cos\theta]_{\theta_1}^{\theta_2}$$

$$B_{net} = \frac{\mu_0 ni}{2} [\cos\theta_2 - \cos\theta_1]$$

$$\boxed{B = \frac{\mu_0 ni}{2} [\cos\theta_2 - \cos\theta_1]}$$

This formula will give magnitude of magnetic field and the direction can be found by using Right hand thumb Rule.

Examples: Determining value of θ_1 and θ_2

$\theta_1 = 30°$

$\theta_2 = 120°$

$$B = \frac{\mu_0 n i}{2}[\cos\theta_2 - \cos\theta_1]$$

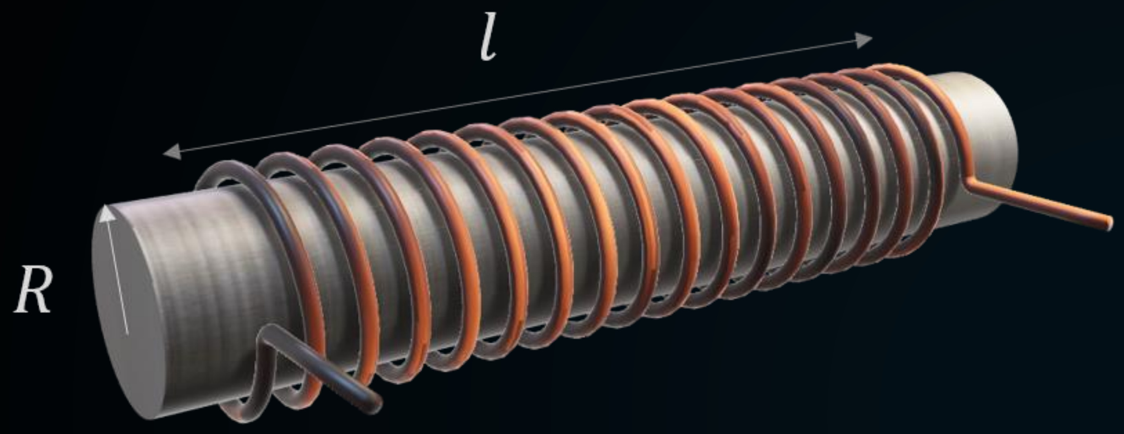
The angles θ_1 and θ_2 should be taken from same side.

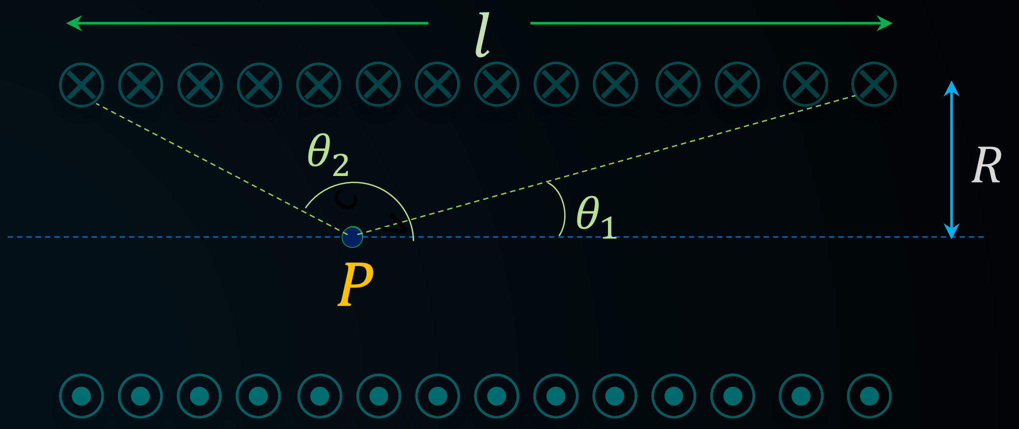
$\theta_1 = 150°$

$\theta_2 = 120°$

IDEAL SOLENOID

- A solenoid can be called as an ideal solenoid if the following conditions are satisfied:





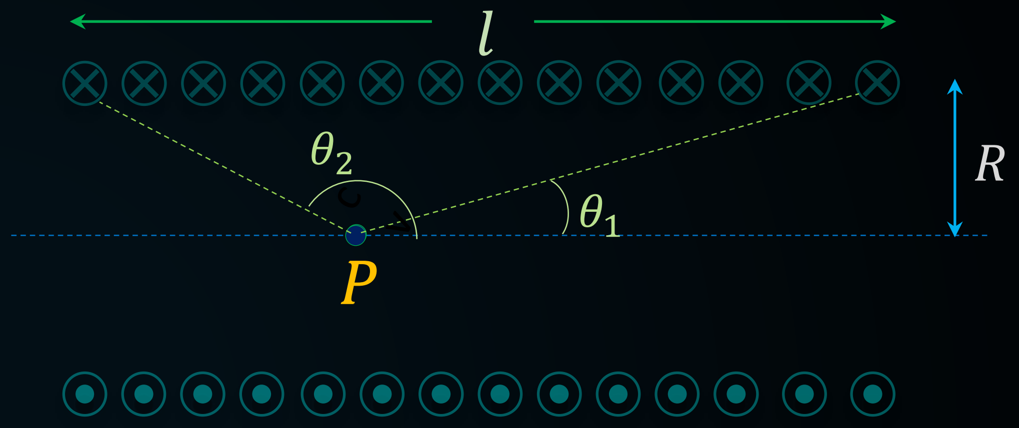
- $l \gg R \Rightarrow$ long solenoid
- $n = N/l$ is a very large number
  $\Rightarrow$ wire is very closely wound.

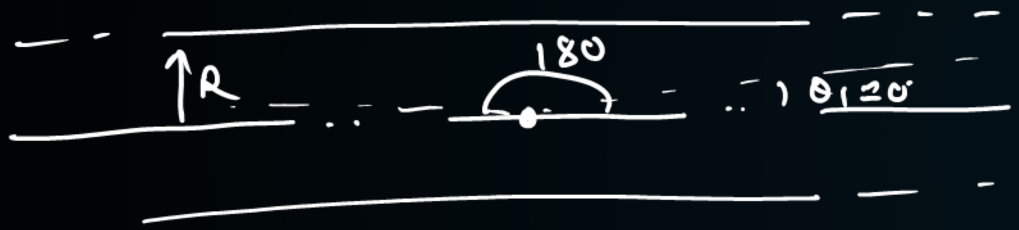
- For ideal solenoid (length: infinite, radius: small, winding of wires: very tight), the field inside the solenoid is uniform and the field outside the solenoid is zero.

IDEAL SOLENOID

Magnetic field due to an Ideal solenoid ($l >> R$)

$$B = \frac{\mu_0 ni}{2}[\cos\theta_2 - \cos\theta_1]$$





Practical $\boxed{\{l >> R\}}$ $l = 10$ m $R = 1$ mm.

$\theta_1 \simeq 0°$

$\theta_2 \simeq 180°$

act as a ideal solenoid

$\boxed{B = \mu_0 ni}$

For ideal solenoid,

$l = \infty$ $\quad \theta_1 \to 0°$

$\theta_2 \to 180°$

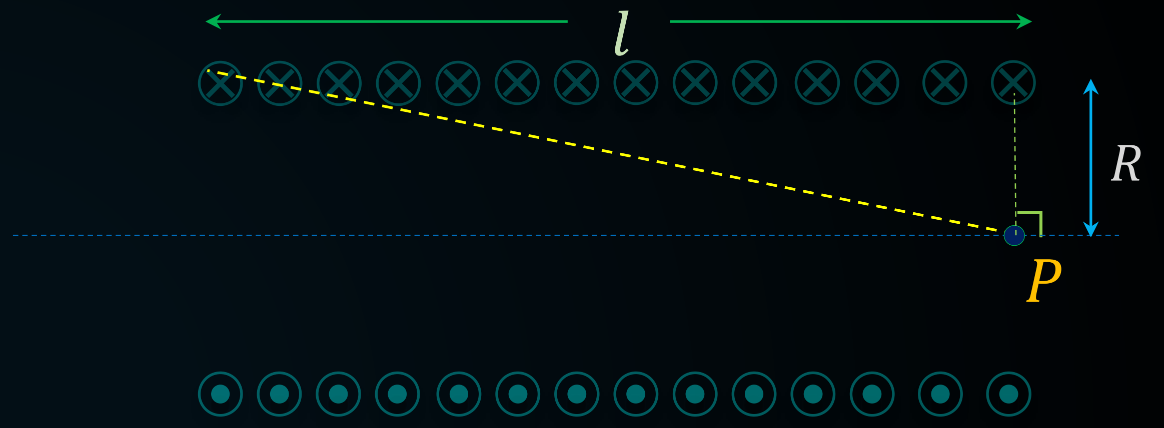
$B = \frac{\mu_0 ni}{2}[\cos 180° - \cos 0°]$

$= \boxed{\mu_0 ni}$ → Yaad Rakhna.

$$\boxed{B = \mu_0 ni}$$

Magnetic field at one end of an ideal solenoid

$$B = \frac{\mu_0 n i}{2}[\cos\theta_2 - \cos\theta_1]$$



$\theta_1 \approx 90°$ and $\theta_2 \approx 180°$

$$B = \frac{\mu_0 n i}{2}[\cos 180 - \underset{0}{\cos 90}]$$

$$= \boxed{\frac{\mu_0 n i}{2}} \text{ — Magnitude}$$

$$B = \frac{\mu_0 n i}{2}$$

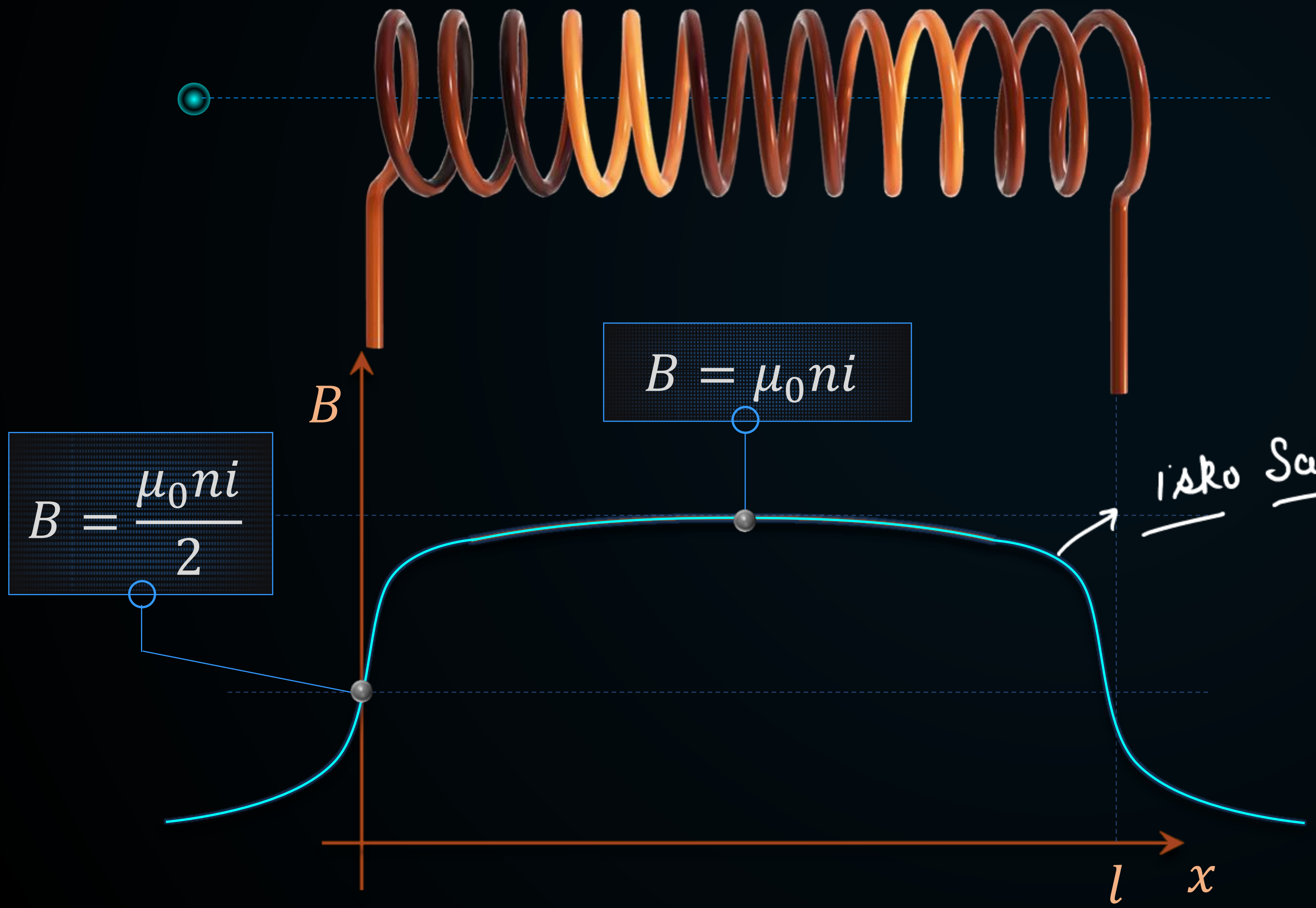
IDEAL SOLENOID

B vs x plot

$B=0$

$B=\mu_0 n i = \text{Constant}$

$$B = \frac{\mu_0 n i}{2}[\cos\theta_2 - \cos\theta_1]$$

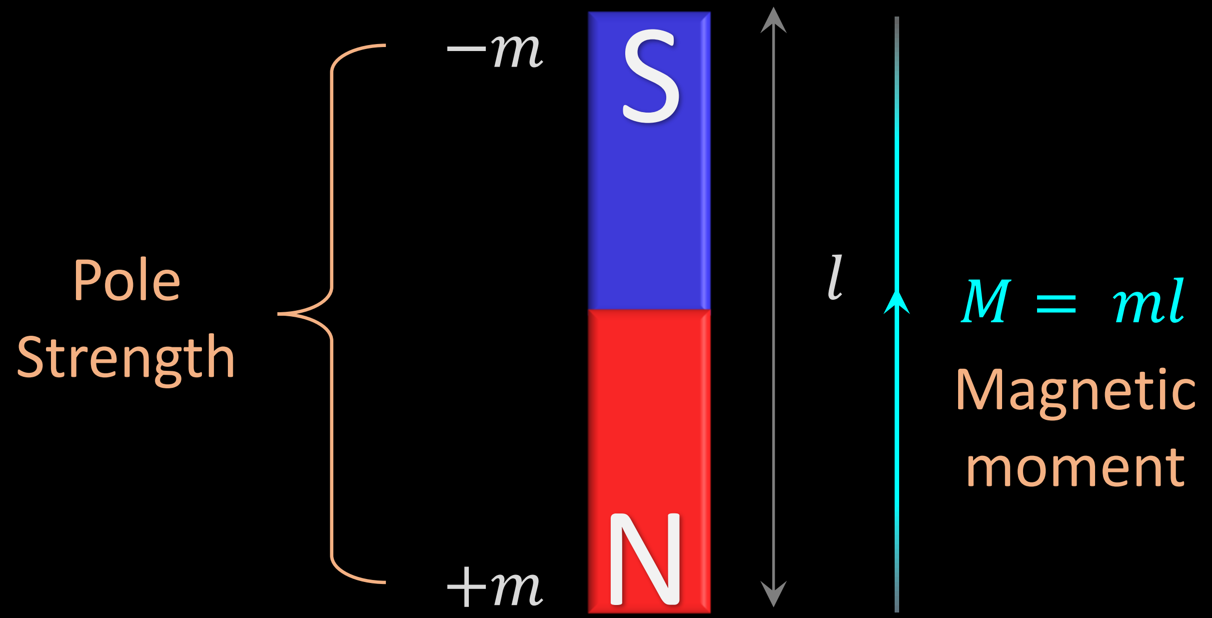


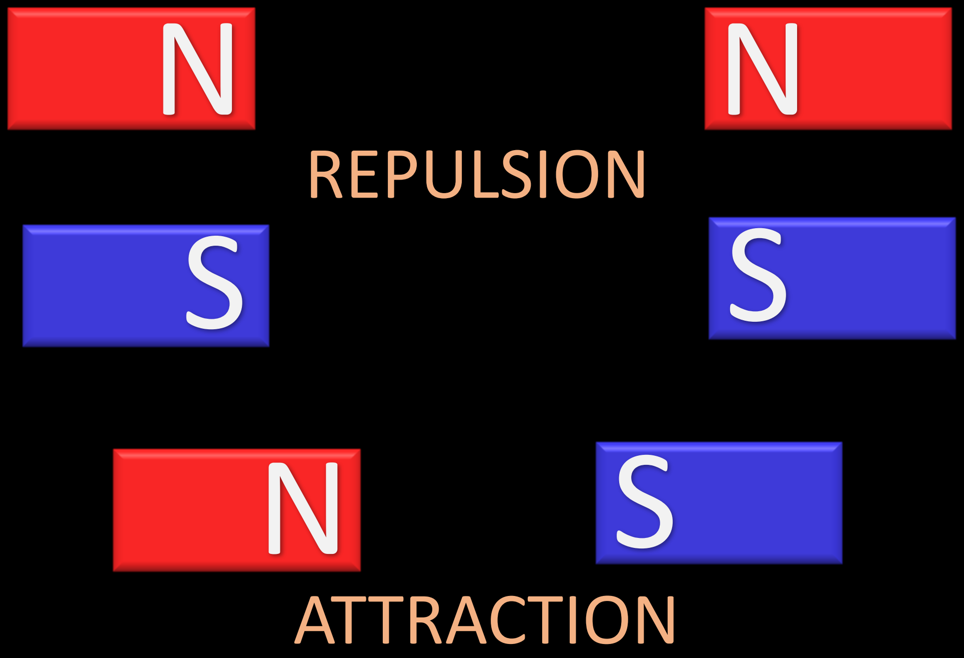
isko Samjho.

For an ideal solenoid, field is almost constant inside. At the points near its end, variation in field is seen and it becomes half at the ends. The variation is shown in adjacent graph.

PROPERTIES OF MAGNETIC FIELD LINES

Magnets





Like poles repel each other and unlike poles attract each other.

$+m$ $-m$.

N S

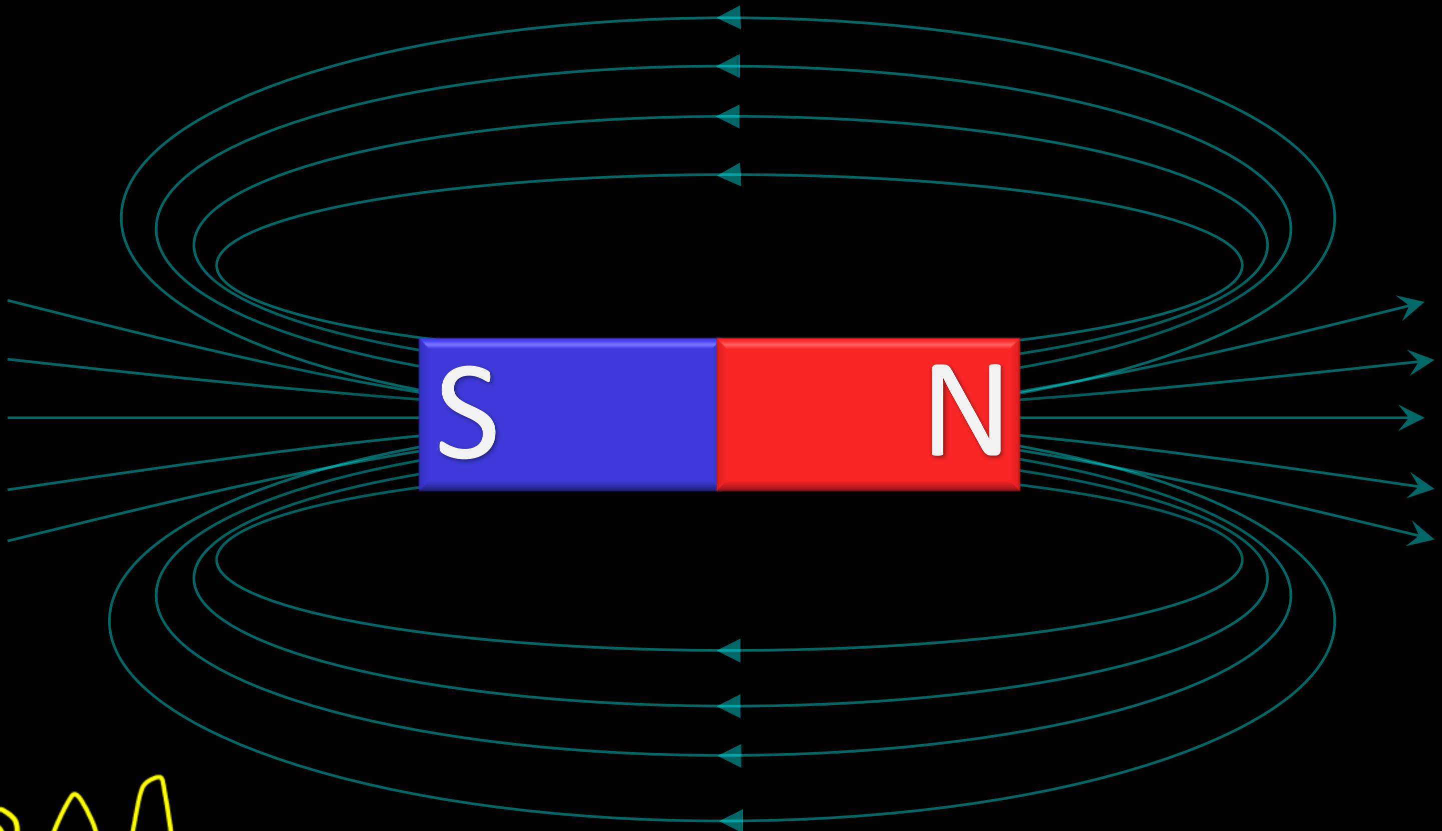
$m =$ Pole Strength.

Magnetic Moment -

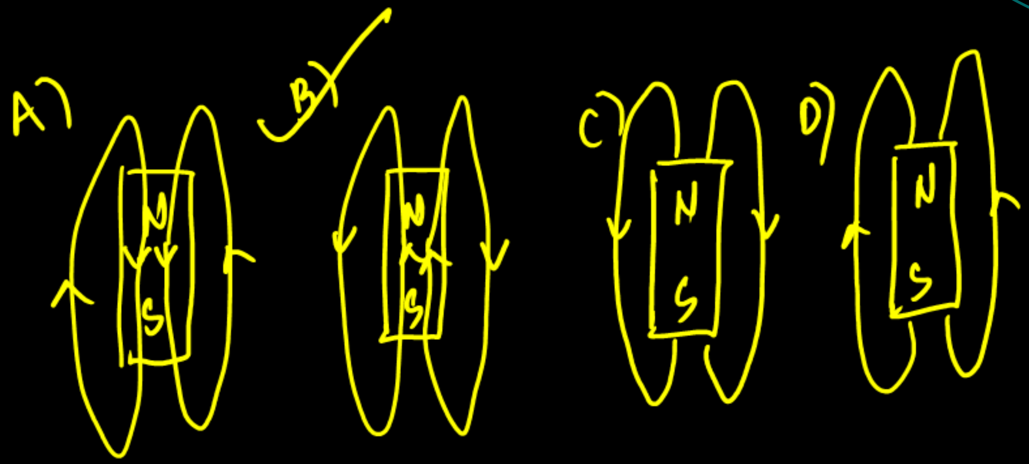
$\vec{M} = m \cdot l$. length of Magnet

PROPERTIES OF MAGNETIC FIELD LINES

- Magnetic field lines originates from North pole and goes in South pole outside the magnet.
- Field lines travel from South pole to North pole inside the magnet.



Eg:-



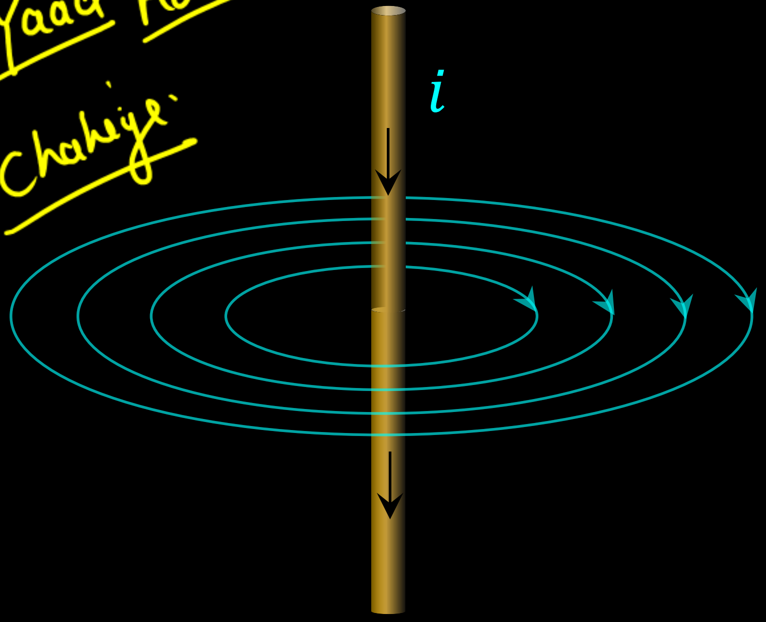
PROPERTIES OF MAGNETIC FIELD LINES

BOARDS

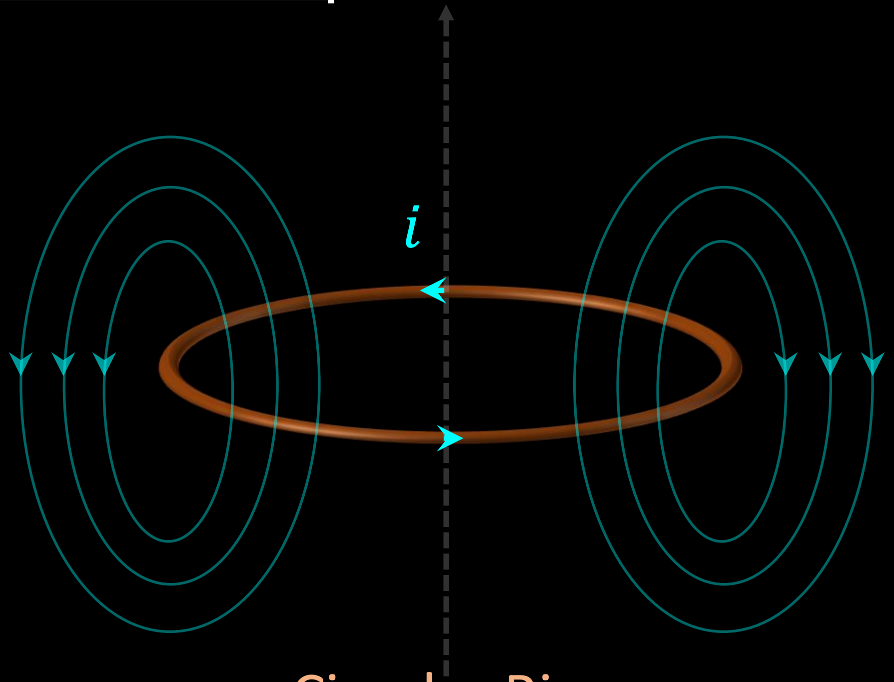
- Outside the magnet, field lines travel from north to south pole.
- Inside the magnet, field lines travels from south to north pole.
- Magnetic field lines always exist in closed loops. (Different from Electric Field lines)
- Tangent at any point on magnetic field line gives the direction of magnetic field only and not the direction of the magnetic force.
- Two magnetic field lines can never intersect each other.
- Number of field lines coming in or going out from a pole are directly proportional to the pole strength.
- Density of the number of field lines represents the intensity of magnetic field at that point.

SHAPE OF MAGNETIC FIELD LINES

Yaad Hona Chahiye

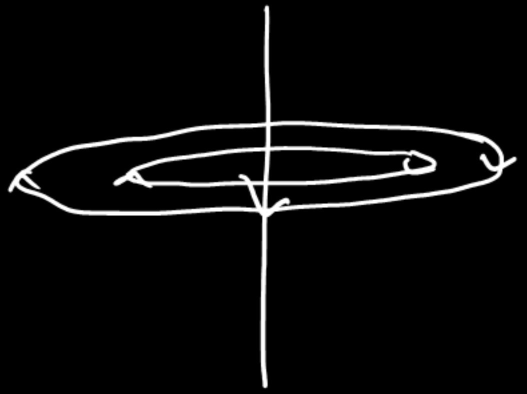


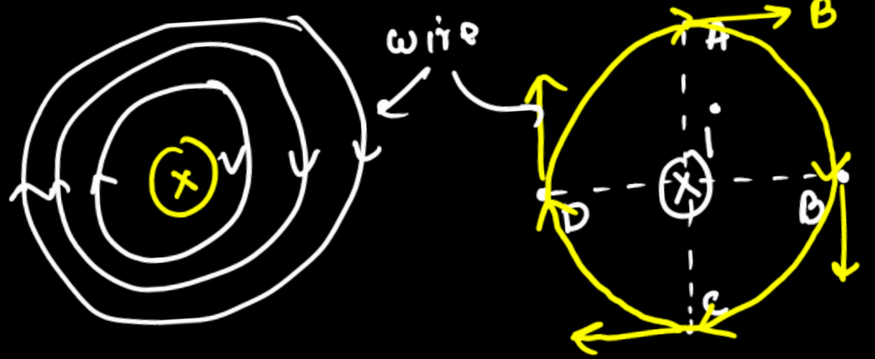
Straight Wire



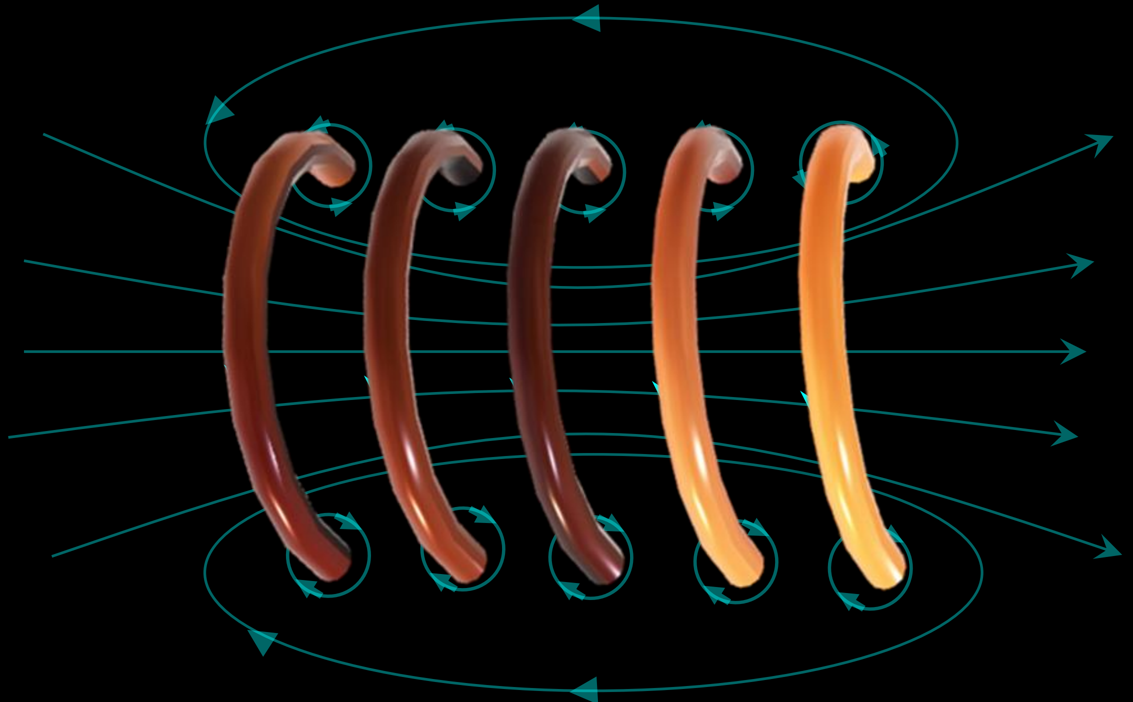
Circular Ring

Apply right hand thumb rule to find the direction of magnetic field.

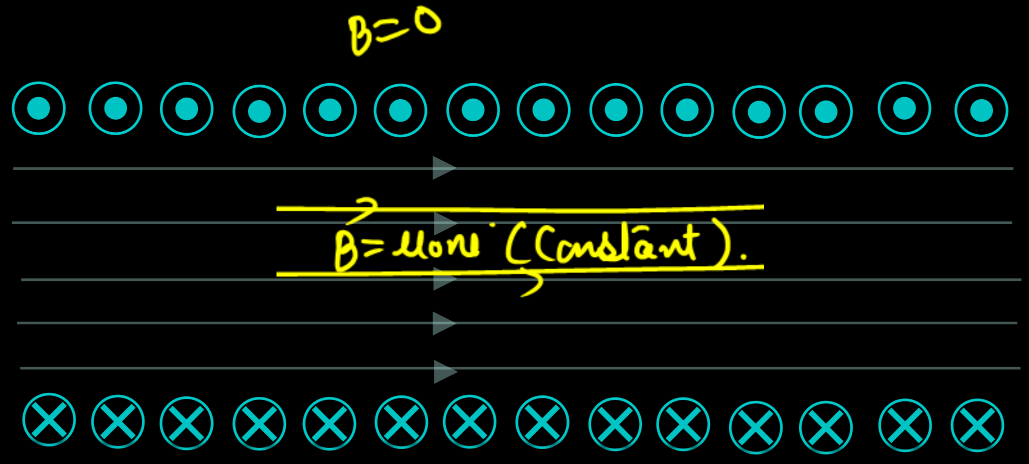




SHAPE OF MAGNETIC FIELD LINES

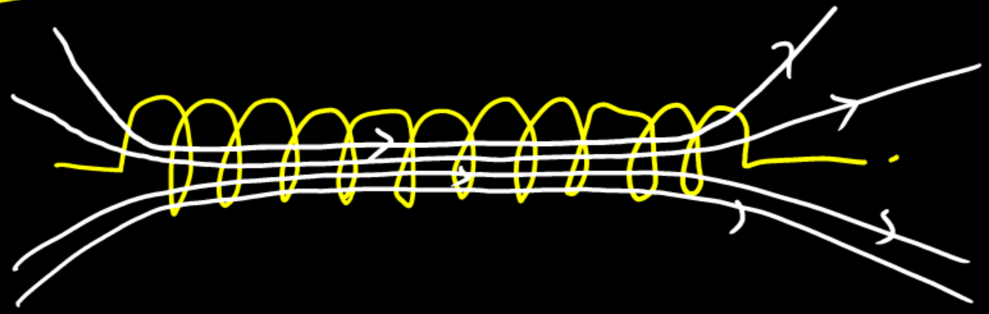


Real solenoid



Ideal solenoid

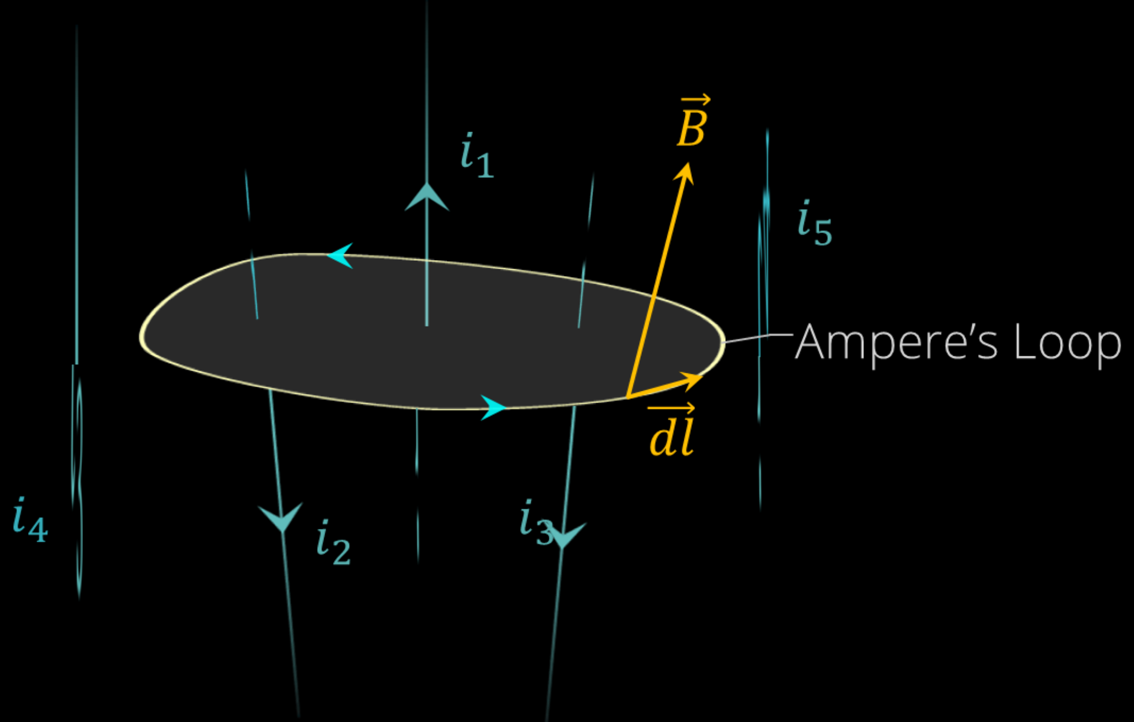
Practical ideal Solenoid $R \ll l$



Ampere's Circuital Law

BOARDS

The magnetic field created by an electric current is proportional to the size of that electric current with a constant of proportionality equal to the permeability of free space.



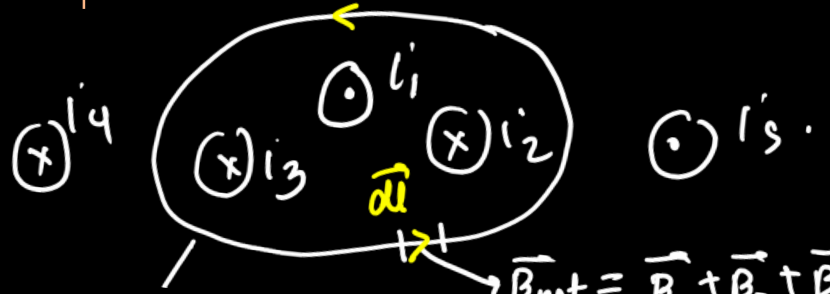
$\vec{B}$: Magnetic field due to all the wires (inside or outside)

$$\oint \vec{B}.\vec{dl} = \mu_0\, i_{in}$$

While assuming the loop, assume the sense of rotation of loop also. This will decide the positive and negative sign of current. Here, the sense of rotation along the loop is chosen in such a way that upward direction becomes positive.

Current i_{in} is only due to the wires passing through loop i.e., (i_1, i_2, i_3)

Example:

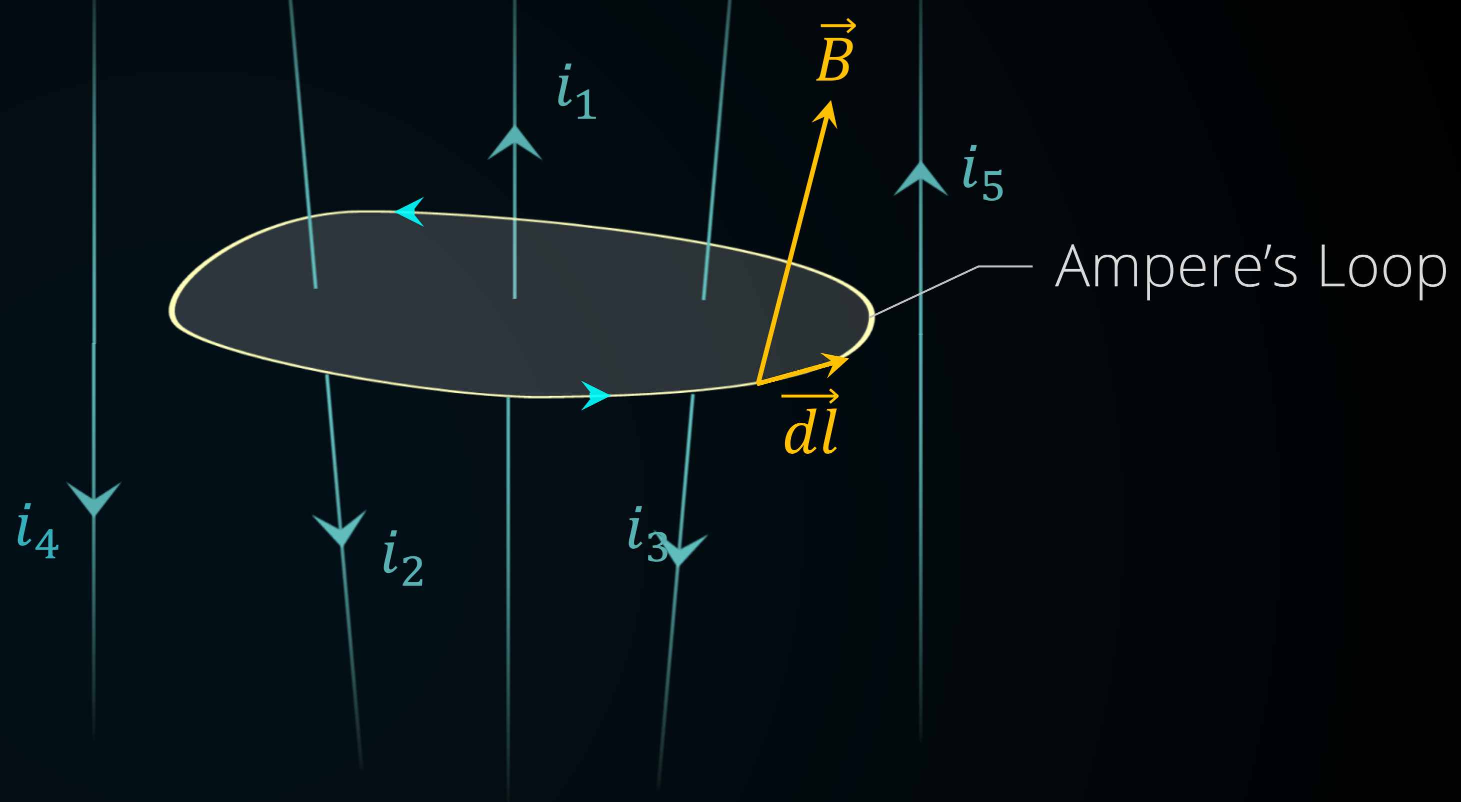


$\vec{B}_{net} = \vec{B_1} + \vec{B_2} + \vec{B_3} + \vec{B_4} + \vec{B_5}$

$i_{in} = i_1 - i_2 - i_3$

Ampere's Circuital Law

$\vec{B}$: Magnetic field due to all the wires (inside or outside)



The line integral of $\vec{B}.\vec{dl}$ along any closed path in a region is equal to μ_0 times the total current crossing the enclosed area.

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 \, i_{in}$$

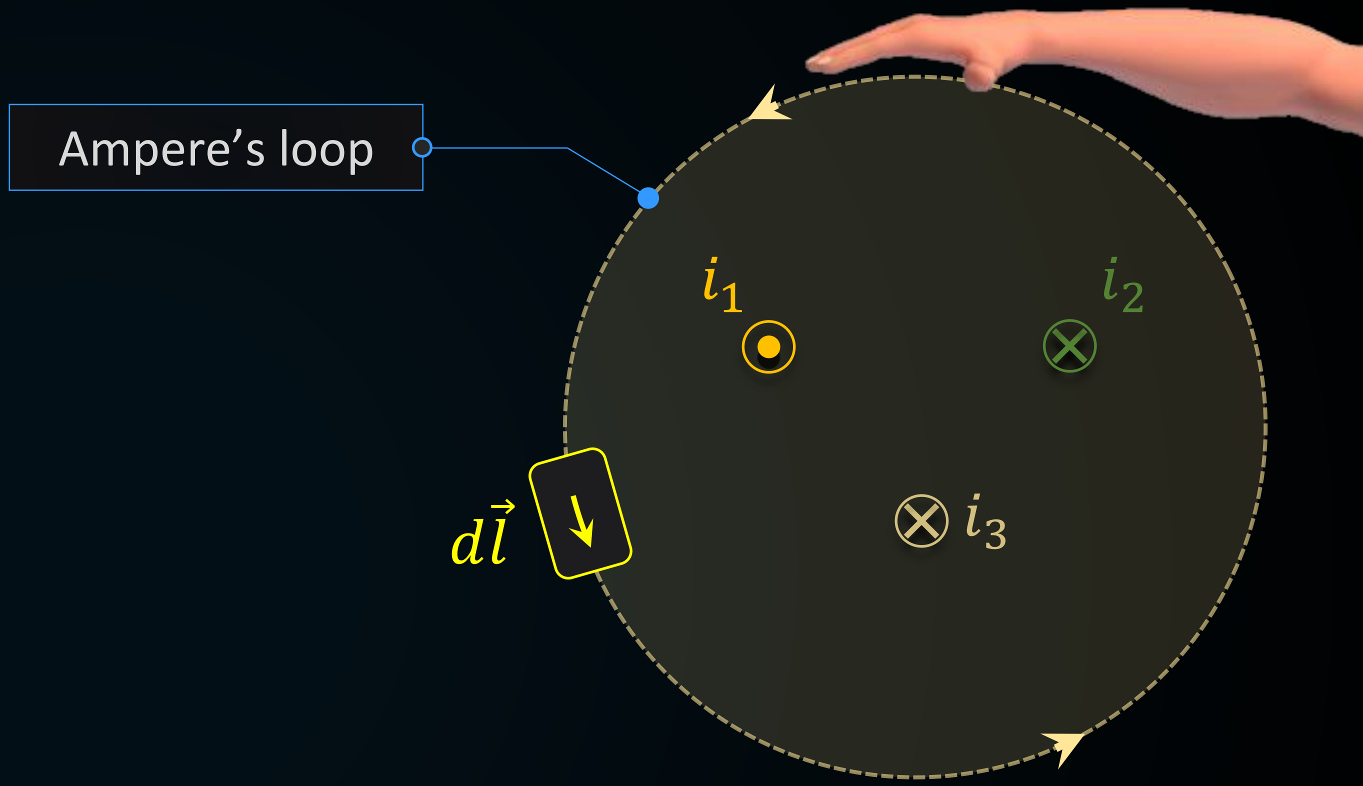
Ampere's Circuital Law

- Draw Ampere's Loop
- Use right hand thumb rule

$i_1 \rightarrow +ve \qquad i_2 \rightarrow -ve \qquad i_3 \rightarrow -ve$

- Consider $d\vec{l}$
- Closed line integral of $\vec{B}.d\vec{l}$ for the loop

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0\,(i_1 - i_2 - i_3)$$



Ampere's Circuital Law

It is valid everywhere, but we can calculate magnetic field using ampere's circuital law only when following conditions are satisfied:

- highly symmetrical current distribution ✓
- Infinite long wire (Thin & Thick)

BOARDS NEET

Magnetic field due to a long straight current carrying wire

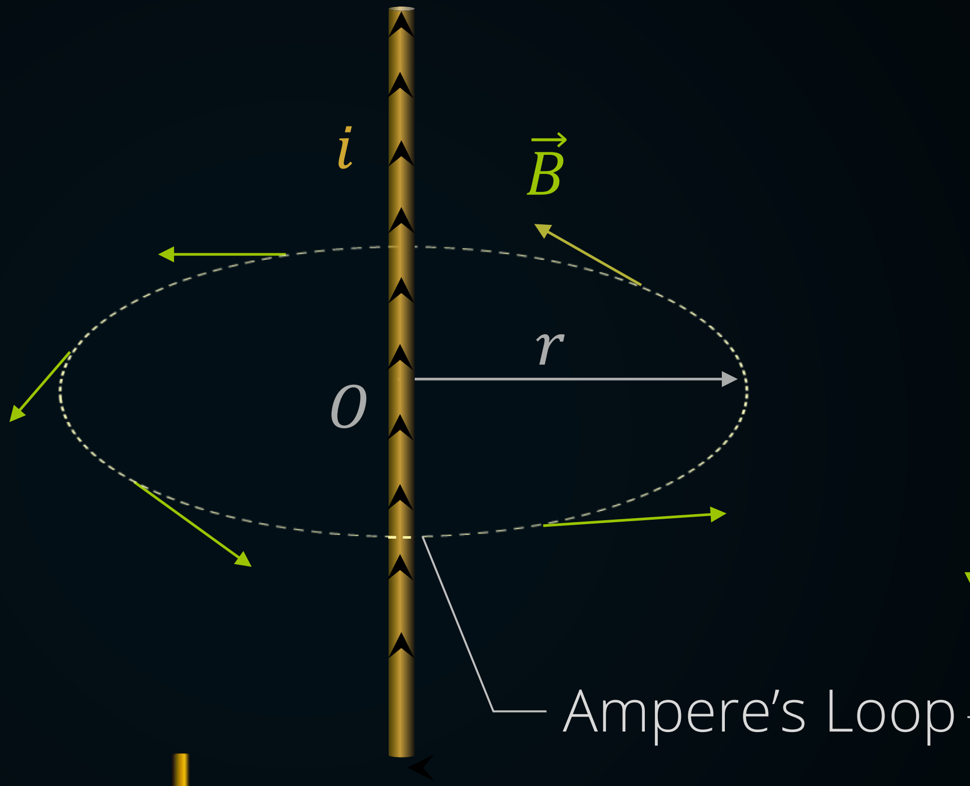
- $\vec{B} \parallel \vec{dl}$

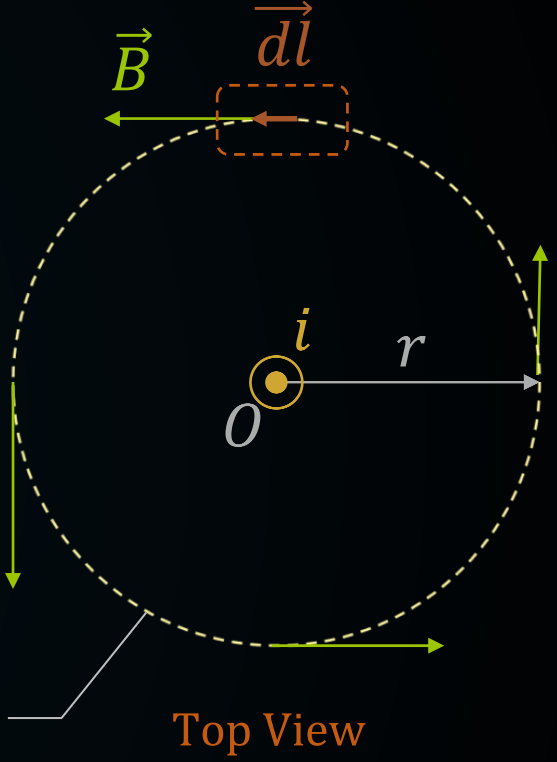
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i_{in}$$

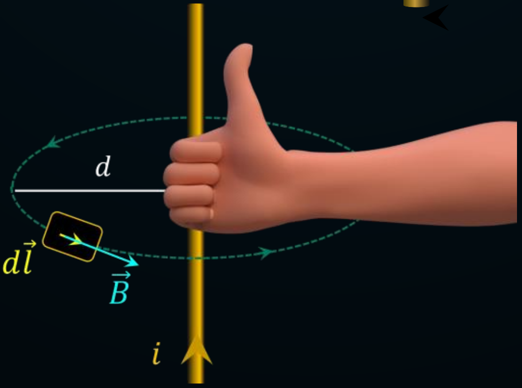
$$\oint B\,dl\cos 0^\circ = \mu_0 i$$

$$B\cdot 2\pi r = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$







$$B = \frac{\mu_0 i}{2\pi r}$$

Ampere's Circuital Law

Magnetic field due to hollow current carrying infinitely long wire

Inside the wire:

$r < R$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{in}$$

$$B \cdot 2\pi r = 0$$

$$\boxed{B = 0}$$

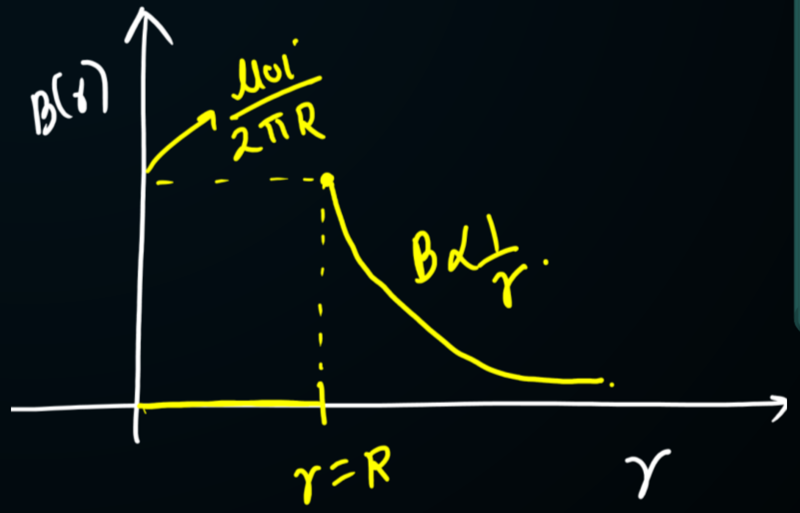
$$B = 0$$

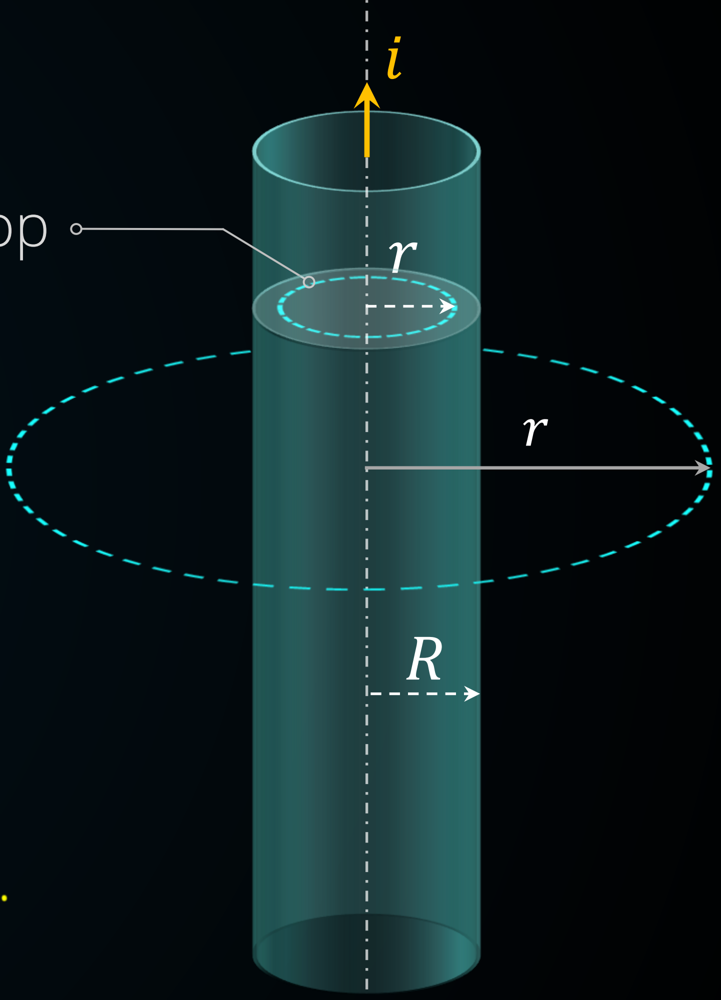
Outside the wire:

outside

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

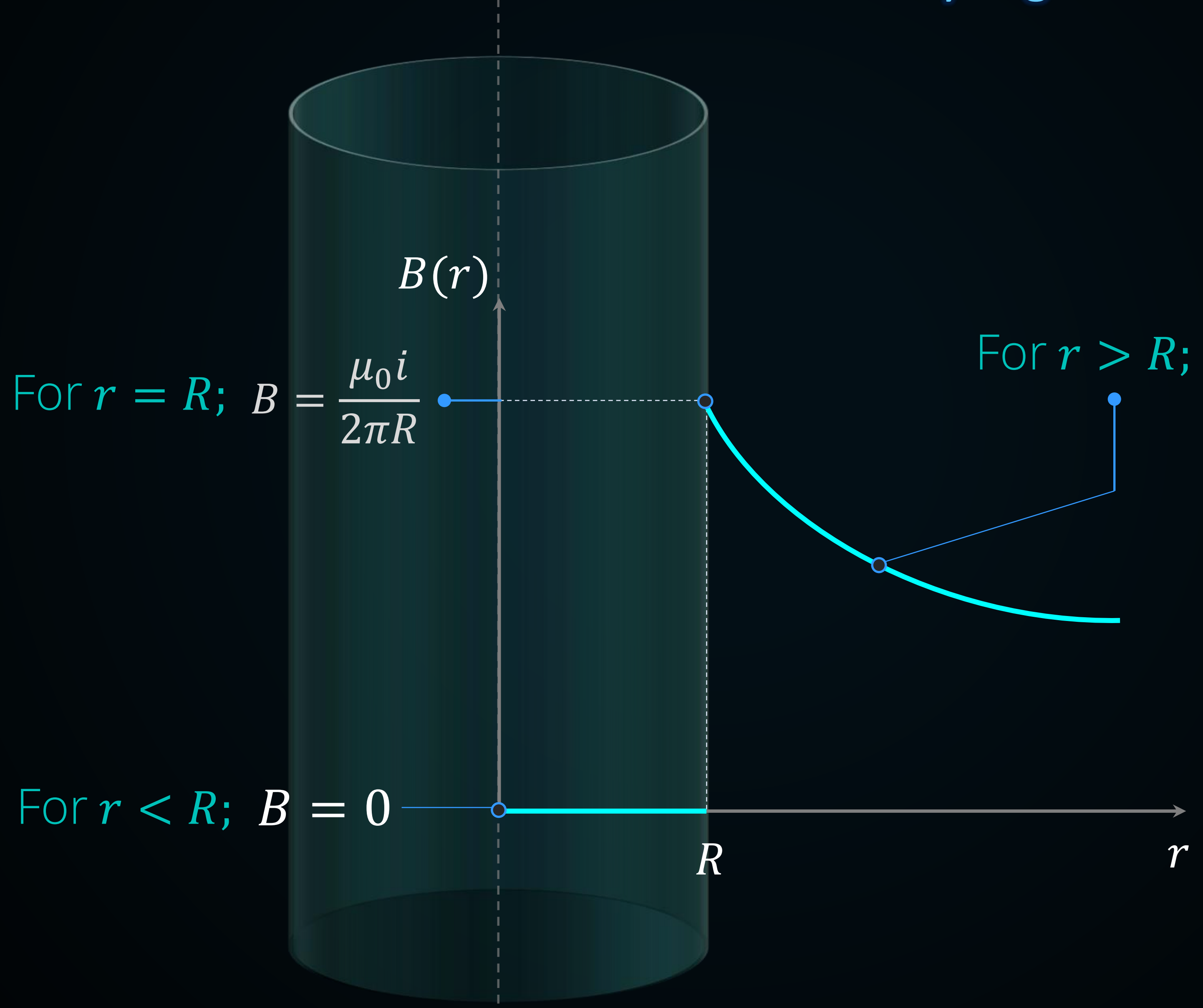
$$B = \frac{\mu_0 i}{2\pi r}$$





Ampere’s Circuital Law

Magnetic field due to hollow current carrying infinitely long wire



Ampere's Circuital Law

Magnetic field due to solid current carrying infinitely long wire

Inside the wire:

$r < R$

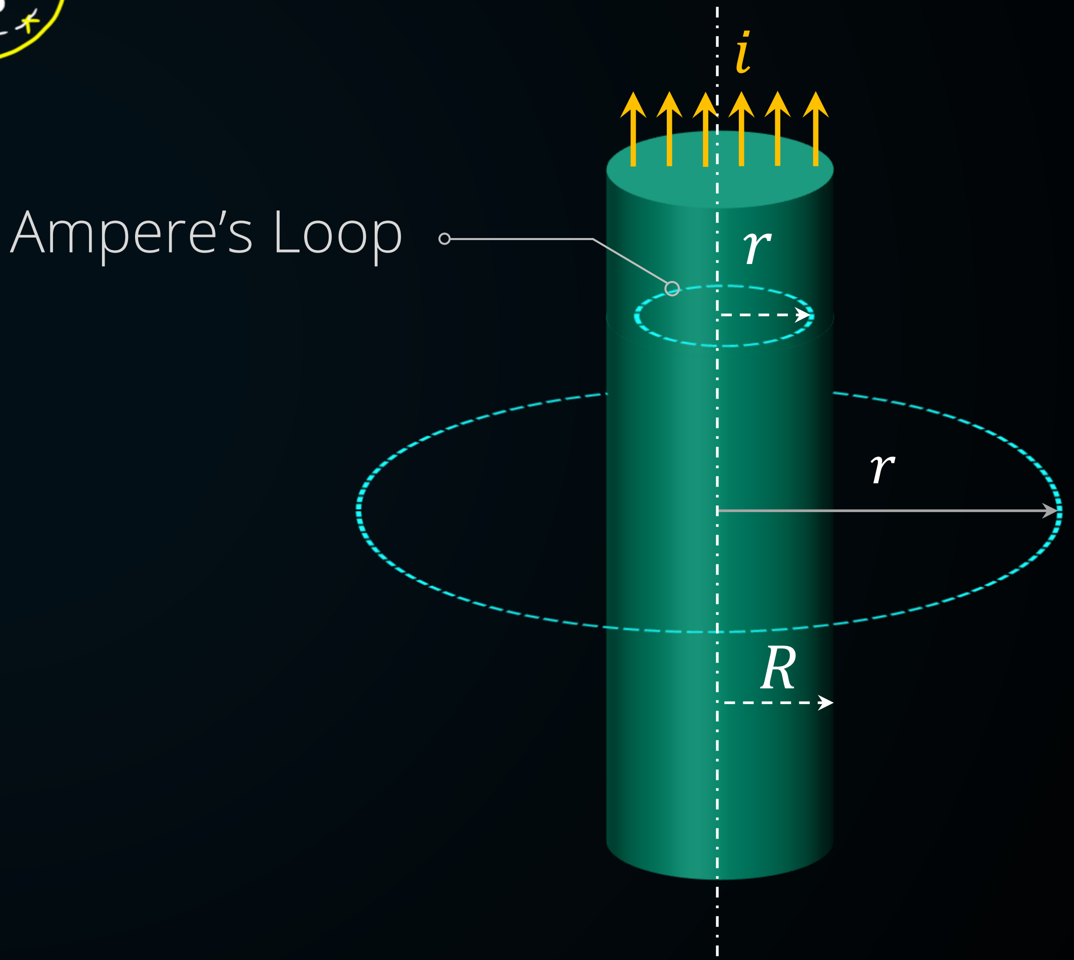
$j = \frac{i}{\pi R^2}$ = Current density

$\oint \vec{B} \cdot \vec{dl} = \mu_0 i_{in}.$

$B \cdot 2\cancel{\pi} \cancel{r} = \mu_0 j \cdot \cancel{\pi} r^{\cancel{2}}$

$$\boxed{B = \frac{\mu_0 j \cdot r}{2}}$$

$$B = \frac{\mu_0 j r}{2}$$



Outside the wire:

outside

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

$$B = \frac{\mu_0 i}{2\pi r}$$

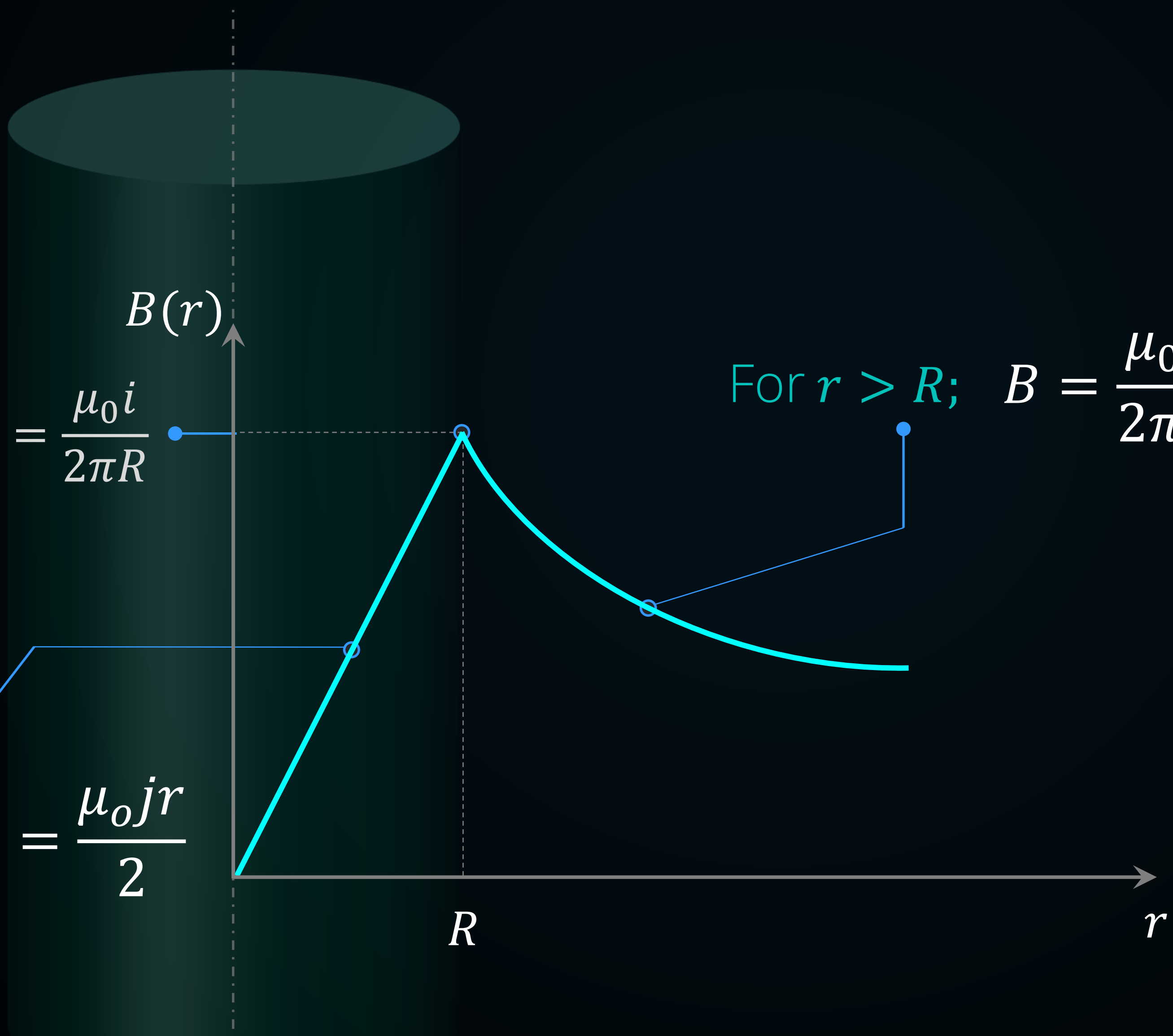
Ampere's Circuital Law

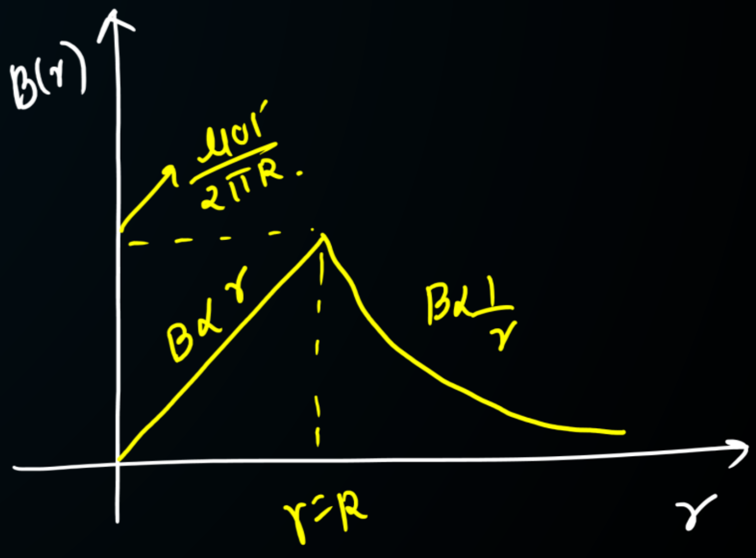
Magnetic field due to solid current carrying infinitely long wire

For $r > R$; $B = \dfrac{\mu_0 i}{2\pi r}$

For $r = R$; $B = \dfrac{\mu_0 i}{2\pi R}$

For $r < R$; $B = \dfrac{\mu_0 j r}{2}$

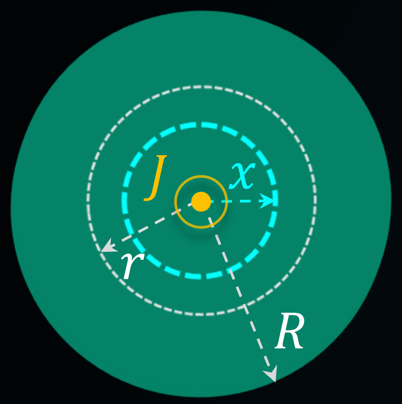




Question Solution

An infinitely long wire of radius R is carrying current in it. If current density inside the wire is varying as $J = J_0(r)$ then calculate the magnetic field at a distance r $(r < R)$.

Question | **Solution**



$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i_{in}$$

$$B\cdot 2\pi r = \mu_0$$

$$di = j(x)\cdot 2\pi x\cdot dx$$

$$i_{in} = \int_0^r j(x)\cdot 2\pi x\,dx$$

$$\oint \vec{B}.\vec{dl} = \mu_0\, i_{in}$$

$$\boxed{B\cdot 2\pi r = \mu_0 \int_0^r j(x)\cdot 2\pi x\,dx}$$

$r > R$

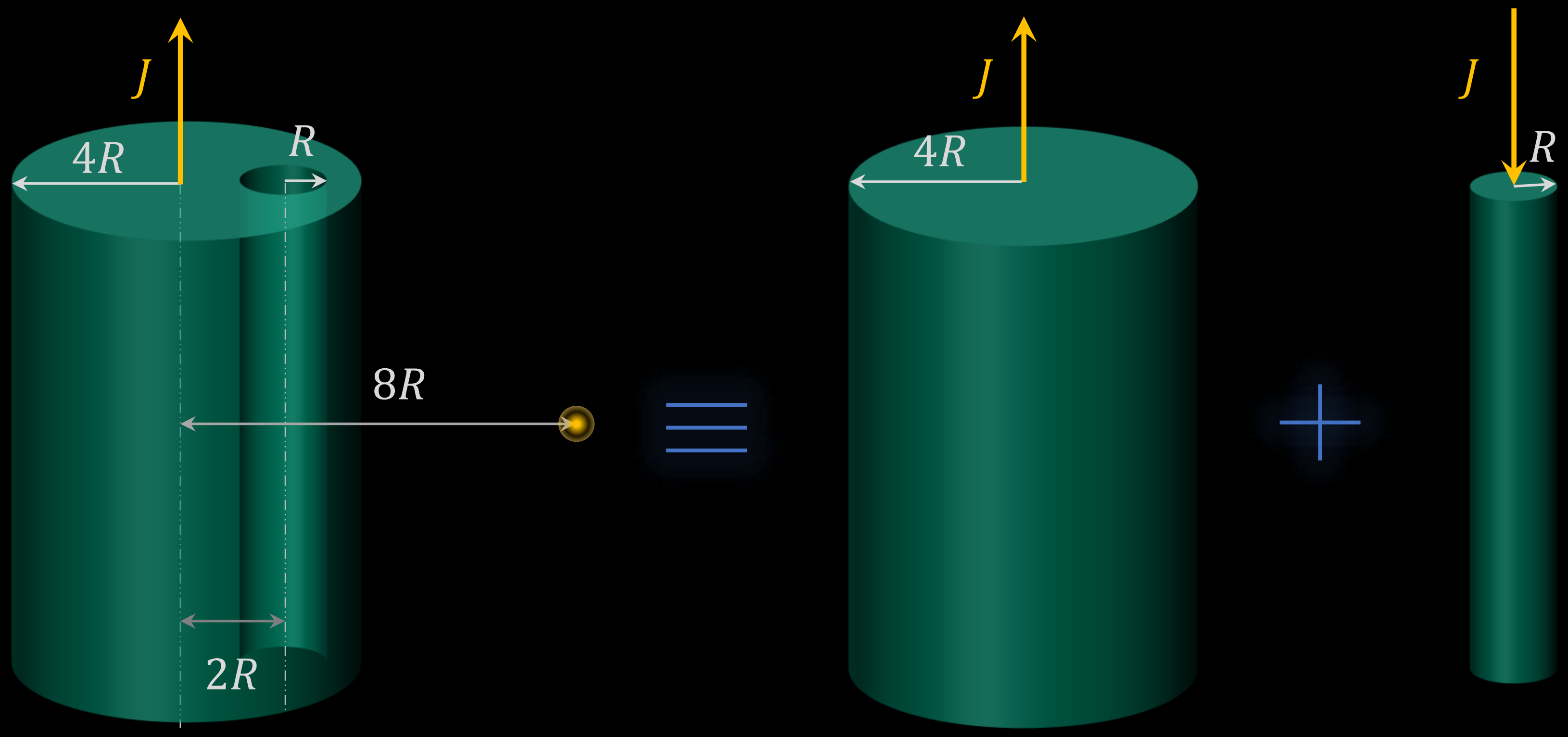
$$B\cdot 2\pi r = \mu_0\, i_T$$

$$B\cdot 2\pi r = \mu_0 \int_0^R j(x)\cdot 2\pi x\,dx$$

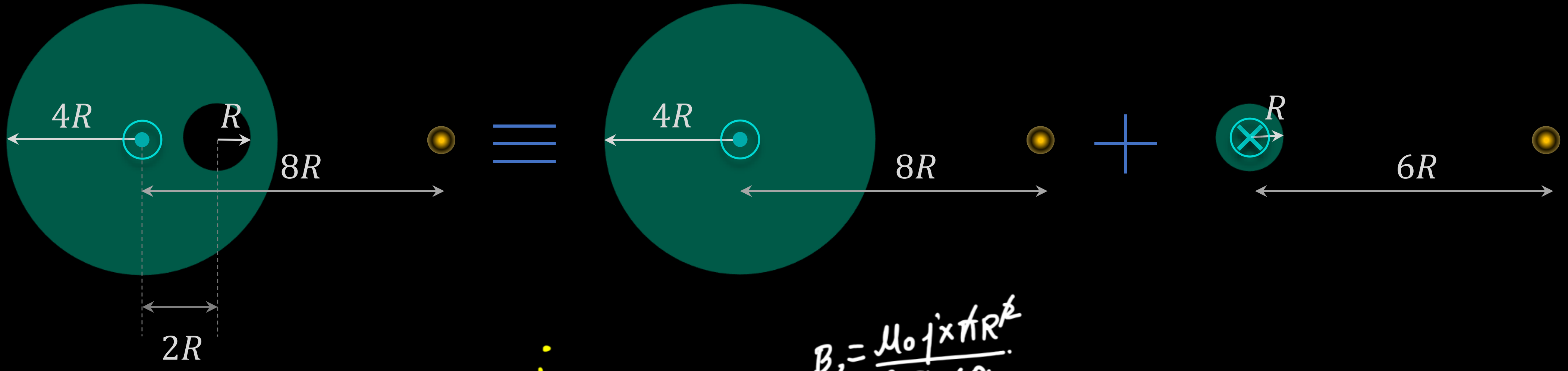
$$B\times(2\pi r) = \mu_0\int_0^r J_0(x)\times 2\pi x\times dx$$

Question Solution

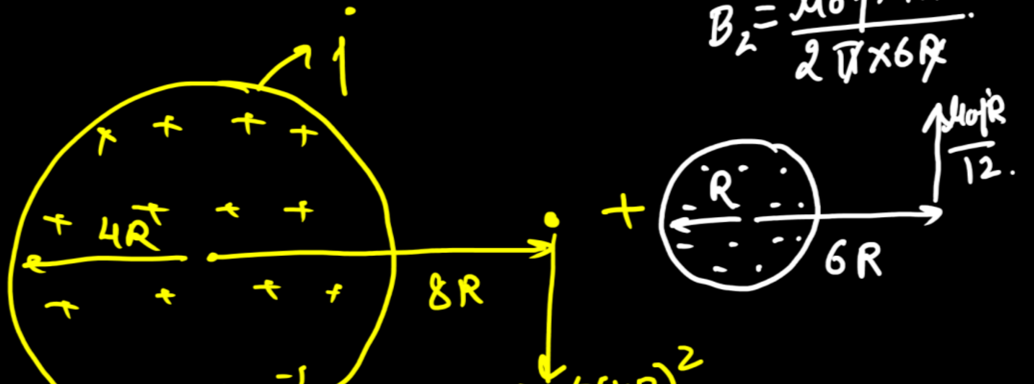
A cylinder of radius $4R$ having cavity of radius R (as shown in the figure) is carrying current of density J. What will be the magnetic field at a distance $8R$ from the axis?



Question | Solution



$$B_2 = \frac{\mu_0 j \times \cancel{\pi} R^{\cancel{2}}}{2\cancel{\pi} \times 6\cancel{R}}$$



$$B_1 = \frac{\mu_0 j \cdot \cancel{\pi}(4R)^2}{2\cancel{\pi} \cdot 8R}$$

$$\boxed{B_1 = \mu_0 j R}$$

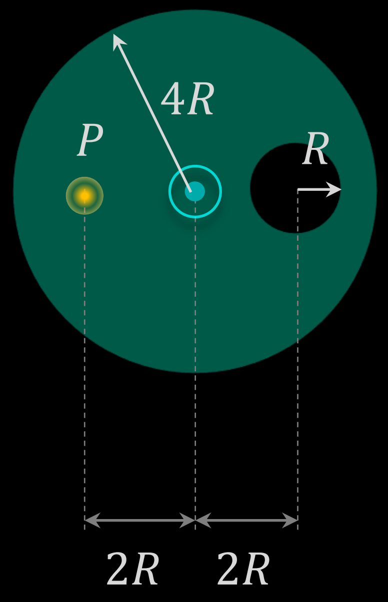
$$B_{net} = \mu_0 j R - \frac{\mu_0 j R}{12}$$

$$= \boxed{\frac{11 \mu_0 j R}{12}} \downarrow$$

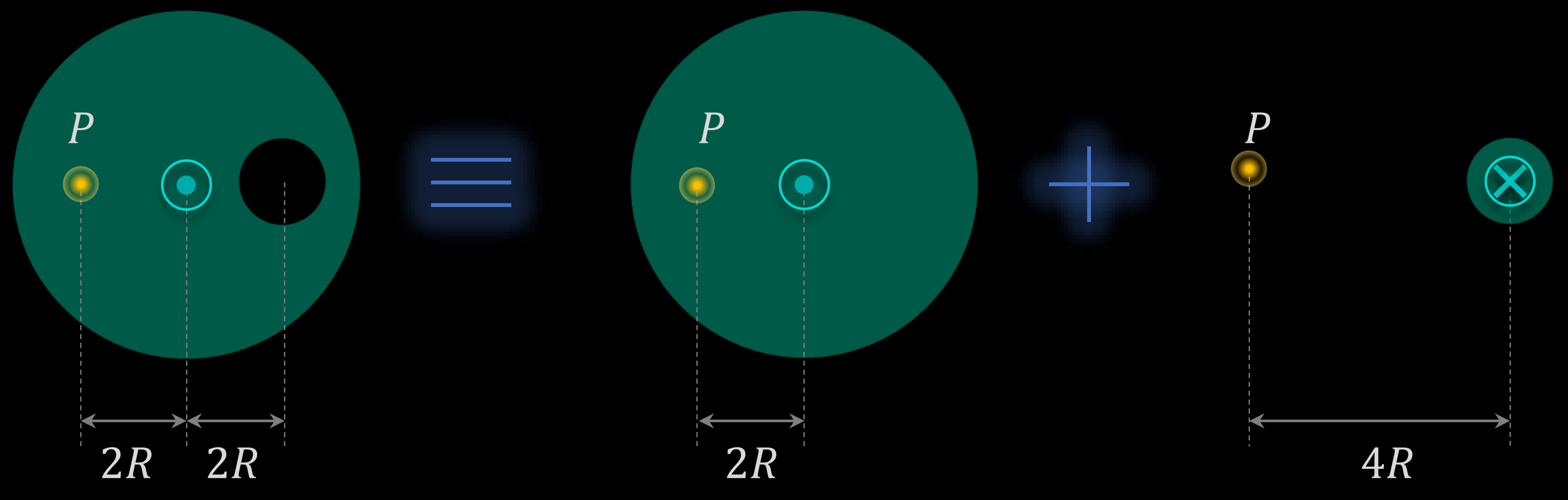
$$B = \frac{11 \mu_0 J R}{12}$$

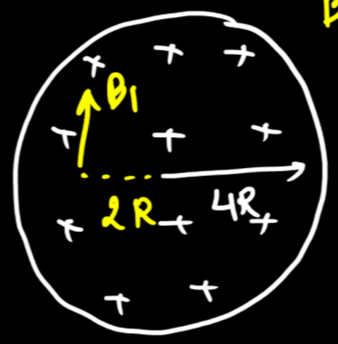
Question Solution

A cylinder of radius $4R$ having cavity of radius R is carrying current density J. What will be the magnetic field at point P which is at a distance $2R$ from the axis as shown in figure?



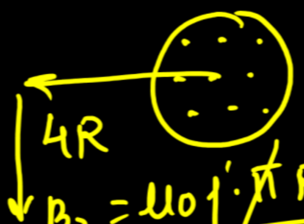
Question | **Solution**





$$B_1 = \frac{\mu_0 J \cdot \cancel{2}R}{\cancel{2}}$$

+



$$B_2 = \frac{\mu_0 J \cdot \cancel{\pi} R^{\cancel{2}}}{2\cancel{\pi} \cdot 4\cancel{R}} = \frac{\mu_0 J R}{8}$$

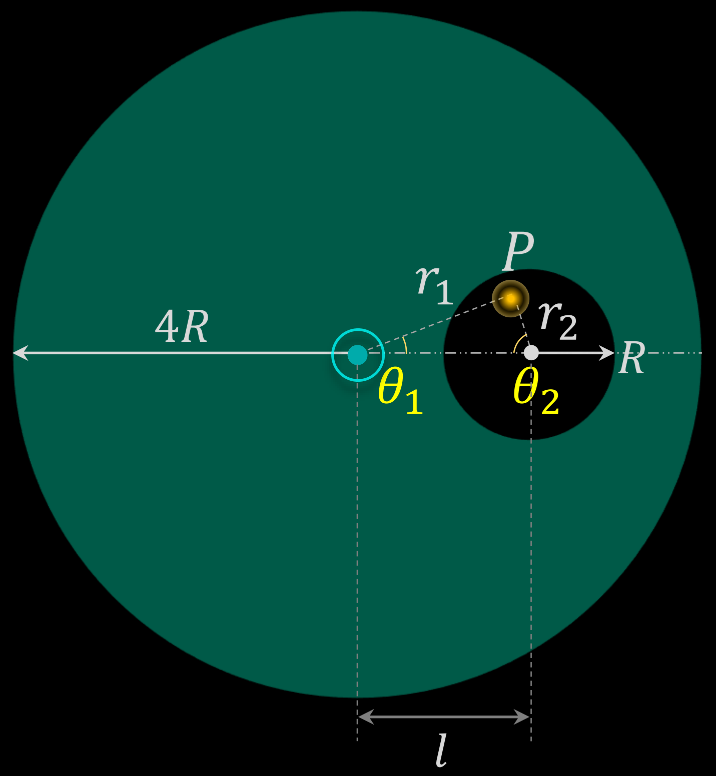
$$\equiv \quad B_{net} = \mu_0 J R - \frac{\mu_0 J R}{8}$$

$$\boxed{= \frac{7\mu_0 J R}{8}}$$

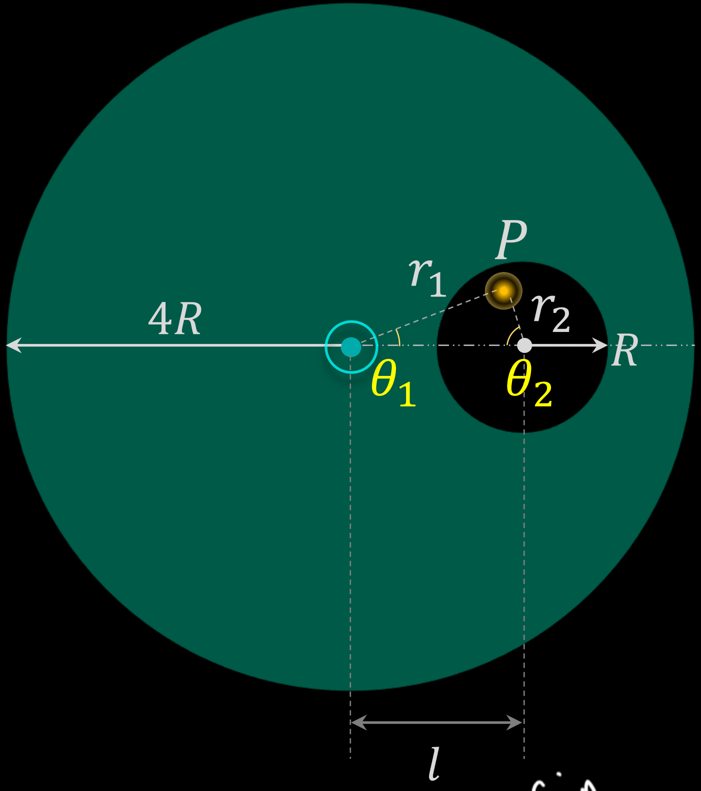
$$B = \frac{7\mu_0 J R}{8}$$

Question Solution

A cylinder of radius $4R$ have a cavity of radius R. It is carrying current of density J. What will be the magnetic field at a point P as shown?



Question | Solution



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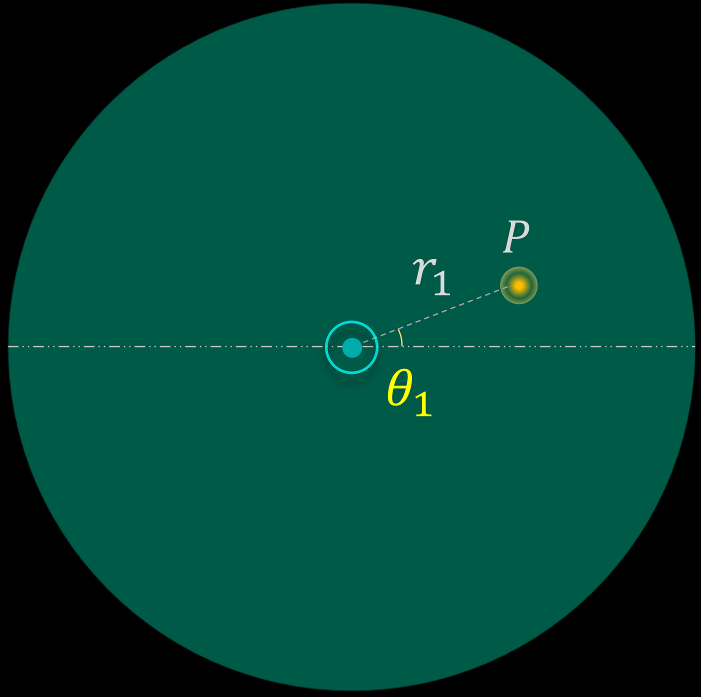
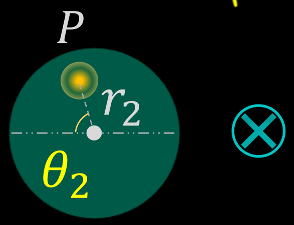
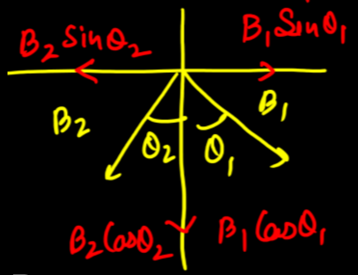


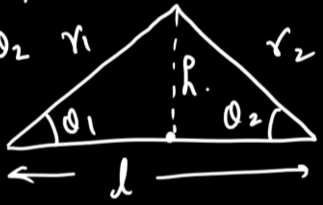
$+$





$h = r_1 \sin\theta_1$

$h = r_2 \sin\theta_2$



x-direction $B_1 \sin\theta_1 - B_2 \sin\theta_2$

$= \frac{\mu_0 j}{2}\left[\underbrace{r_1 \sin\theta_1}_{h} - \underbrace{r_2 \sin\theta_2}_{h}\right]$

$= \boxed{0}$

$B_2 \cos\theta_2 + B_1 \cos\theta_1$

$\frac{\mu_0 j}{2}\left[r_1 \cos\theta_1 + r_2 \cos\theta_2\right]$

$= \boxed{\frac{\mu_0 j l}{2}}$ → Constant

→ direction is $\perp$ to l.

$(B_\parallel)_l = 0$

$(B_\perp)_l = \frac{\mu_0 j l}{2}$

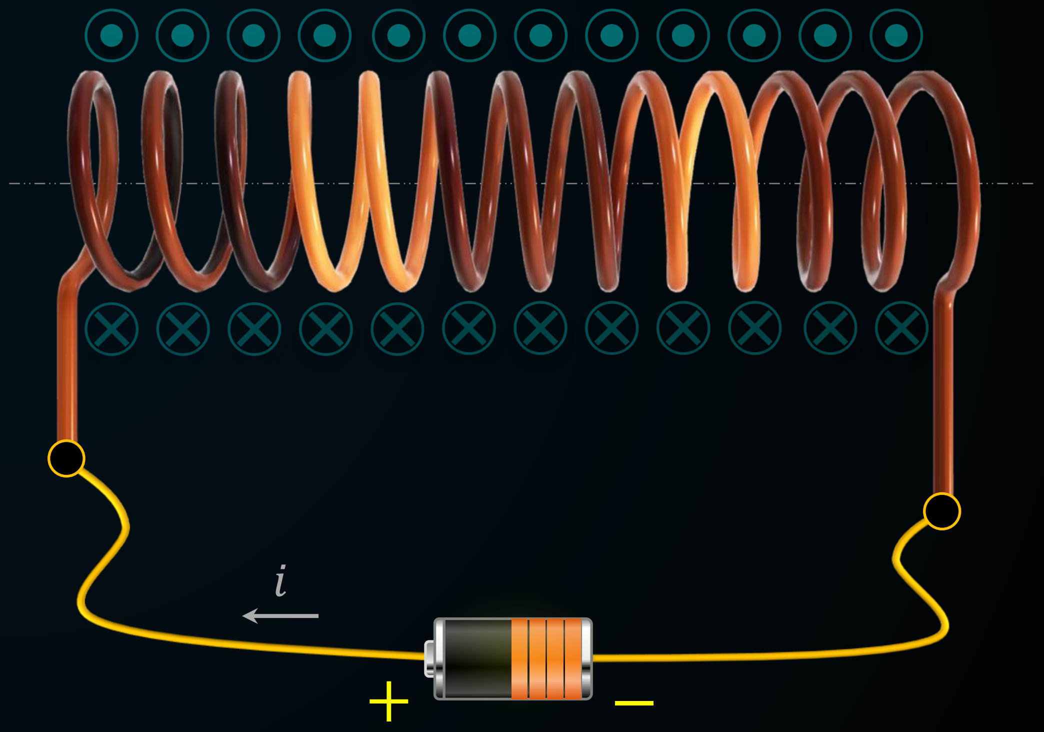
A solenoid is a type of electromagnet, the purpose of which is to generate a controlled magnetic field through a coil wound into a tightly packed helix.

Magnetic field due to ideal Solenoid

A solenoid can be called as an ideal one if the following conditions are satisfied :

- $l \gg R$ $\Rightarrow$ long solenoid
- $n = N/L$ is a very large number

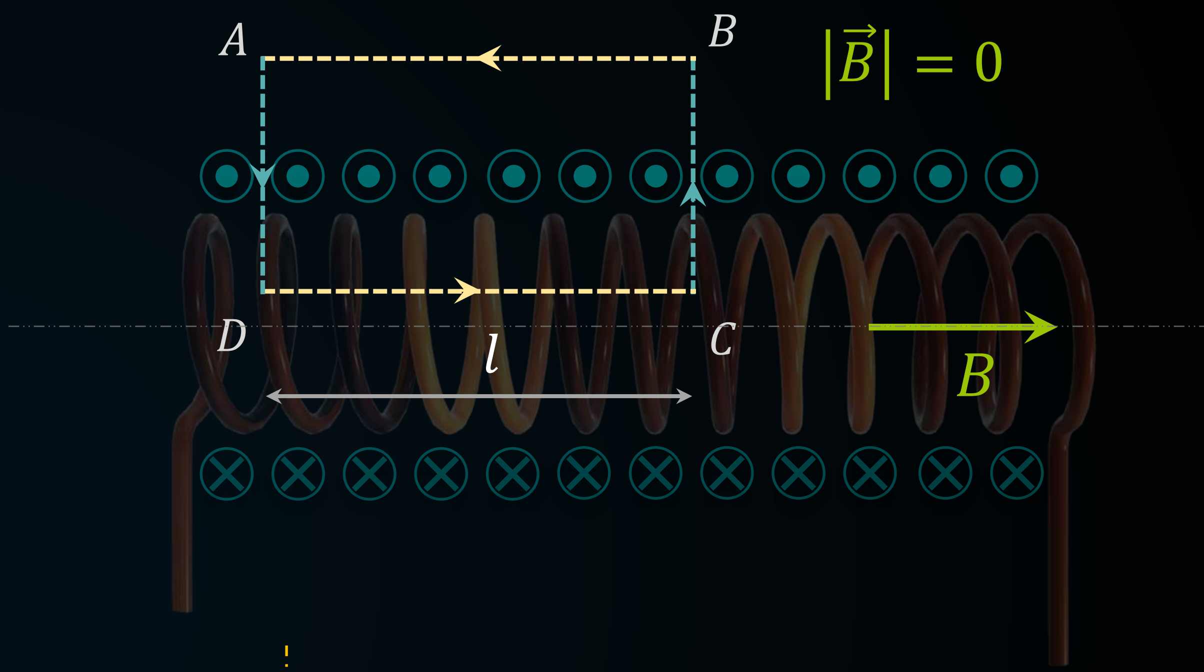
  $\Rightarrow$ turns are very closely wound.



Ampere's Circuital Law

Magnetic field due to Solenoid

$$\boxed{B = \frac{\mu_0 n i}{2}[\cos\theta_1 - \cos\theta_2]}$$



Magnetic field due to ideal Solenoid

(Infinitely long, $l >> R$)

$$\theta_1 \approx 0° \; and \; \theta_2 \approx 180°$$

$$B = \frac{\mu_0 n i}{2}[\cos 0° - \cos 180°] = \frac{\mu_0 n i}{2}[1 - (-1)]$$

$$\boxed{B = \mu_0 n i}$$

n — Number of turns per unit length along the length of the solenoid

$$\int_A^B \vec{B}.\vec{dl} = 0 \quad \text{(Outside solenoid)}$$

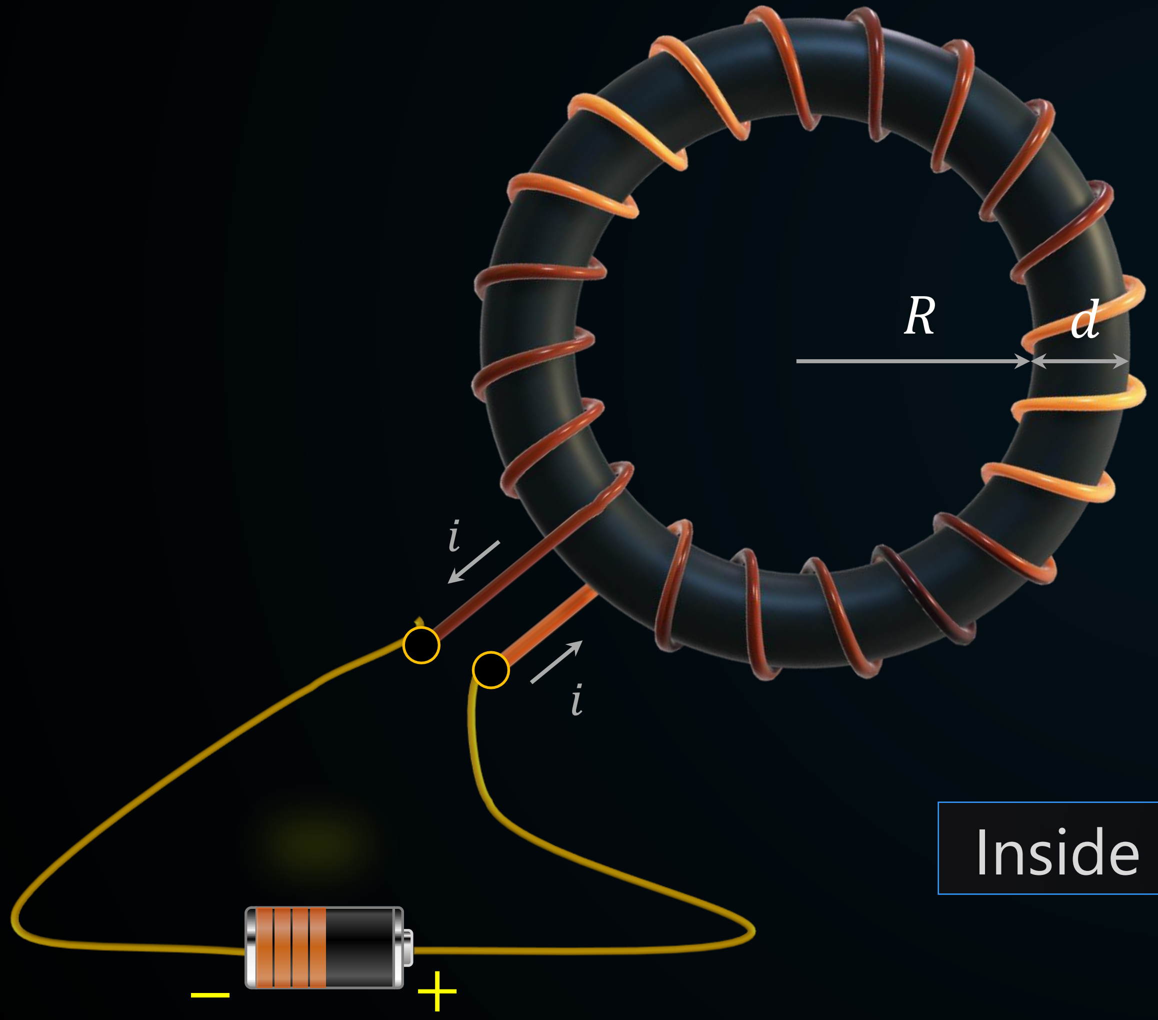
$$\int_D^A \vec{B}.\vec{dl} = 0 \qquad \int_B^C \vec{B}.\vec{dl} = 0$$

$$(B \perp l) \qquad (B \perp l)$$

Ampere's Circuital Law

Magnetic field due to Toroid

A toroid is essentially a solenoid that is bent into the shape of circle.



N = Total turns

$n = \frac{N}{2\pi R}$

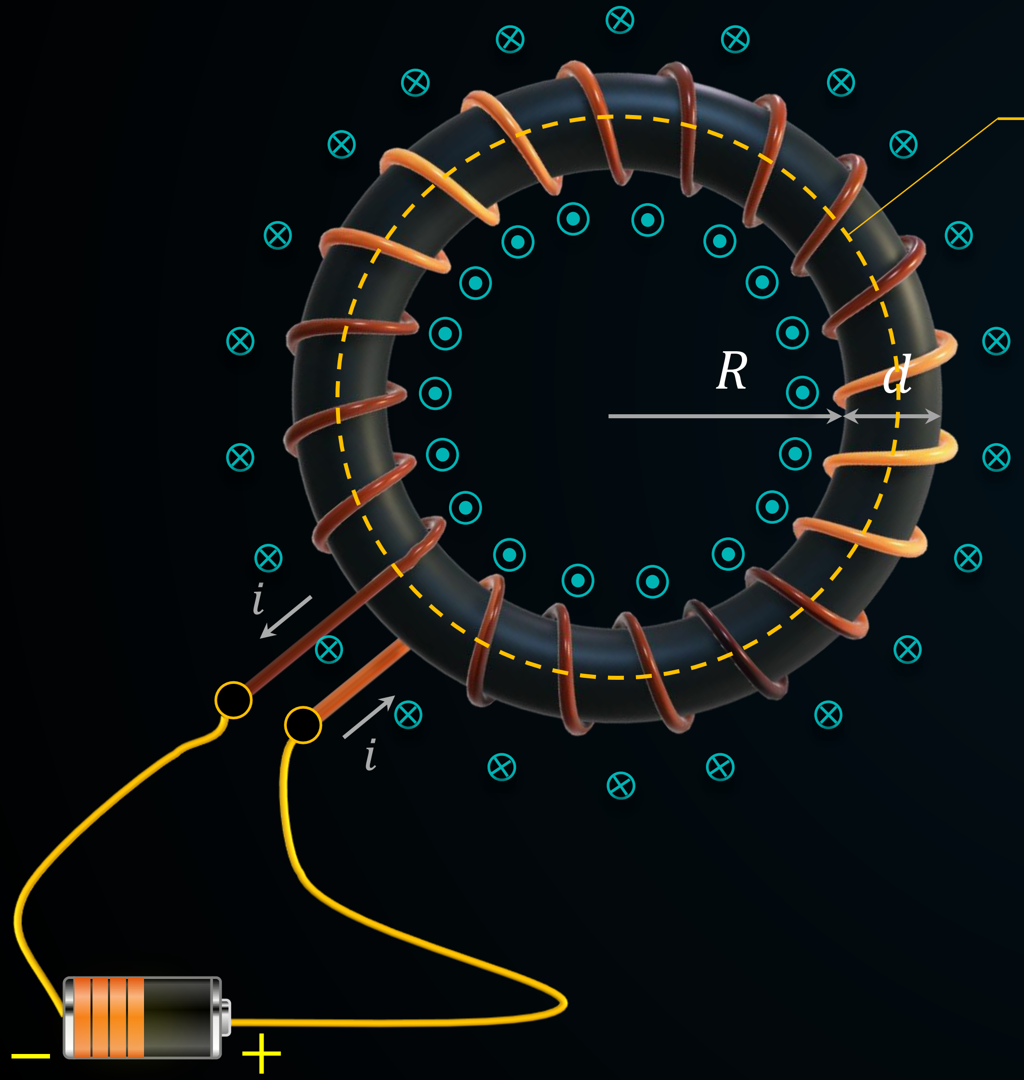
$R >> d$

Inside toroid : $B = 0$

Outside toroid : $B = 0$

Ampere's Circuital Law

Magnetic field due to Toroid



Ampere's loop.

$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{in}$.

$B \cdot 2\pi R = \mu_0 \cdot Ni$

$B = \boxed{\dfrac{\mu_0 Ni}{2\pi R}} = \boxed{\mu_0 ni}$

$$B_O = \frac{\mu_0 Ni}{2\pi R}$$ ⟹ Magnetic field within toroid

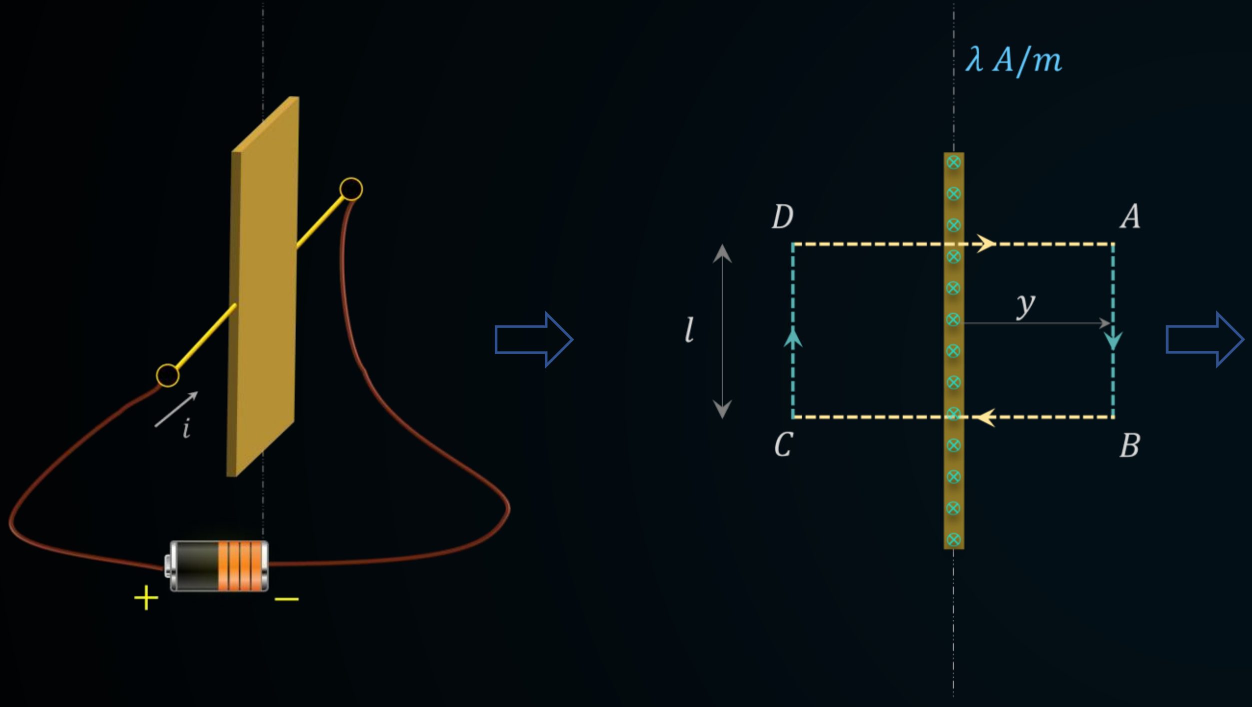
N = Total number of turns

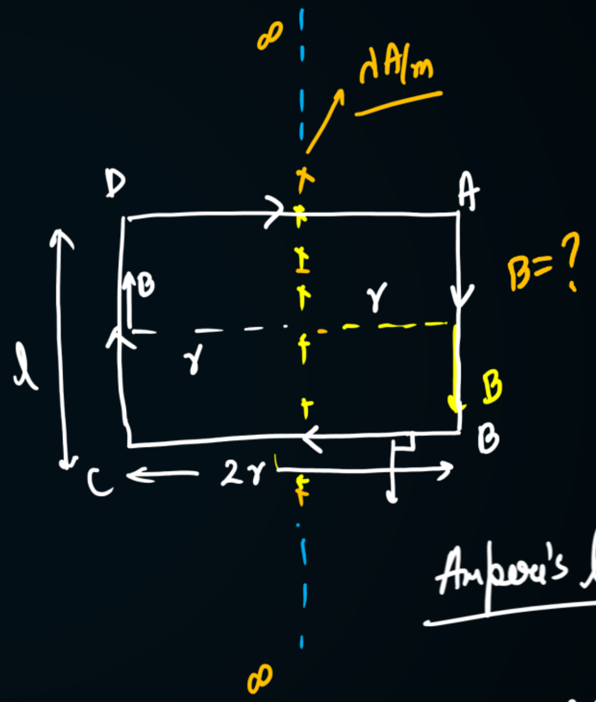
$$B_O = \mu_0 ni$$

n = Number of turns per unit length along the length of the solenoid

Ampere's Circuital Law

Magnetic field due to infinite sheet carrying current





Ampere's loop. $\oint \vec{B}\cdot\vec{dl} = \mu_0 i_{in}$.

AB BC CD DA .

$\vec{l} \perp \vec{B}$

$B\cancel{l} + 0 + B\cancel{l} + 0 = \mu_0 \lambda \cancel{l}$

$\boxed{B = \frac{\mu_0 \lambda}{2}}$ → independent of r.

$$B_O = \frac{\mu_0 \lambda}{2}$$

Force on a moving charge in magnetic field

Observation:

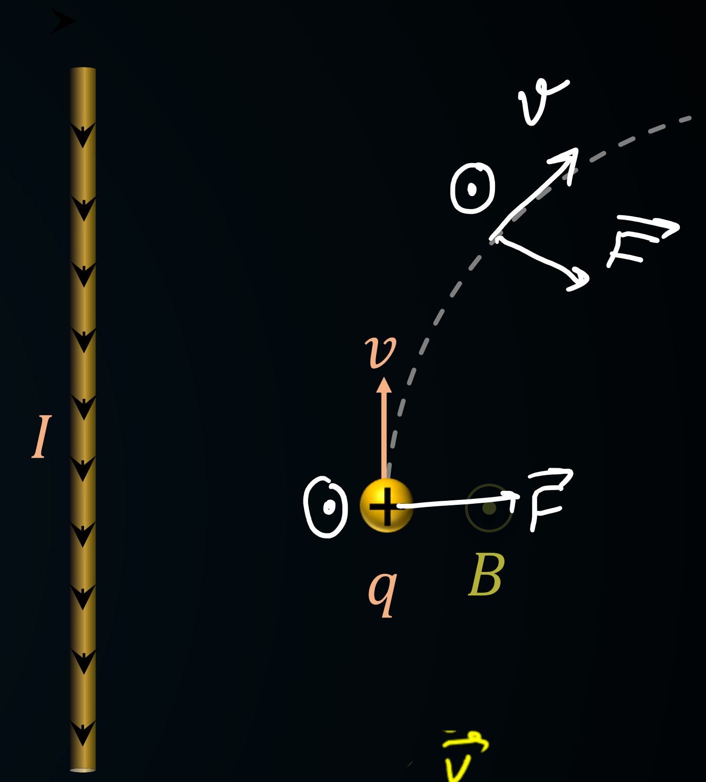
- $|F| \propto q$
- $|F| \propto |\vec{B}|$
- $|F| \propto |\vec{v}|$
- $|F| \propto \sin\theta$

Magnetic force: $\vec{F} = q(\vec{v} \times \vec{B})$

Magnitude: $|\vec{F}| = q|\vec{v}||\vec{B}|\sin\theta$

Here, θ is the angle between $\vec{v}$ and $\vec{B}$.

- As the force is always perpendicular to velocity, work done by it is zero. As the work done is zero, only direction of velocity changes and its magnitude remains same.
- Direction of force is same as $(\vec{v} \times \vec{B})$ if q is $+ve$.
- Direction of force is opposite to $(\vec{v} \times \vec{B})$ if q is $-ve$.



$$\boxed{\vec{F}_m = q(\vec{v} \times \vec{B})}$$

Question

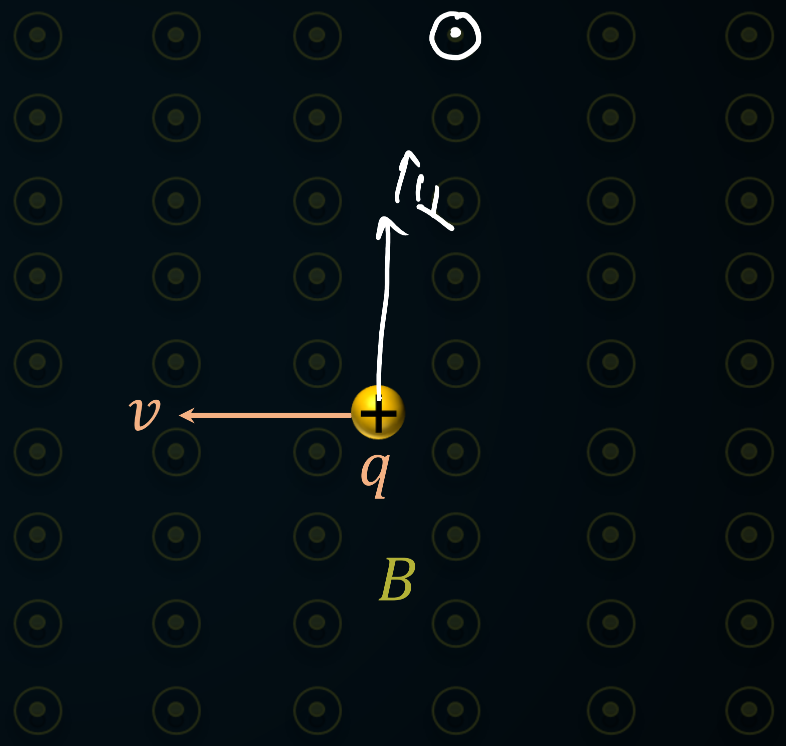
If a charge particle has the velocity in west direction inside the magnetic field as shown, then find the direction of magnetic force.

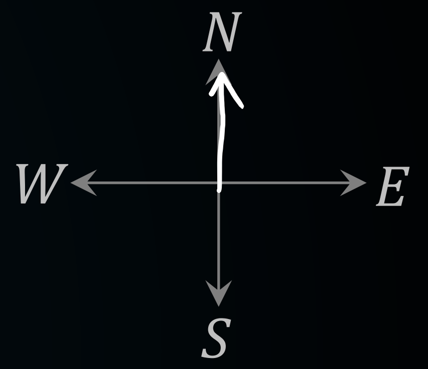
Solution

Here, the charge q is $+ve$.

Thus, direction of force is same as $(\vec{v} \times \vec{B})$.

As $\vec{v}$ is directed towards West and $\vec{B}$ is coming out of the plane of paper, $\vec{v} \times \vec{B}$ is directed towards North.





North direction

Question

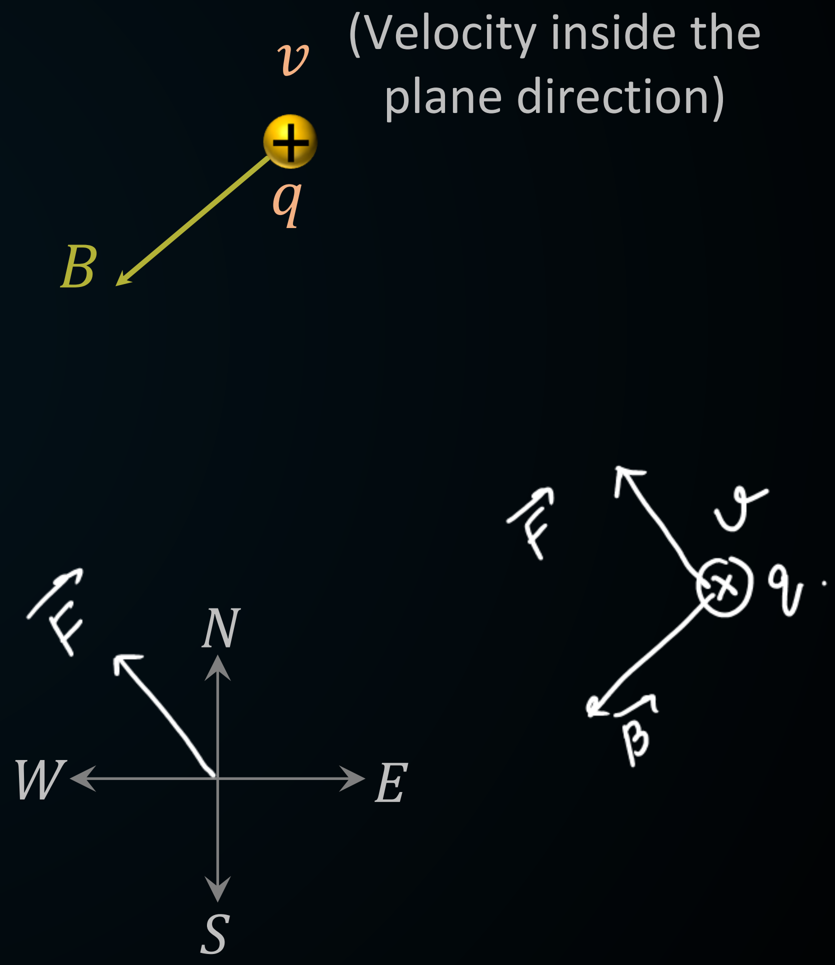
If a magnetic field is acting in the west-south direction and velocity of the charge particle is inside the plane as shown, then find the direction of magnetic force.

Solution

Here, the charge q is $+ve$.

Thus, direction of force is same as $(\vec{v} \times \vec{B})$.

As $\vec{v}$ is directed inside the screen and $\vec{B}$ is towards South-West, $\vec{v} \times \vec{B}$ is directed towards North-West.



North-west direction

Question

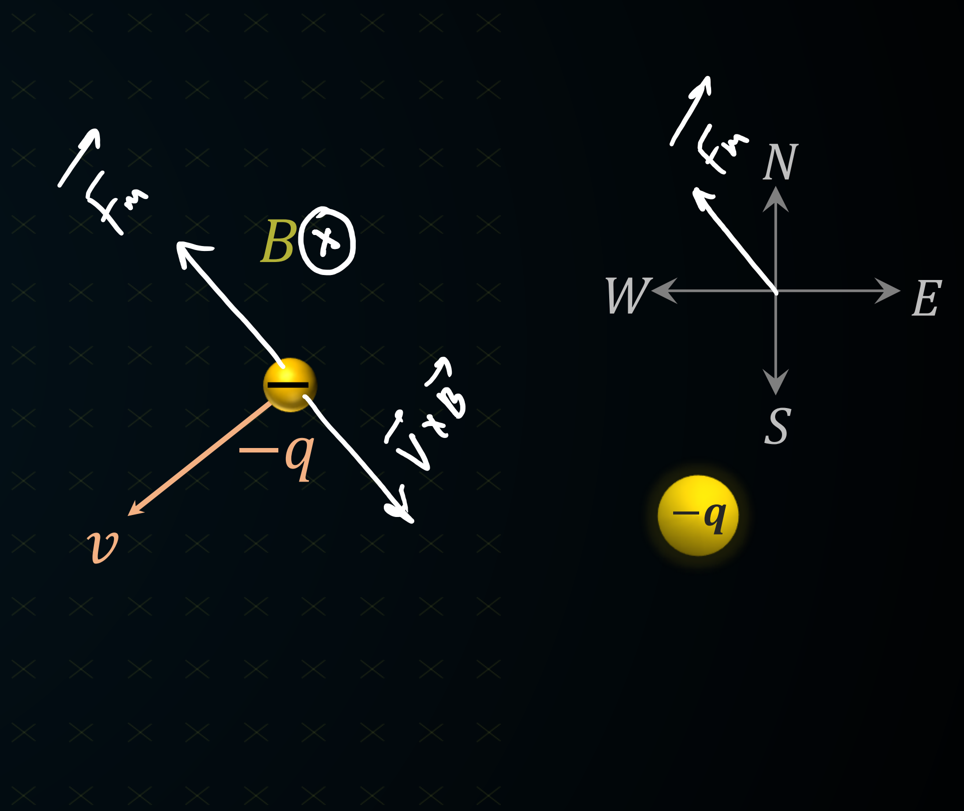
If a magnetic field is acting inside the plane and velocity of the charge particle is in west-south direction as shown, then find the direction of magnetic force.

Solution

Here, the charge q is $-ve$.

Thus, direction of force is opposite to $(\vec{v} \times \vec{B})$.

As $\vec{v}$ is directed towards South-West and $\vec{B}$ is moving in the plane of paper, $\vec{v} \times \vec{B}$ is directed towards South-East. As the force is opposite to $\vec{v} \times \vec{B}$, it will be directed towards North-West.

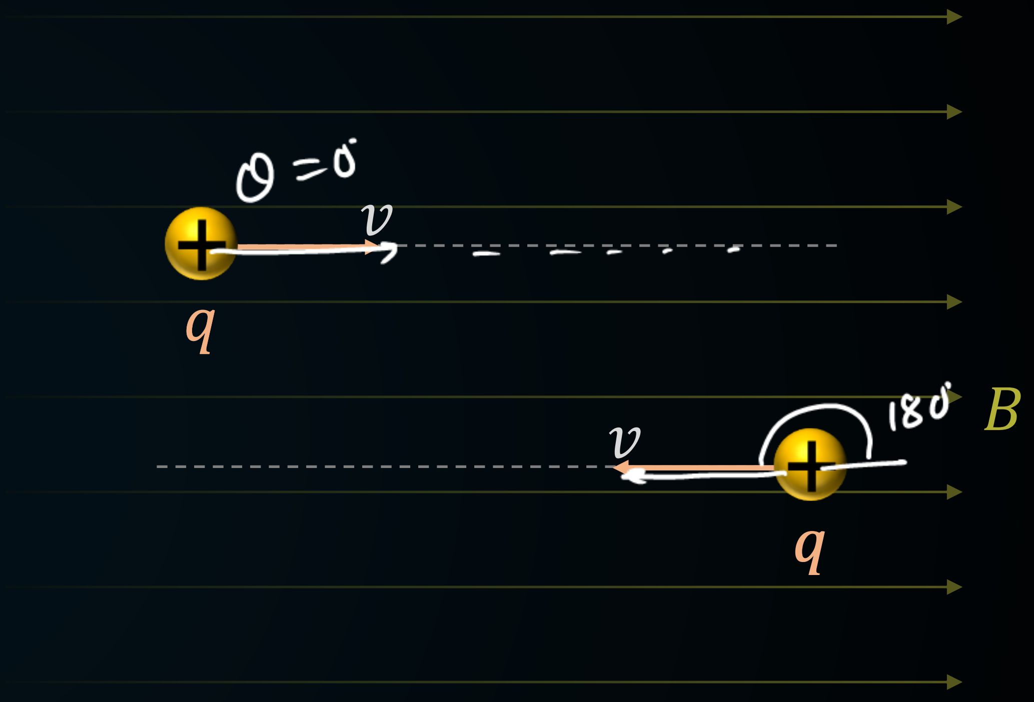


North-west direction

Force on a moving charge in magnetic field

Case 1: When $\vec{v}$ is parallel/antiparallel to $\vec{B}$

- When the velocity of the charge is parallel or antiparallel to the direction of magnetic field, the force acting on it will be zero and the path of the particle will be a straight line.



$$|\vec{F}| = q|\vec{v}||\vec{B}|\sin\theta$$

$\boxed{F = 0}$

$\theta = 0$ or $\theta = 180°$

$\sin\theta = 0$ $\quad$ $\sin 180° = 0$

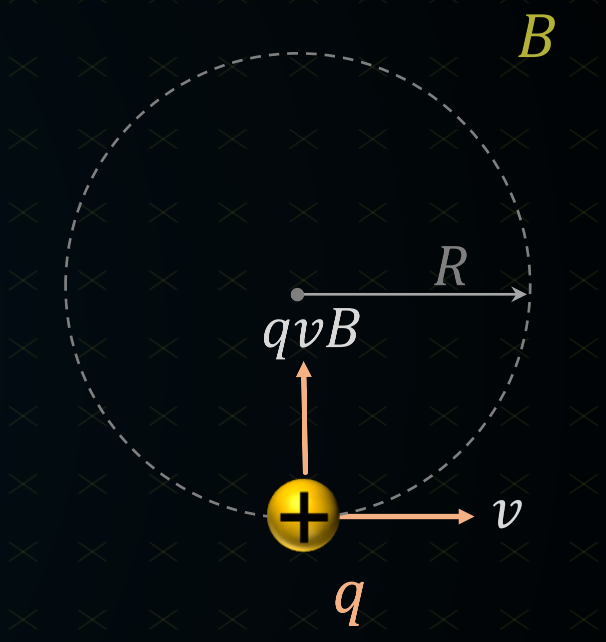
$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

Force on a moving charge in magnetic field

Case 2: When $\vec{v}$ is perpendicular to $\vec{B}$

when $\vec{V} \perp \vec{B}$ $\quad F_m = q\, vB \cdot \sin 90^\circ = \boxed{qvB}$ → Path will be Circular

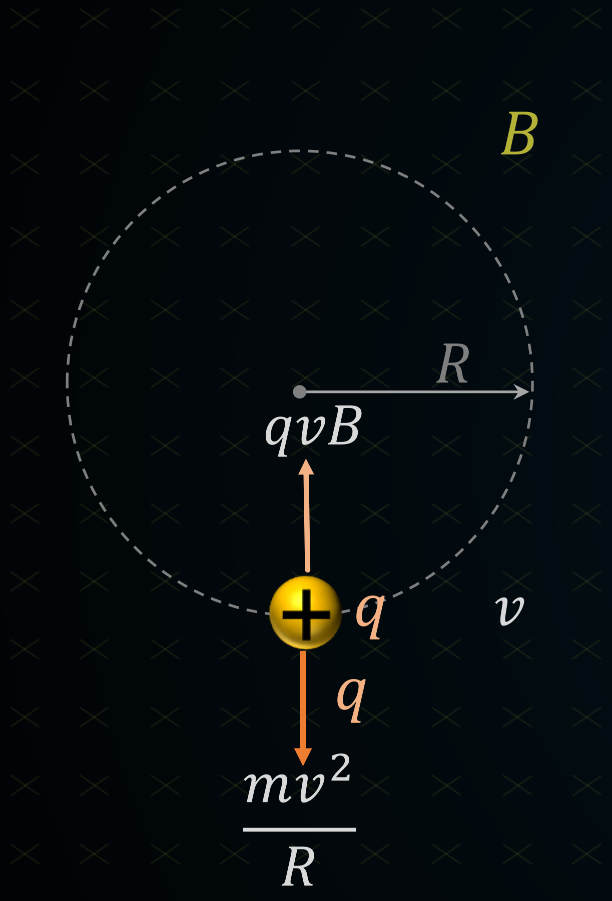
- Magnitude of magnetic force is always fixed and given by $|\vec{F}_m| = qvB$
- Direction of magnetic force will change due to change in direction of velocity.
- Magnetic force is always perpendicular to velocity, therefore charge particle will execute the uniform circular motion.



$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

Force on a moving charge in magnetic field

Case 2: When $\vec{v}$ is perpendicular to $\vec{B}$



- The magnetic force provides necessary centripetal force for UCM. Thus, the radius of circle can be obtained as:

$$qvB = \frac{mv^2}{R} \Rightarrow \boxed{R = \frac{mv}{qB}}$$

- Time period of the uniform circular motion is given by

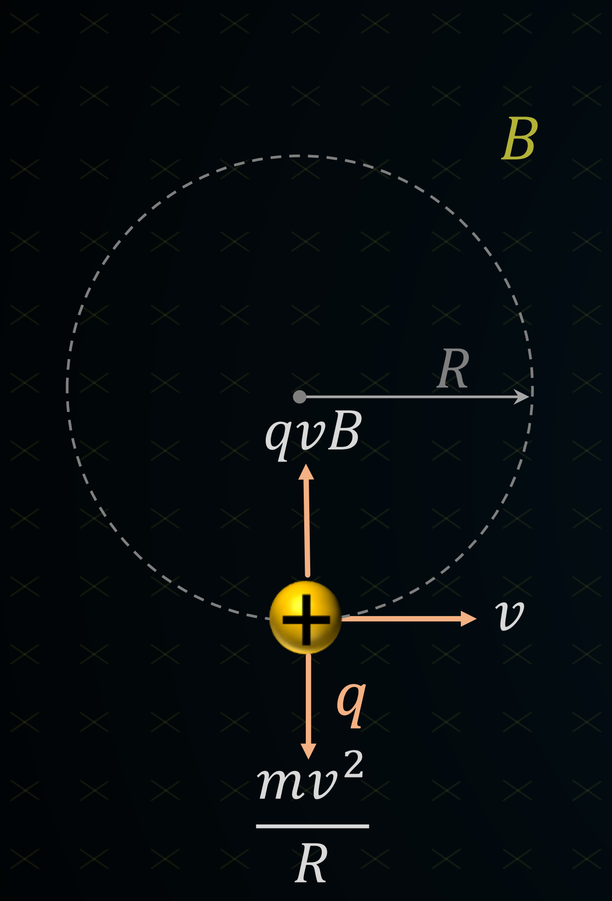
$$T = \frac{2\pi R}{v} \Rightarrow \boxed{T = \frac{2\pi m}{qB}}$$

- Time period is independent of velocity

Force on a moving charge in magnetic field

Case 2: When $\vec{v}$ is perpendicular to $\vec{B}$

$$T = \frac{2\pi m}{qB}$$



Frequency of the uniform circular motion is given by,

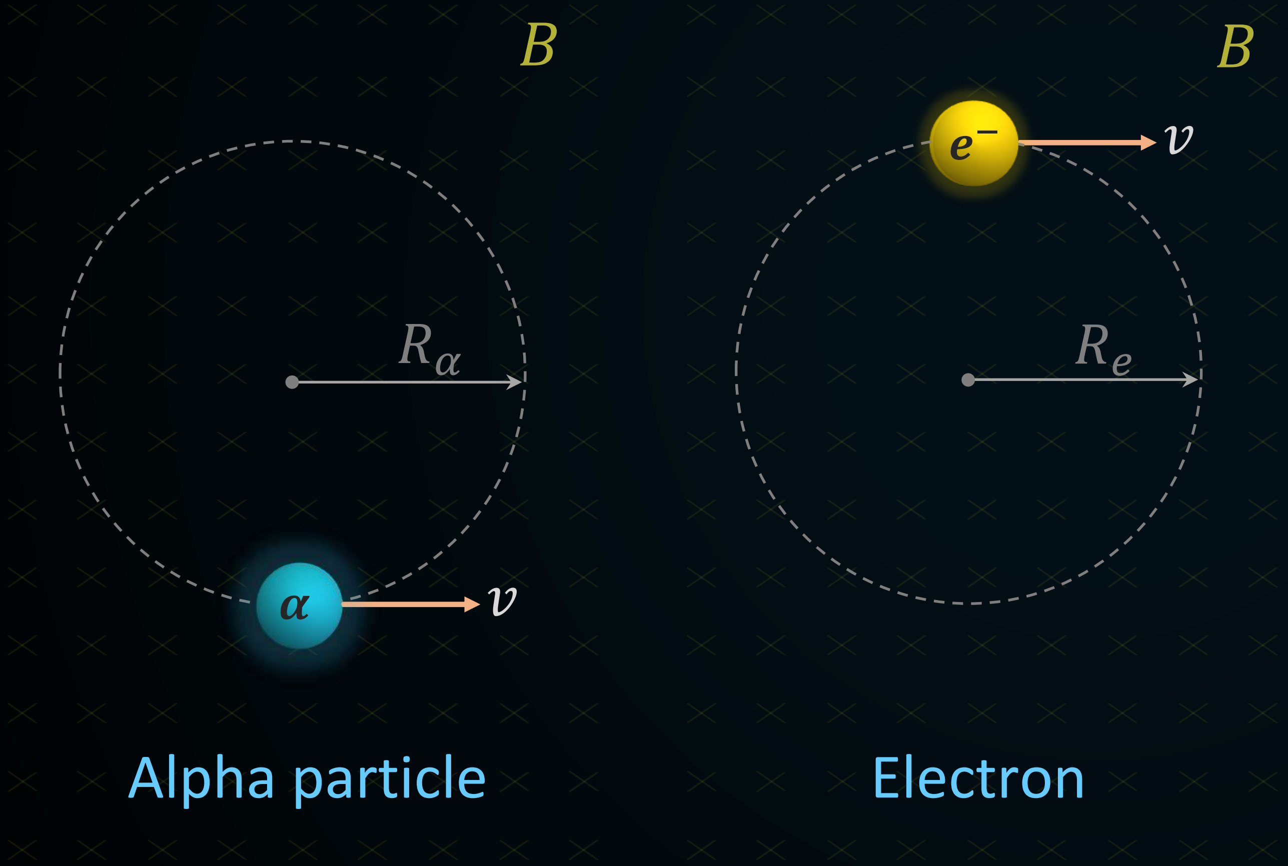
$$f = \frac{1}{T} \Rightarrow \boxed{f = \frac{qB}{2\pi m}}$$

Angular frequency of the uniform circular motion is given by,

$$\omega = \frac{2\pi}{T} \Rightarrow \boxed{\omega = \frac{qB}{m}}$$

Question Solution

In a magnetic field of $2\,T$, an alpha particle and an electron are released with the same velocity. Find the ratio of radii of their circular trajectories.
Given that $m_\alpha = 7294 \times m_e$.



Question | Solution

Given : $B_1 = B_2 = 2\ T$
$m_\alpha = 7294 \times m_e$

$$\boxed{R = \frac{mv}{qB}}$$

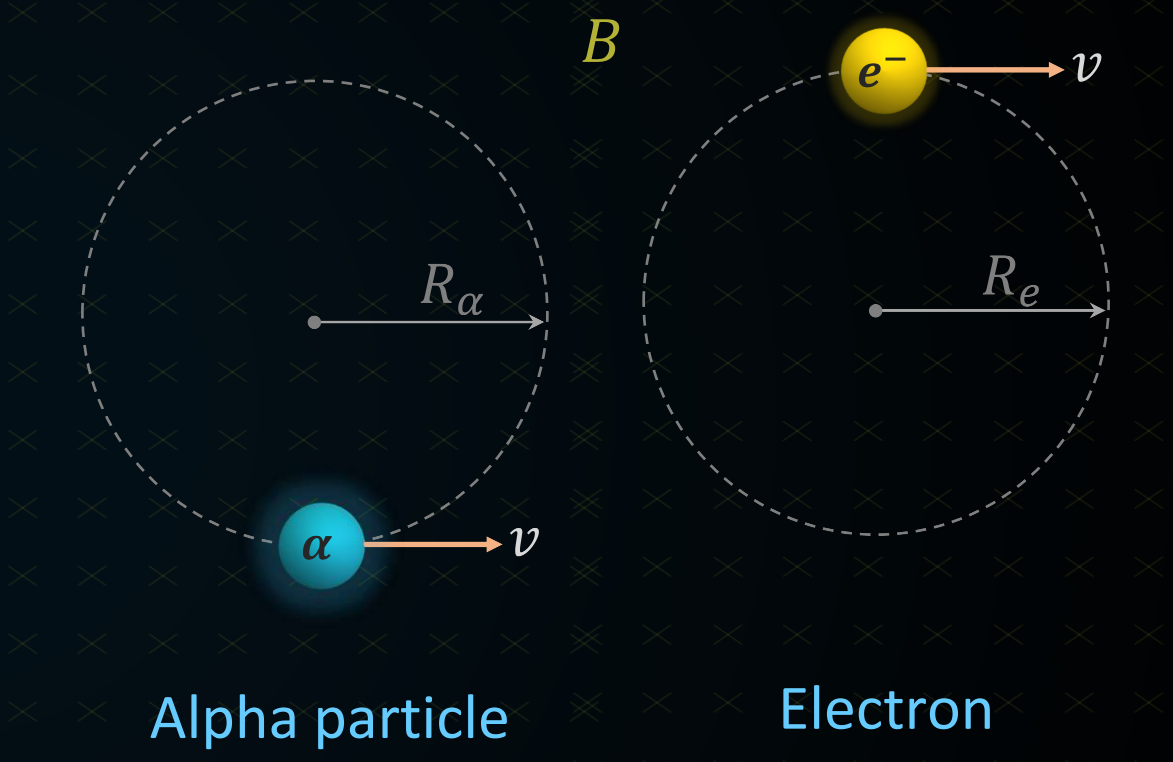
Radius of circle for Alpha particle: $R_\alpha = \dfrac{m_\alpha \cdot v}{+2e \cdot 2}$ .....(1)

Radius of circle for electron: $R_e = \dfrac{m_e \cdot v}{e \cdot 2}$ ....(2)

Dividing equation (1) by (2); $\dfrac{R_\alpha}{R_e} = \dfrac{m_\alpha \cdot \cancel{v}\ \cancel{2e}}{\cancel{4e}_{\,2} \cdot m_e \cdot \cancel{v}}$

$$\frac{R_\alpha}{R_e} = \frac{m_\alpha}{m_e} \times \frac{1}{2} = \frac{\overset{3647}{\cancel{7294}}}{\cancel{2}}$$

$$\boxed{\frac{R_\alpha}{R_e} = 3647}$$



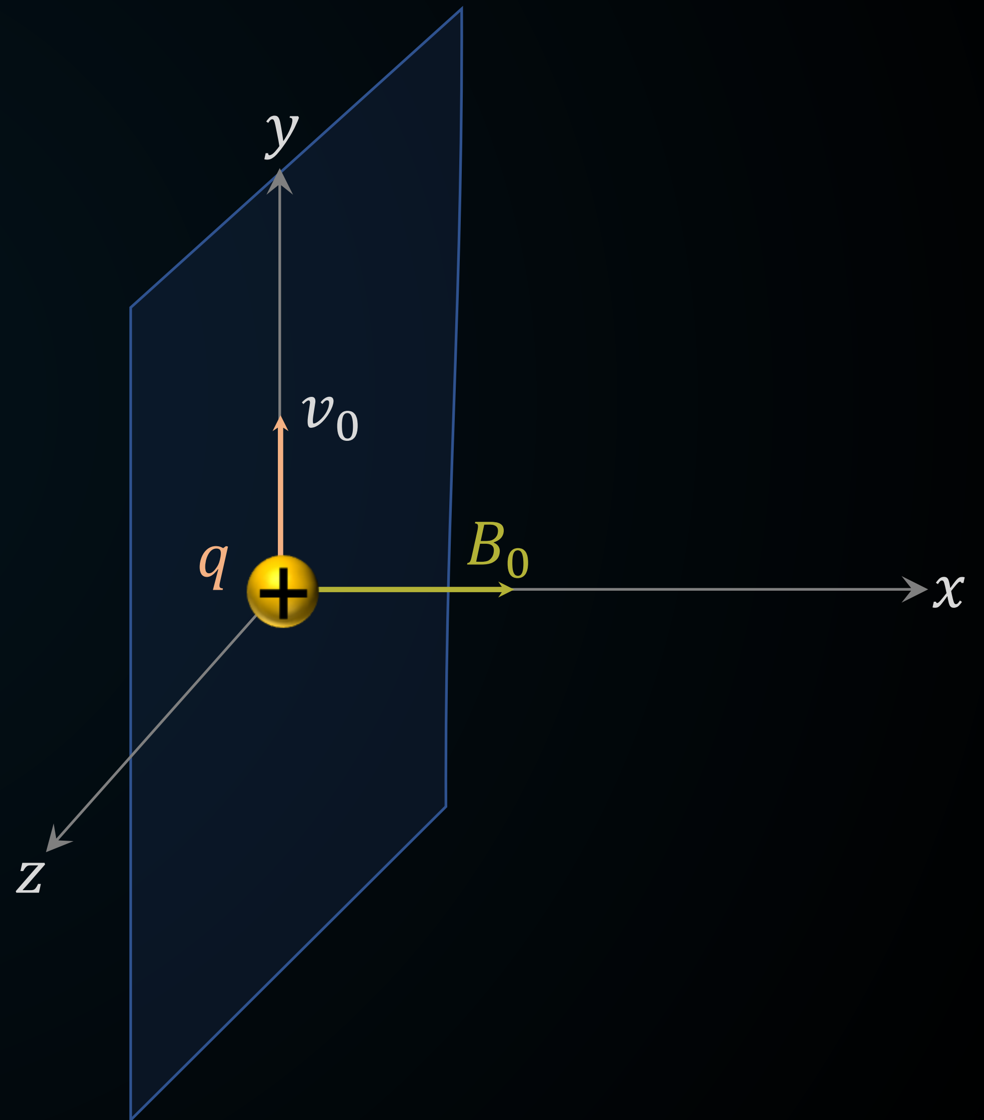
Question

BOARDS NEET

A charge particle of mass m placed at the origin as shown where magnetic field $\vec{B} = B_0\,\hat{\imath}$ and velocity $\vec{v} = v_0\,\hat{\jmath}$. Find the position vector $\vec{r}(t) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ after time t.

Solution

As $(\vec{v} \times \vec{B})$ is directed along $-ve\ z$-axis, the circle will be in $y - z$ plane.



Thus, value of x will always be zero in position vector.

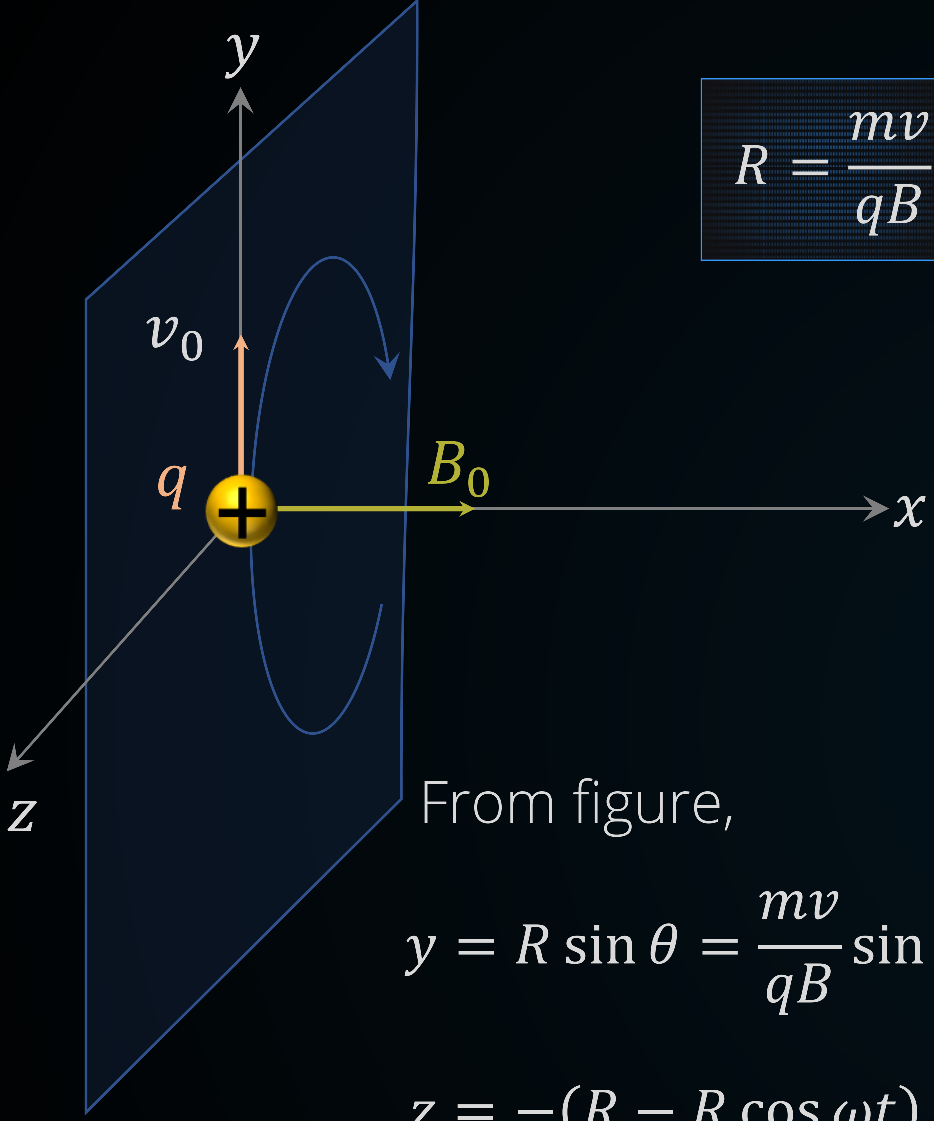
We know, the radius of circle, $R = \frac{mv}{qB}$

Hence, the coordinate of centre of circle, $\left(0, 0, -\frac{mv}{qB}\right)$

At time t, consider the particle moves to $\theta = \omega t$, where $\omega = \frac{qB}{m}$.

$$R = \frac{mv}{qB}$$

$$\omega = \frac{qB}{m}$$



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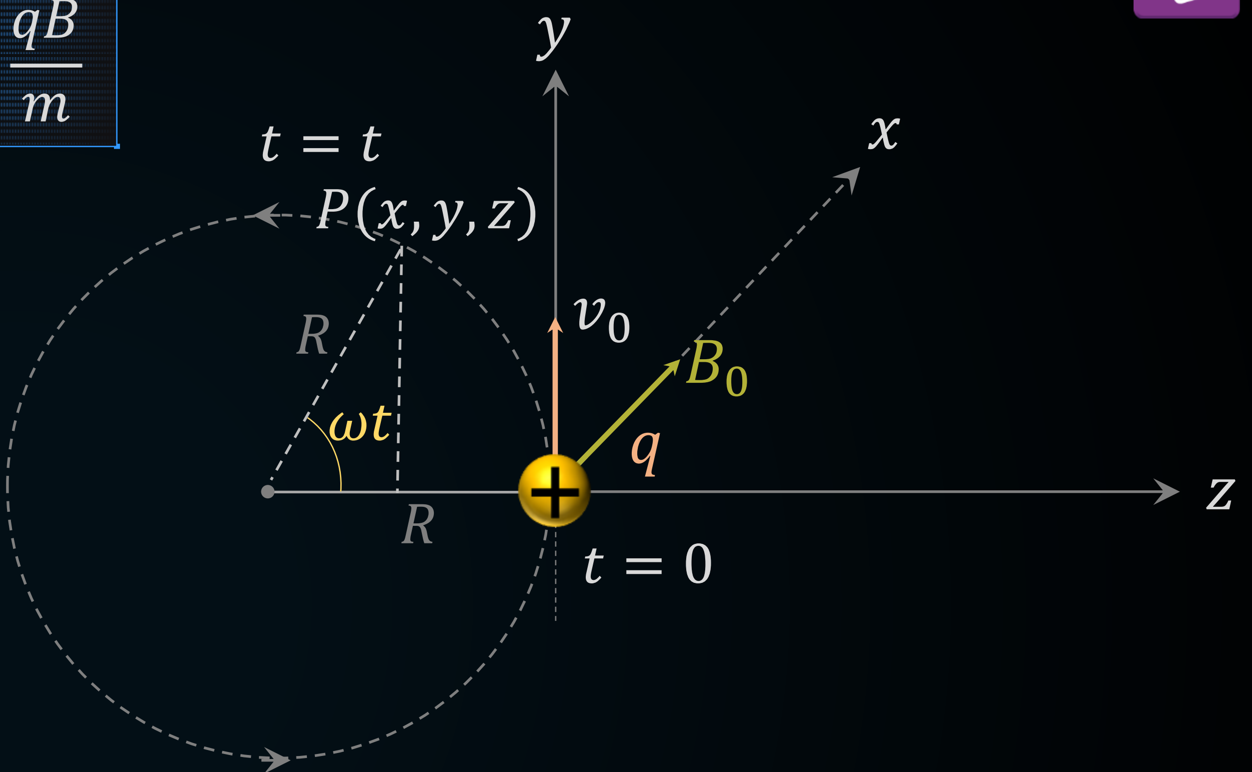


From figure,

$$y = R\sin\theta = \frac{mv}{qB}\sin\left(\frac{qB}{m}\right)t$$

$$z = -(R - R\cos\omega t) = -\frac{mv}{qB}[1 - \cos\left(\frac{qB}{m}\right)t$$

$$\boxed{x = 0,\ y = \frac{mv}{qB}\sin\left(\frac{qB}{m}\right)t,\ z = -\frac{mv}{qB}\left(1 - \cos\left(\frac{qB}{m}\right)t\right)}$$

Question

A charge particle of mass m placed at the origin as shown where magnetic field $\vec{B} = -B_0\,\hat{j}$ and velocity $\vec{v} = v_0\,\hat{k}$. Find the position vector $\vec{r}(t) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ after time t.

Solution

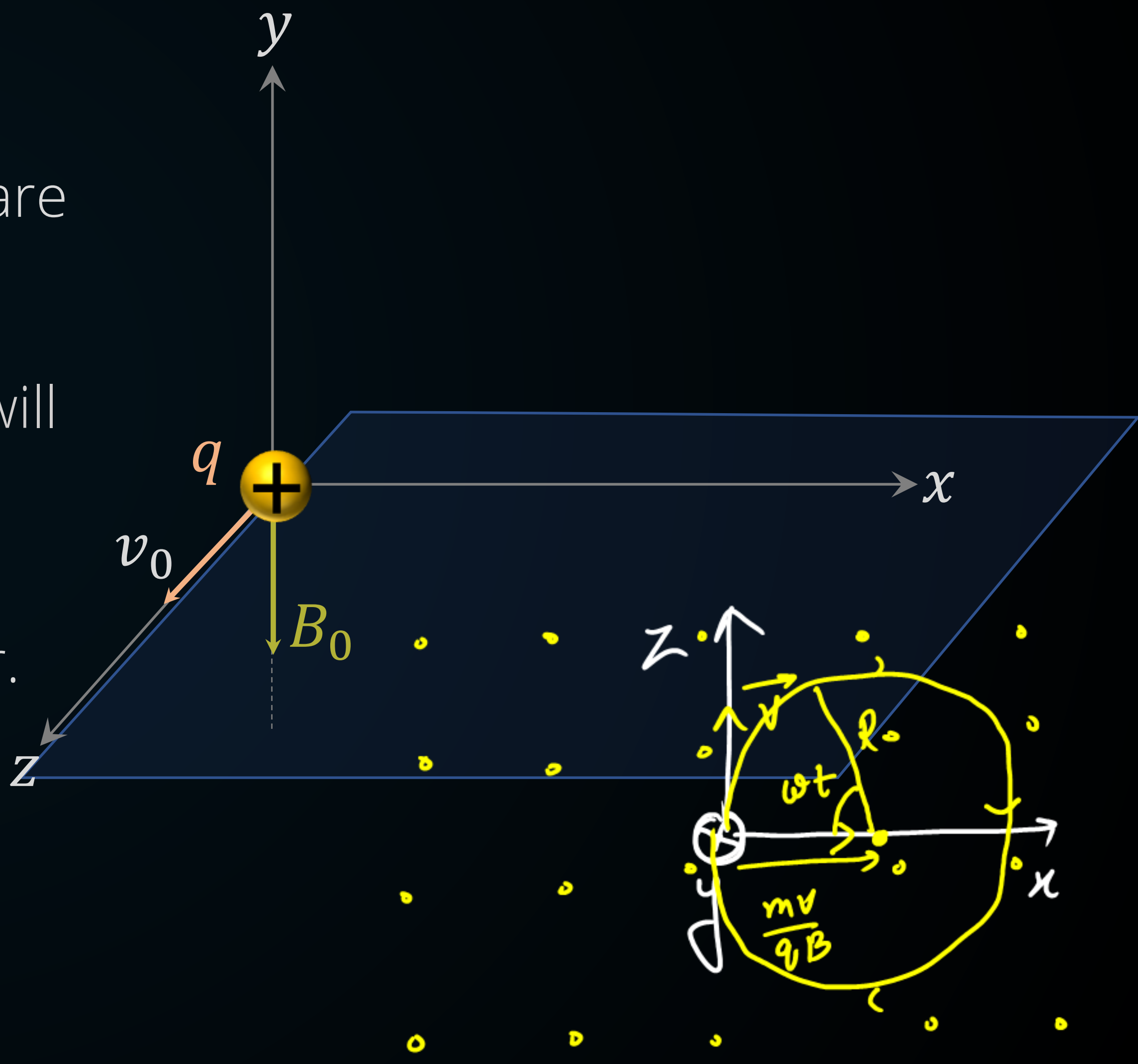
Here, the directions of magnetic field and velocity are changed. Rest of the problem is same as previous.

As $(\vec{v} \times \vec{B})$ is directed along $+ve\ x$ -axis, the circle will be in $x - z$ plane.

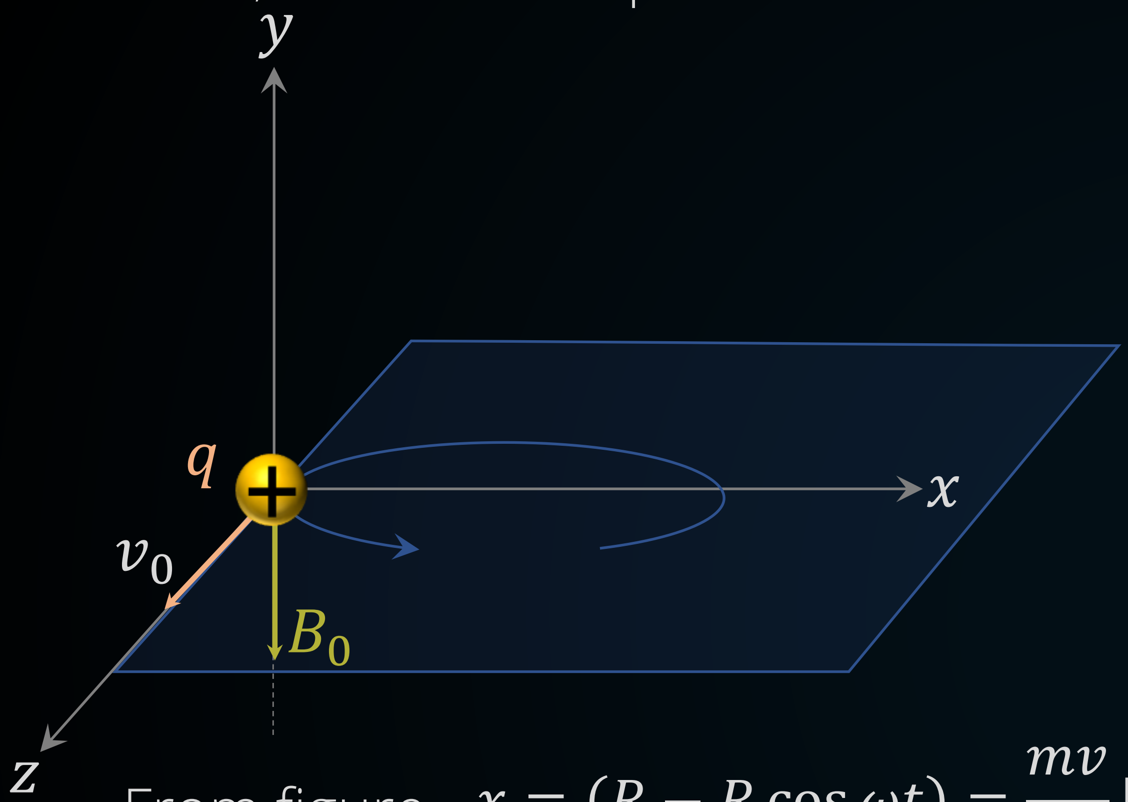
Thus, value of y will always be zero in position vector.

We know, the radius of circle, $R = \frac{mv}{qB}$

Hence, the coordinates of centre of circle, $(\frac{mv}{qB}, 0, 0)$



At time t, consider the particle moves to $\theta = \omega t$, where $\omega = \frac{qB}{m}$.



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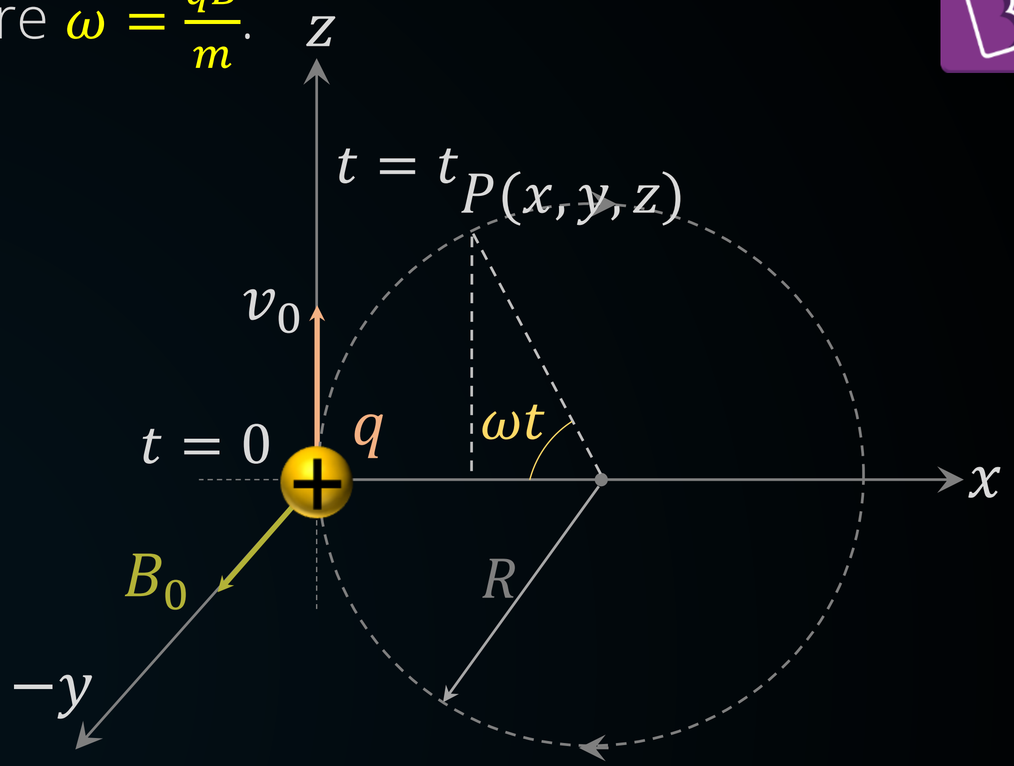


From figure, $x = (R - R\cos\omega t) = \frac{mv}{qB}\left[1 - \cos\left(\frac{qB}{m}\right)t\right.$

$$z = R\sin\omega t = \frac{mv}{qB}\sin\left(\frac{qB}{m}\right)t$$

$$\boxed{x = \frac{mv}{qB}\left(1 - \cos\left(\frac{qB}{m}\right)t\right),\ y = 0,\ z = \frac{mv}{qB}\sin\left(\frac{qB}{m}\right)t}$$

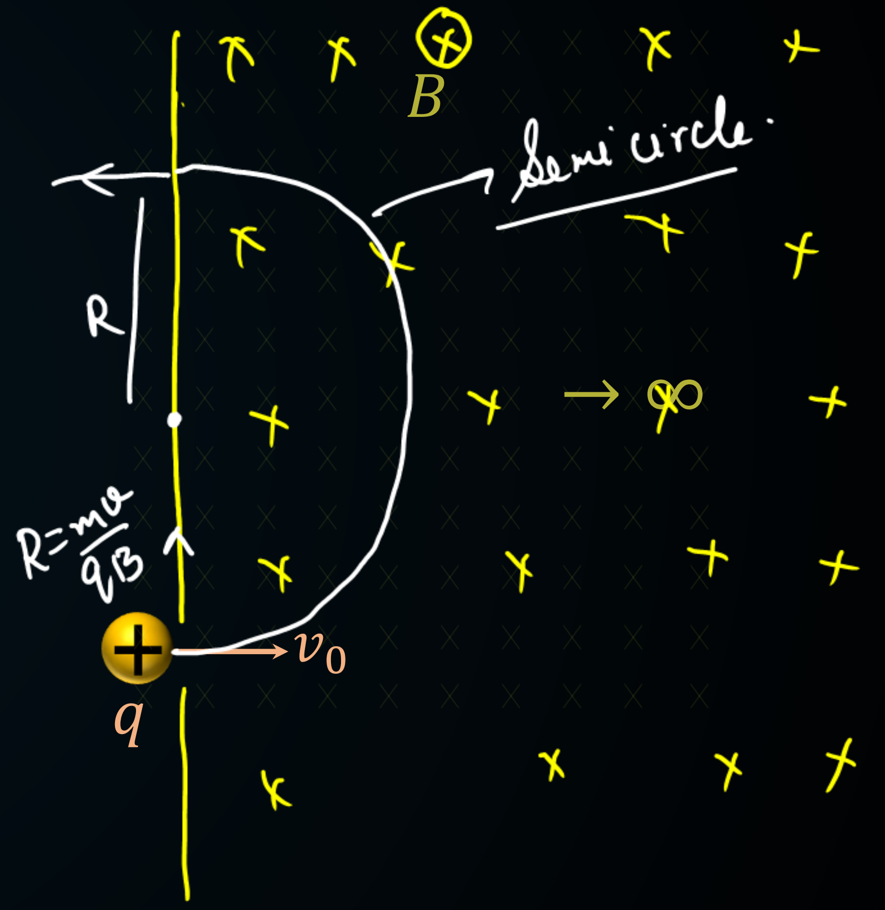
Question

A charged particle of mass m enters a magnetic field B with velocity v_0 as shown. Find the deviation and time spent by the charged particle in the magnetic field.

Solution

Velocity of particle is perpendicular to magnetic field.

The force acts on in upward direction. Hence, particle moves in circular path in magnetic field. It completes a semi-circle and then moves in straight line as magnetic field in not present.



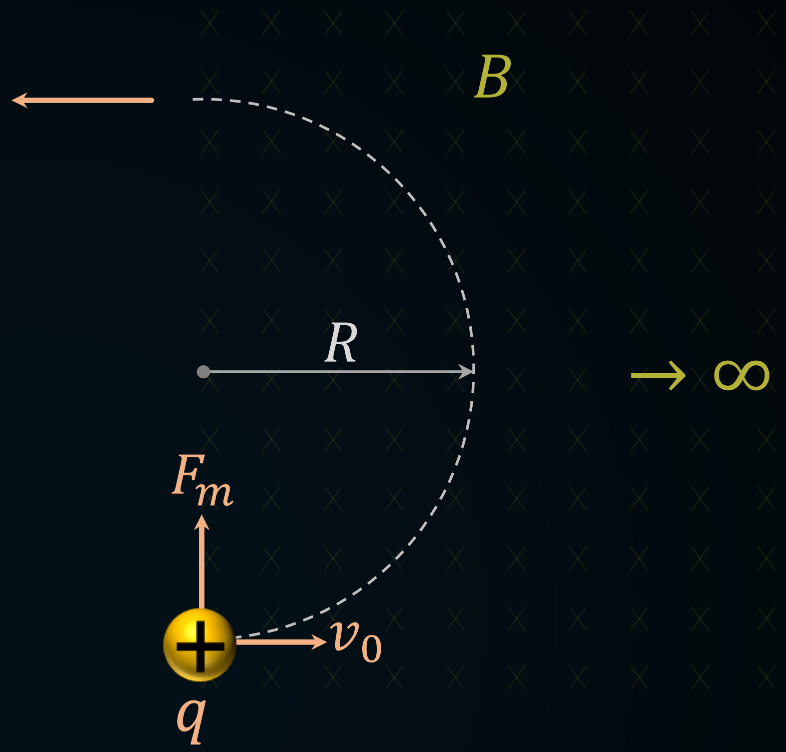
As the particle completes semicircular path in magnetic field, the deviation is $180°$ or π rad.

We have, $\underline{\theta = \omega t}$.

$$\pi = \frac{qB}{m} \cdot t$$

Thus, time spent by particle,

$$\boxed{t = \frac{\pi m}{qB}}$$



$t = \dfrac{\pi m}{qB}$, 180° devaition

Question

A charged particle of mass m enters a magnetic field B with velocity v_0 at an angle θ as shown. Find the time spent by the charged particle in the magnetic field.

Solution

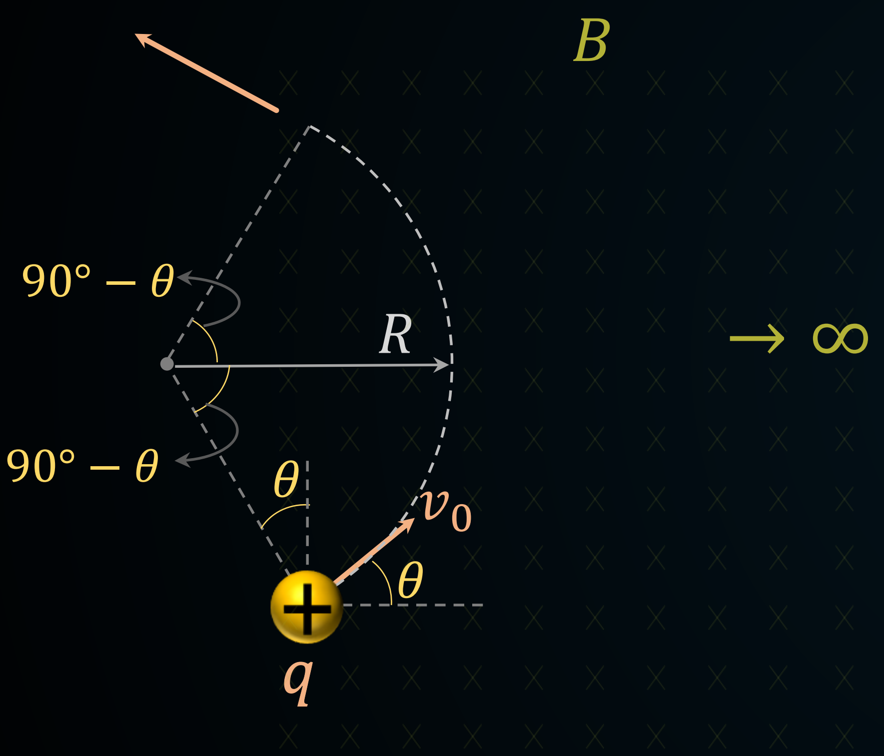
Velocity of particle is perpendicular to magnetic field.

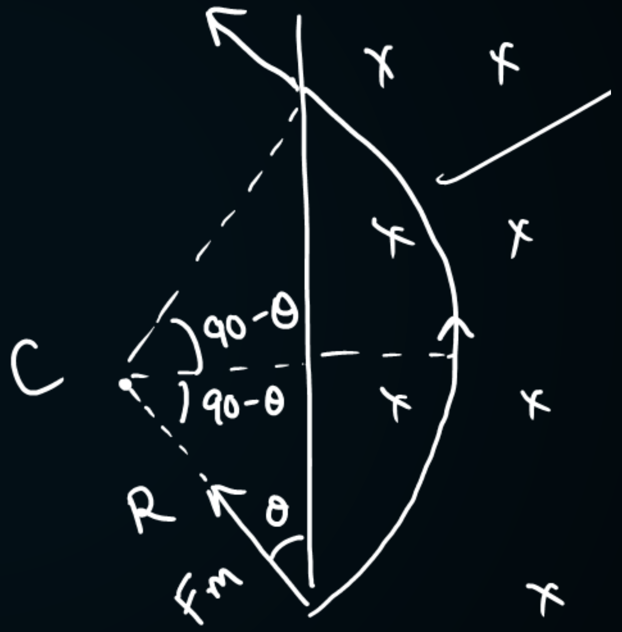
Hence the force acts along $\vec{v} \times \vec{B}$ as shown in the figure. Centre of the circle lies on the line along the direction of force at radius,

$$R = \frac{mv}{qB}.$$

From figure, angular displacement of particle inside magnetic field is $(\pi - 2\theta)$.

We have, $\pi - 2\theta = \frac{qB}{m} \cdot t \Rightarrow t = \frac{(\pi - 2\theta)m}{qB}$





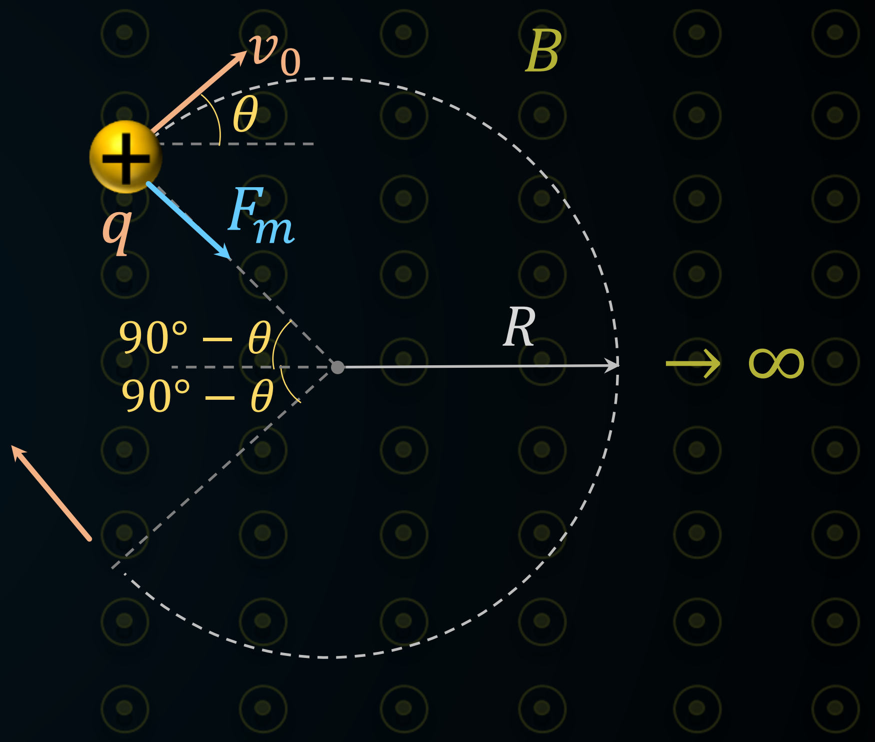
$$\boxed{t = \frac{m}{qB}(\pi - 2\theta)}$$

Question Solution

A charge particle of mass m enter in the magnetic field B with velocity v_0 at an angle θ as shown. Find the time spent in the magnetic field by the charge particle.

Question Solution

Velocity of particle is perpendicular to magnetic field. Hence the force acts along $\vec{v} \times \vec{B}$ as shown in the figure. Centre of the circle lies along the direction of force with radius, $R = \frac{mv}{qB}$.



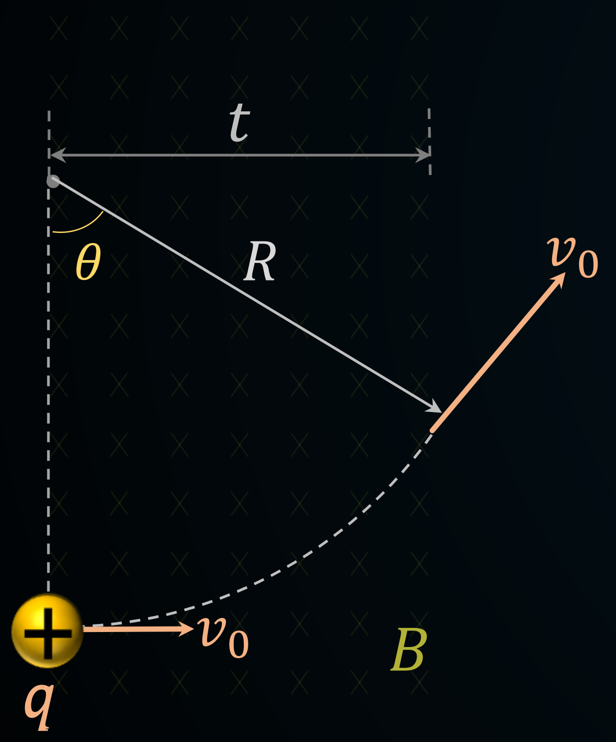
From figure, angular displacement of particle inside magnetic field is, $2\pi - (\pi - 2\theta) = (\pi + 2\theta)$.

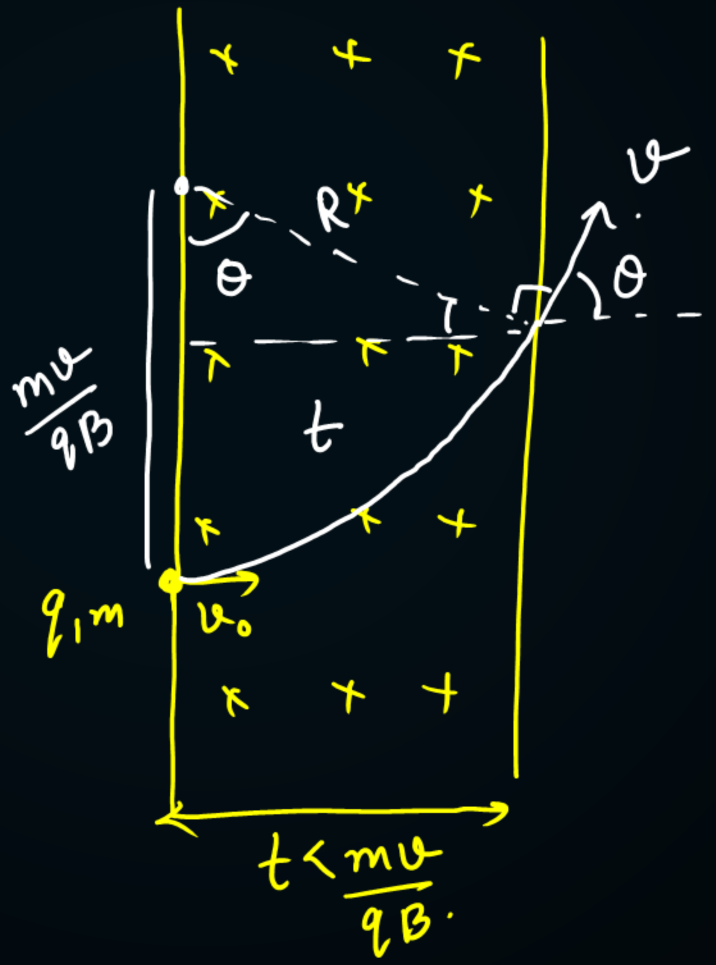
We have, $(\pi + 2\theta) = \frac{qB}{m} \cdot t \Rightarrow t = \frac{m(\pi + 2\theta)}{qB}$

$$\boxed{t = \frac{m}{qB}(\pi + 2\theta)}$$

Question Solution

A charge particle of mass m enter in the magnetic field B with velocity v_0 as shown. Find the deviation of the charge particle. Given that $t < R$.

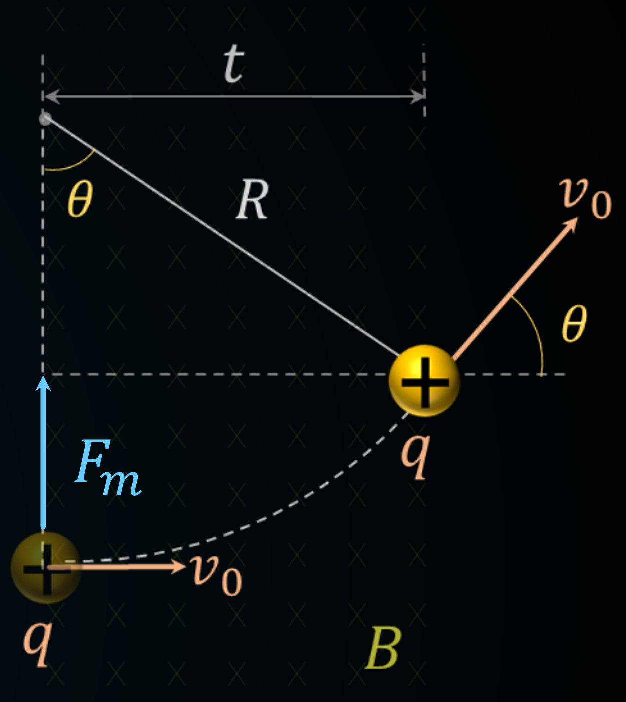




Question Solution

Velocity of particle is perpendicular to magnetic field.

Hence the force acts along $\vec{v} \times \vec{B}$ as shown in the figure. Centre of the circle lies on the direction of force at radius, $R = \frac{mv}{qB}$.



As the width of magnetic field space (t) is less than the radius of circle, particle can not complete the semi-circular path. It exit from magnetic field at angle θ with horizontal as shown in figure.

From figure, when particle exits magnetic field,

$$\sin\theta = t/R$$

$$\Rightarrow \theta = \sin^{-1}(t/R)$$

$$\boxed{\theta = \sin^{-1}\left(\frac{qBt}{mv_0}\right)}$$

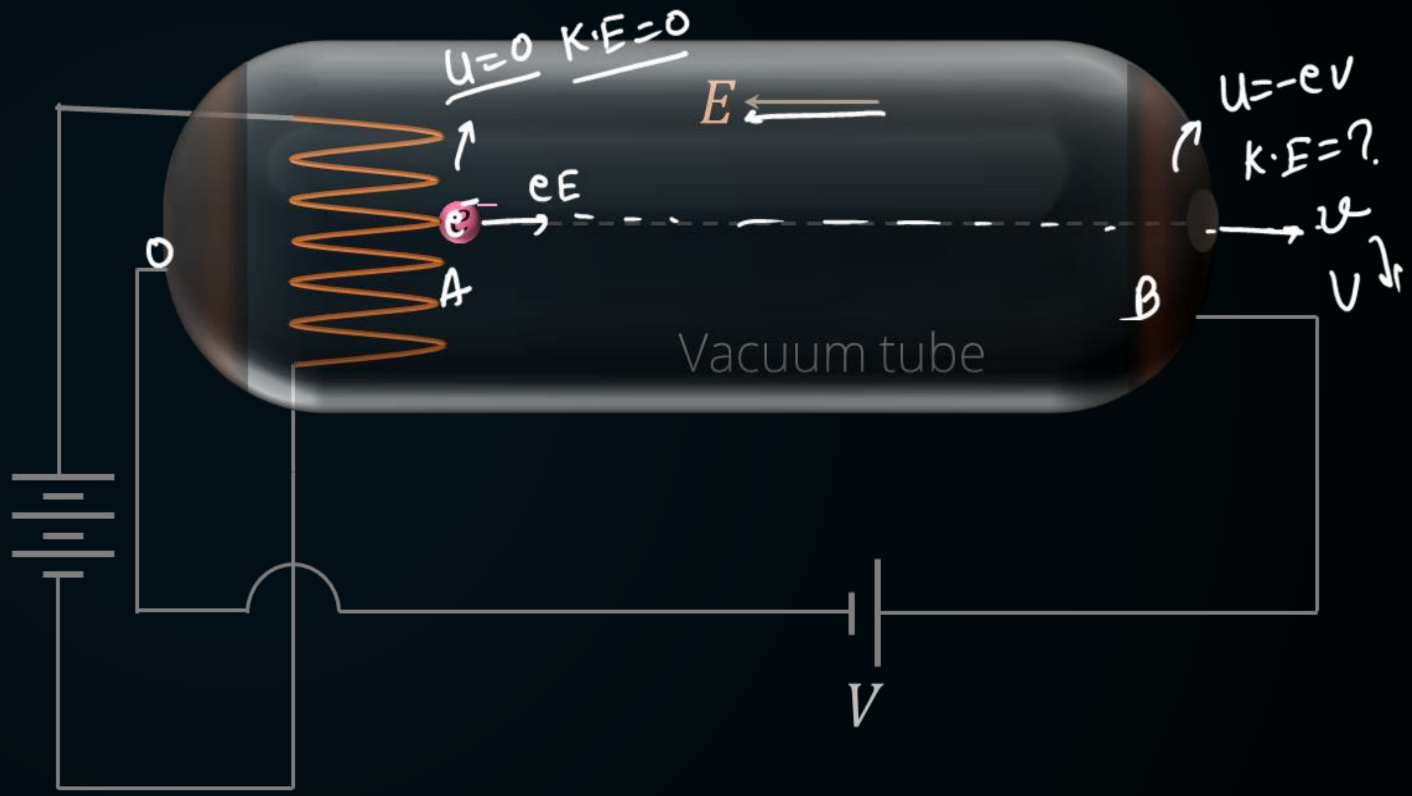
Force on a moving charge in magnetic field

Electron Gun

- An electron gun is a source of focused and accelerated electron beam

- Power source heats up the filament coil and electrons are emitted by thermionic emission.

- A battery is connected across the vacuum tube through which accelerated electrons move out.



From energy balance between points A and B,

$K \cdot E_B + U_B = K \cdot E_A + U_A$

$K \cdot E - eV = 0 + 0$

$\boxed{K \cdot E_B = eV}$ $\boxed{eV = \frac{1}{2} m_e v^2}$

- Thus, electron gun provides highly accelerated electrons that are used for experiments.

Electron Gun

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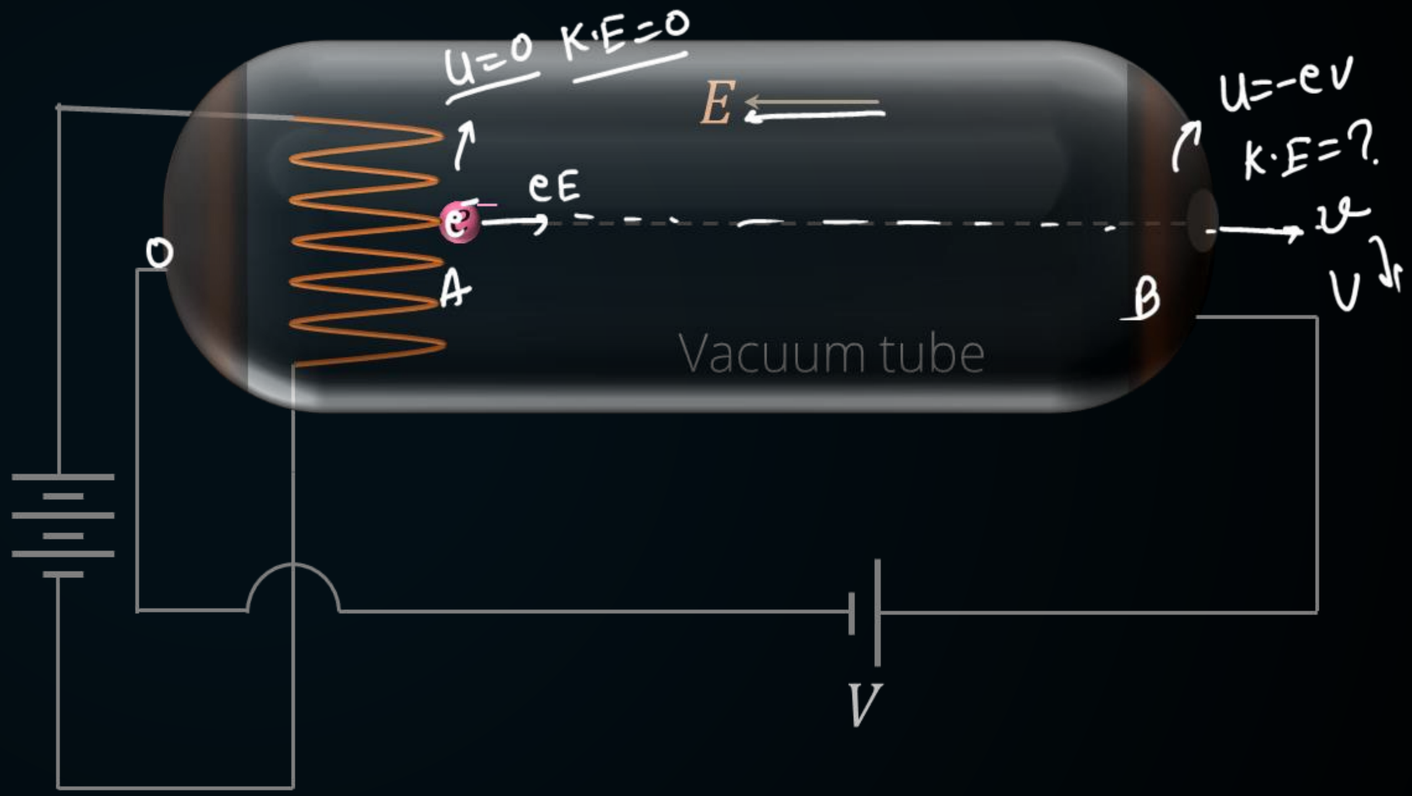

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$$\boxed{K \cdot E_B = eV} \qquad \boxed{eV = \frac{1}{2} m_e v^2}$$

- Thus, electron gun provides highly accelerated electrons that are used for experiments.

Electron Gun

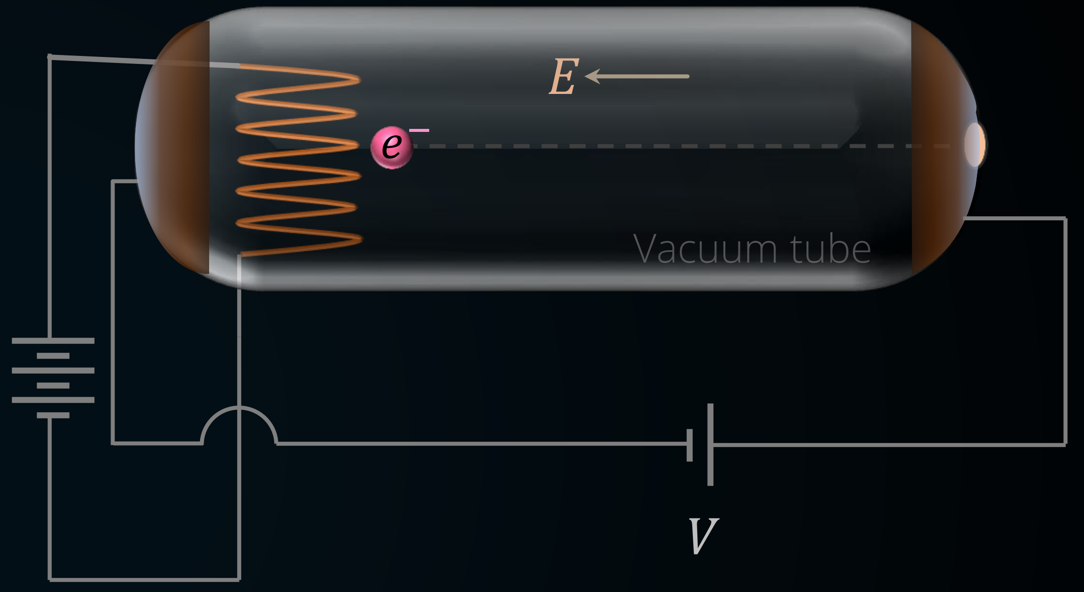
- Velocity of an electron of mass m accelerated through the potential V is given by,

$$eV = \frac{1}{2} m v^2$$

$$\boxed{v = \sqrt{\frac{2eV}{m}}}$$

Speed

P.D

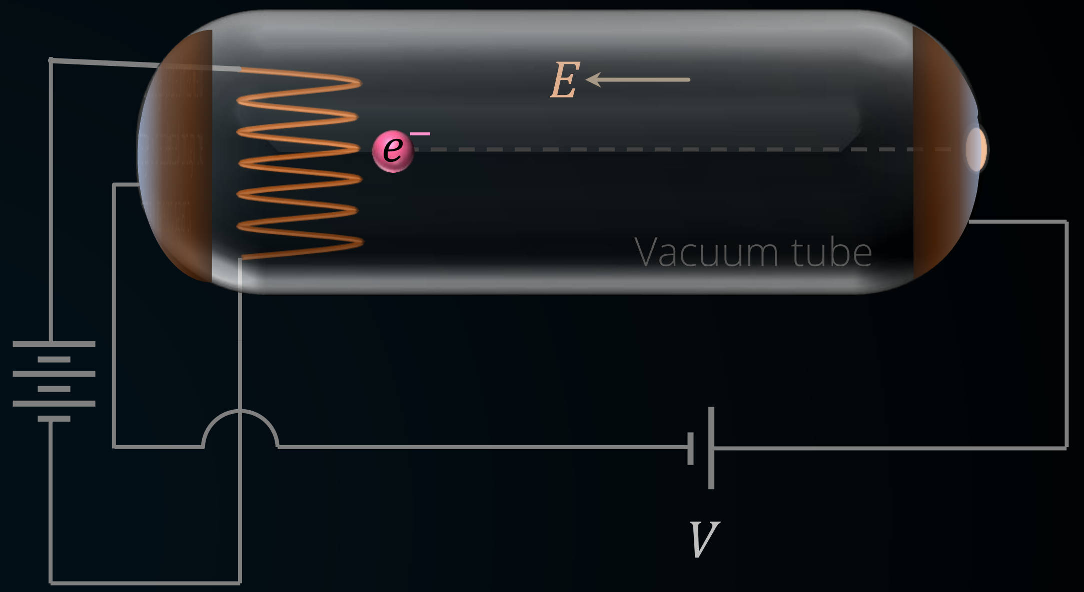


$$v = \sqrt{\frac{2eV}{m}}$$

Force on a moving charge in magnetic field

Electron Gun

- When an electron is accelerated through the potential V, its energy become eV.



- When a charged particle of mass m and charge q is accelerated through the potential V, its velocity is given by,

$$qV = \frac{1}{2}mv^2 \;\ldots\ldots (1)$$

$$\boxed{v = \sqrt{\frac{2qV}{m}}}$$

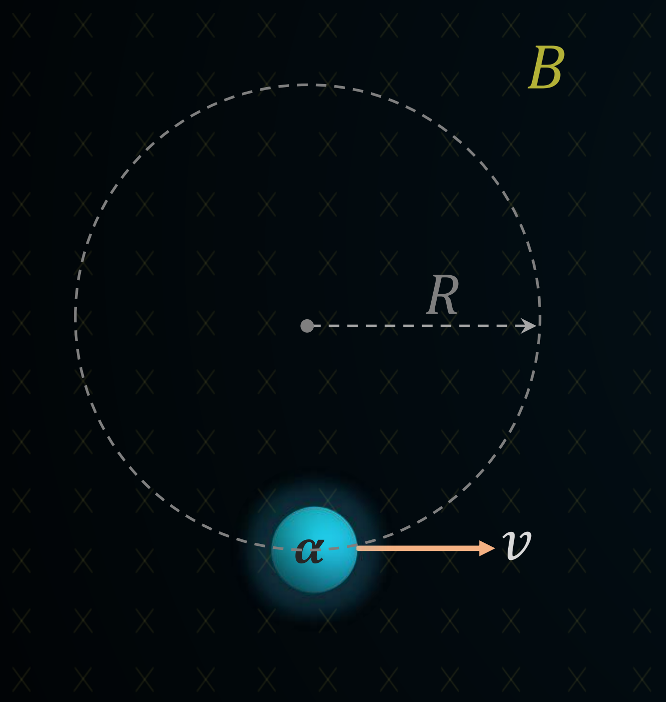
- Momentum of the particle, $P = mv$

From equation (1), $qV = K \cdot E = \frac{P^2}{2m} = \frac{1}{2}mv^2$

$$\Rightarrow P = \sqrt{2m\,K\cdot E} \quad \boxed{= \sqrt{2mqV}}$$

Question Solution

An alpha particle of mass m is accelerated through a potential V and falls perpendicularly in a magnetic field B with velocity v. Find the radius of its trajectory.



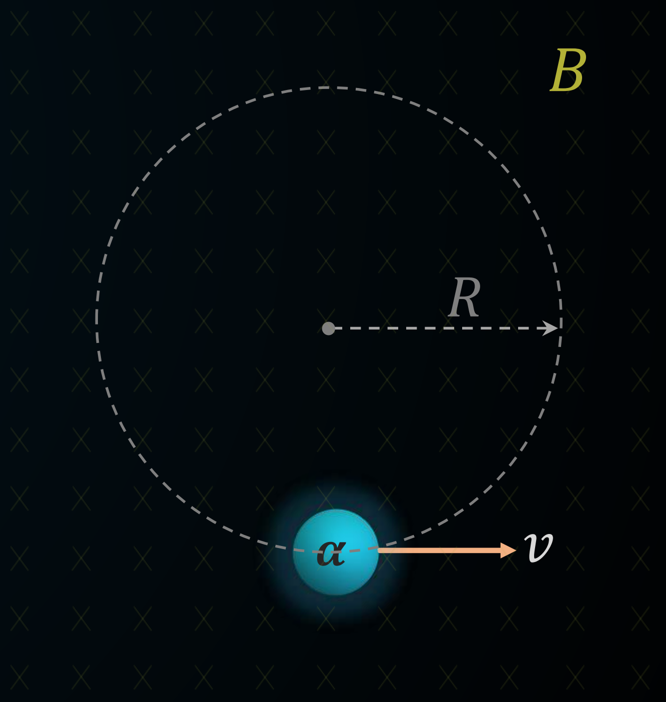
Question Solution NEET

We know that an Alpha particle is a nucleus of Helium atom. When two electrons are removed from the He atom, it becomes positively charged alpha particle. ${}_{2}^{4}He^{+2e}$

Charge on Alpha particle, $q = 2e$

As the particle is projected perpendicular to magnetic field, it moves in circular trajectory.

Momentum of the particle, $p = \sqrt{2m \cdot 2eV} = 2\sqrt{meV}$



Radius of the trajectory, $R = \frac{mv}{qB}$

$= \frac{p}{qB}$

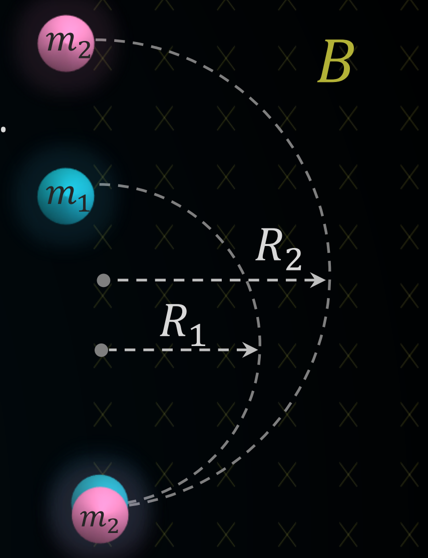
$= \frac{\cancel{2}\sqrt{meV}}{\cancel{2}e \cdot B}$

$= \sqrt{\frac{mV}{eB^2}}$

$$R = \frac{1}{B}\sqrt{\frac{mV}{e}}$$

Force on a moving charge in magnetic field

Mass spectrometry



- Mass spectrometry is used to segregate the two isotopes of same material.

Isotopes have different no. of neutrons and same no. of protons.

- When the isotopes are accelerated by same potential and projected perpendicular to the B, they travel in circular path with different radii.

As the potential is same and masses are different,

$$qV = \frac{1}{2} m_1 v_1^2 = \frac{p_1^2}{2m_1} \Rightarrow p_1 = \sqrt{2 m_1 q V}$$

$$qV = \frac{1}{2} m_2 v_2^2 = \frac{p_2^2}{2m_2} \Rightarrow p_2 = \sqrt{2 m_2 q V}$$

Radius of trajectories,

$$R_1 = \frac{\sqrt{2 m_1 q V}}{qB} = \sqrt{\frac{2 m_1 \cancel{q} V}{q^{\cancel{2}} B^2}}$$

$$R_2 = \frac{\sqrt{2 m_2 q V}}{qB} = \sqrt{\frac{2 m_2 V}{q B^2}}$$

$$\boxed{\frac{R_1}{R_2} = \sqrt{\frac{m_1}{m_2}}}$$

- Isotopes are segregated as they travel in different paths.
- Mass spectrometry is an analytical technique that is used to measure the mass-to-charge ratio of ions by measuring radius practically.

Mass spectrometry

Practically, a magnetic field space of thickness t and a ZnS screen is used. The particle is slightly deviated in the field and then moves in straight line to screen. Distance between screen (D) and the magnetic field space is more compared to thickness of field.

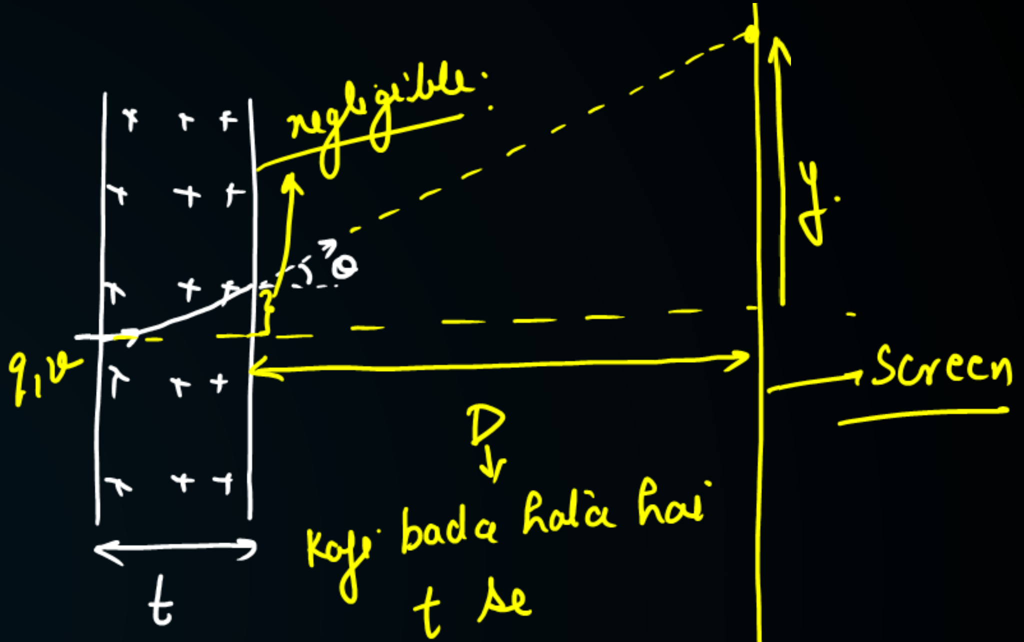
As the deviation (θ) is small, $\sin\theta$ is also small.

We have, $\sin\theta = t/R$ $\quad R = \frac{mv}{qB}$ $\quad R = \frac{\sqrt{2mqV}}{qB} = \boxed{\frac{1}{B}\sqrt{\frac{2mV}{q}}}$

From figure, $\tan\theta \simeq y/D \simeq \theta \simeq \sin\theta$

$$\Rightarrow \frac{y}{D} = \frac{t}{R} \qquad R = \frac{Dt}{y}$$

$$\boxed{\frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{Dt}{y}}$$



D ↓ Koi bada hota hai t se

negligible duri kaise nikali hai

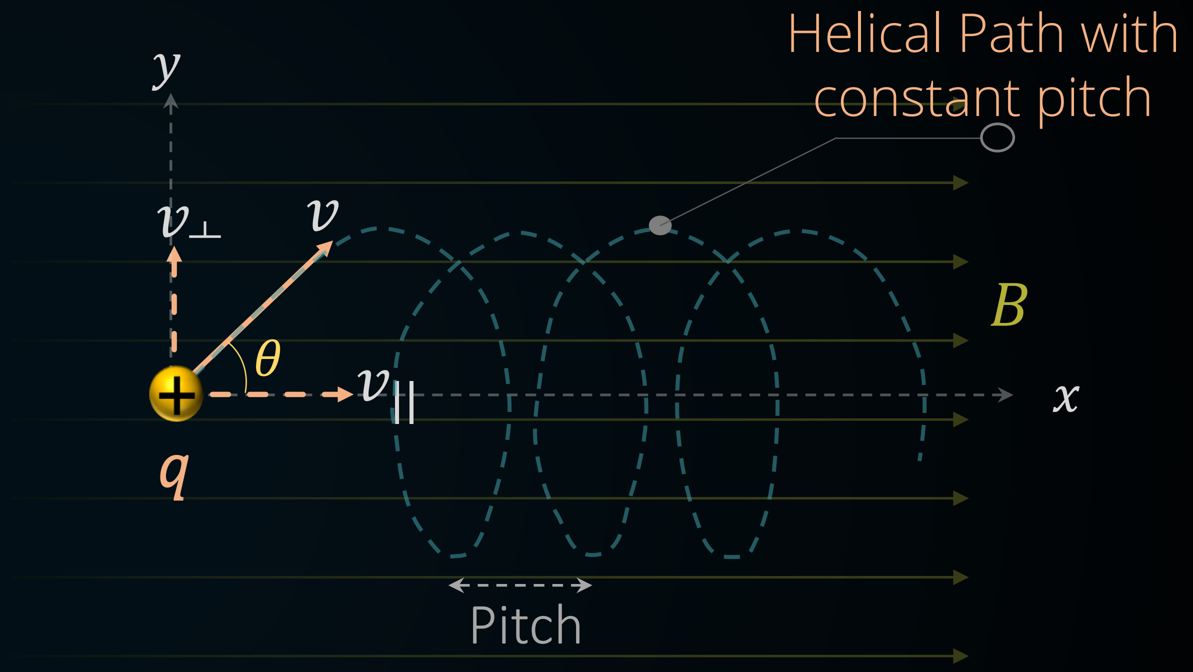
$a = \frac{qvB}{m}$ ↑

θ itna small tha ki qvB ki direction same man sakti hain.

When angle between B and v is θ

Components of velocity:

$$v_{\perp} = v\sin\theta \quad v_{||} = v\cos\theta$$



- $v_{\perp}$ imparts circular motion whereas the $v_{||}$ helps for linear motion. The resultant path becomes helical with constant pitch. Particle touches x-axis after every helix.

- Magnetic force on the charge particle is

$$\boxed{F_m = qvB\sin\theta}$$

- Radius of the circle is given by

$$R = \frac{mv_{\perp}}{qB} \qquad \boxed{R = \frac{mv\sin\theta}{qB}}$$

Force on a moving charge in magnetic field

When angle between B and v is θ

$v_{\perp} = v\sin\theta \quad v_{||} = v\cos\theta$

Time period of the particle is

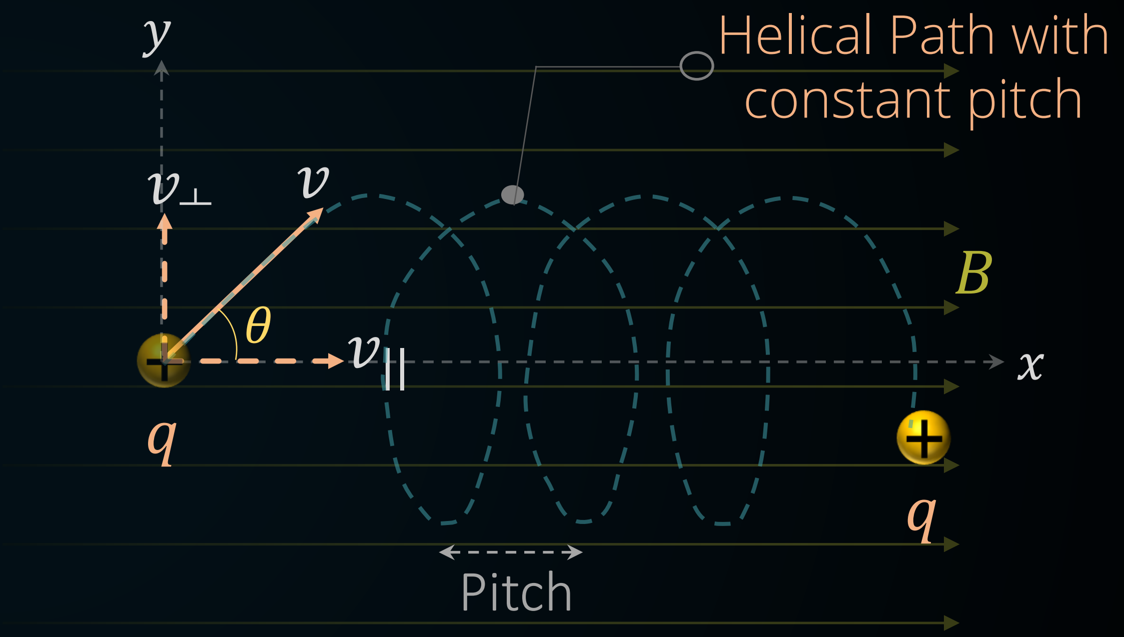
$$\boxed{T = \frac{2\pi m}{qB}}$$

T is independent of θ

Pitch (P) of the circle is given by

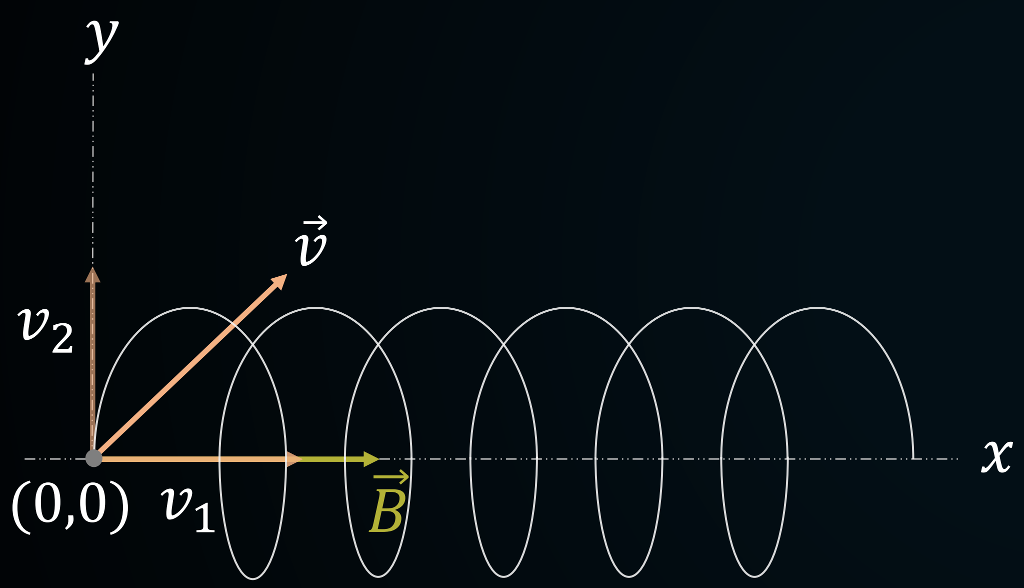
$P = v_{||} \times T$

$$\boxed{P = v\cos\theta\,\frac{2\pi m}{qB}}$$



Question Solution

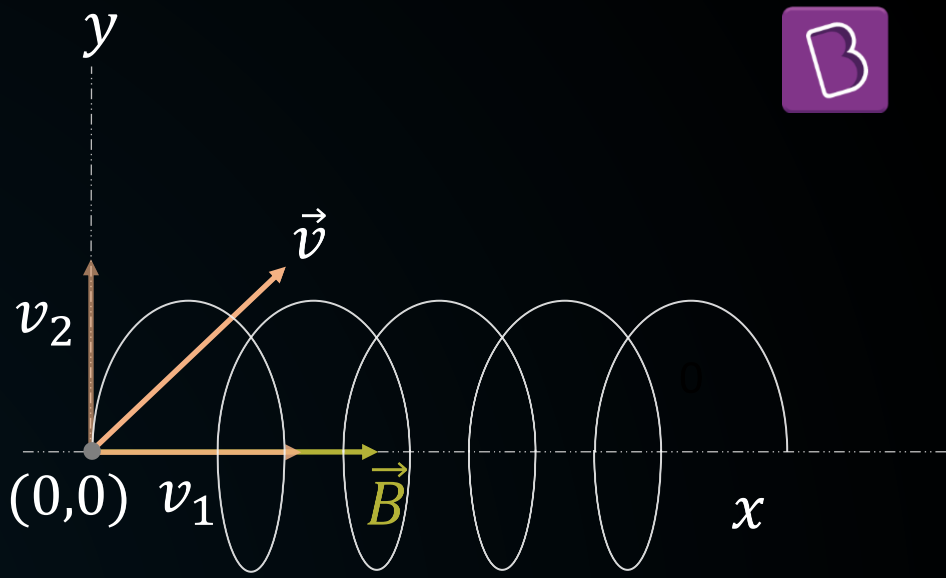
A charged particle q starts from origin $(0,0)$ with a velocity of $\vec{v} = v_1\hat{\imath} + v_2\hat{\jmath}$ in uniform magnetic field $\vec{B} = B_o\hat{\imath}$. Find the position vector $\vec{r}(t) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ as a function of time t.



Question Solution

Here, the magnetic field is along x – axis.

Given, $\vec{v} = v_1\hat{i} + v_2\hat{j}$

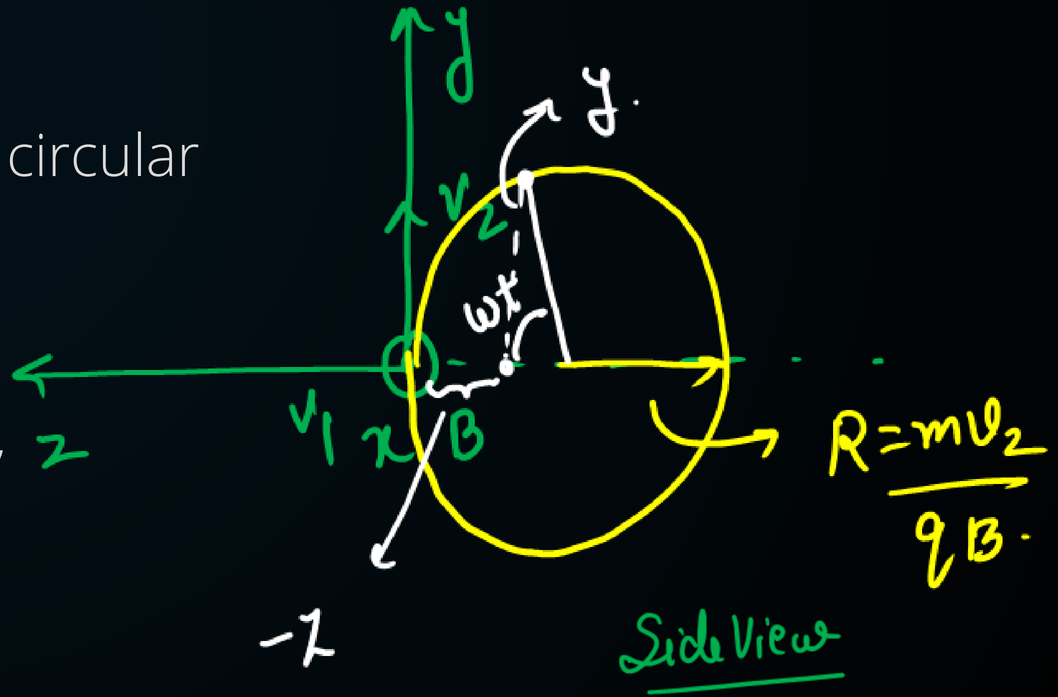


As v_1 is parallel to magnetic field, it leads to linear motion

At time t, the distance covered by particle in x- direction,

$$\boxed{x = v_1 t}$$

v_2 is perpendicular to magnetic field which leads to circular motion with radius, $R = \dfrac{mv_2}{qB}$



For the helical motion of particle in given conditions,

$$\omega = \frac{qB}{m}.$$

$$T = \frac{2\pi m}{qB}$$

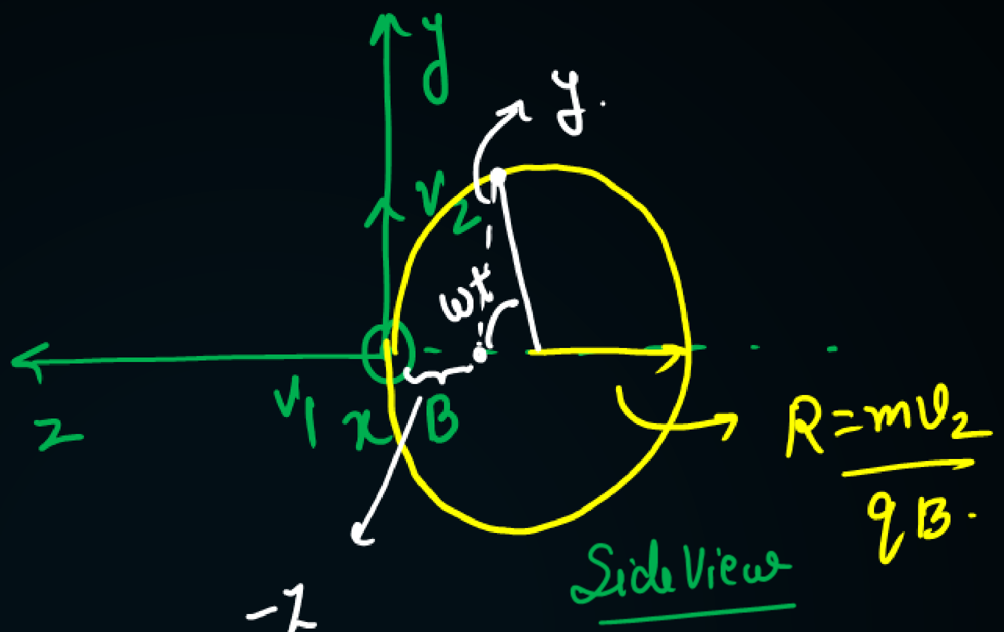
From the figure,

y-coordinate at time t,

$$y = R \sin \omega t = \frac{mv_2}{qB} \sin\left(\frac{qB}{m} t\right)$$

z-coordinate at time t,

$$z = -R(1 - \cos\omega t) = -\frac{mv_2}{qB}\left(1 - \cos\left(\frac{qB}{m}t\right)\right)$$

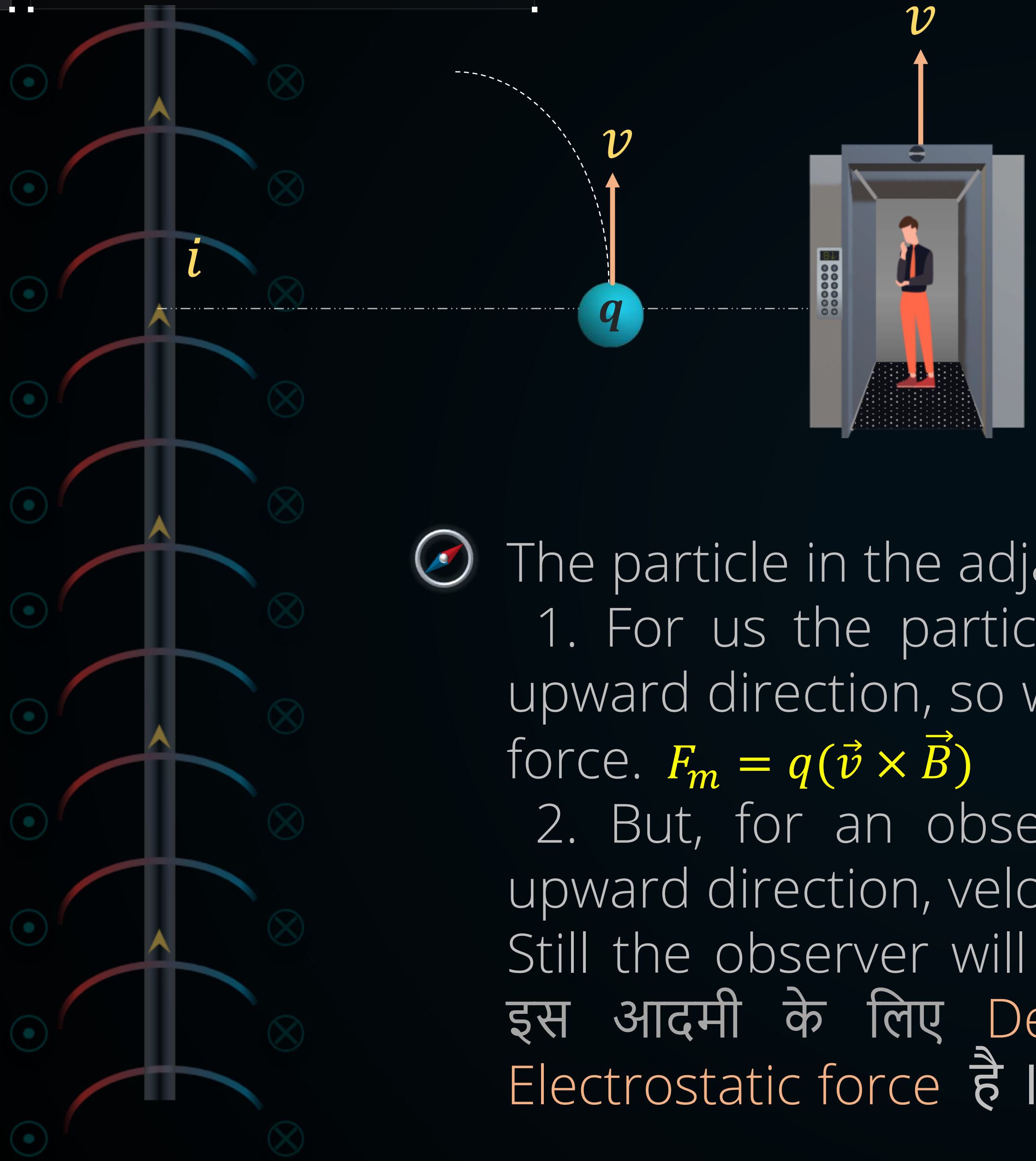


$\vec{v} \parallel \vec{B}$ → St. line

$\vec{v} \perp \vec{B}$ → Circle

$\vec{v}$ & $\vec{B}$ have some angle θ → Helical with Constant Pitch.

$$x = v_1 t\ ;\ \ y = \frac{mv_2}{qB_o}\sin\left(\frac{qB_o}{m}t\right)\ ;\ \ z = \frac{-mv_2}{qB_o}\left[1 - \cos\left(\frac{qB_o}{m}t\right)\right]$$



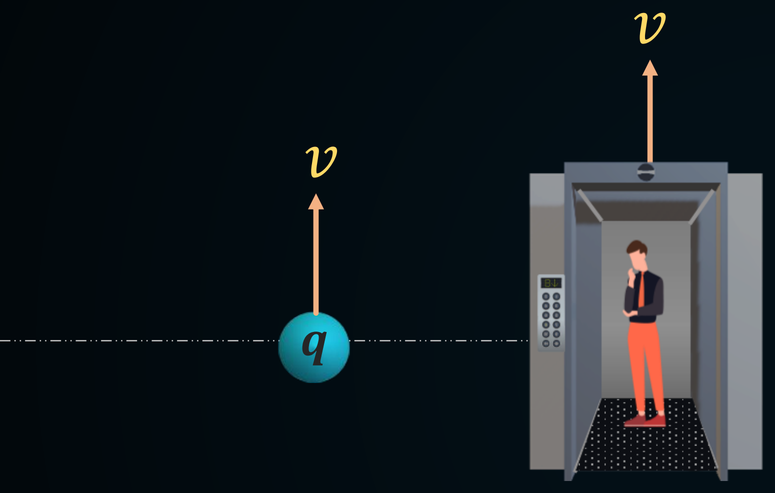
The particle in the adjacent setup deviates due to a force.

1. For us the particle seems to be moving with velocity v in upward direction, so we will say particle is experiencing magnetic force. $F_m = q(\vec{v} \times \vec{B})$
2. But, for an observer in lift moving with same velocity in upward direction, velocity of particle is zero (in upward direction). Still the observer will see particle moving away from him. Why?

इस आदमी के लिए Deviation का कारण Magnetic force नहीं है Electrostatic force है । $F_E = q\vec{E}$

LORENTZ FORCE

Lorentz Force is a force on electrically charged particles due to electromagnetic field. It is simply the sum of electric and magnetic force.



$$\vec{F}_{net} = q\vec{E} + q(\vec{v} \times \vec{B})$$

- Electrostatic Force and Magnetic Force depends on reference frame.
- Lorentz Force does not depend on reference frame.

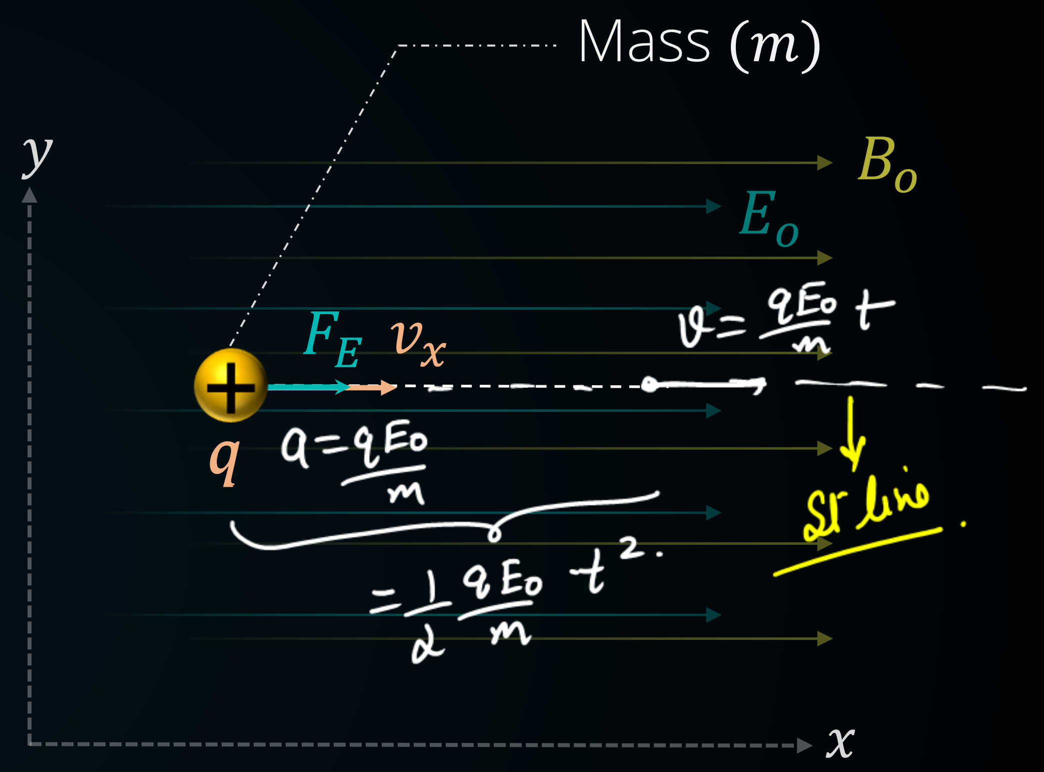
LORENTZ FORCE WHEN $\vec{E} \parallel \vec{B}$ q at rest

When the particle is at rest, magnetic force is zero. As only electrostatic force acts on it, the particle moves in straight line motion (along x).

As it starts moving in x direction, B will be parallel to v, magnetic force is zero.

$$\vec{F}_E = q\vec{E}_o$$

$$\vec{F}_m = 0$$



Acceleration of particle, $a = \frac{F}{m} = \frac{qE_o}{m}$

Velocity of particle, $v = at = \frac{qE_o}{m} t$

Distance travelled in time t,

$$v = 0 + \frac{1}{2} at^2 = \frac{1}{2} \frac{qE_o}{m} t^2$$

$$\boxed{\vec{F}_{net} = q\vec{E}_o}$$

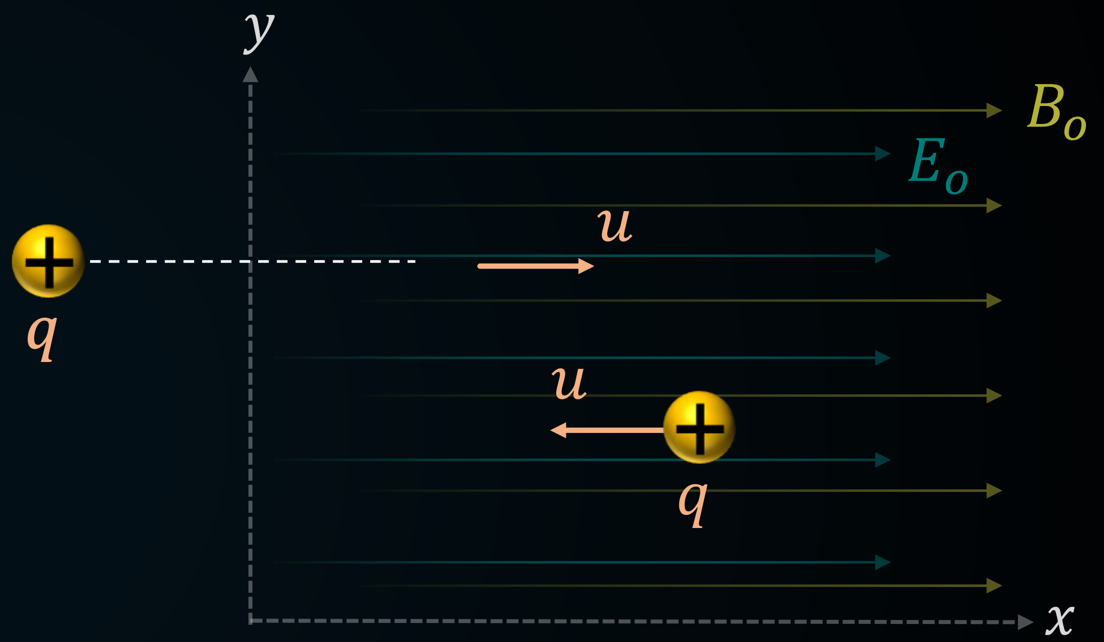
q moves in a straight line along the direction of electric field with an acceleration.

LORENTZ FORCE WHEN $\vec{E} \parallel \vec{B}$

NEET

q moving in parallel or anti-parallel to both $\vec{E}$ & $\vec{B}$

As velocity is parallel to magnetic field, magnetic force is zero. As only electrostatic force acts on it, the particle moves in straight line motion



Acceleration of particle, $a = \frac{F}{m} = \frac{qE_o}{m}$

Velocity of particle, $v = u + at = u + \frac{qE_o}{m}t$

Distance travelled in time t, when it is projected parallel to electric field initially, $s = ut + \frac{1}{2}at^2$

When particle is project in anti-parallel direction, velocity becomes zero due to negative acceleration and particle changes the direction. Let the particle changes direction at time t'.

As the velocity becomes zero when direction is changed,

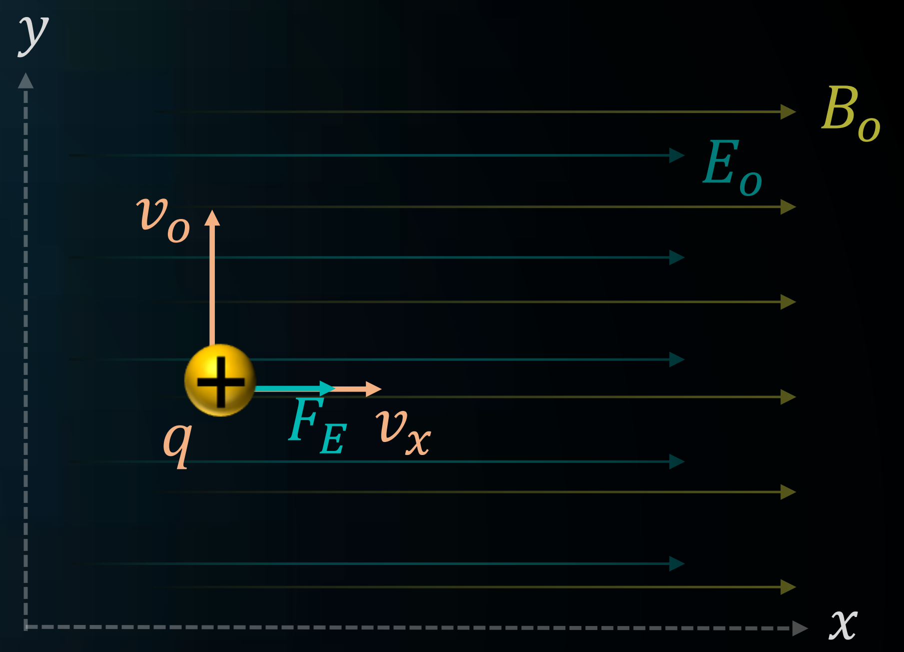
$0 = u - \frac{qE_o}{m} \cdot t'$

$t' = \boxed{\frac{mu}{qE_o}}$

$\vec{F}_{net} = q\vec{E}_o$

LORENTZ FORCE WHEN $\vec{E} \parallel \vec{B}$

q moving in perpendicular to both $\vec{E}$ & $\vec{B}$

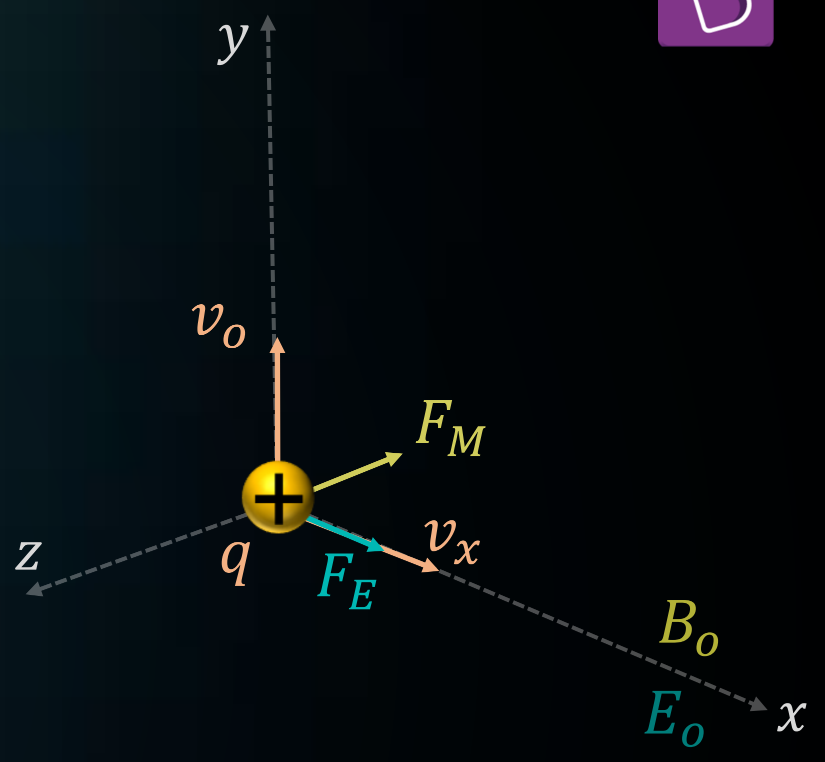


- The electrostatic force acting on the charge: $\vec{F}_E = qE_0\hat{\imath}$
- At $t = 0$, since the velocity of the charged particle and magnetic field are perpendicular to each other, $(\vec{v} \times \vec{B}) = |\vec{v}||\vec{B}| \sin 90° \left(-\hat{k}\right) = -qv_0B\hat{k}$.
- The velocity of the charged particle will remain v_0 for the whole process because the velocity generated due to the electrostatic force always acts parallel to magnetic field in this case.
- Therefore, the net force on the charged particle is, $\boxed{\vec{F}_{net} = qE_o\hat{\imath} - qv_oB_o\hat{k}}$
- Since v_0 is constant, the particle will execute UCM under the action of magnetic field alone with constant radius and the radius is given by,

$$\boxed{R = \frac{mv_o}{qB_o}}$$

LORENTZ FORCE WHEN $\vec{E} \parallel \vec{B}$

q moving in perpendicular to both $\vec{E}$ & $\vec{B}$



- The acceleration of the charged particle along the x-axis is produced by the electrostatic force on it. Therefore,

$$\vec{a} = \frac{qE_o}{m}\,\hat{i}$$

- Since the initial velocity of the charged particle along the x-axis is zero i.e., $u_x = 0$, the velocity of the particle at any instant of time t is given by,

$$\boxed{v_x = \frac{qE_o}{m}\cdot t}$$

- Hence, the displacement of the particle along the x-axis in time t is given by, $x = \frac{1}{2}\frac{qE_o}{m}\cdot t^2$.

- Thus, the motion of the particle due to electric field alone will be a straight line motion with acceleration a.

LORENTZ FORCE WHEN $\vec{E} \parallel \vec{B}$

q moving in perpendicular to both $\vec{E}$ & $\vec{B}$

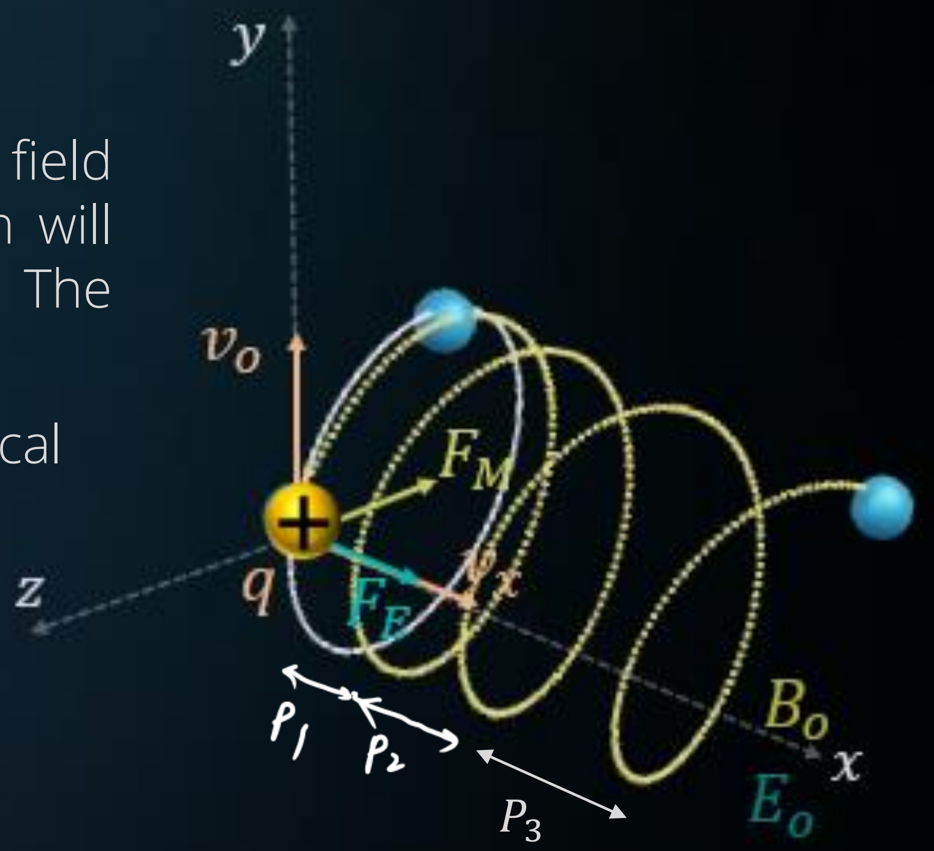
- If we combine the two types of motion due to magnetic field and electric field individually, then the resultant motion will be a helix with increasing pitch, as shown in the figure. The pitch increases due to the acceleration of the particle.
- At any instant of time, the y and z coordinate of the helical path followed by the particle will be:

$$y = \frac{mv_0}{qB_0} \sin\left(\frac{qB_0}{m}\right)t \qquad z = -R(1 - \cos\omega t)$$

Where $R = \dfrac{mv_0}{qB_0}$ and $\omega = \dfrac{qB_0}{m}$

- The P_1 i.e., the 1st pitch of helical path is nothing but the distance covered by the particle in the direction of electric field in time 0 to T. Therefore,

$$P_1 = \frac{1}{2}\frac{qE_0}{m} \cdot T^2$$



LORENTZ FORCE WHEN $\vec{E} \parallel \vec{B}$

q moving in perpendicular to both $\vec{E}$ & $\vec{B}$

- The P_2 is the 2nd pitch of helical path and it denotes the distance covered by the particle in the direction of electric field in time T to $2T$. Therefore,

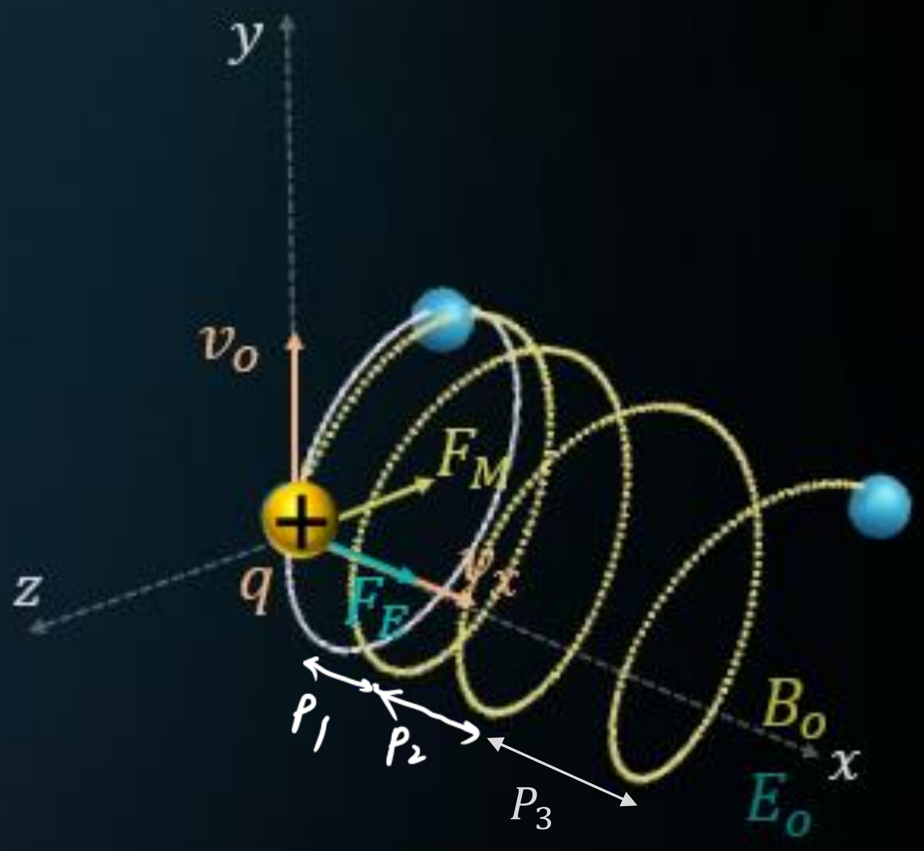
$$P_2 = \frac{1}{2}\frac{qE_0}{m}\left[(2T)^2 - T^2\right] = \frac{3qE_0}{2m}T^2$$

- The P_2 is the 3rd pitch of helical path and it denotes the distance covered by the particle in the direction of electric field in time $2T$ to $3T$. Therefore,

$$P_3 = \frac{1}{2}\frac{qE_0}{m}\left[(3T)^2 - (2T)^2\right]$$

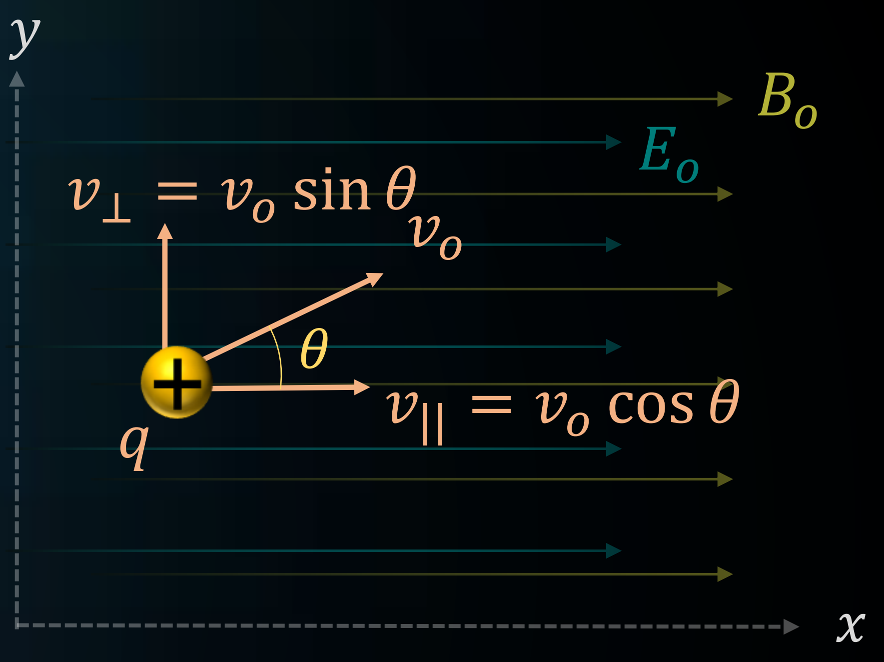
$$= \frac{5qE_0}{2m}\cdot T^2$$

$$\boxed{P_1 : P_2 : P_3 = 1 : 3 : 5}$$



LORENTZ FORCE WHEN $\vec{E} \parallel \vec{B}$

q moving at an angle θ with both $\vec{E}$ & $\vec{B}$



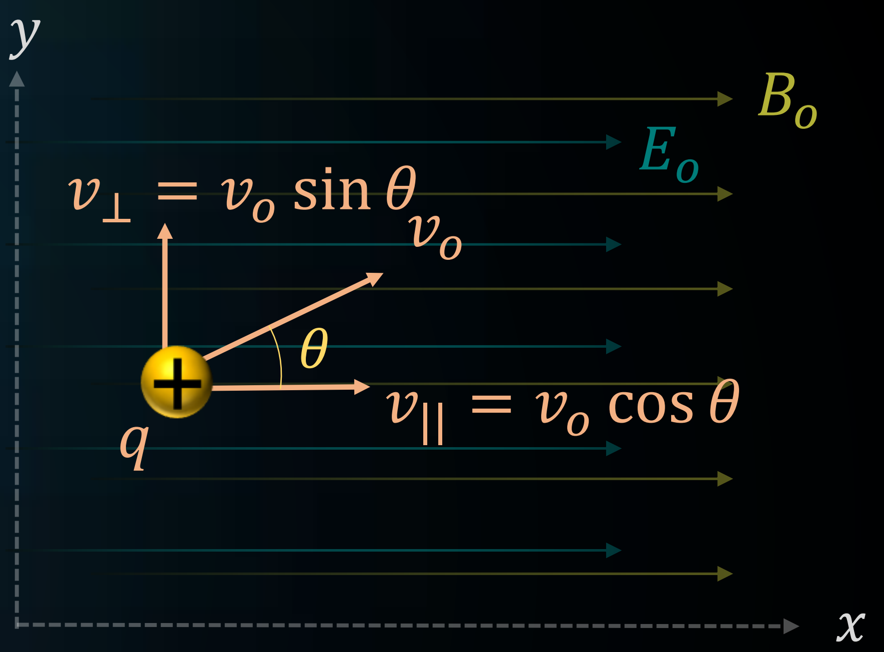
- Let a charge particle is projected in the presence of both electric and magnetic field at an angle θ with x-axis, as shown in the figure. For simplicity, lets assume for an instant that the electric field is absent.
- The perpendicular component of the velocity i.e., $v_\perp = v_o \sin\theta$ is responsible for UCM because this component of the velocity is perpendicular to the magnetic field B_o.
- The parallel component of the velocity i.e., $v_{||} = v_o \cos\theta$ is responsible for linear translational motion because this component of the velocity is parallel to the magnetic field.
- The magnetic force on the charge particle, $|F_m| = qv_0B_0 \sin\theta$

In absence of electric field, if we combine these two types of motion, then the resultant motion will be a helix with uniform pitch.

LORENTZ FORCE WHEN $\vec{E} \parallel \vec{B}$

q moving at an angle θ with both $\vec{E}$ & $\vec{B}$

Now, let us apply the electric field parallel to the magnetic field.



- The electrostatic force on the particle is, $\vec{F}_E = q\vec{E}$ and this force reinforces $v_{||}$ and produces acceleration $\vec{a} = \frac{qE_0}{m}\hat{\imath}$ in the particle. Consequently, the pitch of the helix increases.
- Since $v_\perp = v_o \sin\theta$ is perpendicular to both $\vec{F}_E$ and $\vec{F}_M$, it remains same for the whole process but $v_{||}$ changes due to the acceleration produced by the electric field.
- If we combine the two types of motion due to magnetic field and electric field individually, then the resultant motion will be a helix with increasing pitch.
- Since the initial velocity of the particle along the x-axis is $v_{||} = v_o \cos\theta$, thus, the velocity of the particle along x-axis at any instant will be:

$$V_x = u_x + a_x t$$

$$V_x = \underbrace{(V_0 \cos\theta)}_{V_{||}} + a_x t$$

LORENTZ FORCE WHEN $\vec{E} \parallel \vec{B}$

q moving at an angle θ with both $\vec{E}$ & $\vec{B}$

- Hence, the displacement of the particle along the x-axis in time t is given by,

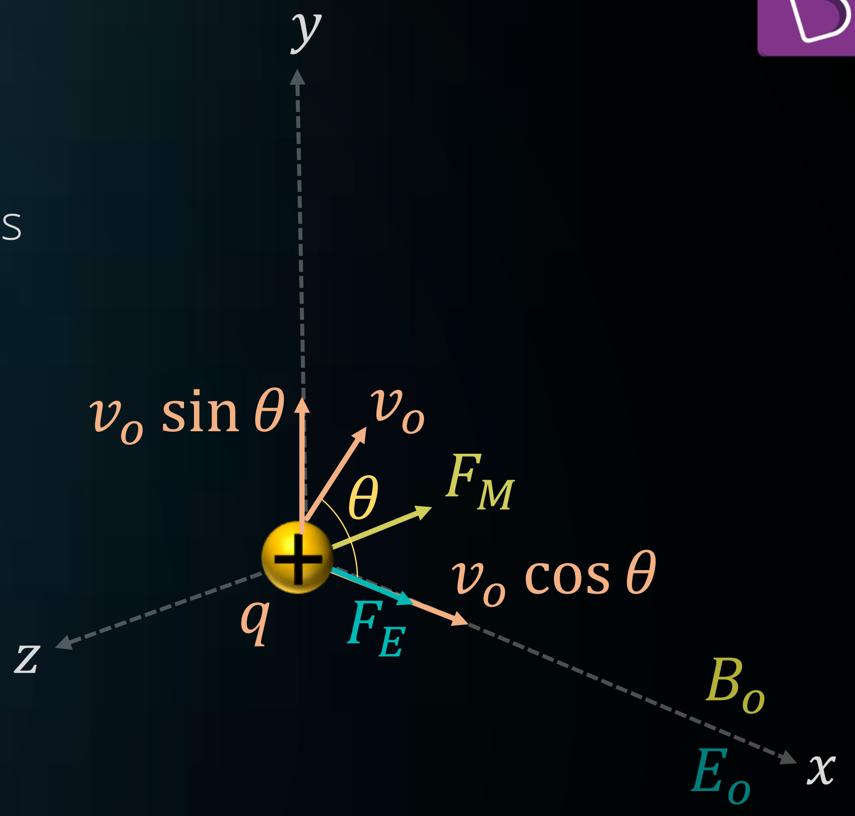
$$v_0 \cos\theta \cdot t + \frac{1}{2}\frac{qE_0}{m}\cdot t^2 = x$$

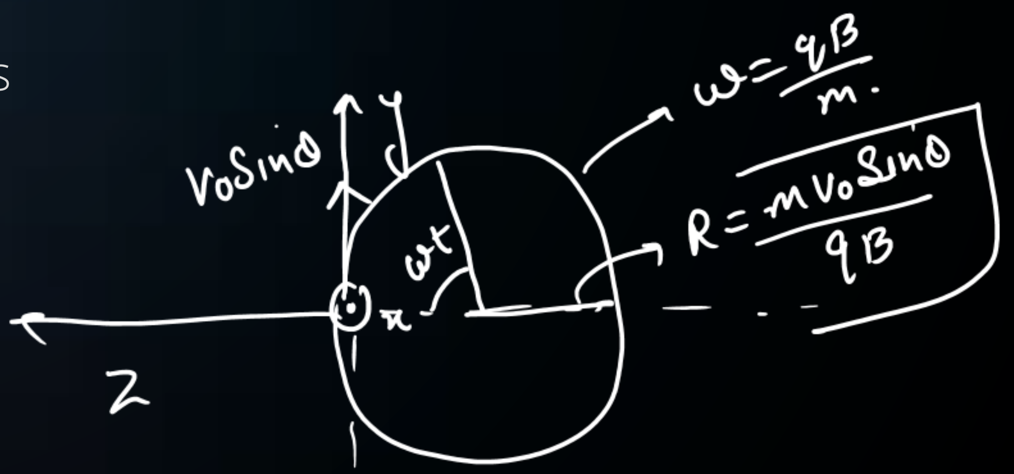
- Since the perpendicular component of the velocity i.e., $v_\perp = v_o \sin\theta$ is responsible for UCM, the radius of the circle will be:

$$\boxed{R = \frac{mV_o \sin\theta}{qB}}$$

- Since the time period doesn't not depend upon the velocity, it is given by,

$$\boxed{T = \frac{2\pi m}{qB}}$$

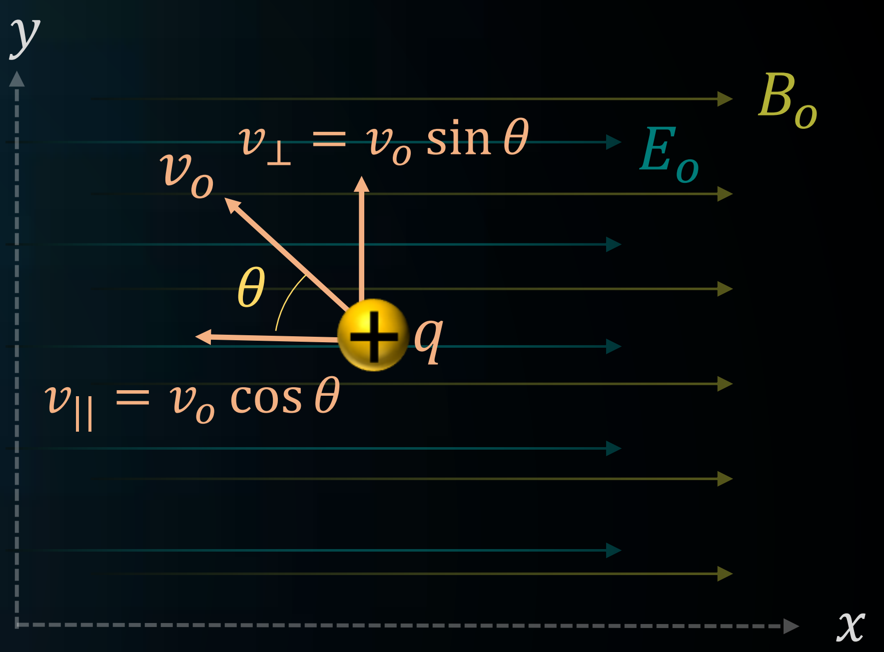




LORENTZ FORCE WHEN $\vec{E} \parallel \vec{B}$

q moving at an angle θ with both $\vec{E}$ & $\vec{B}$

- If the charge particle is projected in the presence of both electric and magnetic field at an angle θ with $-ve$ x-axis, as shown in the figure, then $v_{||} = v_o \cos\theta$ and the acceleration produced by the electric field will be opposite of each other. Thus, the pitch of the helical path will decrease at first along the $-ve$ x-axis and then due to the dominance of the acceleration produced by electric force, the pitch starts increasing along $+ve$ x-axis.



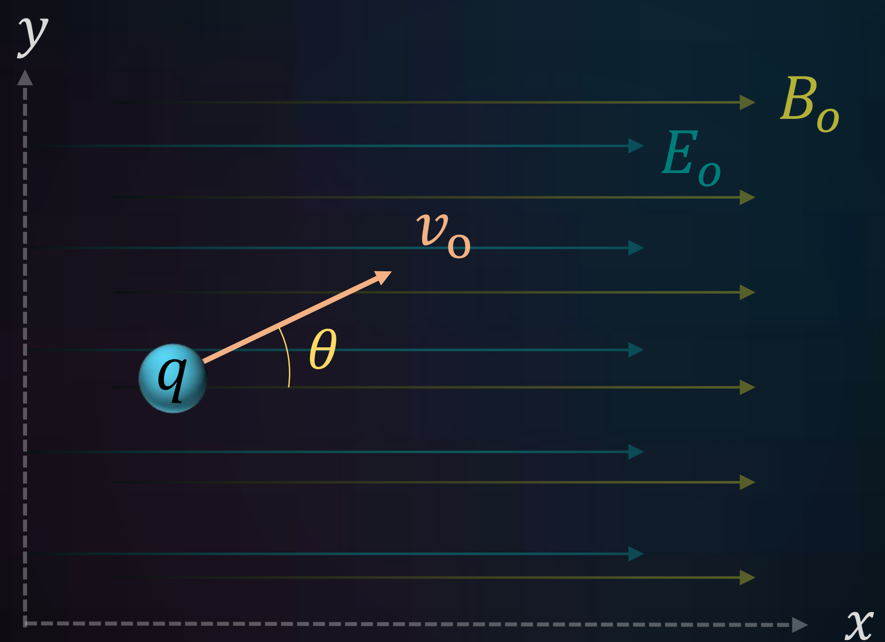
Extra bol -

$\vec{E}$ & $\vec{B}$

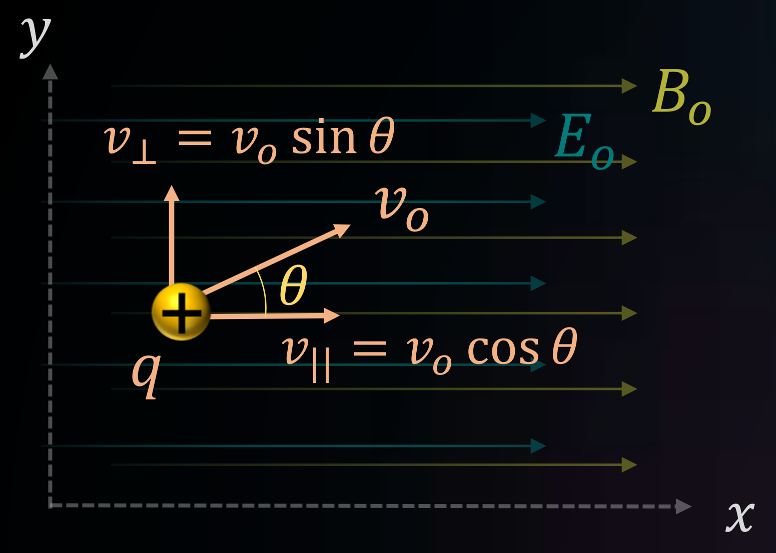
Pehle Piche Pitch ↓ Karenge.
then Palat Kare wapas ↑ Pitch
with Helical Path { Socho }

Question Solution

A charged particle of mass m starts moving with an initial velocity v_o at an angle θ with both electric and magnetic field as shown. Find speed as a function of x.

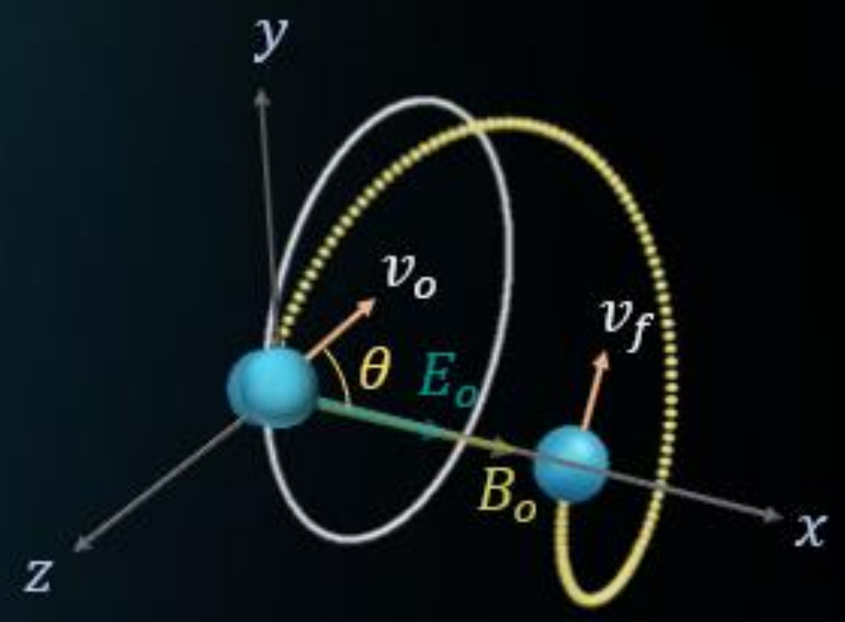


Question Solution



We know that the path of the charge particle will be a helix with increasing pitch, aa shown in the figure.

Along x-axis the velocity will change due to the action of electrostatic force on the particle.



Applying work energy theorem, we get,

$$WD_M + WD_E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$$

Now, work done by the magnetic field is always zero i.e., $WD_M = 0$

And, if the particle gets displaced by x in the direction of electric field, then the work done by the electric field is always zero i.e., $WD_E = qE_0x$

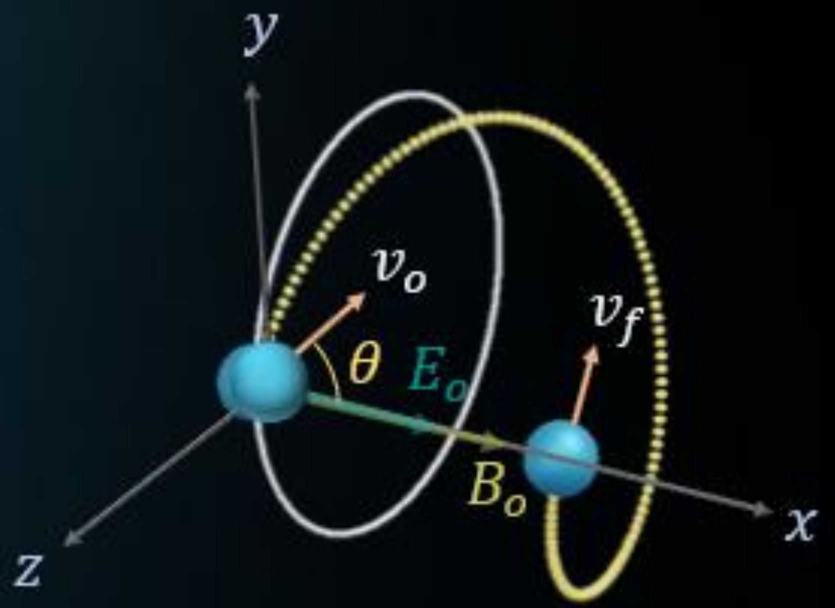
Question

Solution

Substituting the values of work done by magnetic field and electric field in the work energy theorem, we get,

$$qE_0 \cdot x + 0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

$$v_f = \sqrt{\frac{2qE_0x}{m} + v_0^2}$$



$$\boxed{v_f = \sqrt{\frac{2\left(qE_ox + \frac{1}{2}mv_o^2\right)}{m}}}$$

LORENTZ FORCE WHEN $\vec{E} \perp \vec{B}$

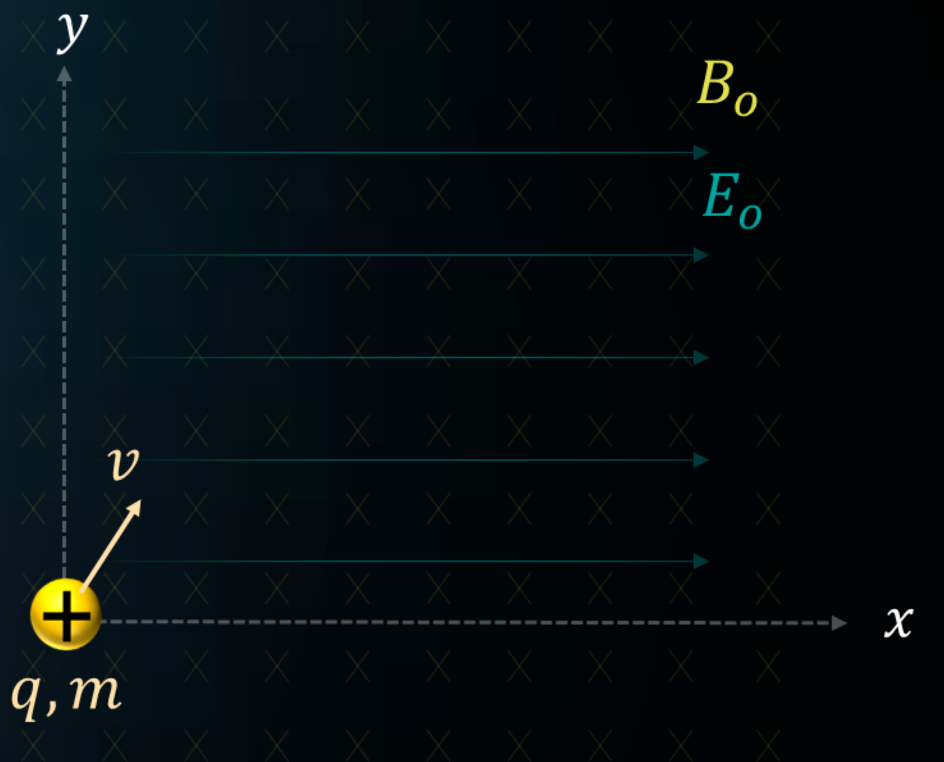
q moving with velocity $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$ $\quad \vec{E} = E_o\hat{\imath}$ $\quad \vec{B} = -B_o\hat{k}$

- The net force acting on the charge particle:

$$\vec{F} = qE_0\hat{\imath} + q(v_x\hat{\imath} + v_y\hat{\jmath}) \times (-B_0)\hat{k}$$

$$\vec{F} = qE_0\hat{\imath} - qv_xB_0(-\hat{\jmath}) - qv_yB_0(\hat{\imath})$$

$$= (qE_0 - qv_yB_0)\hat{\imath} + qv_xB_0\hat{\jmath}$$

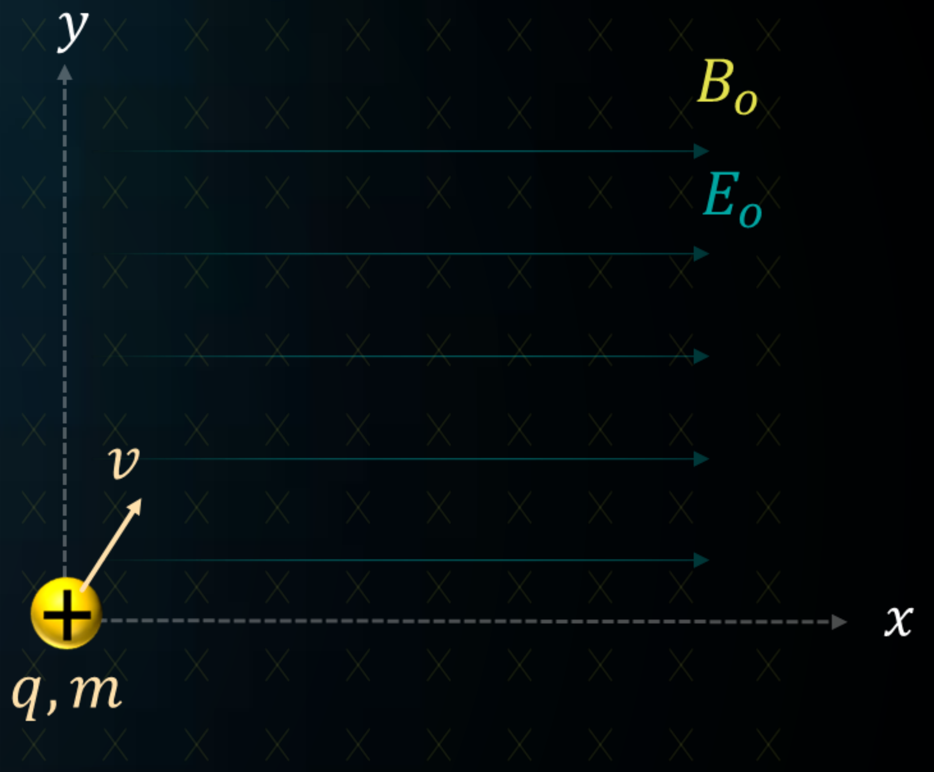


Direction	Force	Acceleration
Along x-axis	$F_x = qE_0 - qv_yB_0$	$a_x = \dfrac{qE_0 - qv_yB_0}{m}$
Along y-axis	$F_y = qv_xB_0$	$a_y = \dfrac{qv_xB_0}{m}$

- Therefore, x – component of acceleration depends on y – component of velocity and vice-versa.

LORENTZ FORCE WHEN $\vec{E} \perp \vec{B}$

q moving with velocity $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$ $\quad \vec{E} = E_o\hat{\imath}$ $\quad \vec{B} = -B_o\hat{k}$



We have: $a_x = \frac{qE_0 - qv_yB_0}{m}$ and $a_y = \frac{qv_xB_0}{m}$. Thus,

$$\frac{dv_x}{dt} = \frac{qE_0 - qv_yB_0}{m} \qquad \frac{dv_y}{dt} = \frac{qv_xB_0}{m}$$

Differentiating $\frac{dv_x}{dt}$ w.r.t t, we get,

$$\frac{d^2v_x}{dt^2} = -\frac{qB_0}{m} \cdot \frac{dv_y}{dt}$$

$$\frac{d^2v_x}{dt^2} = -\frac{q^2B_0^2}{m^2} \cdot V_x$$ [Substituting the value of $\frac{dv_y}{dt}$]

$$\frac{d^2v_x}{dt^2} + \left(\frac{qB_0}{m}\right)^2 v_x = 0$$

It represents an equation of SHM with angular velocity $\omega = \frac{qB_0}{m}$

LORENTZ FORCE WHEN $\vec{E} \perp \vec{B}$

q moving with velocity $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$ $\quad \vec{E} = E_o\hat{\imath}$ $\quad \vec{B} = -B_o\hat{k}$

$$\frac{d^2 v_x}{dt^2} + \left(\frac{qB_0}{m}\right)^2 v_x = 0$$

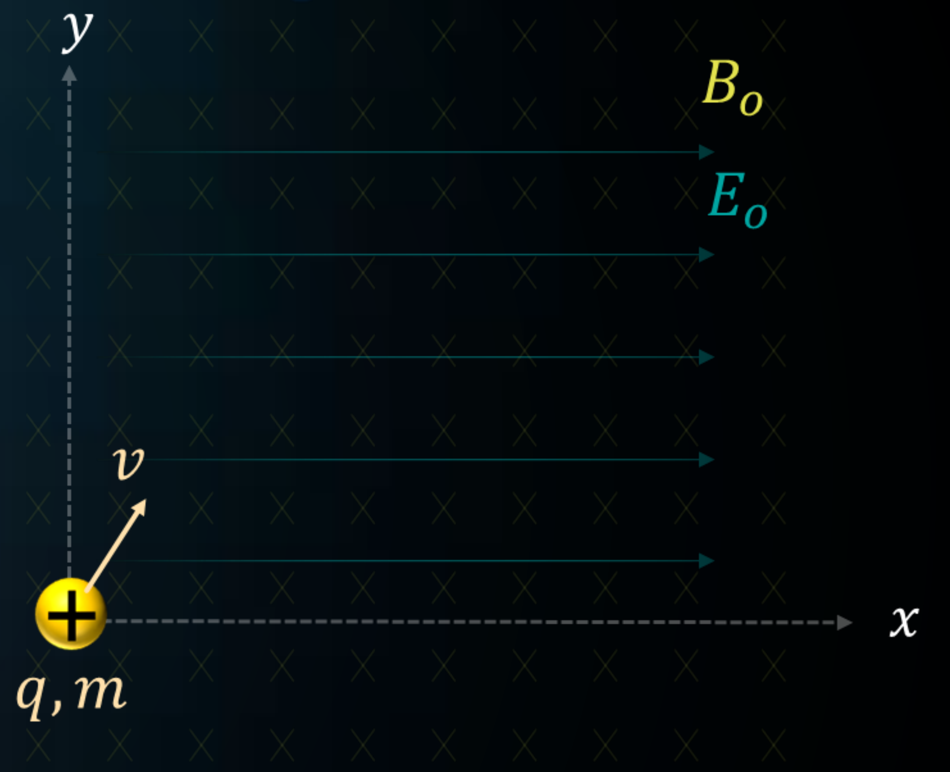
Steps to find x and y coordinate

Find v_x ⟶ Find x

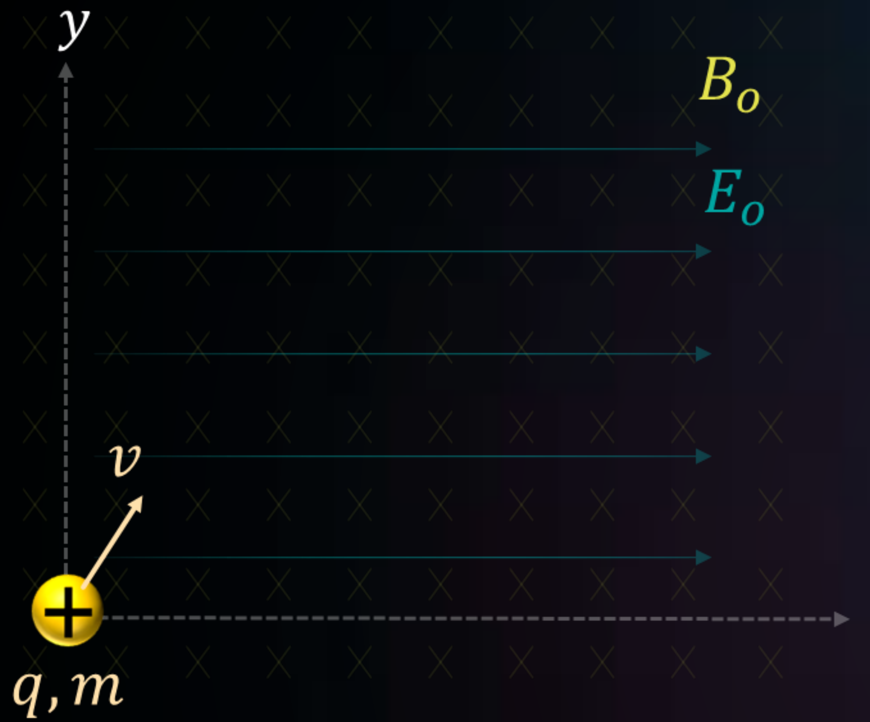
Substitute v_x in a_y

Find a_y

Find v_y ⟶ Find y



LORENTZ FORCE WHEN $\vec{E} \perp \vec{B}$

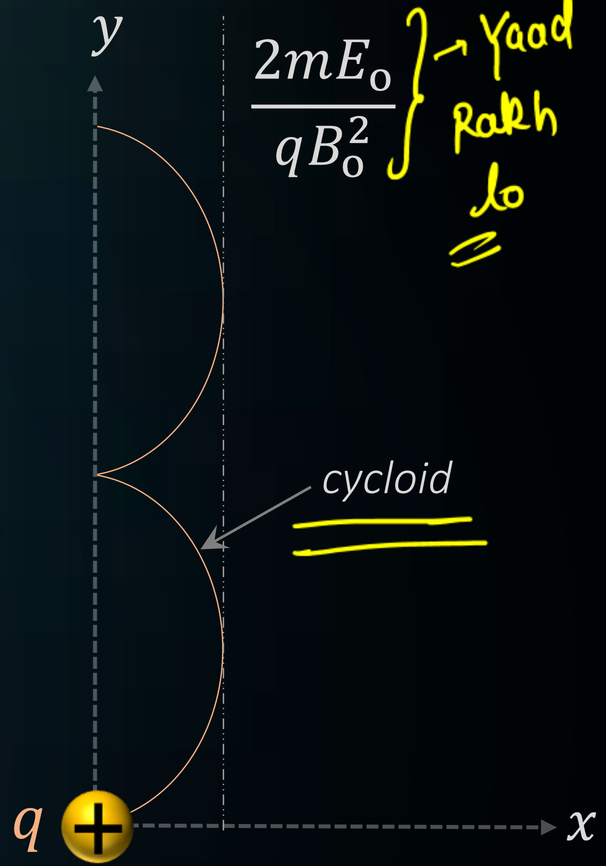


$$x = \frac{mE_o}{qB_o^2}(1 - \cos\omega t)$$

$$y = \frac{qE_o t}{m\omega} - \frac{qE_o}{m\omega^2}\sin\omega t$$

Cycloid

$x_{max} = \dfrac{2mE_o}{qB_o^2}$ $x_{min} = 0$



Rotation

cycloid

Question Answer

In a non-zero uniform Electric field $\vec{E}$ and Magnetic field $\vec{B}$, a particle (q, m) is moving with constant velocity. Which of the following is/are true ?

a $\vec{B}$ must be $\perp$ $\vec{E}$

b $\vec{v}$ must be $\perp$ $\vec{E}$

c $\vec{v}$ must be $\perp$ $\vec{B}$

d All the three must be $\perp$ to each other

Question | Answer

Since the particle is moving with constant velocity, we can say that the net force on the particle must be zero.

The net force on the particle is given by,

$$\vec{F}_{net} = q\vec{E} + q(\vec{v} \times \vec{B})$$

$$q\vec{E} + q(\vec{v} \times \vec{B}) = 0$$

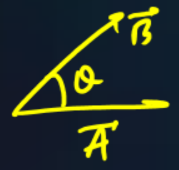
$$\boxed{\vec{E} = -(\vec{v} \times \vec{B})}$$

$$\Downarrow$$

$\vec{E} \perp \vec{B}$ → must

$\vec{E} \perp \vec{v}$ → must

$\vec{v} \perp \vec{B}$ → Maybe.



$\vec{C} = \vec{A} \times \vec{B}$

$\vec{C} \perp \vec{B}$ ✓

$\vec{C} \perp A$ ✓

$\vec{A} \perp \vec{B}$ → Kar zaruri nahi hai.

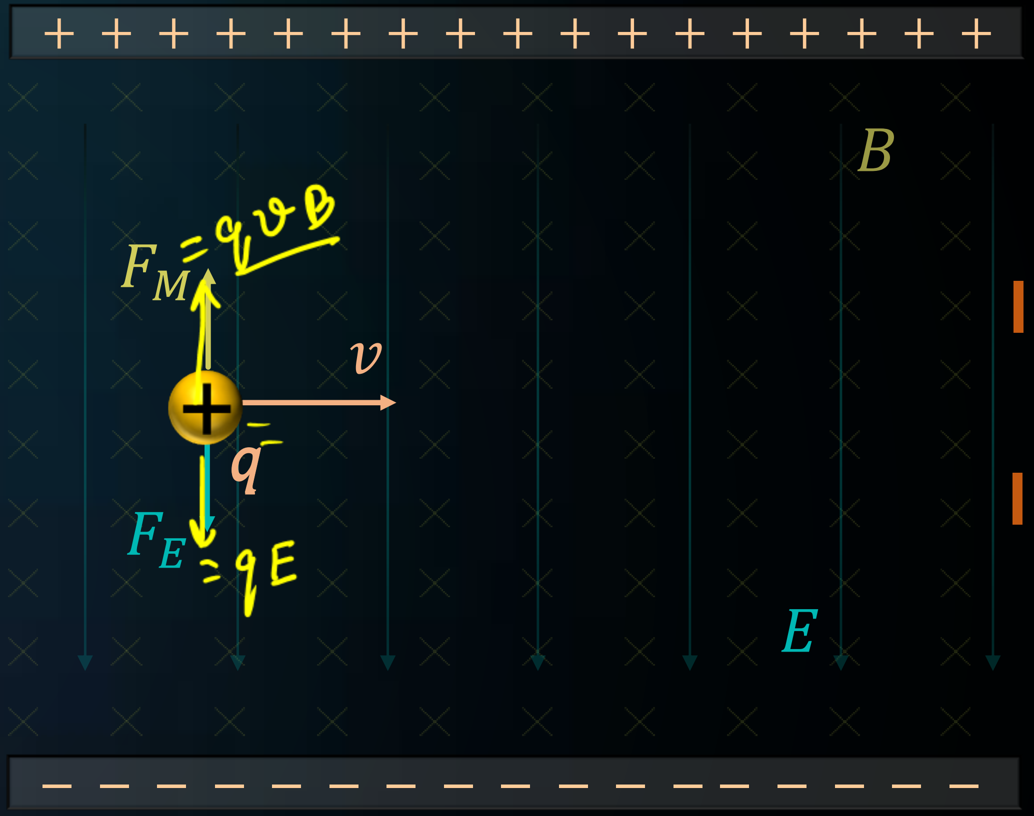
Thus, option a and option b are correct answers.

Velocity Selector

If $F_{net} = 0$, both the forces will be equal and opposite to each other and hence, the charged particle will move undeflected through the fields. Therefore,

$\cancel{q}E = \cancel{q}vB$ $\boxed{v = \frac{E}{B}}$

$$|\vec{v}|_{req} = \frac{|\vec{E}|}{|\vec{B}|}$$



This condition is used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds (irrespective of their charge and mass).

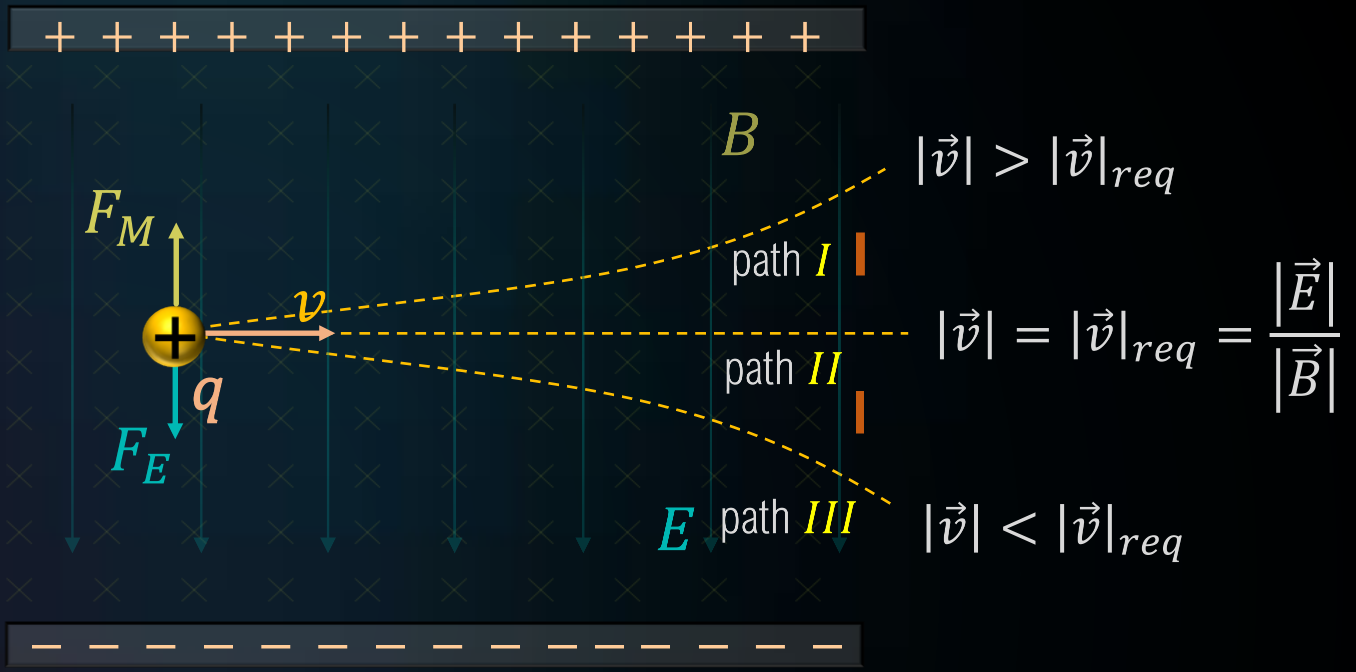
Velocity Selector

- If $\vec{F}_M > \vec{F}_E$, the net force on the particle is in upward direction. Therefore, the condition of velocity so that the particle follow path I can be obtained as follows:

$$\vec{F}_M > \vec{F}_E \Rightarrow q|\vec{v}||\vec{B}| > q|\vec{E}| \Rightarrow |\vec{v}| > \frac{|\vec{E}|}{|\vec{B}|}$$

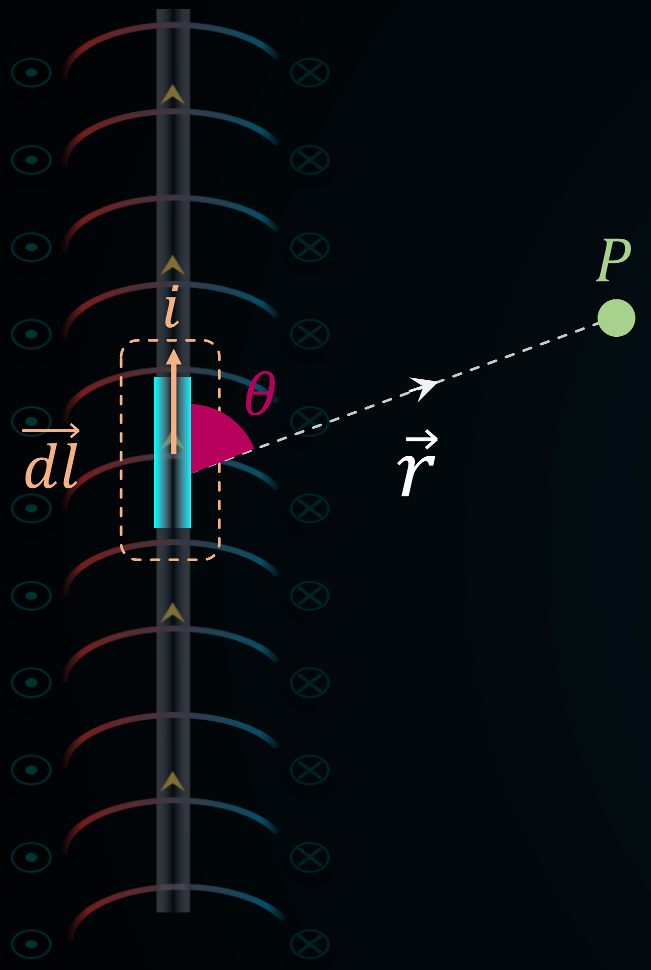
- If $\vec{F}_E = \vec{F}_M$, the net force on the particle is zero. Therefore, the condition of velocity so that the particle follow path II can be obtained as :

$$\vec{F}_E = \vec{F}_M \Rightarrow q|\vec{E}| = q|\vec{v}||\vec{B}| \Rightarrow |\vec{v}| = \frac{|\vec{E}|}{|\vec{B}|}$$

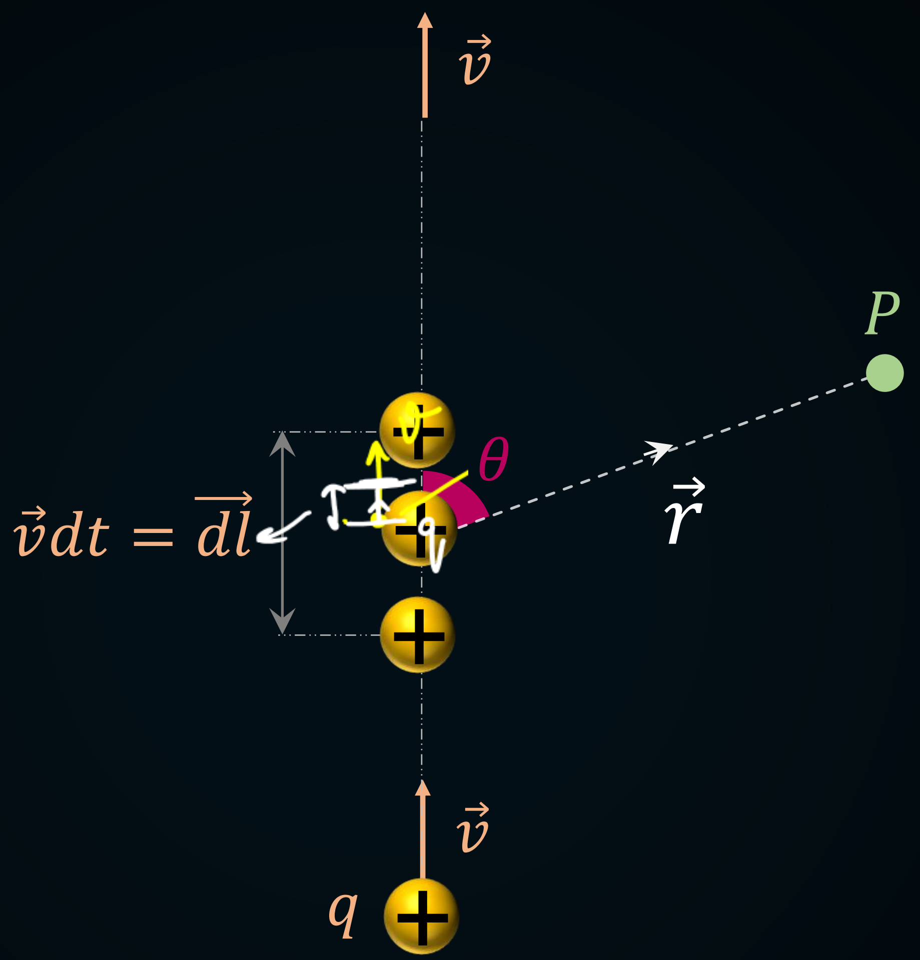


- If $\vec{F}_E > \vec{F}_M$, the net force on the particle is in downward direction. Therefore, the condition of velocity so that the particle follow path III can be obtained as : $\vec{F}_E > \vec{F}_M \Rightarrow q|\vec{E}| > q|\vec{v}||\vec{B}| \Rightarrow |\vec{v}| < \frac{|\vec{E}|}{|\vec{B}|}$

Current Carrying Conductor vs Moving Charge



$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{i(\overrightarrow{dl} \times \vec{r})}{|\vec{r}|^3}$$



$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{q(\vec{v} \times \vec{r})}{|\vec{r}|^3}$$

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{i(\overrightarrow{dl} \times \vec{r})}{|\vec{r}|^3}$$

$i = \frac{q}{dt}$

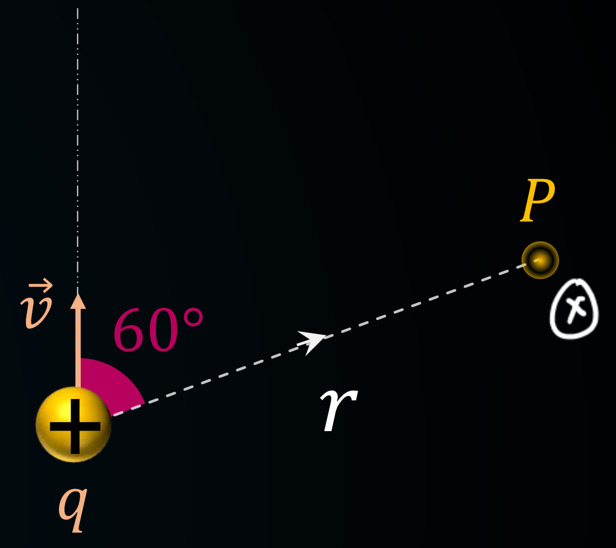
$\overrightarrow{dl} = \vec{v}dt$

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{\cancel{q/dt}[\vec{v}\cancel{dt} \times \vec{r})}{|\vec{r}|^3}$$

$$\boxed{= \frac{\mu_0}{4\pi}\frac{q(\vec{v} \times \vec{r})}{|\vec{r}|^3}}$$

Question

Calculate the magnetic field at point P due to a charge q moving with velocity v.



Solution

Applying 'Right hand thumb rule' according to the direction of the velocity of the charge, we have found that the magnetic field at P is directed into the plane of the paper.

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q(\vec{v} \times \vec{r})}{|\vec{r}|^3}$$

$$\Rightarrow B = \frac{\mu_0 q (v \cancel{r} \cdot \sin 60^\circ)}{4\pi r^{\cancel{3}}}$$

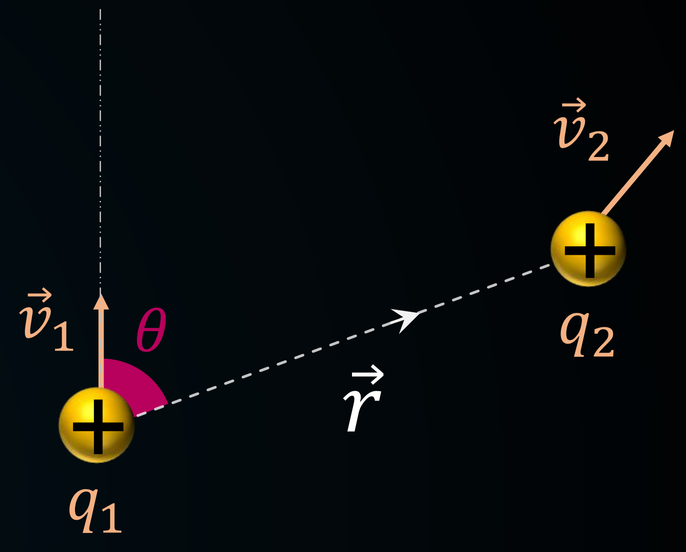
$$= \boxed{\frac{\mu_0 q v \cdot \sqrt{3}}{8\pi r^2}}$$

$$\vec{B} = \frac{\sqrt{3}\mu_0}{8\pi} \frac{qv}{r^2} \otimes$$

Force on One Moving Charge due to Another Moving Charge

The magnetic field produced by the charge q_1 moving with velocity $\vec{v}_1$ at the location of q_2 is given by,

$$\vec{B}_1 = \frac{\mu_0 q_1 (\vec{v}_1 \times \vec{r})}{4\pi |\vec{r}|^3}.$$



Therefore, the force on the charge q_2 due to $\vec{B}_1$ is given by,

$$\vec{F}_{21} = q_2(\vec{v}_2 \times \vec{B}_1)$$

$$\underbrace{\vec{F}_{21} = q_2\left(\vec{v}_2 \times \left(\frac{\mu_0 q_1 (\vec{v}_1 \times \vec{r})}{4\pi |\vec{r}|^3}\right)\right)}$$

Ratna nahi hai Concept dhyan me Rakhna hai.

$$\boxed{\vec{F}_{21(m)} = q_2\left(\vec{v}_2 \times \frac{\mu_0}{4\pi}\frac{q_1(\vec{v}_1 \times \vec{r})}{|\vec{r}|^3}\right)}$$

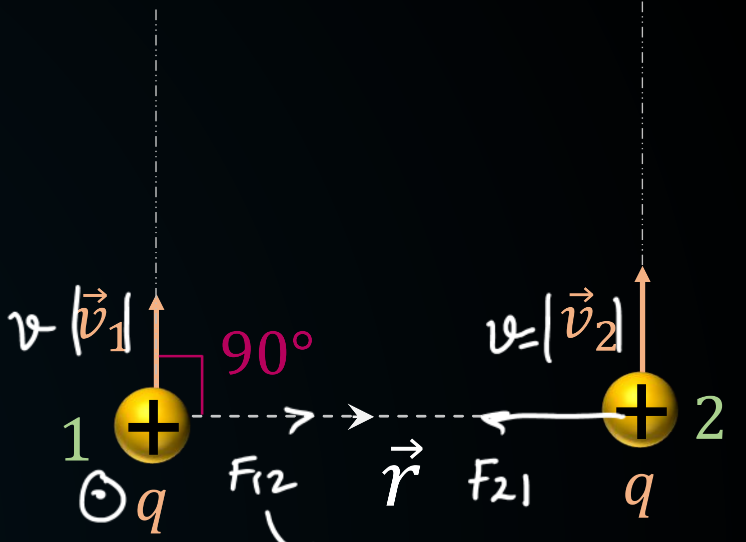
Force on One Moving Charge due to Another Moving Charge

Electric Force vs Magnetic Force:

- $\vec{F}_{21(E)}$ = Electrostatic force on 2 due to 1 $= \boxed{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}}$

- The magnetic field due to charge 1 at the location of 2 is, $B = \frac{\mu_0 q v r}{4\pi r^3}$

$$B_{21} = \boxed{\frac{\mu_0 q v}{4\pi r^2}}$$



$$\vec{B}_{12} = \frac{\mu_0 q v}{4\pi r^2} \rightarrow \frac{\mu_0 q^2 v^2}{4\pi r^2}$$

- $\vec{F}_{21(m)}$ = Magnetic force on 2 due to 1 $= q\cdot v\cdot\frac{\mu_0 q v}{4\pi r^2} = \boxed{\frac{\mu_0 q^2 v^2}{4\pi r^2}}$

$$\text{Ratio} = \frac{F_{21}(m)}{F_{21}(E)} = \frac{\mu_0 \cancel{q^2} v^2\, \cancel{r^2}\cdot \cancel{4\pi}\varepsilon_0}{\cancel{4\pi}\cancel{r^2}\cdot \cancel{q^2}} = \mu_0\varepsilon_0 v^2 = \frac{v^2}{c^2} = \left(\frac{v}{c}\right)^2$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad \mu_0\varepsilon_0 = \frac{1}{c^2}$$

Generally $v << c$ $\therefore$ Ratio is very small $\Rightarrow$ $\{F_{magnetic} << F_{electrostatic}\}$

$$\frac{\vec{F}_{21(E)}}{\vec{F}_{21(m)}} = \left(\frac{c}{v}\right)^2$$

$$\vec{F}_{21(E)} \gg \vec{F}_{21(m)}$$

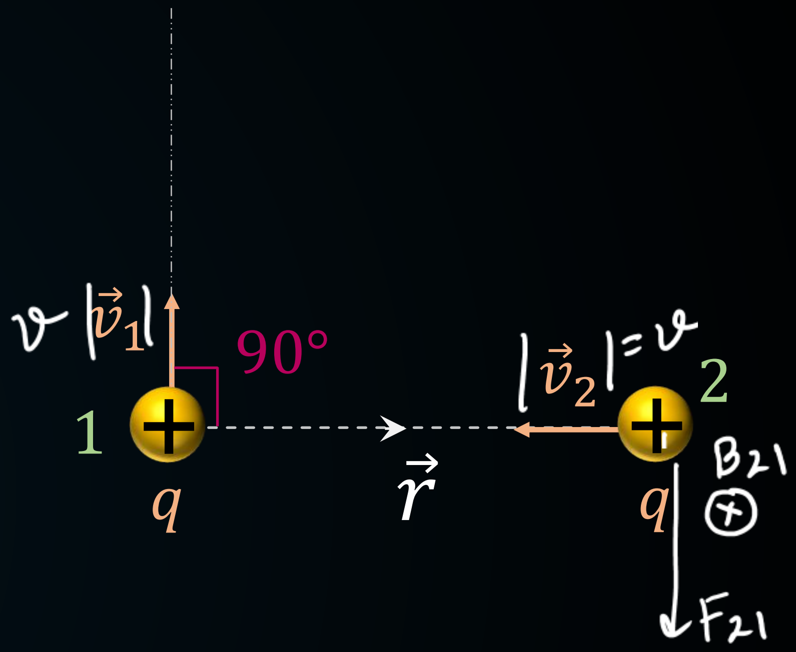
Force on One Moving Charge due to Another Moving Charge

- The magnetic field due to charge 1 at the location of charge 2 is,

$$B_{21} = \frac{\mu_0 q v}{4\pi r^2}$$

- Magnetic force on charge 2 due to charge 1 is,

$$F_{21} = \frac{\mu_0 q^2 v^2}{4\pi r^2}$$



- The magnetic field due to charge 2 at the location of charge 1 is,

$$\vec{B}_{12} = 0$$

Because $\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v} \times \vec{r})}{|\vec{r}|^3}}$ and the direction of both $\vec{v}$ and $\vec{r}$ are same for charge 2.

- Magnetic force on charge 1 due to charge 2 is, $\vec{F}_{12_m} = 0$

- Hence, magnetic force is not mandatorily an action-reaction force.

Cyclotron

- Very high energy particles are required for experiments. As very high voltage have practical limitations, a special setup known as cyclotron is used for generating high speed charge particles.

$$Energy = qV = K.E = \frac{1}{2}mv^2$$

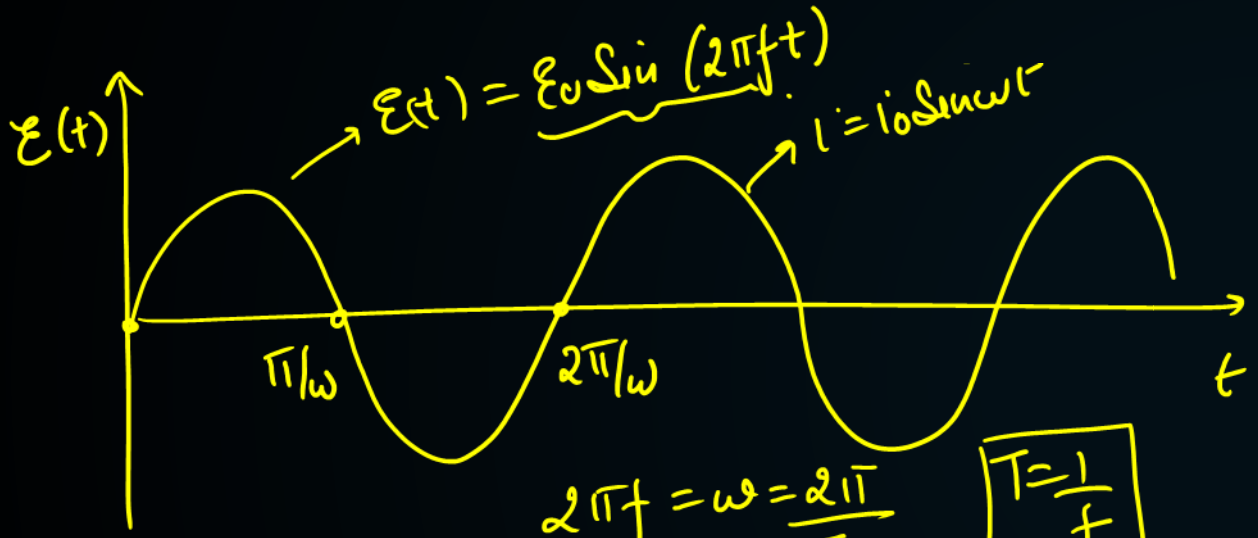
- Energy of charged particle on left plate is 0.
- Energy of charged particle on right plate is $1\ eV$.

World's largest Cyclotron

University of British Columbia in Vancouver

Cyclotron

AC supply: It is a current supply in form of sinusoidal wave wherein polarity changes with a constant frequency.



$$2\pi f = \omega = \frac{2\pi}{T}$$

$$\boxed{T = \frac{1}{f}}$$

Representation of AC current

$$i = i_0 \sin(\omega t)$$

$$V = V_0 \sin(\omega t)$$

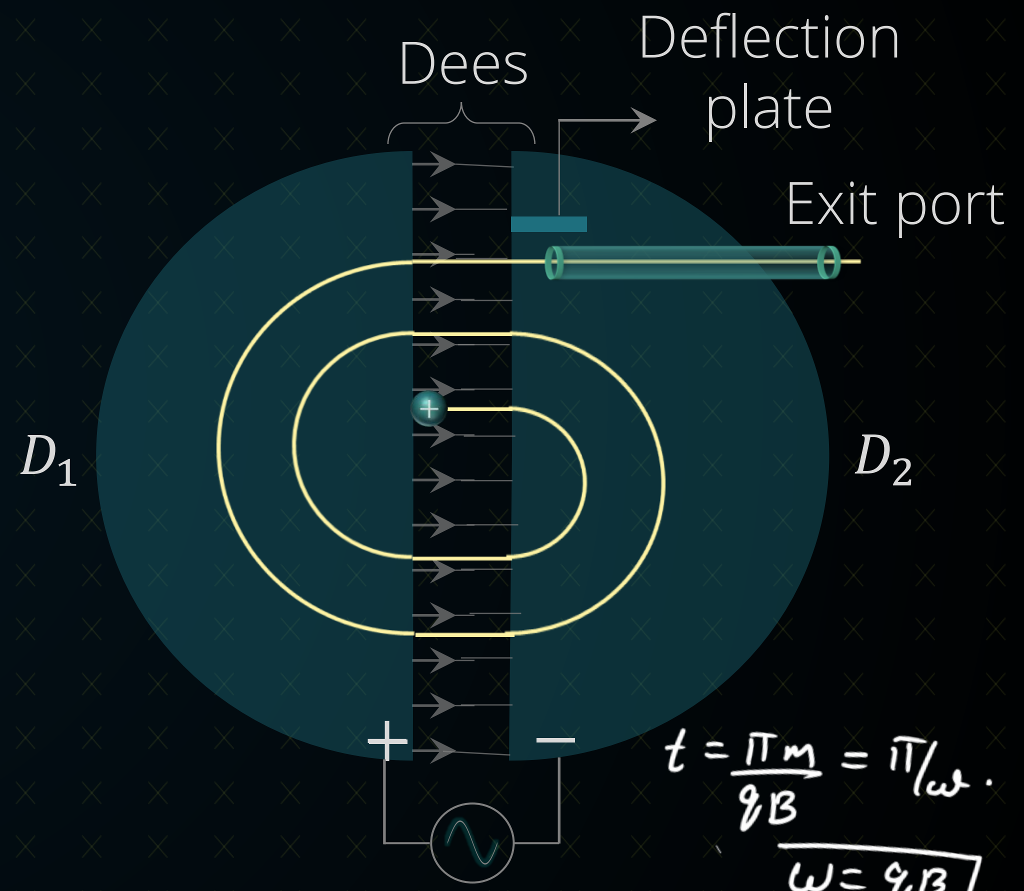
- Frequency of AC supply in our house is $50\ Hz$.
- Intensity of AC current becomes zero twice in a cycle.

Cyclotron

BOARDS NEET

Cyclotron is a machine that uses both electric field and magnetic field in combination to accelerate charged particles or ions to high energy.

The electric field accelerates the particles whereas magnetic field rotates it through Dees (D_1 and D_2). The particles travel from the edge of one Dee to other and gets accelerated due to the electric field. As soon as the charge particles enter into the other Dee, it gets rotated due to the magnetic field and come to the edge of the same Dee again. Instantly, the polarity of electric field is reversed (for this reason AC supply is used) and particle is accelerated again. This cycle is repeated till particle gains required energy. With each cycle, radius of rotation increases and with the help of deflection plate, particles are taken out for experiments.



$t = \frac{\pi m}{qB} = \pi/\omega$

$\boxed{\omega = \frac{qB}{m}}$

Energy gained by particle,

$qV = \frac{1}{2} m v^2 = K \cdot E$

Cyclotron

Radius of Trajectory:

$$R_1 = \frac{p}{qB} = \frac{\sqrt{2m \cdot qV}}{qB} = \sqrt{\frac{2mV}{qB^2}}$$

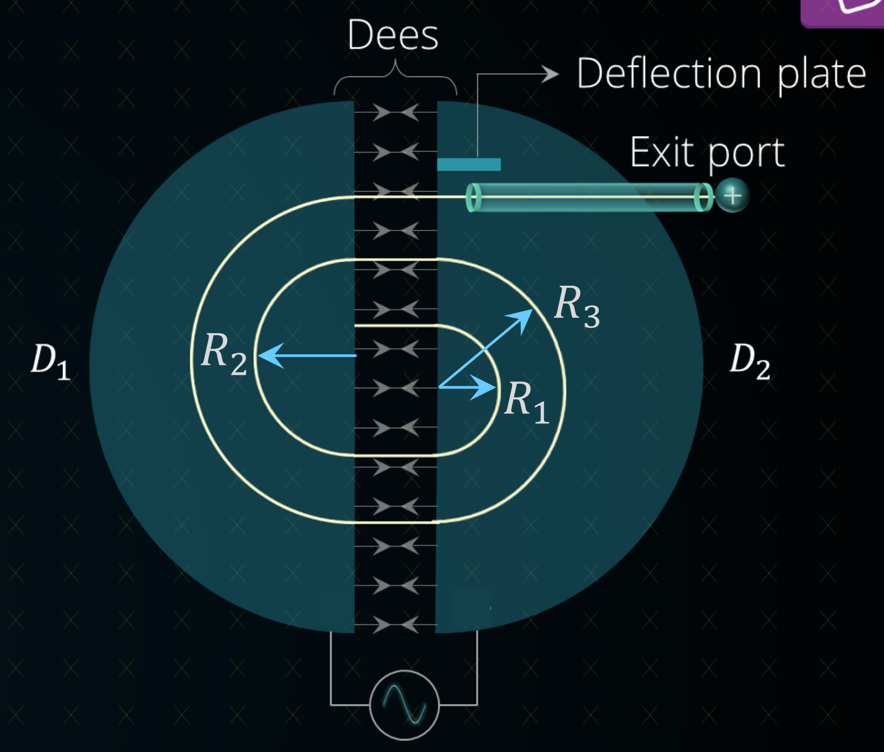
$$R_2 = \sqrt{\frac{2m \cdot 2V}{qB^2}} = \sqrt{2}\,R_1$$

$$R_3 = \sqrt{\frac{2m \cdot 3V}{qB^2}} = \sqrt{3}\,R_1$$

$$R_2 - R_1 = (\sqrt{2}-1)R_1$$

$$R_3 - R_2 = (\sqrt{3}-\sqrt{2})R_1$$

diffⁿ decreases.



- The gap $(R_{n+1} - R_n)$ between every successive trajectory decreases in the process.
- Radius (R) of every successive trajectory increases in the process.
- As speed of particle increases to very high value, its mass decreases. Synchrotron is used to cover this limitation.

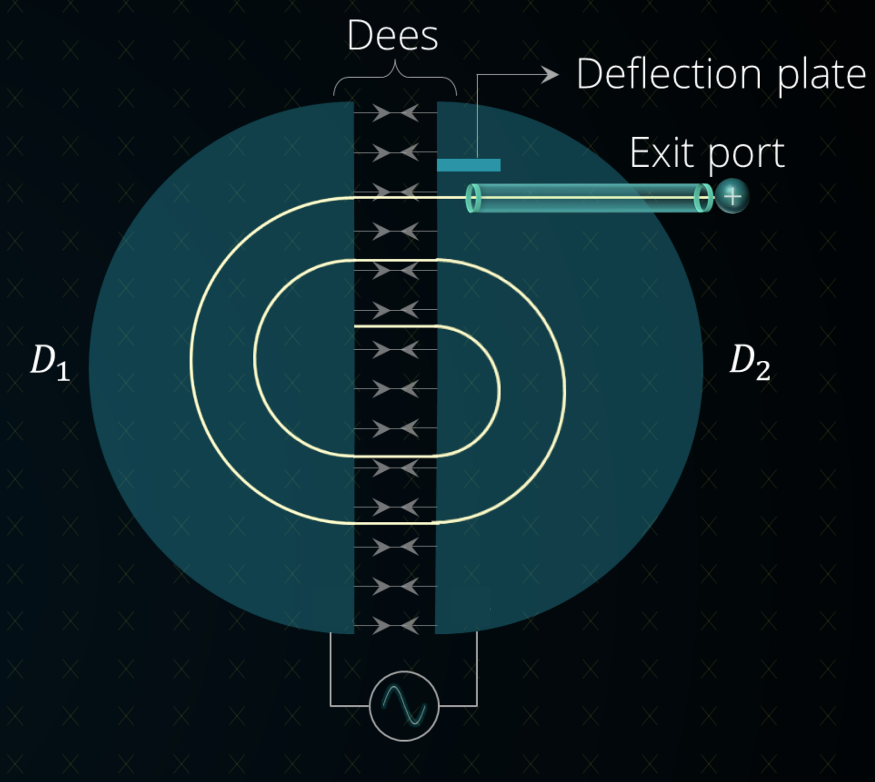
Cyclotron

- Time period of revolution of charged particle in uniform magnetic field:

$$T = \frac{2\pi m}{qB}$$

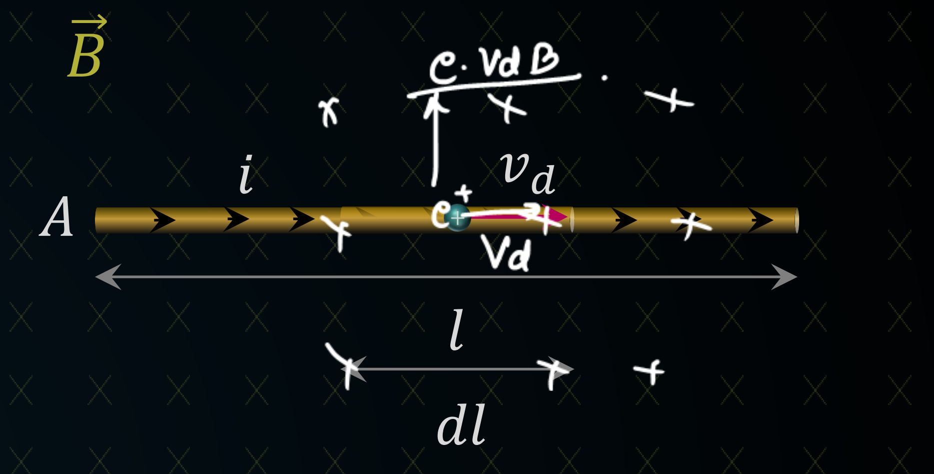
- Time period of AC current:

$$T = \frac{2\pi m}{qB}.$$



Time period of revolution of charged particle within cyclotron and time period of AC current are kept same to design cyclotron.

Angle between drift velocity and magnetic field is 90°



When the velocity of charge is perpendicular to magnetic field, force acting on it is, $F = ev_dB$. For a wire kept in magnetic field, force acts on each electron and thus, the wire experiences a force.

Now, consider small length of wire dl. Number of free electrons in this section is, $nAdl$.

Force acting on small length of wire dl,

$$F = nAdl \times eV_dB \cdot \qquad \boxed{F_{net} = idlB} \qquad \because (i = neAv_d)$$

v_d = Drift velocity of electron.

n = Number of free electrons per unit volume

A = Cross sectional area of wire

General case: For any angle θ between $d\vec{l}$ and $\vec{B}$

$$\text{Force on wire} = nAdl \cdot (e(\vec{V_d} \times \vec{B}))$$

$$= \underbrace{neAV_d}_{} \cdot (d\vec{l} \times \vec{B})$$

$$\vec{F} = \boxed{i(d\vec{l} \times \vec{B})}$$

The direction of dl should be taken along the direction of current

Force on Current carrying Wire due to magnetic Field

Random shaped wire in uniform magnetic field

Consider a wire with three segments l_1, l_2 and l_3 perpendicular to the magnetic field as shown in the figure.

Force on the wire, $\vec{F}_{net} = i(\vec{l}_1 \times \vec{B}) + i(\vec{l}_2 \times \vec{B}) + i(\vec{l}_3 \times \vec{B})$

As the magnetic field is constant,

Constant

$$\vec{F}_{net} = i\left(\underbrace{(\vec{l}_1 + \vec{l}_2 + \vec{l}_3)}_{\text{Displacement vector from initial point to final point}} \times \vec{B}\right)$$

$\Rightarrow \vec{F}_{net} = i(\vec{l} \times \vec{B})$

$\vec{F} = i(\vec{l} \times \vec{B})$

$$\boxed{\vec{F}_{net} = i(\overrightarrow{dl} \times \vec{B})}$$

For random shaped wire and variable magnetic field, net force is calculated by integration of force on a small element.

Question

Calculate the magnetic force ($\vec{F}_{net}$) on current carrying wire of length l placed in a magnetic field $\vec{B}$ at an angle $37°$ as shown.

Solution

As the magnetic field is constant and wire is straight,

$$F_{net} = ilB\sin\theta = ilB\sin 37°$$

$$F = ilB \times 3/5 = \boxed{\frac{3\,ilB}{5}}$$

The direction of force is inwards of the screen.

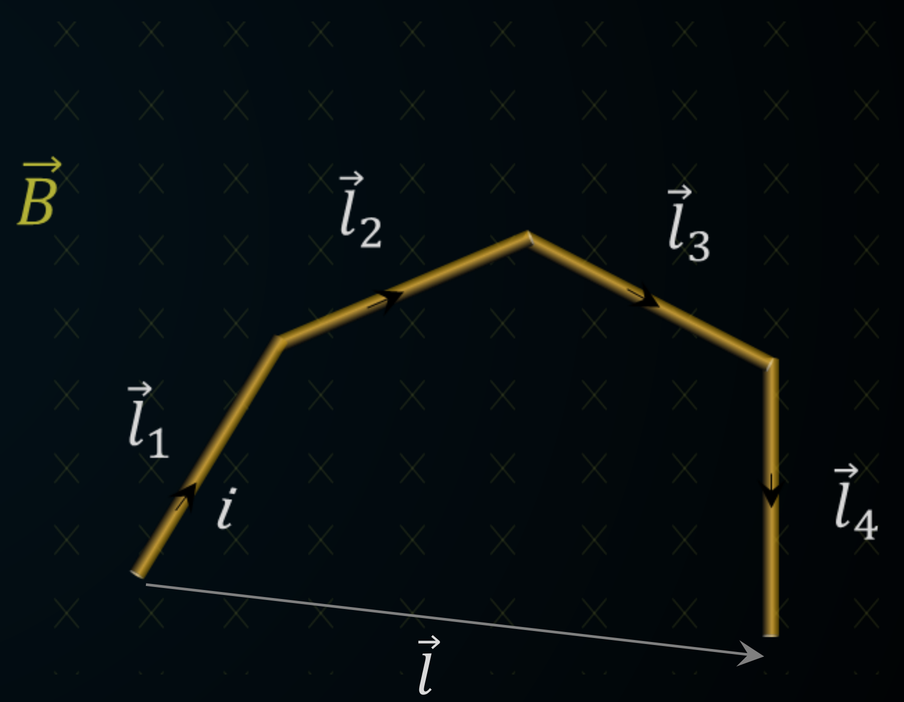
$$\boxed{\vec{F}_{net} = ilB\sin 37° \otimes}$$

Question

Calculate the magnetic force ($\vec{F}_{net}$) on current carrying wires of length l_1, l_2, l_3 and l_4 joined together and placed in a magnetic field $\vec{B}$ as shown.

Solution

As the magnetic field is constant, force is calculated from displacement vector.



$$\vec{F}_{net} = i(\vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \cdots) \times \vec{B}$$

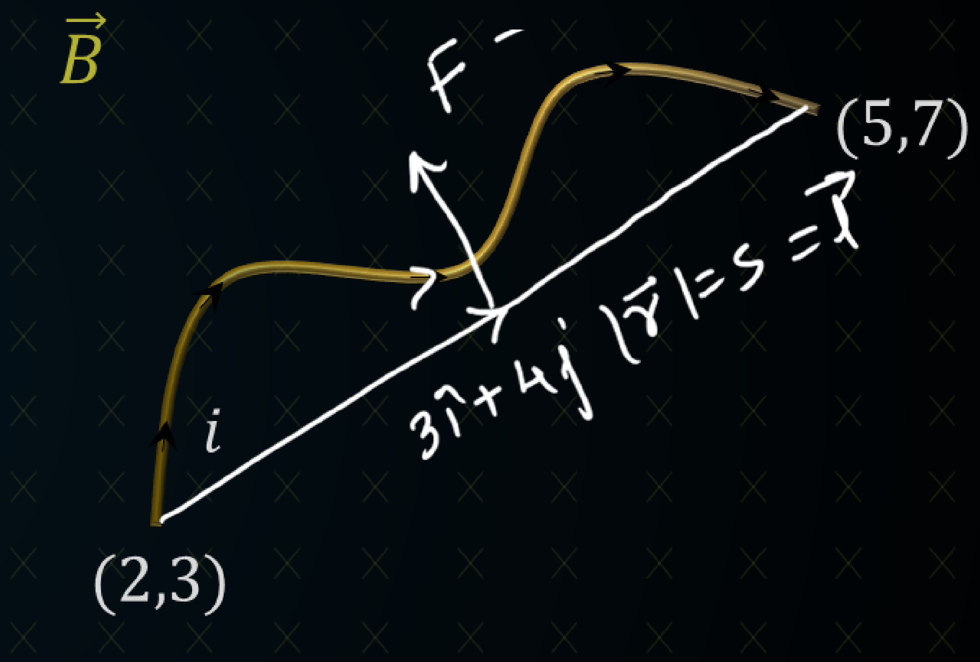
$$\vec{F}_{net} = i(\vec{l} \times \vec{B})$$

It is valid when $\vec{B}$ and i are constant.

Question

Calculate the magnetic force ($\vec{F}_{net}$) on a current carrying wire placed in a magnetic field $\vec{B}$ as shown. (Consider $\vec{B}$ is constant)

Solution



Net force,

$$\vec{F}_{net} = i(\vec{l} \times \vec{B})$$

Here,

$$\vec{l} = (5-2)\,\vec{i} + (7-3)\,\vec{j} = 3\vec{i} + 4\vec{j}$$

$$|\vec{l}| = \sqrt{\{3^2 + 4^2\}} = 5$$

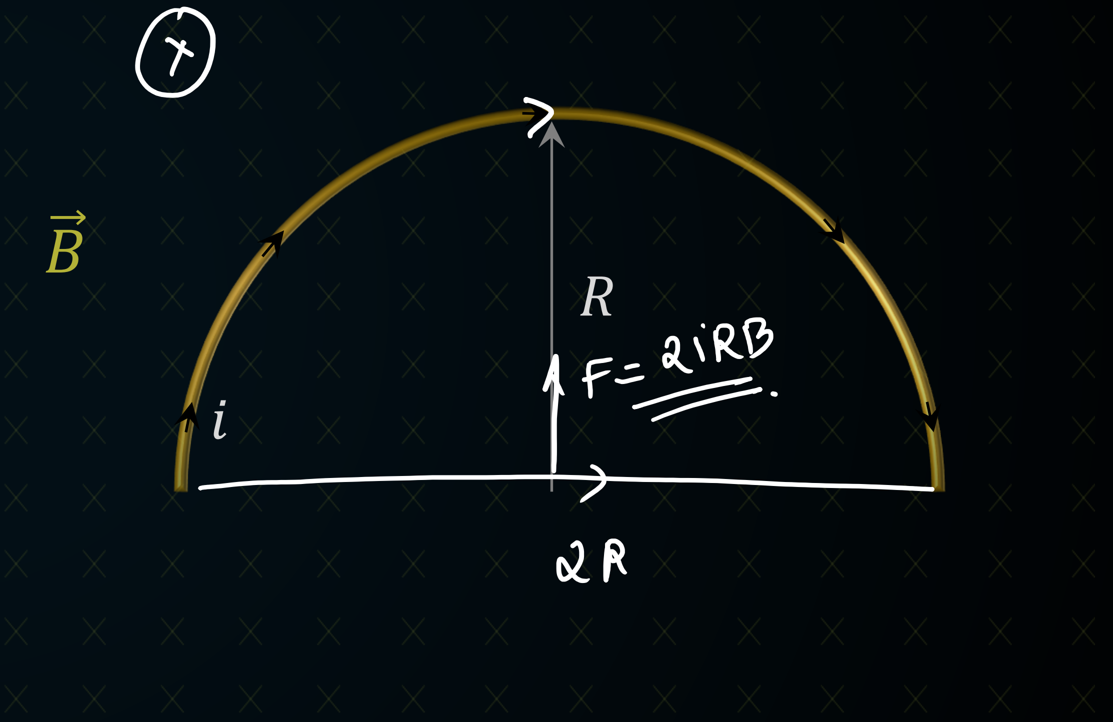
$F = i \times 5 \times B = \underline{\underline{5iB}}$ [Since the angle between $\vec{l}$ and $\vec{B}$ is 90°]

$$\boxed{|\vec{F}_{net}| = 5iB}$$

Question

Calculate the magnetic force ($\vec{F}_{net}$) on a current carrying semicircular wire of radius R placed in a magnetic field $\vec{B}$ as shown. (Consider $\vec{B}$ is constant)

Solution



Net force,

$$\vec{F}_{net} = i(\vec{l} \times \vec{B})$$

Here,

$$|\vec{l}| = 2R$$

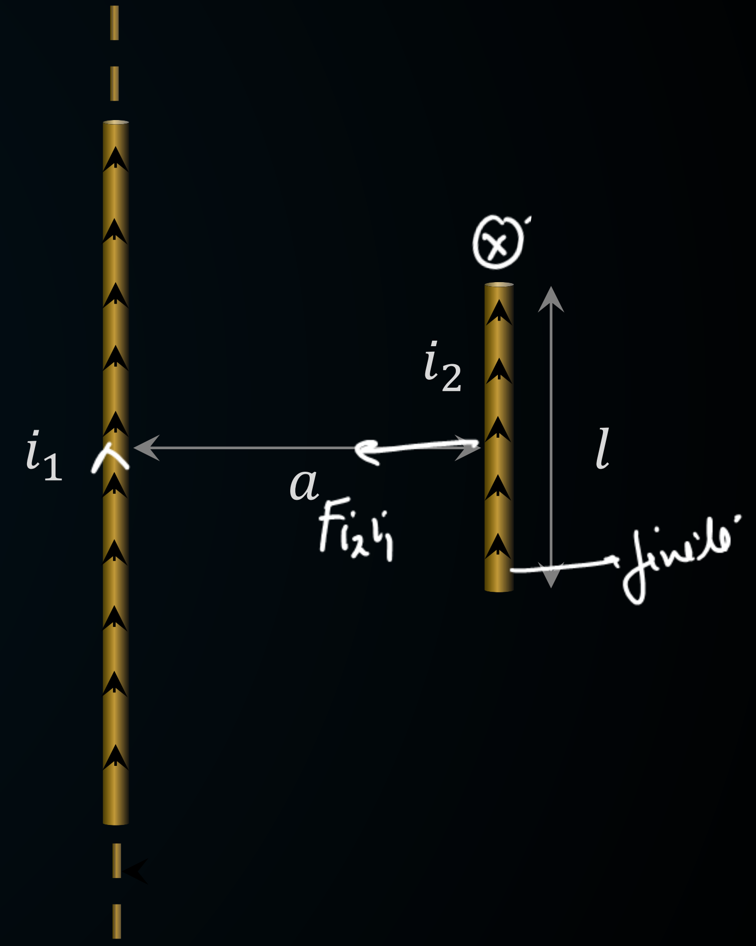
$$|\vec{F}_{net}| = i2RB$$

$$\boxed{\vec{F}_{net} = 2iRB \uparrow}$$

Question

Calculate the magnetic force ($\vec{F}_{net}$) on a current carrying wire of length l placed at a distance a from another infinite current carrying wire as shown.

Solution



Magnetic field at finite wire due to infinite wire,

$$\otimes \rightarrow B_1 = \frac{\mu_0 i_1}{2\pi a} \Bigg\} \rightarrow \text{for } i_2 \text{ this is Constant}$$

As the field is constant, force on finite wire is, $F_{12 i_1} = i_2 \cdot l \dfrac{\mu_0 i_1}{2\pi a}$

$$\boxed{= \frac{\mu_0 i_1 i_2 l}{2\pi a}}$$

$$\boxed{\vec{F}_{net} = i_2 l \frac{\mu_0 i_1}{2\pi a} \leftarrow}$$

Force between Two Infinite Current Carrying Wires

Magnetic field at wire 2 due to wire 1, $B_{21} = \frac{\mu_0 i_1}{2\pi d} \otimes$

Force per unit length on wire 2 due to this magnetic field,

$$\frac{F_{21}}{l} = i_2 \times 1 \times \frac{\mu_0 i_1}{2\pi d} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

Magnetic field at wire 1 due to wire 2, $B_{12} = \frac{\mu_0 i_2}{2\pi d} \odot$

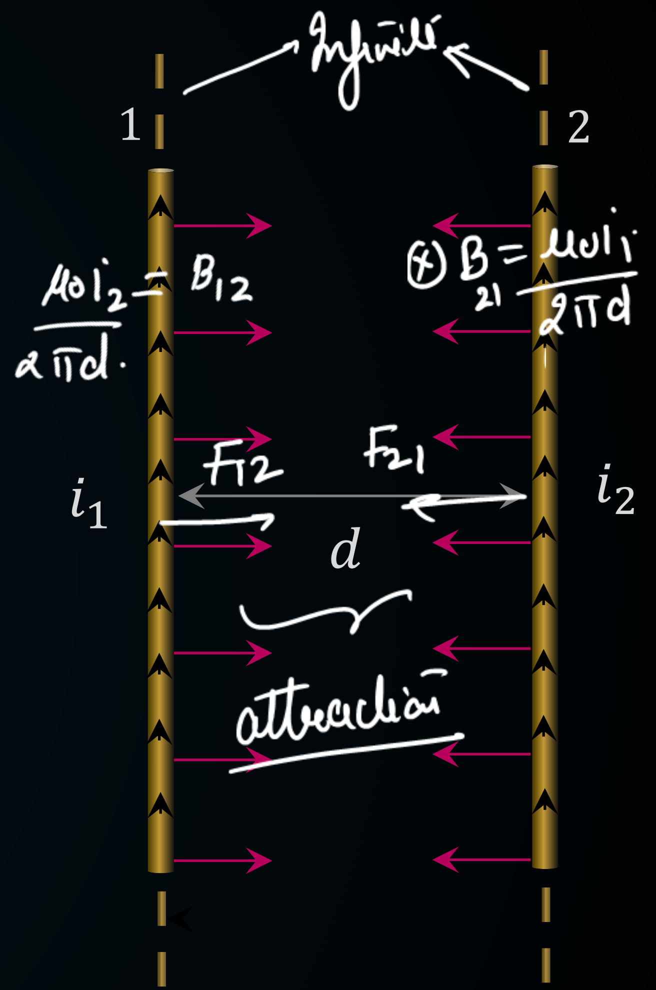
Force per unit length on wire 2 due to this magnetic field,

$$\frac{F_{12}}{l} = i_1 \times 1 \times \frac{\mu_0 i_2}{2\pi d} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

The magnitude of force on both the wires is same.

$$\boxed{\frac{\text{Force}}{\text{length}} = \frac{\mu_0 i_1 i_2}{2\pi d}}$$

$$\frac{Force}{Length} = \frac{\mu_0 i_1 i_2}{2\pi d}$$



» Due to this force, two parallel current carrying wires having current i in the same direction attract each other. Example: Electric wires between the poles. An insulator is placed between two high tension power lines to counter the attractive force between them.

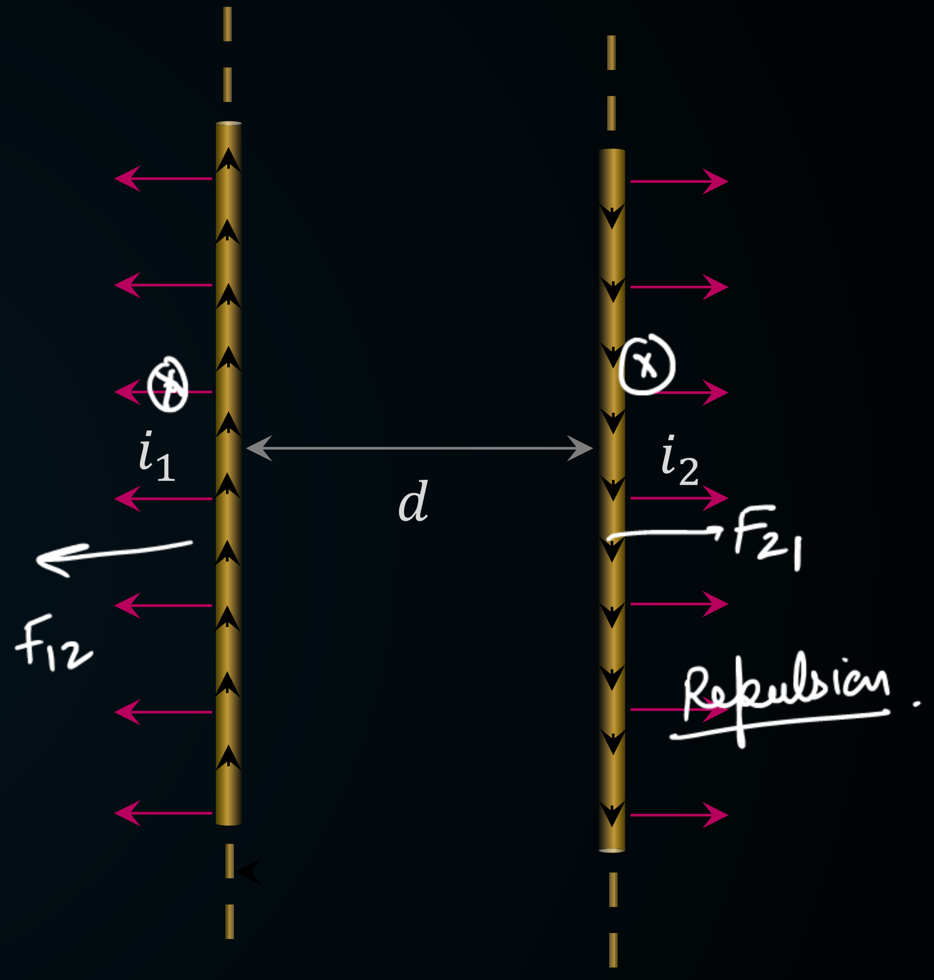
Force between Two Infinite Current Carrying Wires

- Two parallel current carrying wires having current i in the opposite direction repel each other.

$$\frac{F_{21}}{l} = \frac{F_{12}}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

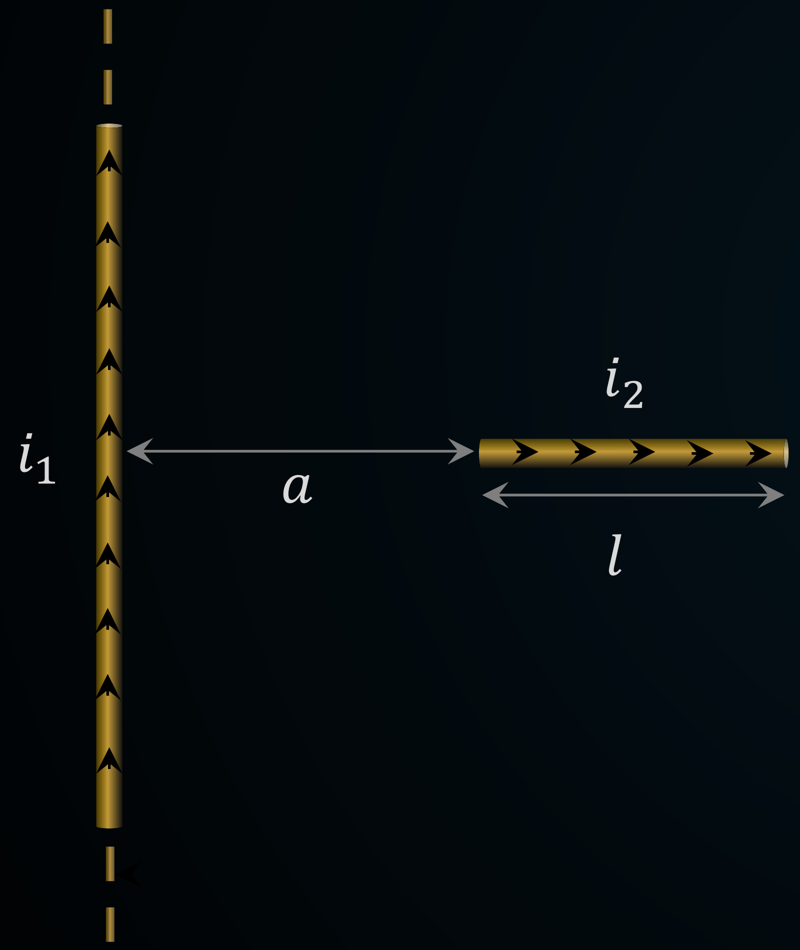
The magnitude of force is same as previous case but the direction is opposite.

$$\boxed{\frac{Force}{Length} = \frac{\mu_0 i_1 i_2}{2\pi d}}$$



Question Solution

Calculate the magnetic force $(\vec{F}_{net})$ on a current carrying wire of length l placed at a distance a from another infinite current carrying wire as shown.



Question Solution

Consider a small element of length dx on the finite wire at distance x from the infinite wire.

Magnetic field at this location, $\otimes\ B_{i_1} = \frac{\mu_0 i_1}{2\pi x}$

Thus, the magnetic field along the length of the finite wire is variable.

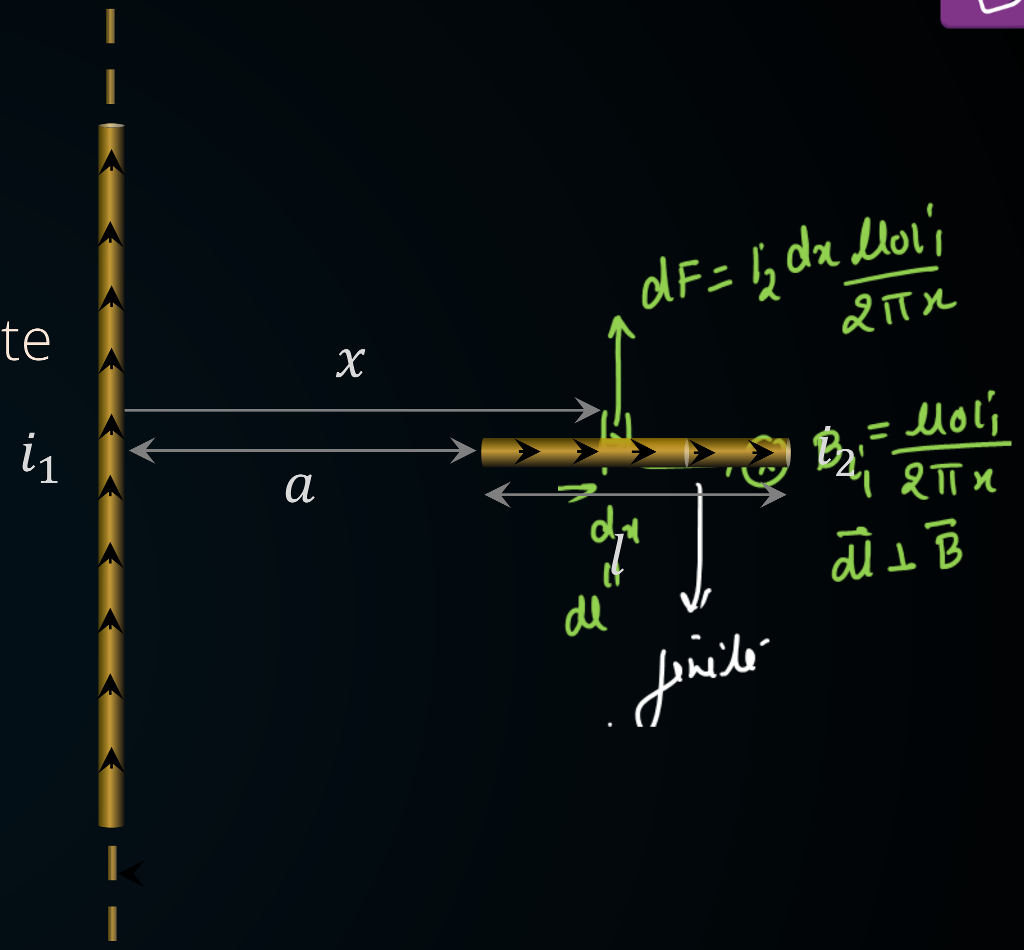
Force on the element of the finite wire, $dF = i_2\, dx \frac{\mu_0 i_1}{2\pi x}$

The direction of force remains same as element is changed. Hence, Integrating for net force on the wire,

$$\therefore\ F_{net} = \int_a^{a+l} d\vec{F}$$

$$F_{net} = \frac{\mu_0 i_1 i_2}{2\pi}\int_a^{a+l}\frac{dx}{x} = \frac{\mu_0 i_1 i_2}{2\pi}\left[\ln x\right]_a^{a+l}$$

$$\boxed{= \frac{\mu_0 i_1 i_2}{2\pi}\ln\left[\frac{a+l}{a}\right]}$$



$$\vec{F}_{net} = \frac{\mu_0 i_1 i_2}{2\pi}\ln\frac{(l+a)}{a}\ \uparrow$$

Question

Calculate the magnetic force ($\vec{F}_{net}$) on current carrying closed square loop placed in a magnetic field $\vec{B}$ as shown. (Consider $\vec{B}$ is constant)

Solution

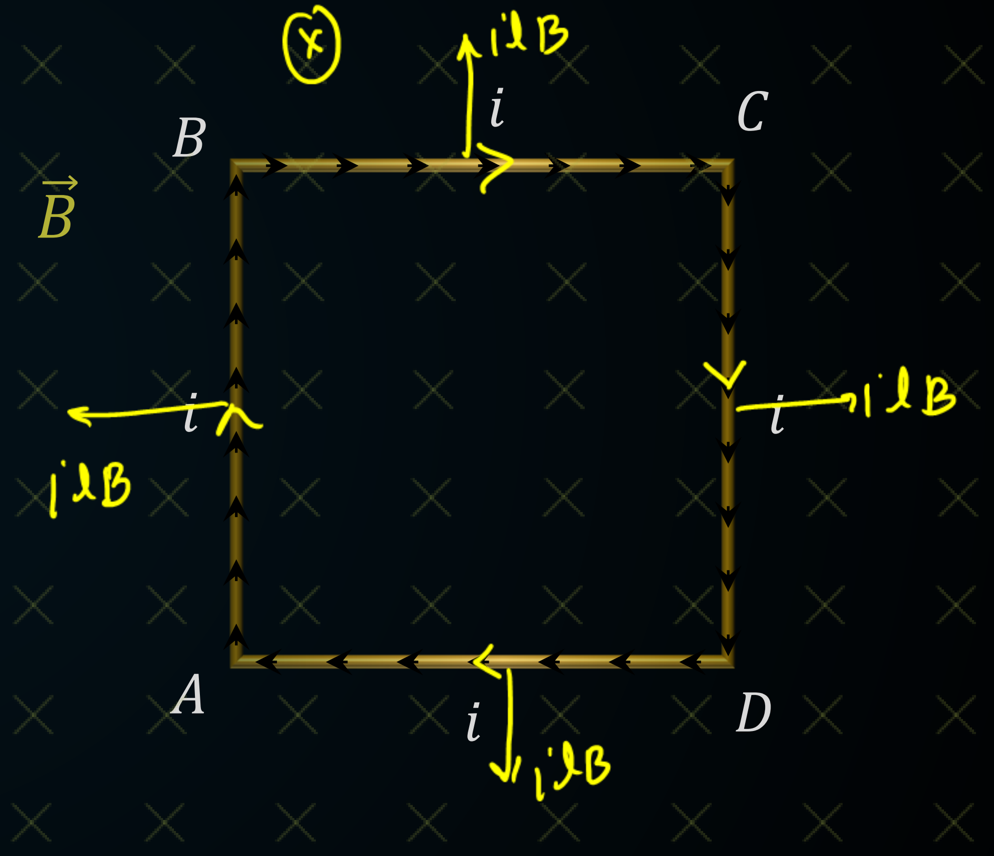
As the wire is closed, the displacement vector is a null vector. Thus, force on the wire is zero.

$\vec{B}$ = Constant

$\vec{dl} = 0$

$\therefore \vec{F}_{net} = 0$

F_{net} on a closed current carrying loop is zero in a constant M.f.

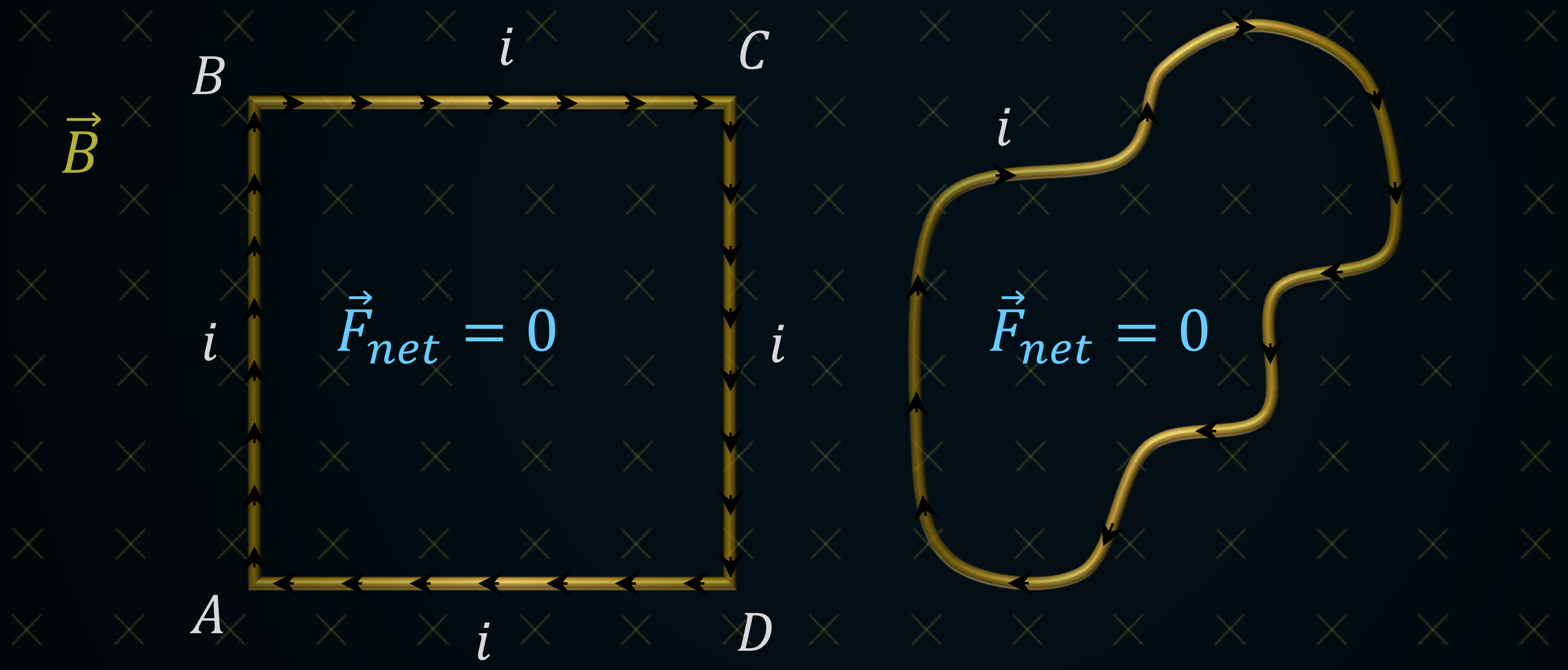


$F_{net} = 0$

Though the net force on loop is zero, it need not to be zero on individual segments.

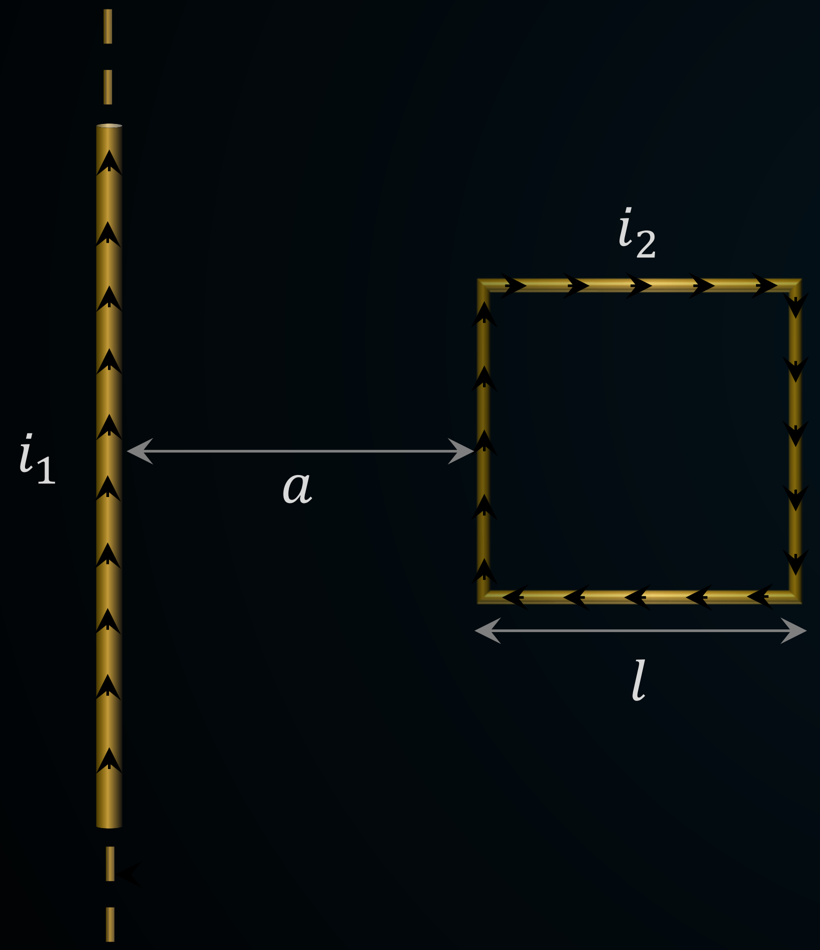
Force on Closed Current Carrying Loop

Net magnetic force ($\vec{F}_{net}$) is zero in a closed current carrying loop placed in uniform magnetic field, but not zero in any segment of the loop.



Question | Solution

Calculate the magnetic force ($\vec{F}_{net}$) on a current carrying closed square loop of side l placed at a distance a from another infinite current carrying wire as shown.



Question Solution

Here, the loop is closed but magnetic field is not constant.

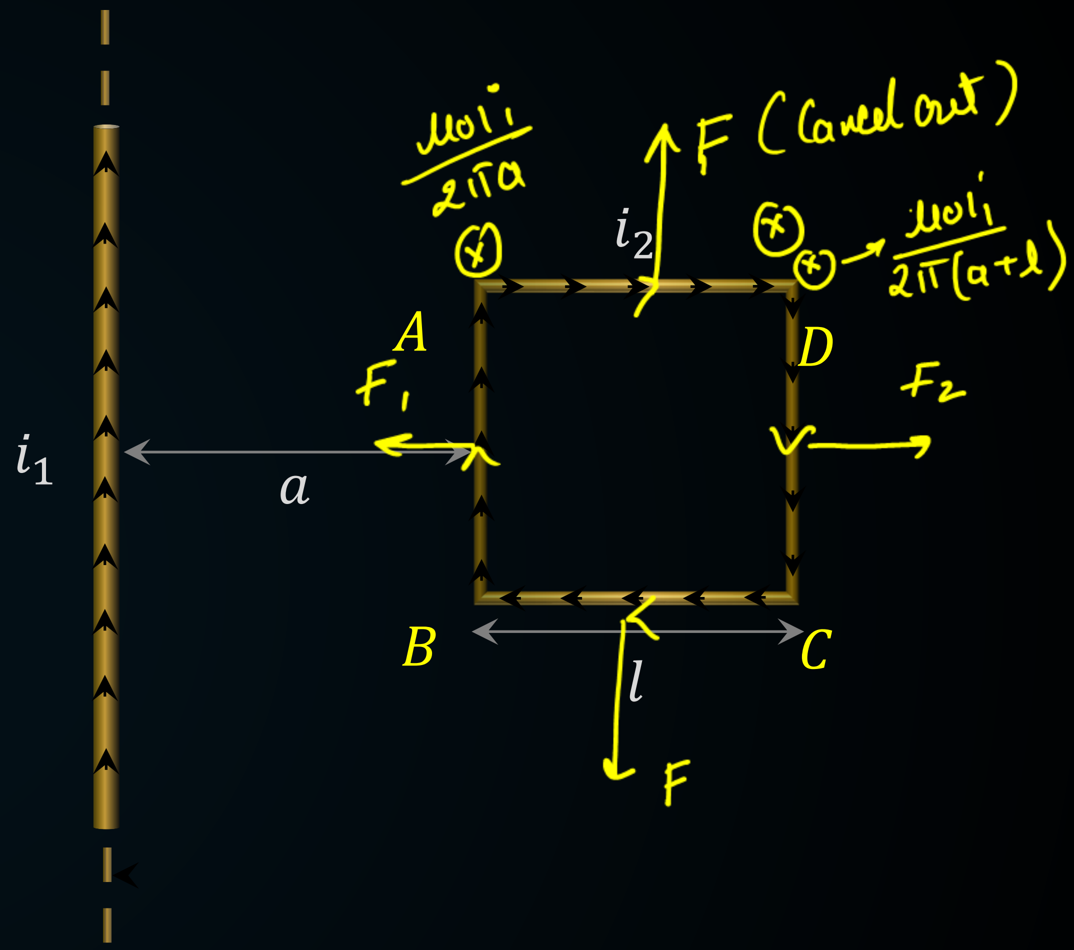
Magnetic field at wire *AB*: $\dfrac{\mu_0 i_1}{2\pi a}$

Magnetic field at wire *CD*: $\dfrac{\mu_0 i_1}{2\pi(a+l)}$

As the field on wires *AD* and *BC* is same, force acting on them is equal and opposite. Hence this force cancel out. As the magnetic field at wire *AB* is more than wire *CD*, and the force acting on these wires is in opposite direction, net force is acting leftwards.

Force on the wire *AB*: $F_1 = i_2 \cdot l \cdot \dfrac{\mu_0 i_1}{2\pi a}$

Force on the wire *CD*: $F_2 = i_2 \cdot l \cdot \dfrac{\mu_0 i_1}{2\pi(a+l)}$



Net force on the loop:

$$\underbrace{F_1 - F_2}_{} = \frac{\mu_0 i_1 i_2 l}{2\pi}\left[\frac{1}{a} - \frac{1}{a+l}\right]$$

$$|\vec{F}_{net}| = \frac{\mu_0 i_1 i_2 l}{2\pi}\left(\frac{1}{a} - \frac{1}{a+l}\right)$$

Question | Solution

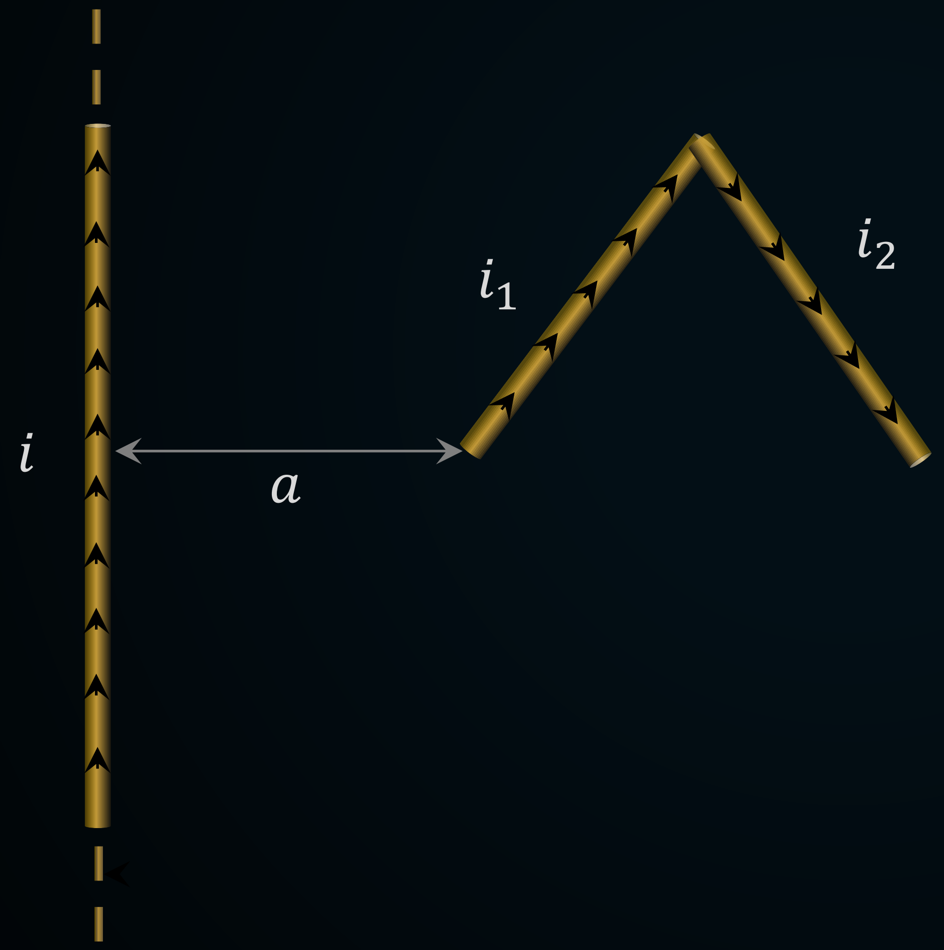
Find the direction of net magnetic force ($\vec{F}_{net}$) on a current carrying wire placed at a distance a from another infinite current carrying wire as shown.

a $\uparrow$

b $\rightarrow$

c $\nwarrow$

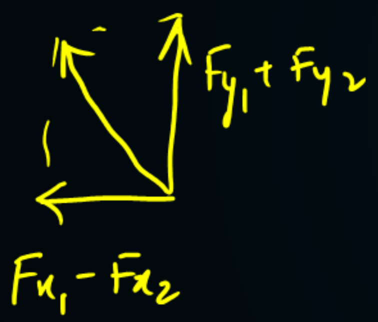
d $\nearrow$



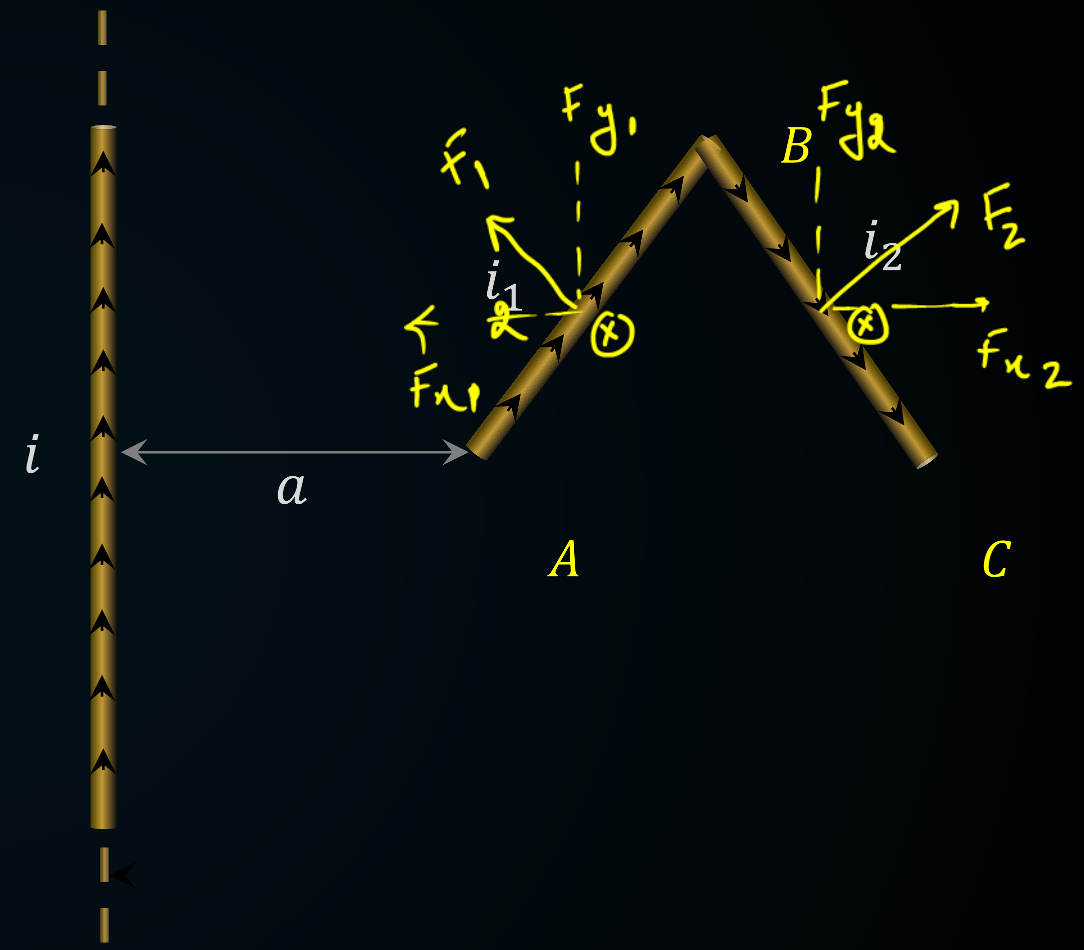
Question | Solution

The direction of magnetic field is same for both segments of wire, but its magnitude is more for wire segment AB. Thus, $F_1 > F_2$.

From figure, y-component of force adds whereas x-components is facing opposite for the two segments.

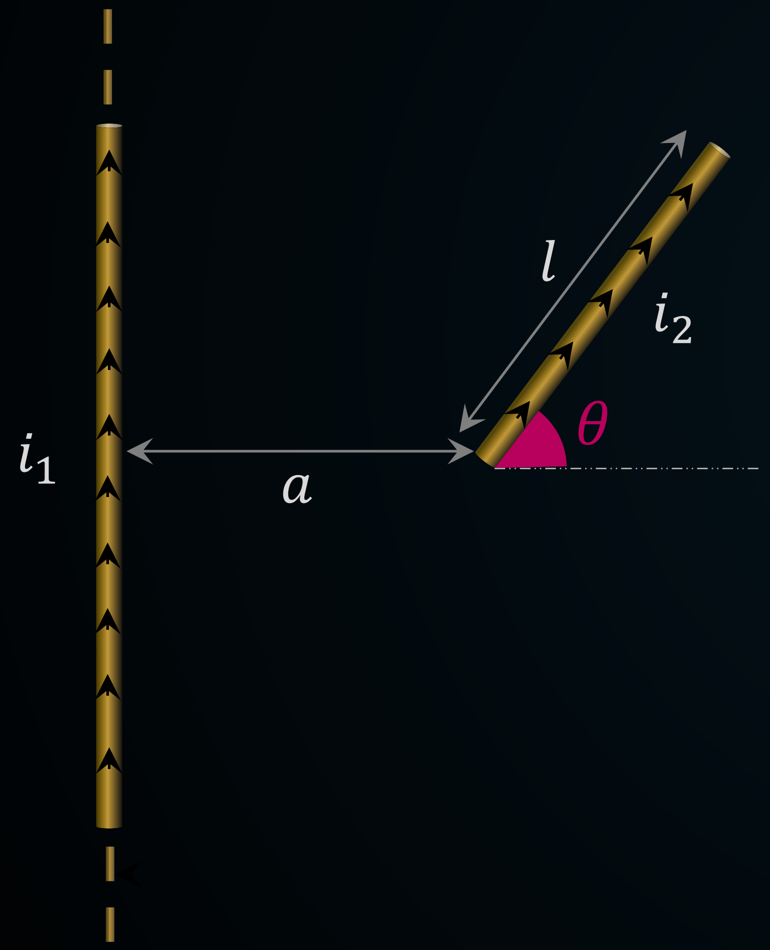


Thus, if upper is north, net force will be directed towards North-West direction. Thus, option c is correct.



Question Solution

Calculate the magnetic force ($\vec{F}_{net}$) on a current carrying wire of length l placed at a distance a from another infinite current carrying wire as shown.



Question Solution

$\vec{i}$ can be broken into components when magnetic field is uniform in the region.

Here, magnetic field is non uniform. Take a small element of length dx at length x from lower end of finite wire as shown in figure.

Magnetic field at the element: $B = \frac{\mu_0 i_1}{2\pi(a+x\cos\theta)}$

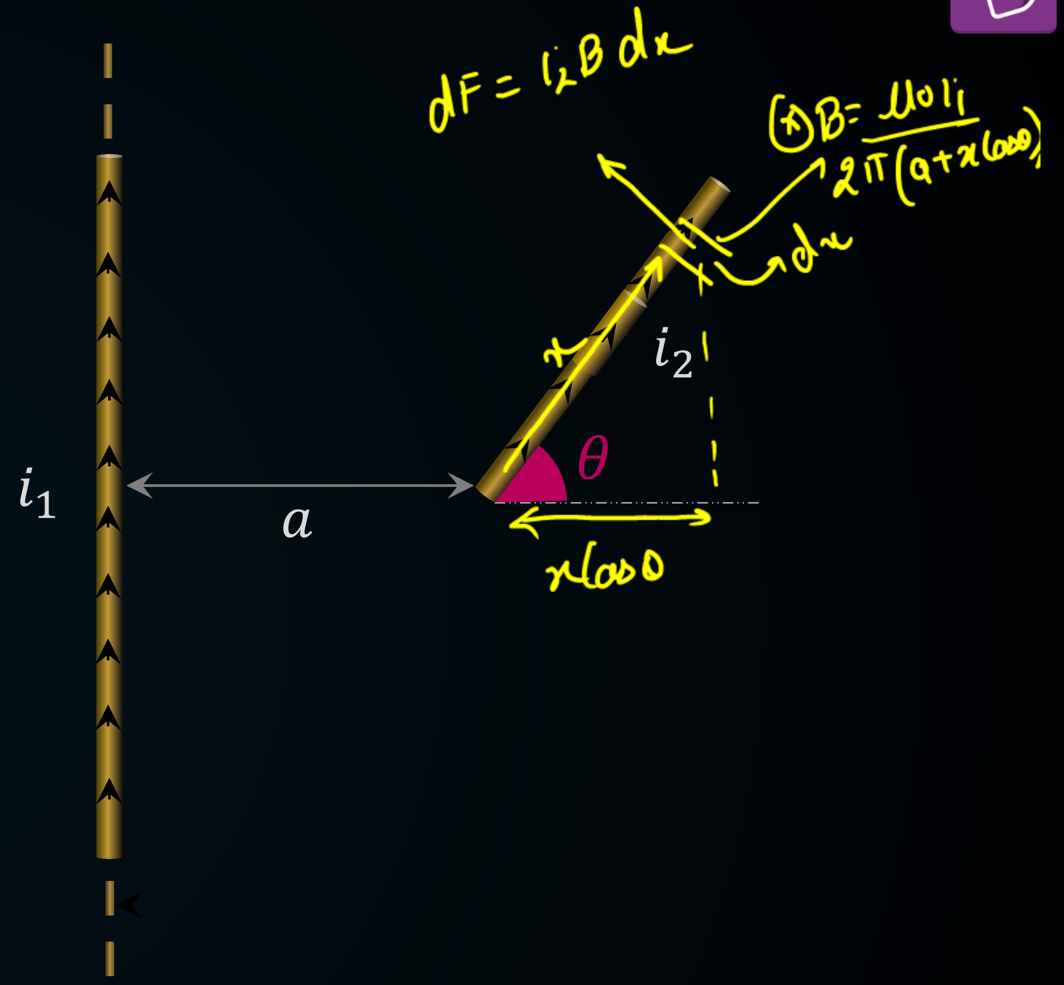
Force on the element dx: $dF = i_2 B\, dx$

Integrating for the wire,

$$F_{net} = \int dF = \int i_2 \frac{\mu_0 i_1}{2\pi(a+x\cos\theta)} dx.$$

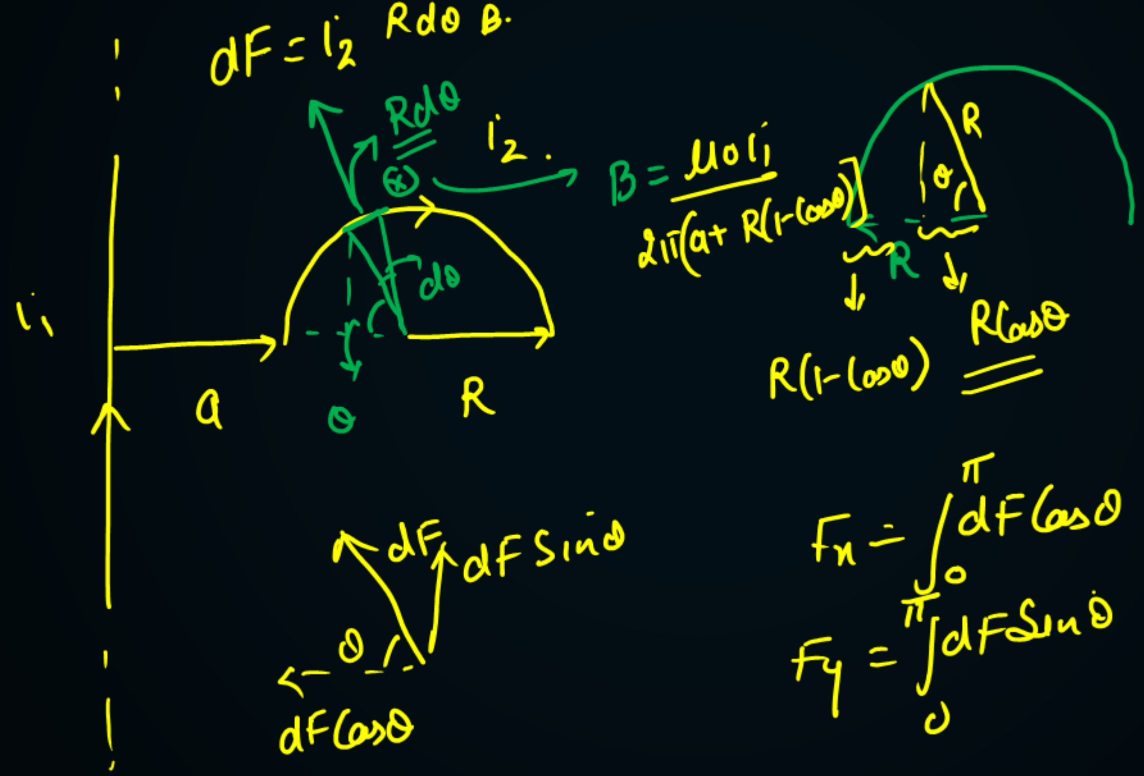
$$= \frac{\mu_0 i_1 i_2}{2\pi}\int_0^l \frac{dx}{(a+x\cos\theta)}.$$

$$F_{net} = \frac{\mu_0 i_1 i_2}{2\pi\cos\theta} \ln\left[\frac{a+l\cos\theta}{a}\right]$$



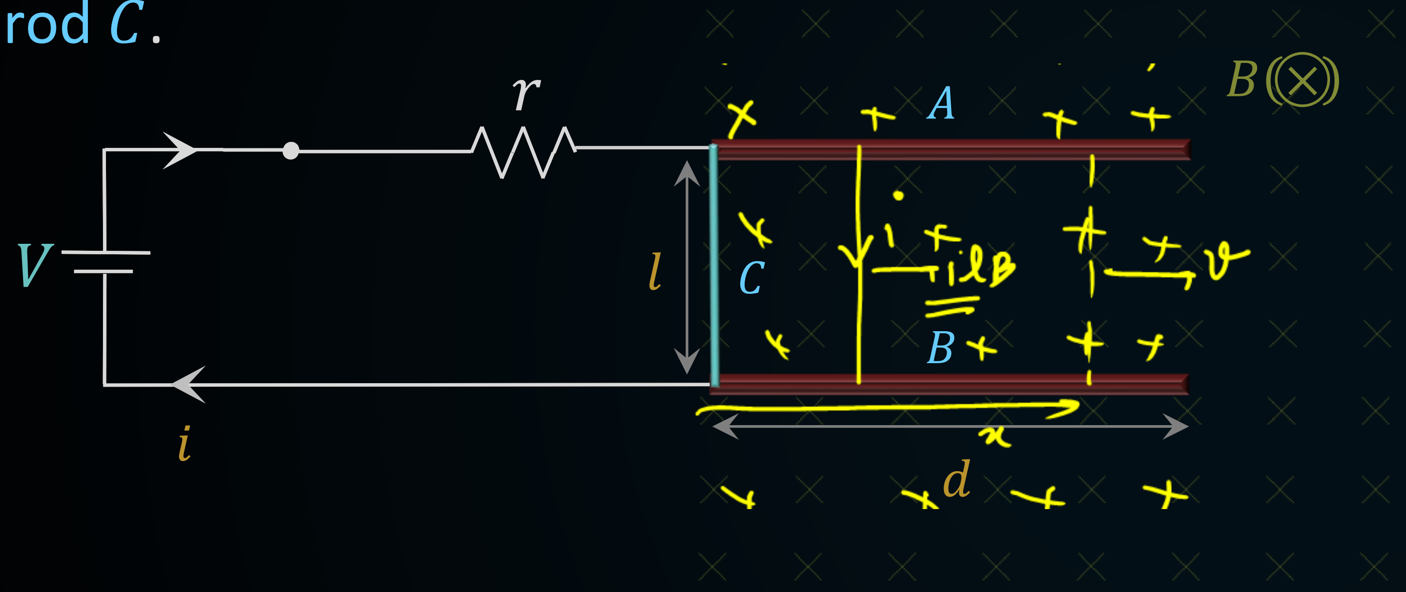
$$\vec{F}_{net} = \frac{\mu_0 i_1 i_2}{2\pi}\int_0^l \frac{dx}{(a+x\cos\theta\ )}$$

$\vec{i}$ can be broken into components when magnetic field is uniform in the region. For varying magnetic field, we can write the components of force on an element and then integrate it as shown in the example below:



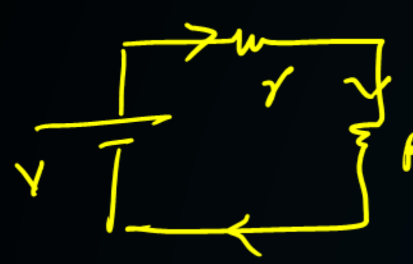
Question Solution

Two frictionless rods A and B with zero resistance are separated by a distance l and placed in a magnetic field. These rods are connected to an external circuit as shown. If another rod C of mass m, length l and resistance R is placed on these rods, then find the force on the rod C.

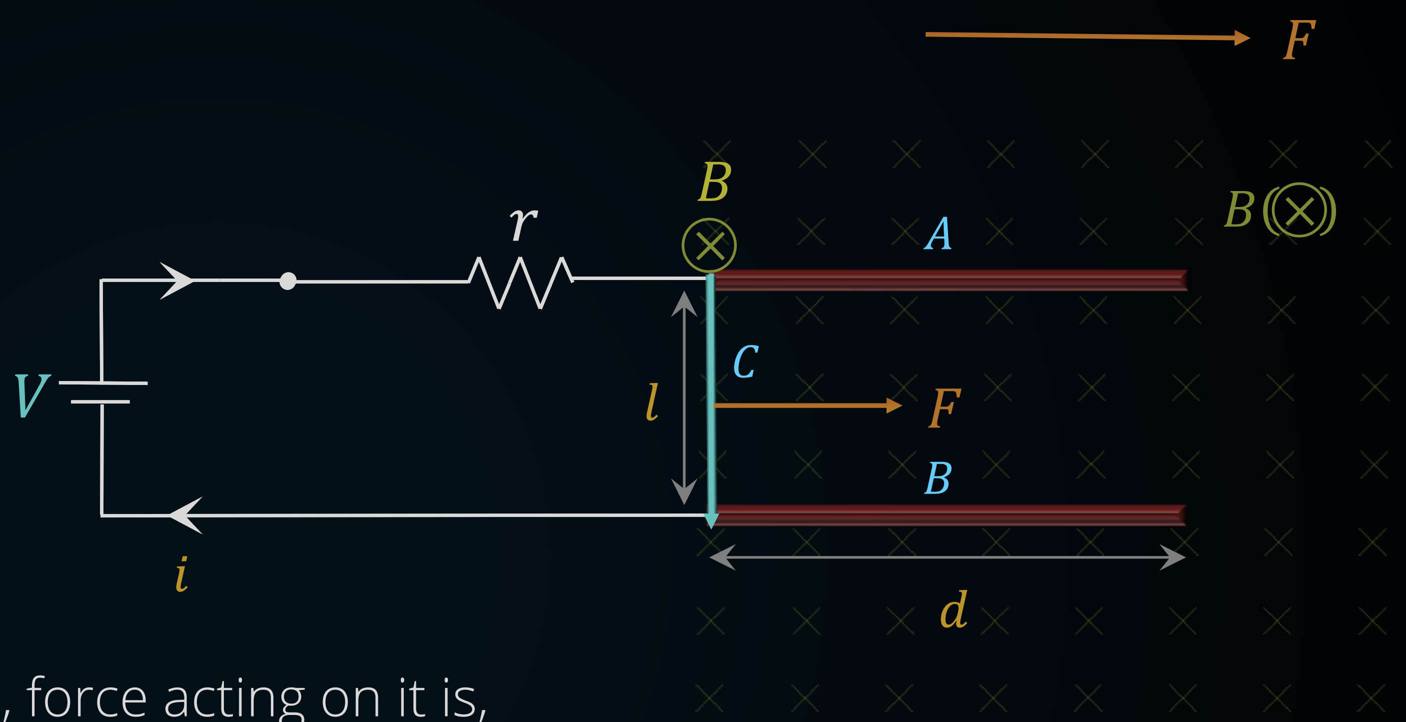


Question | Solution

The equivalent circuit is shown below.



$$i' = \frac{V}{R+r}$$



Current flowing through the circuit,

$$i' = \frac{V}{R+r}$$

As the rod is kept in uniform magnetic field, force acting on it is,

$$F = ilB = \frac{VlB}{R+r}$$

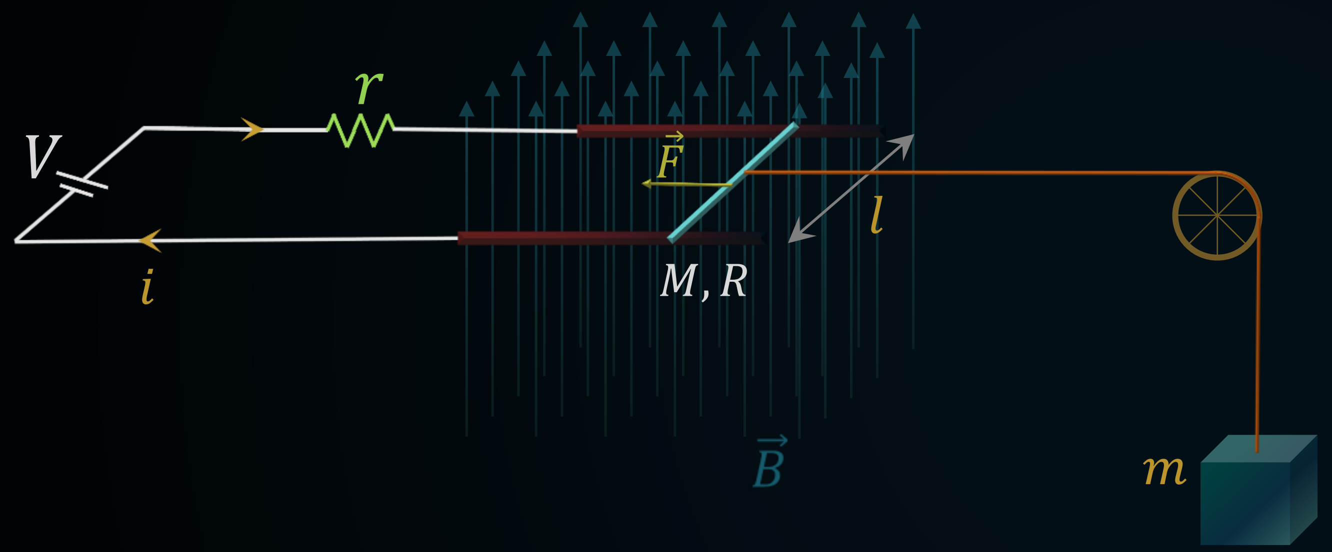
Acceleration of the rod: $a = \frac{i'lB}{m}$

Velocity at distance x: $v^2 - 0^2 = 2\frac{i'lB}{m} \cdot x \Rightarrow v = \sqrt{\frac{2i'lBx}{m}}$

$$F = \frac{VlB}{R+r}$$

Question Solution

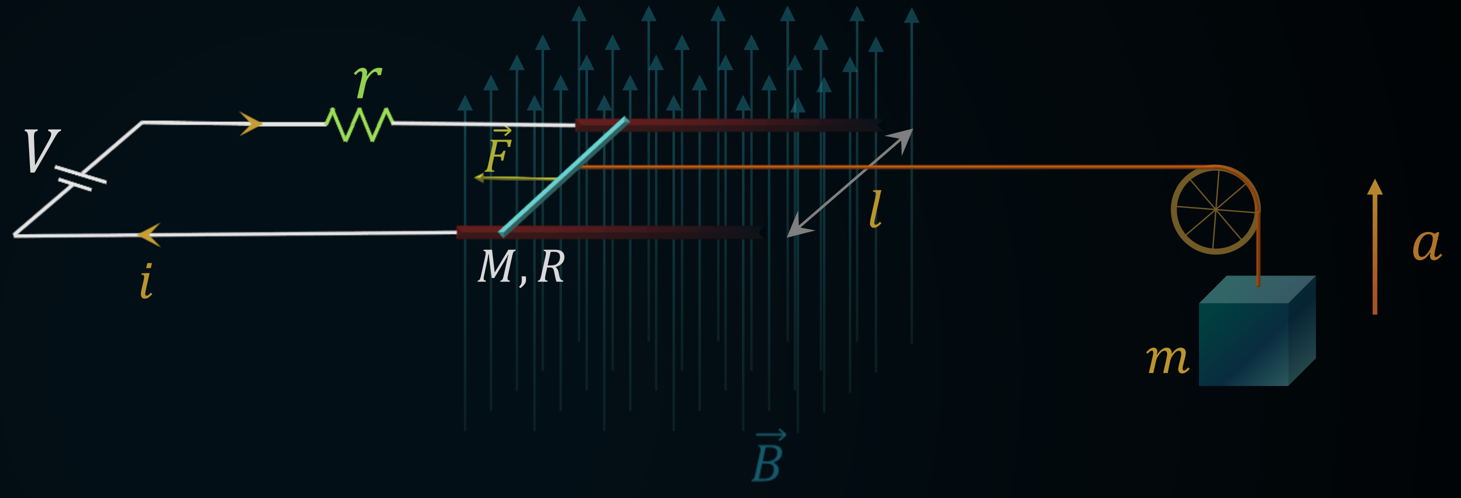
A rod of mass M and resistance R is connected to a block of mass m as shown. What should be the strength of magnetic field so that the given arrangement is able to pick up the mass m with an acceleration a.



Question | Solution

The magnetic force acts on the rod and weight force on the mass. Let the net acceleration of system be a as shown.

Writing force equations in vertical and horizontal direction,



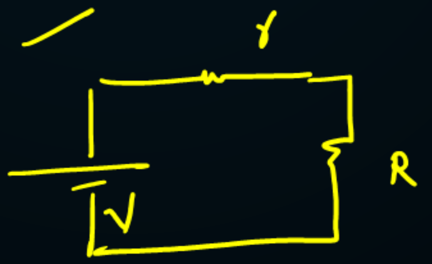
$$i'lB - T = M\cdot a \quad \text{—(1)}$$

$$T - mg = m\cdot a \quad \text{—(2)}$$

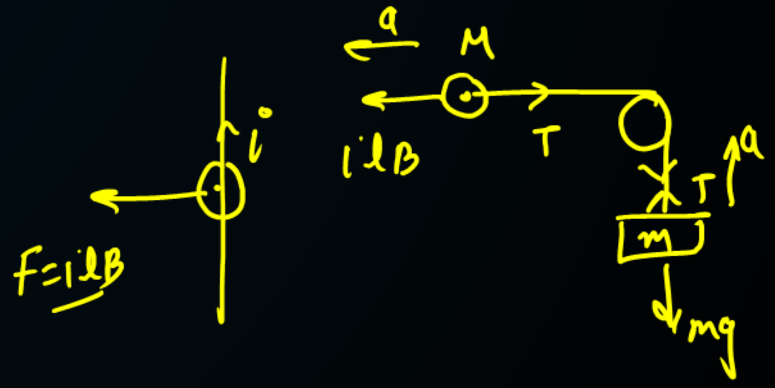
Adding equation (1) and (2),

$$\frac{i'lB - mg}{M+m} = a \quad .....(3)$$

Where, current in the circuit is, $i = \dfrac{V}{R+r}$



$$i = \frac{V}{R+r}$$

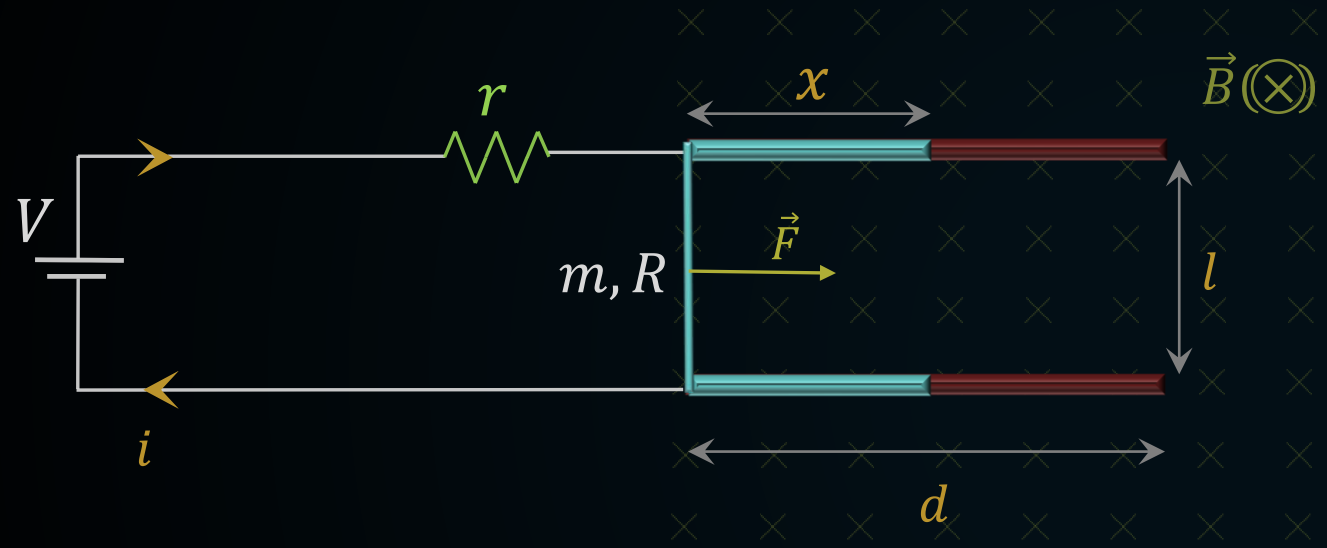


Putting in equation (3), magnetic field is,

$$B = \frac{[m(a+g) + Ma](R+r)}{Vl}$$

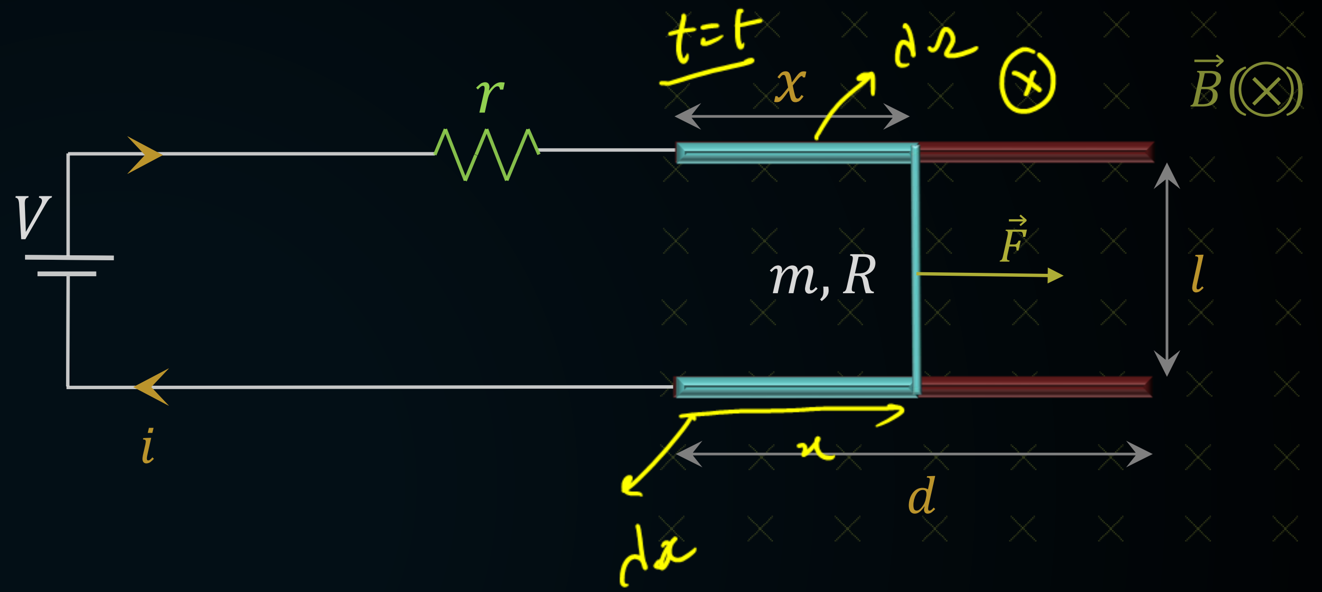
Question Solution

A frictionless rod is placed over two rails of resistance $\lambda \frac{\Omega}{m}$. This arrangement is connected to an external circuit as shown. Find the value of velocity in terms of x.



Question Solution NEET

At a time t, let the rod is moved through distance x. The equivalent circuit is shown in figure.



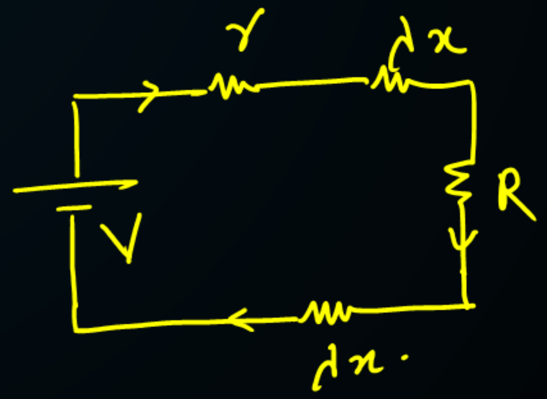
Force on the rod: $\frac{V\,lB}{R+r+2\lambda x} = F(x)$

Acceleration: $a(x) = \frac{V l B}{m(R+r+2\lambda x)} = v\frac{dv}{dx}$

Rearranging and integrating for velocity,

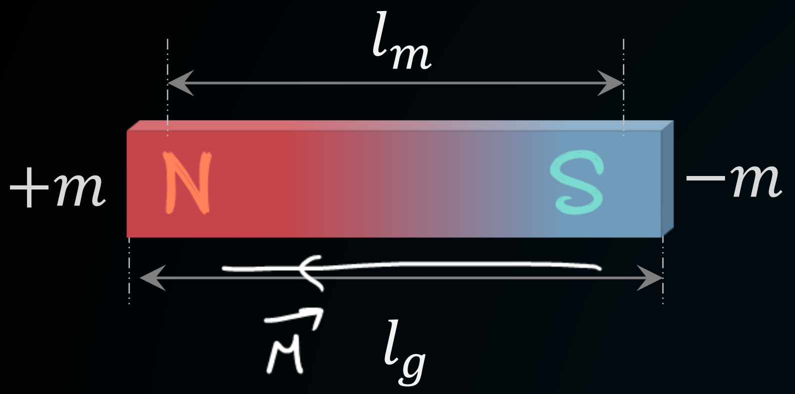
$$\int_0^v v\,dv = \frac{VlB}{m}\int_0^x \frac{dx}{R+r+2\lambda x}$$

$$\boxed{\frac{v^2}{2} = \frac{VlB}{m\,2\lambda}\ln\left[\frac{R+r+2\lambda x}{R+r}\right]}$$



$$v = \sqrt{\frac{BlV}{m\lambda}\ln\left|\frac{(R+r+2\lambda x)}{R+r}\right|}$$

Magnetic Moment [$\vec{M}$]



l_g- Geometric length

l_m- Magnetic length

l_m is slightly less than l_g.

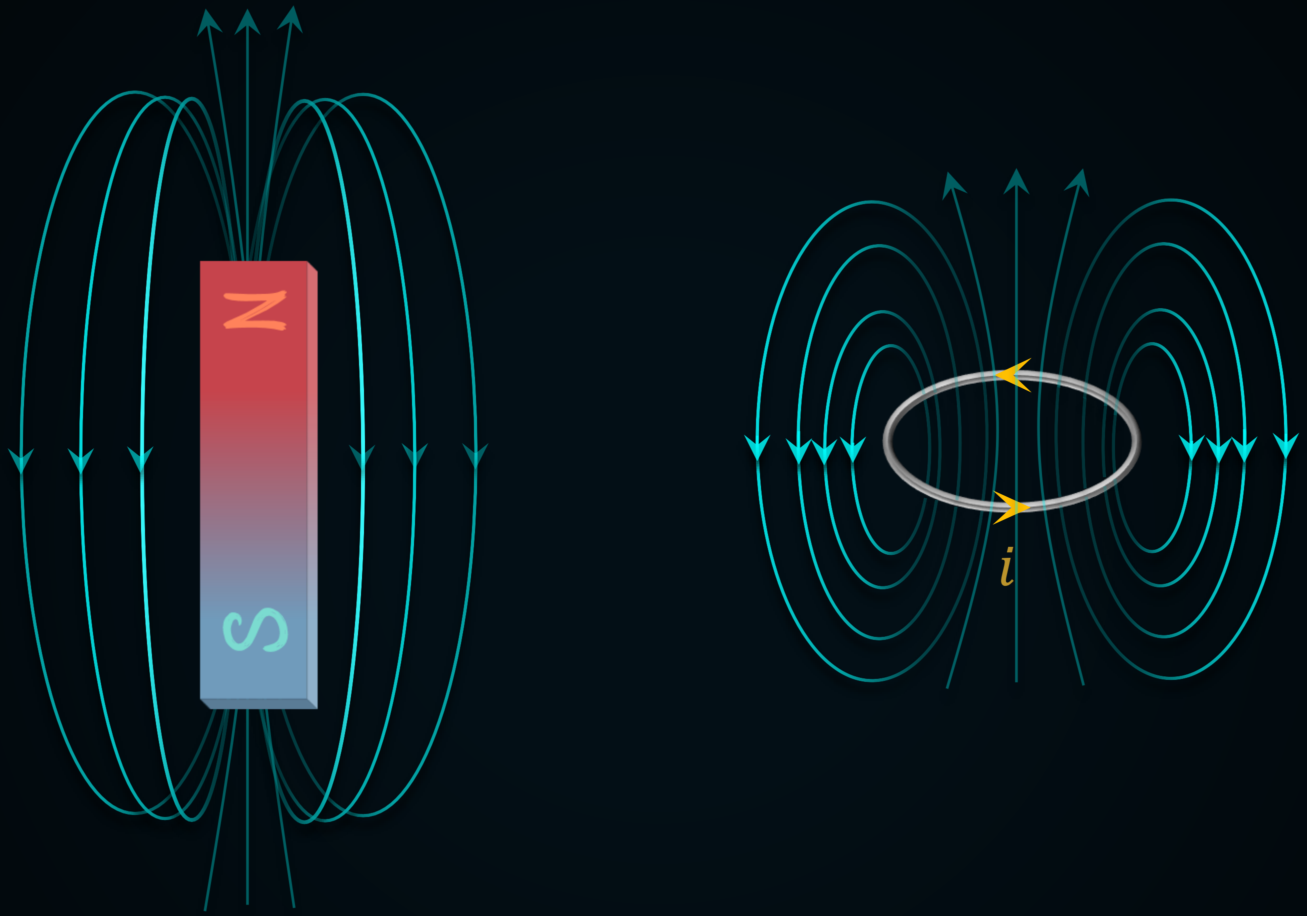
m- Pole strength

$M = m \cdot l_m$ but generally we consider $l_m \approx l_g$ {bcoz diffⁿ is very less}.

$$M = ml_g$$

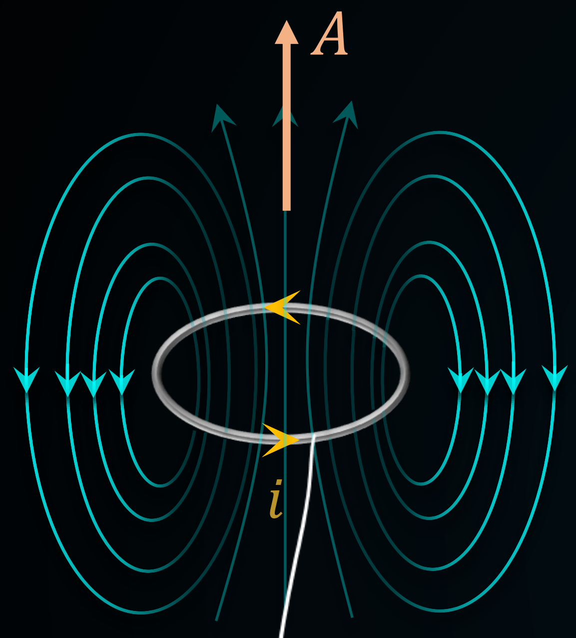
- Direction of magnetic moment (M) is from South to North
- Magnetic moment indicates the power of the magnet
- We define magnetic moment for current carrying loops because current carrying loops acts as magnets,

Magnetic Moment



Current carrying loop is equivalent to a magnet

Magnetic Moment For a Current Carrying Loop



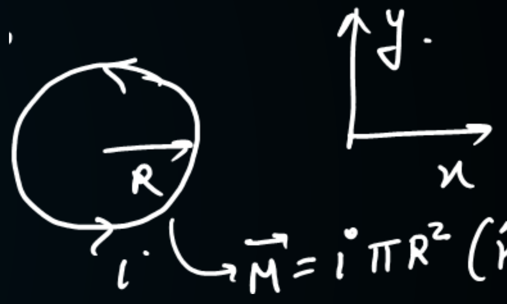
$\boxed{\vec{M} = i\vec{A}}$ Yaad rakhna hai

Area vector

$\vec{A} \perp$ to the loop plane

To find the direction of area vector $\vec{A}$, curl fingers of right-hand in the direction of current. The thumb gives the direction of $\vec{A}$.

Examples:

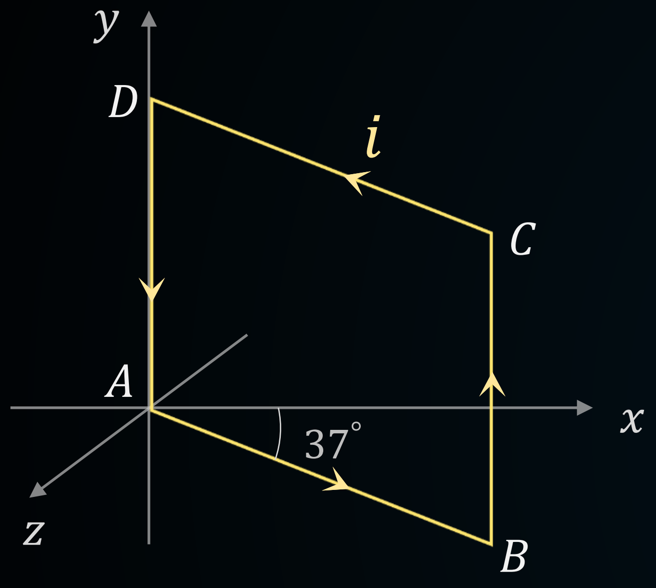


$\vec{M} = i\pi R^2 (\hat{k})$.

$$\vec{M} = i\vec{A}$$

Question | Solution

A square loop of side $5\ m$ is carrying current i as shown. Find the magnetic moment of the loop.



Question Solution NEET

Given, $\vec{AB} = 4\hat{i} + 3\hat{k}$

$\vec{AD} = 5\hat{j}$

From the figure, area vector is,

$$\vec{A} = \vec{AB} \times \vec{AD}$$

$$\vec{A} = (4\hat{i} + 3\hat{k}) \times 5\hat{j}$$

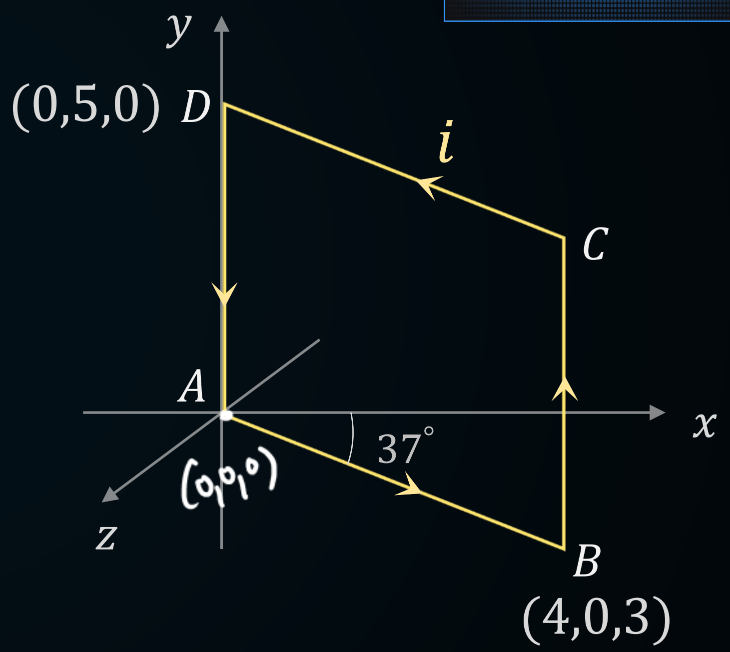
$$= 20\hat{k} - 15\hat{i}$$

We have, $\vec{M} = i\vec{A}$

$$\vec{M} = i[20\hat{k} - 15\hat{i}]$$

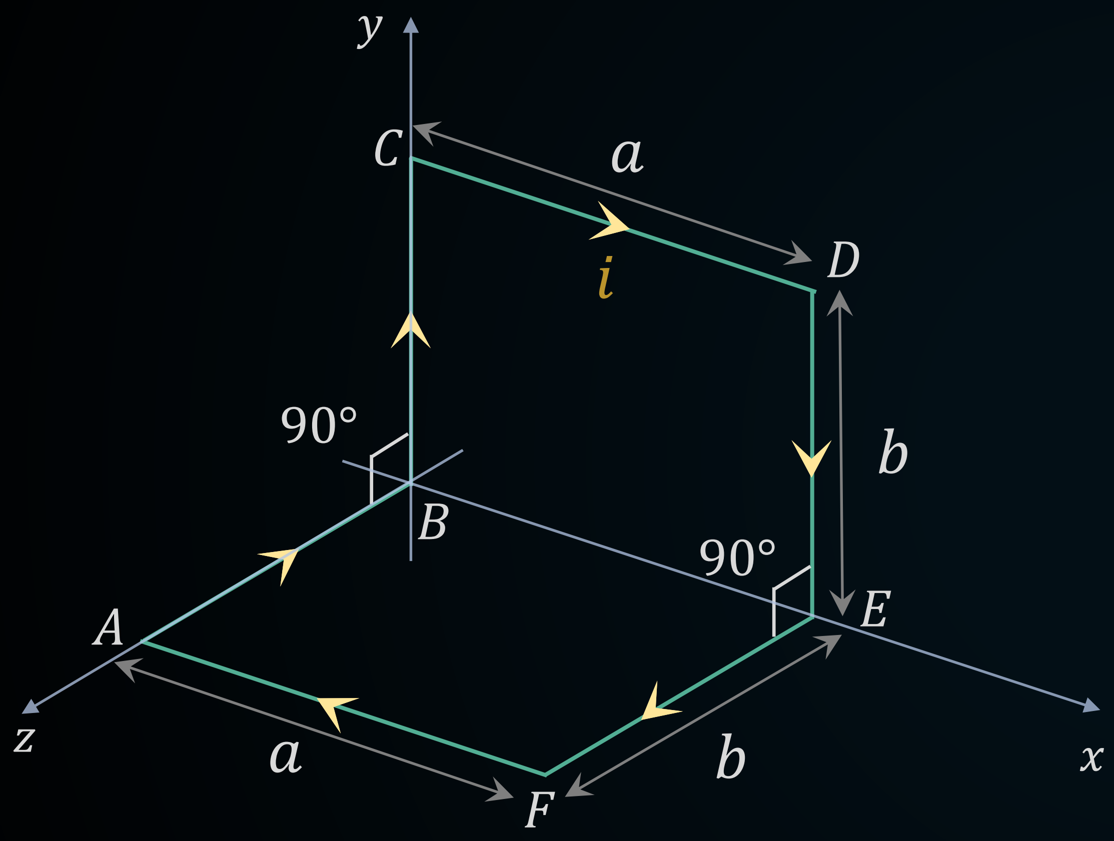
$$\boxed{\vec{M} = i(20\hat{k} - 15\hat{i})}$$

$$\boxed{\vec{M} = i\vec{A}}$$



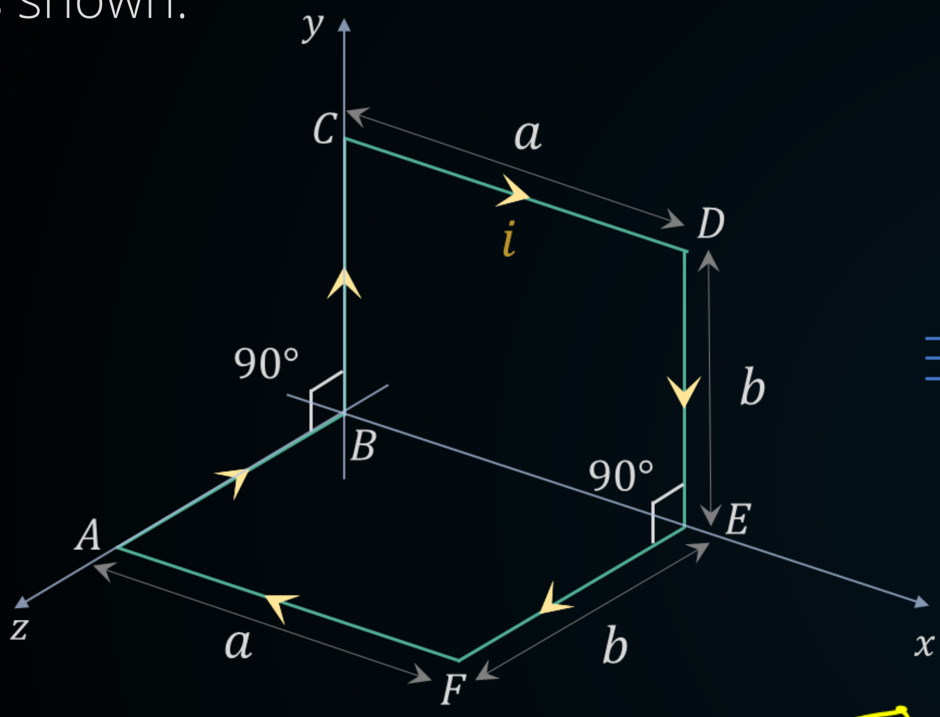
Question Solution

Find the magnetic moment of the current carrying loop as shown.



Question Solution

We can rearrange loop in two planes by adding current elements in opposite direction between BE as shown:



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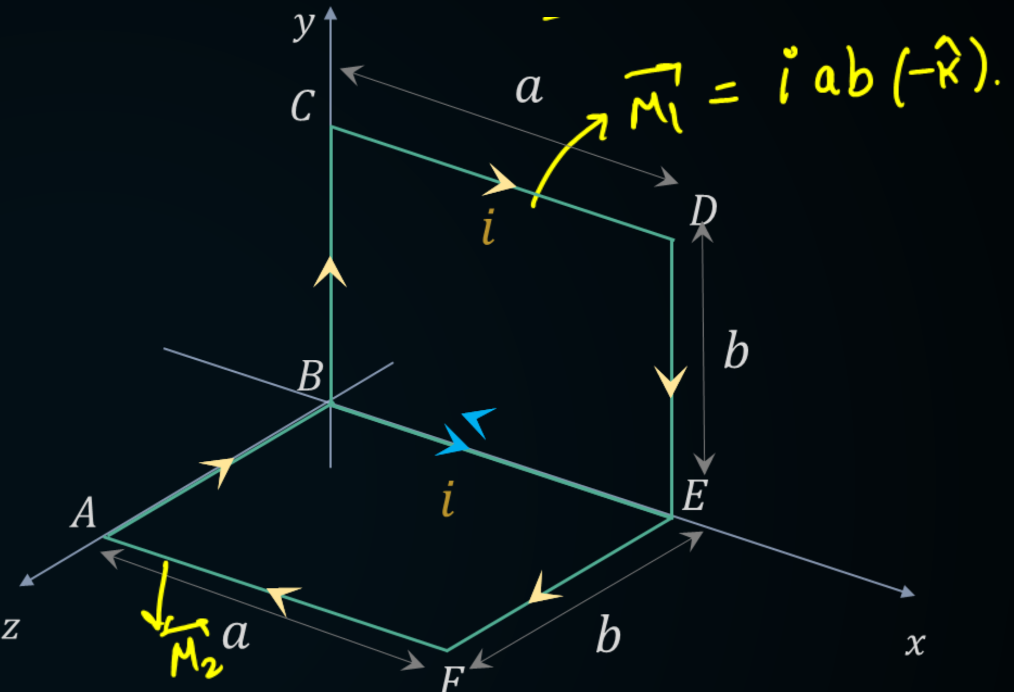


$\vec{M_1} = i\,ab\,(-\hat{k})$.

$\vec{M_2} = i\,ab\,(-\hat{j})$

Net moment is the vector sum of both components.

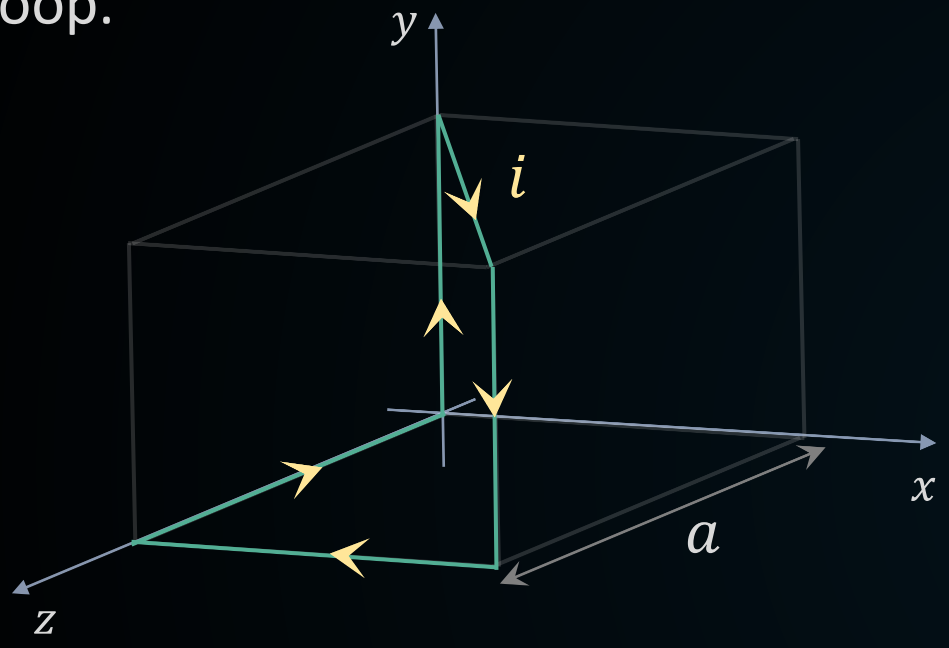
$$\vec{M}_{net} = \vec{M_1} + \vec{M_2}$$

$$= \boxed{i\,ab\,[-\hat{j} - \hat{k}]}$$

$$\boxed{\vec{M}_{net} = iab(-\hat{j} - \hat{k})}$$

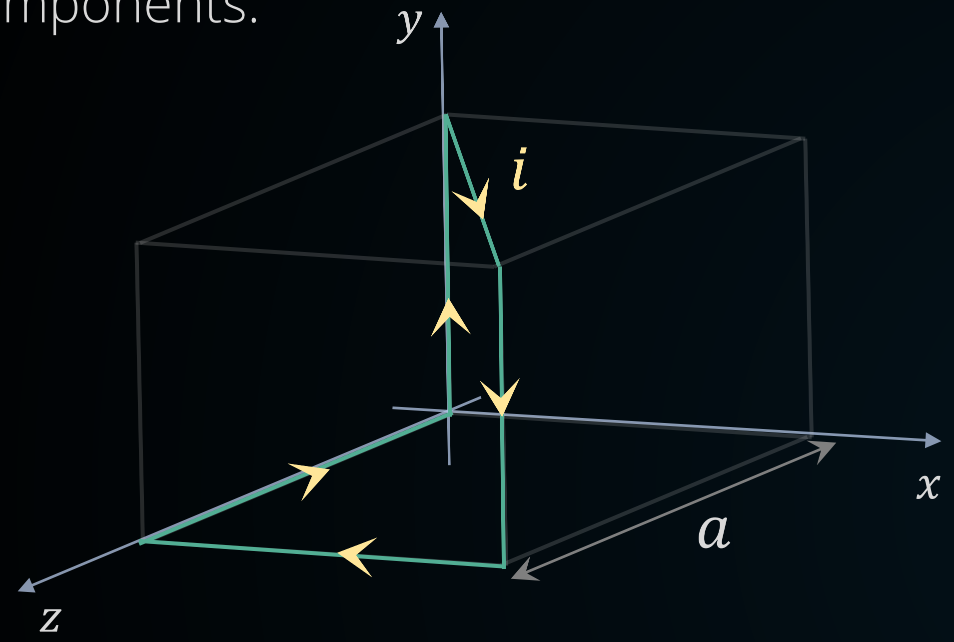
Question Solution

A cube of side a has a current carrying loop as shown. Find the magnetic moment of the loop.

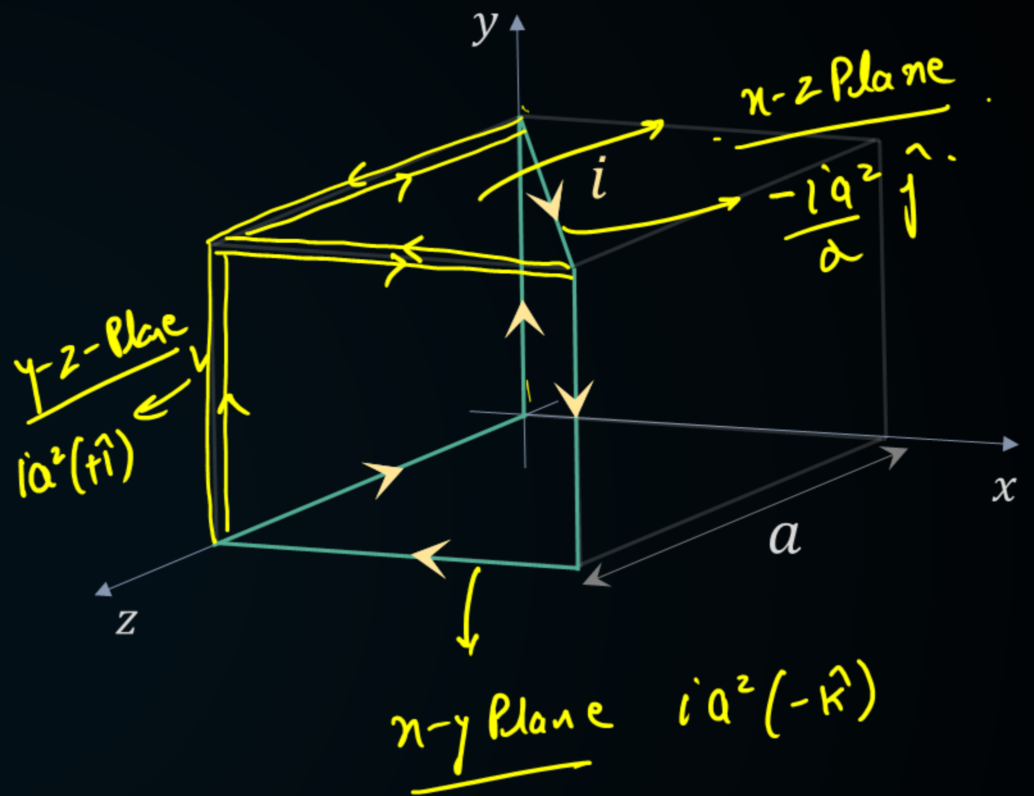


Question Solution

Rearrange the circuit as shown. Three loops are formed. Net moment is the sum of three components.



$\equiv$



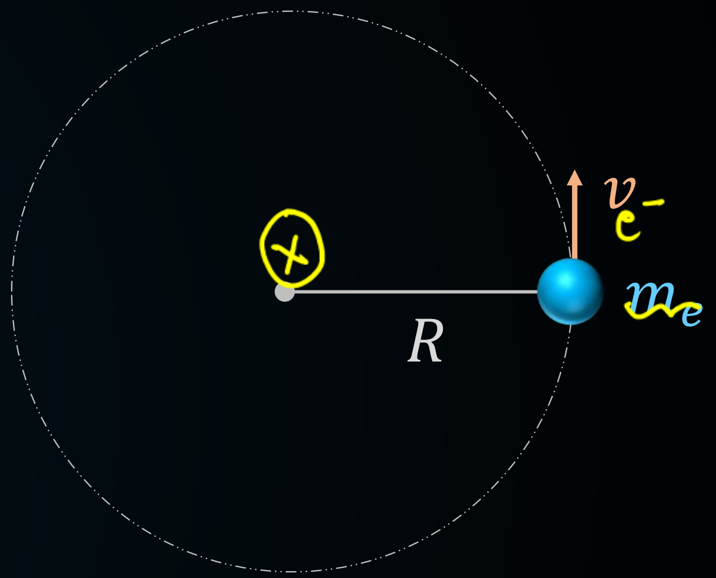
$$\vec{M} = \vec{M}_{yz} + \vec{M}_{xy} + \vec{M}_{xz} = ia^2\hat{\imath} - \frac{ia^2}{2}\hat{\jmath} - ia^2\hat{k}$$

$$\boxed{\vec{M} = ia^2\hat{\imath} - \frac{ia^2}{2}\hat{\jmath} - ia^2\hat{k}}$$

Question

An electron is moving in circular path of radius R with speed v. Find the magnetic field at the center of the circular path.

Solution



I^{st} Method

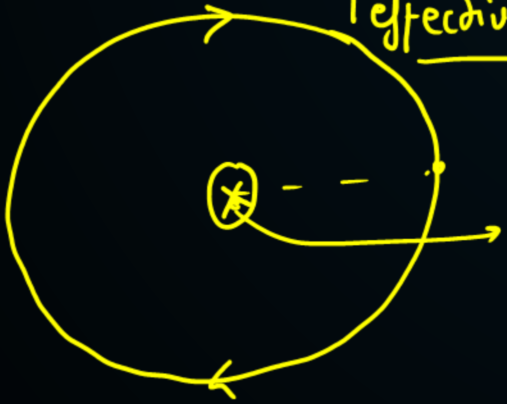
$$B = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{|\vec{r}|^3}$$

$\vec{v} \perp \vec{R}$

$|\vec{r}| = R$

$$= \frac{\mu_0 e v \cancel{R}}{4\pi R^{\cancel{3}}} = \boxed{\frac{\mu_0 e v}{4\pi R^2}}$$

II^{nd} Method



$$i_{effective} = \frac{e}{T} = \boxed{\frac{e \cdot v}{2\pi R}}$$

$$= ef$$

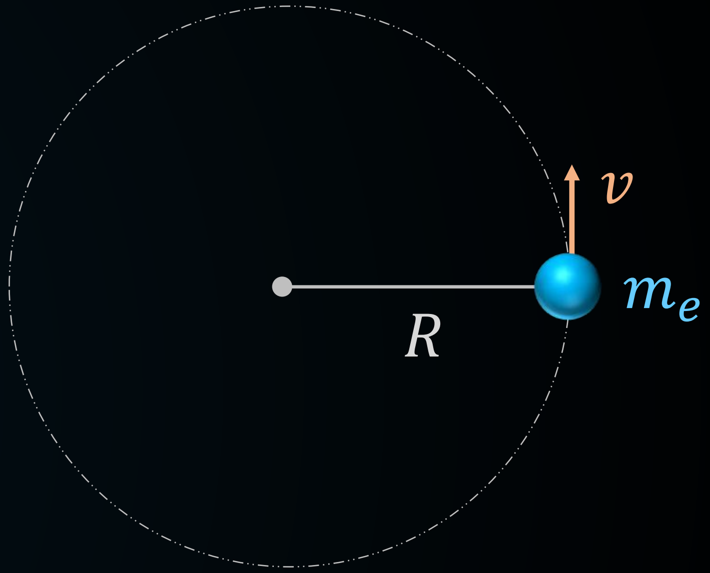
$$B = \frac{\mu_0 i}{2R} = \boxed{\frac{\mu_0 e v}{4\pi R^2}}$$

$$\boxed{B = \frac{\mu_0 e v}{4\pi R^2}}$$

Question

An electron is moving in circular path of radius R with speed v. Find the magnetic field at the center of the circular path.

Solution



1st method:

We know that the magnetic field created by a charge moving with velocity v at a distance r is,

$$B = \frac{\mu_0}{4\pi} \frac{q(\vec{v} \times \vec{r})}{|\vec{r}|^3}$$

Since we are required to find the magnetic field at the centre of the circular path,

$|\vec{r}| = R$

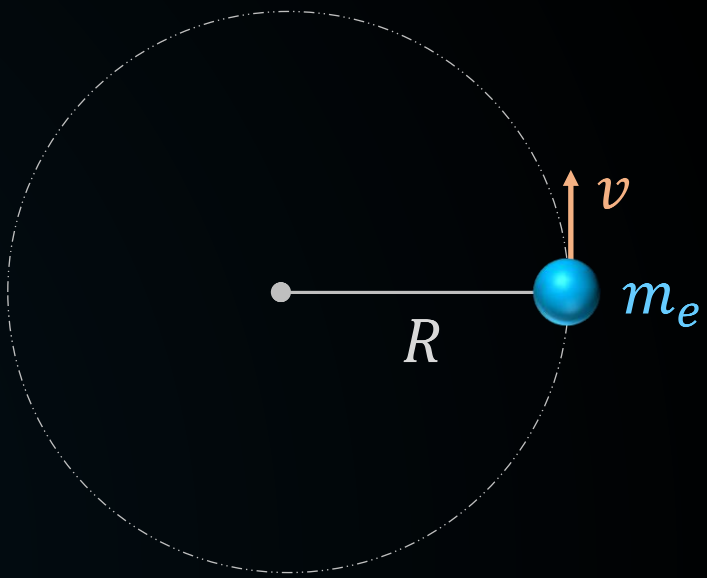
For the given case, $\vec{v} \perp \vec{R}$

Therefore, the magnetic field at the centre of the circular path becomes,

$$B = \frac{\mu_0 e v \cancel{R}}{4\pi R^{\cancel{3}}} = \boxed{\frac{\mu_0 e v}{4\pi R^2}}$$

Question Solution

Since the moving charge is electron, its charge is negative and hence, $[q(\vec{v} \times \vec{B})] = [-e(\vec{v} \times \vec{B})]$ will be directed into plane of motion of the electron.



2nd method:

Since the electron is moving with velocity v, the time period of the period motion of the electron will be, $T = \frac{2\pi R}{v}$

Therefore, current formed by the electron due to its motion is,

$$i_{effective} = \frac{e}{T} = \boxed{\frac{e \cdot v}{2\pi R}}$$

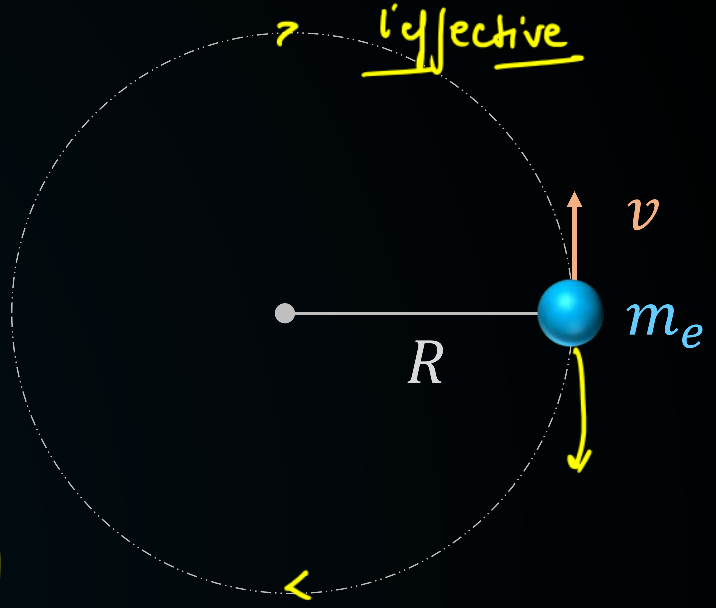
We know that the magnetic field at the centre of a current carrying loop of radius R is, $B = \frac{\mu_0 i}{2R}$

Therefore, the magnetic field at the centre of the circular path will be, $B = \frac{\mu_0 i}{2R} = \boxed{\frac{\mu_0 e v}{4\pi R^2}}$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{ev}{R^2} \otimes}$$

Gyromagnetic ratio

Due to revolution of a charged particle



- It is defined as the ratio of Magnetic moment and Angular Momentum.
- Since the electron is moving with velocity v in anticlockwise direction, current is established in the clockwise direction and the magnitude of the current is given by,

$$i_e = e\cdot f = \boxed{\frac{e\cdot v}{2\pi R}}$$

- The magnetic moment is given by, $\vec{M} = i\cdot\vec{A} \Rightarrow |\vec{M}| = \frac{ev}{2\cancel{\pi}\cancel{R}}\cdot\cancel{\pi}R^{\cancel{2}} = \boxed{\frac{evR}{2}}$
- The angular momentum is, $|\vec{L}| = m\vec{r}\times\vec{v} = \boxed{mRv}$ [Since $\vec{r}\perp\vec{v}$ and $|\vec{r}| = R$]
- The ratio of magnetic moment to angular momentum is given by, $\frac{M\cdot M}{A\cdot M} = \frac{ev\cancel{R}}{2\cdot mv\cancel{R}} = \boxed{\frac{e}{2m}} = \boxed{\frac{q}{2m}}$

independent of v, R
only depends on q/m ratio. Yani q/m ki charge.

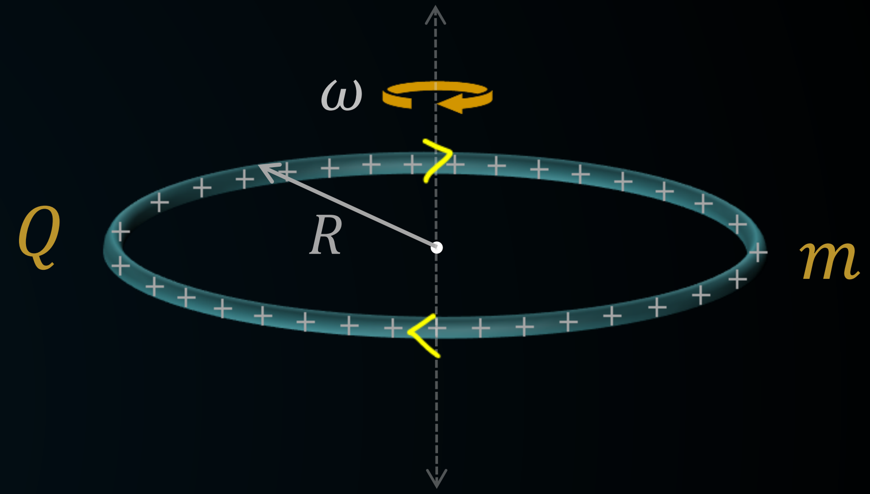
$$\boxed{\frac{M}{L} = \frac{q}{2m}}$$

M: Magnetic moment
L: Angular momentum

Question

A ring of radius R carrying charge Q is rotating about its axis with an angular velocity ω.

Find the magnetic field at the center.



Solution

Total charge of the ring = Q

Angular velocity of the ring = ω ⇨ Frequency, $f = \frac{\omega}{2\pi}$

Therefore, the current established by the electron is,

$i_e = Q \cdot f = \frac{Q \cdot \omega}{2\pi}$

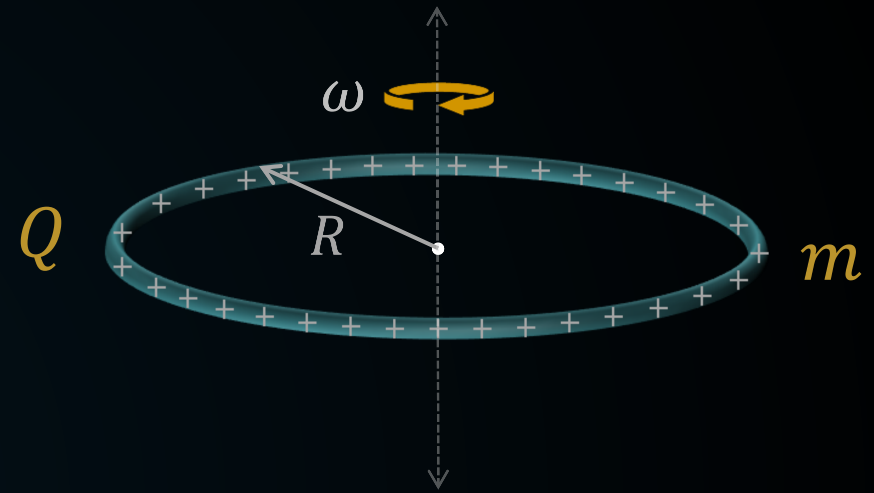
We know that the magnetic field at the centre of a current carrying loop of radius R is, $B = \frac{\mu_0 i}{2R}$

Therefore, the magnetic field at the centre of the ring will be, $B = \frac{\mu_0 Q \omega}{4\pi R}$

$$\boxed{B = \frac{\mu_0 Q \omega}{4\pi R}}$$

Gyromagnetic ratio

Due to revolution of a charged ring



Total charge of the ring = Q

Angular velocity of the ring = ω

Thus, frequency, $f = \frac{\omega}{2\pi}$

Therefore, the current established by the electron is,

$$I_e = Q\cdot f = \frac{Q\omega}{2\pi}$$

Magnetic moment: $M\cdot M = \frac{Q\omega\cdot\cancel{\pi}R^2}{2\cancel{\pi}} = \boxed{\frac{Q\omega R^2}{2}}$

Angular momentum: $A\cdot M = I\omega = \underline{mR^2\cdot\omega}$

$$\frac{M\cdot M}{A\cdot M} = \frac{Q\cancel{\omega}\cancel{R^2}}{2\cdot m\cancel{R^2}\cancel{\omega}} = \boxed{\frac{Q}{2m}}$$

$$\boxed{\frac{M}{L} = \frac{Q}{2m}}$$

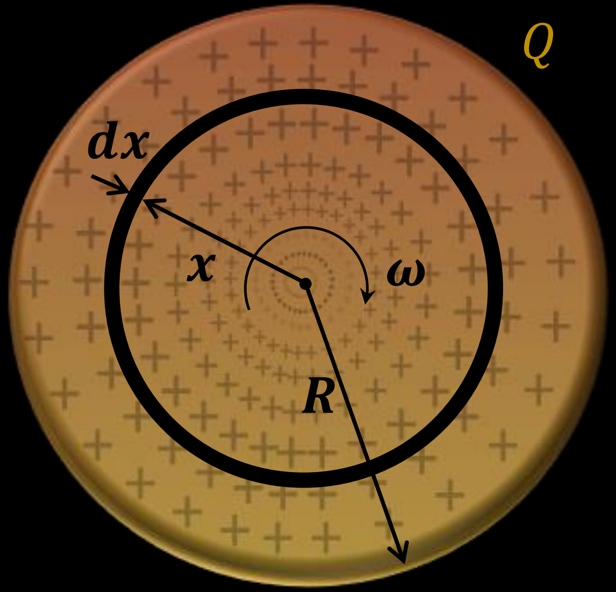
M: Magnetic moment

L: Angular momentum

Question

Calculate the ratio of magnetic moment and angular momentum of a charged non-conducting disc rotating with an angular velocity ω about its center.

Solution



Consider an elemental ring of radius x and thickness dx, as shown in the figure.

The circular plate has radius R and charge Q. Thus, the surface charge density of the plate is, $\sigma = \frac{Q}{\pi R^2}$

The angular momentum of the disc: $A.M = I\omega = \boxed{\frac{mR^2}{2}\cdot\omega}$

Therefore, the charge of the elemental ring is,

$$dq = \sigma \cdot 2\pi x \cdot dx$$

$$= \frac{Q}{\cancel{\pi} R^2} \cdot 2\cancel{\pi} x\, dx = \frac{2Q}{R^2} x\, dx$$

Since the ring is rotated with angular velocity ω, therefore, the current produced by the elemental ring is given by,

$$di_e = \frac{\cancel{2}Q \cdot x\, dx}{R^2} \frac{\omega}{\cancel{2}\pi} \Rightarrow di_e = \frac{Q}{R^2}\frac{\omega}{\pi} x\, dx$$

Question | Solution

The magnetic moment of the elemental ring will be,

$$dM = di_e \cdot \text{Area.}$$

$$= \frac{Q\omega}{R^2}\,\frac{x\,dx}{\cancel{\pi}} \cdot \cancel{\pi}x^2.$$

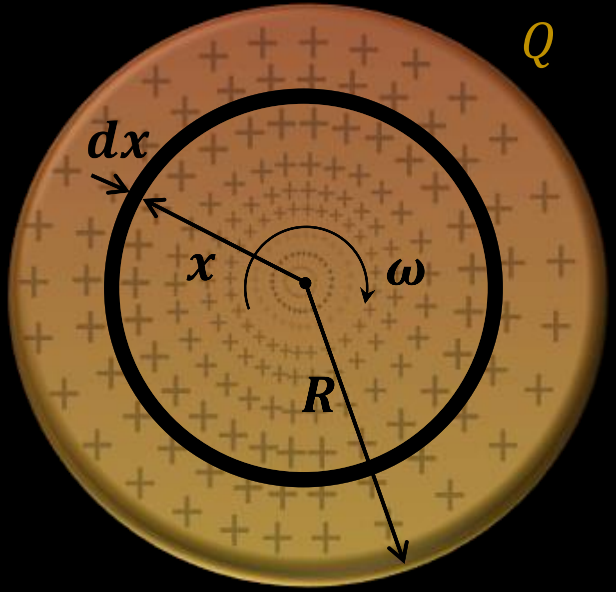
Hence, total magnetic moment of the plate will be,

$$\int_0^M dM = \frac{Q\omega}{R^2}\int_0^R x^3\,dx$$

$$M\cdot M = \frac{Q\omega}{R^2}\cdot\frac{R^4}{4} = \boxed{\frac{Q\omega R^2}{4}}$$

Therefore, the ratio $\frac{M}{L}$ becomes: $\frac{M\cdot M}{A\cdot M} = \frac{Q\omega \cancel{R^2}\,2}{4\cdot m\cancel{R^2}\omega} = \boxed{\frac{Q}{2m}}$

$$\boxed{\frac{M}{L} = \frac{Q}{2m}}$$



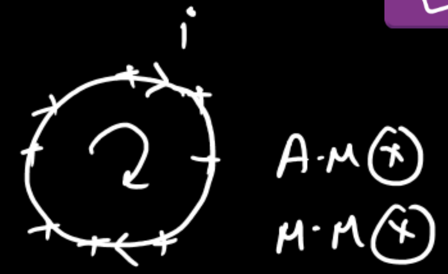
Short trick to find the magnetic moment of the given disc:

$$M\cdot M = \frac{Q}{2m} \times A\cdot M$$

$$= \frac{Q}{2\cancel{m}} \times \frac{\cancel{m}R^2\cdot\omega}{2} = \boxed{\frac{Q\omega R^2}{4}}$$

Gyromagnetic ratio

- The ratio of magnetic moment to angular momentum $= \frac{Q}{2m}$.
- This ratio is a constant.
- If the charge is positive, then directions of $\vec{M}$ and $\vec{L}$ is same.
- If the charge is negative, then the directions of $\vec{M}$ and $\vec{L}$ are opposite.

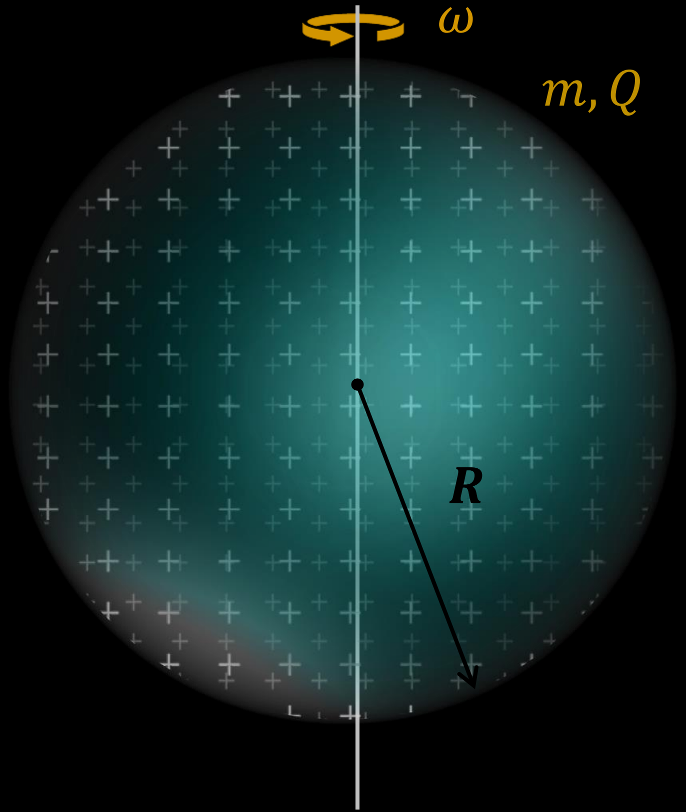


Question

Calculate the magnetic moment of a charged non-conducting sphere rotating with an angular velocity ω about its center.

Solution

NEET



Total charge of the sphere = Q

Angular velocity of the sphere = ω

Moment of ine2rtia of the sphere about the rotation axis passing through its diameter, $I = \frac{2}{3}mR^2$

Therefore, angular momentum of the sphere, $L = I\omega = \frac{2}{5}mR^2\omega$

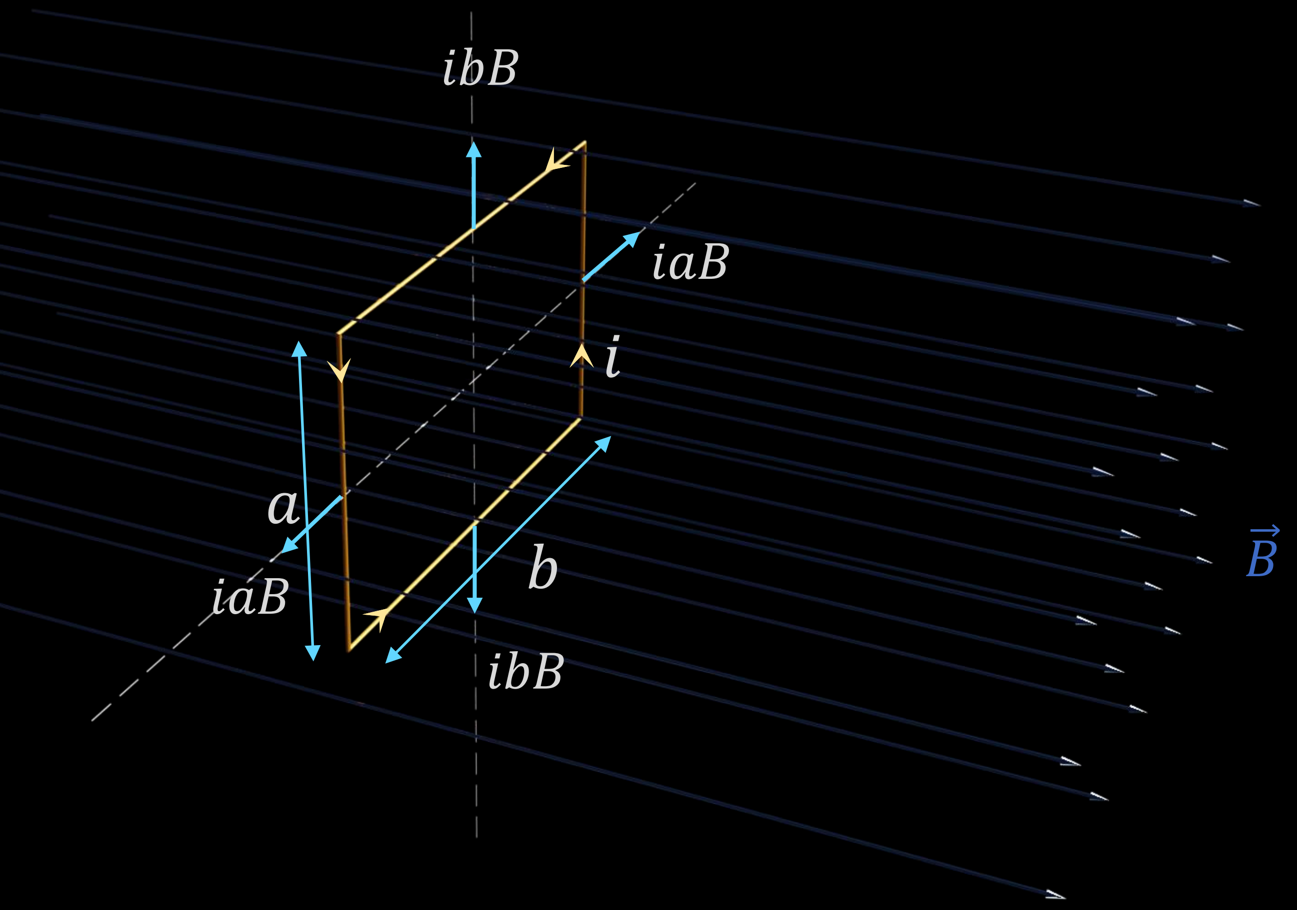
If the magnetic moment of the sphere is denoted by M, then,

$$\frac{Q}{\cancel{2}\cancel{m}} = \frac{M}{\frac{\cancel{2}}{5}\cancel{m}R^2 \cdot \omega}$$

$$\boxed{M = \frac{Q\omega R^2}{5}}$$

$$\boxed{M = \frac{QR^2\omega}{5}}$$

Torque on a Current Carrying Loop

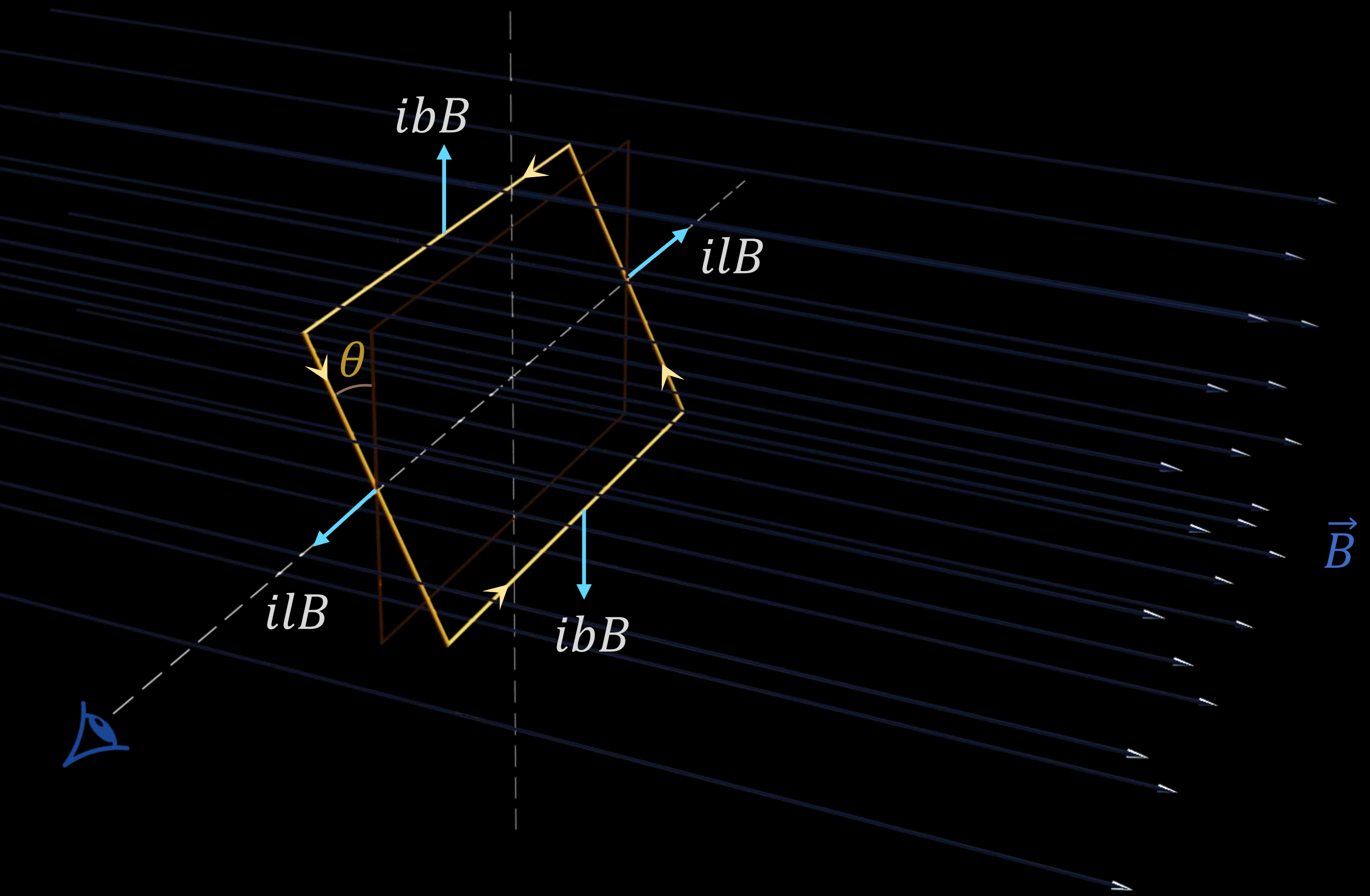


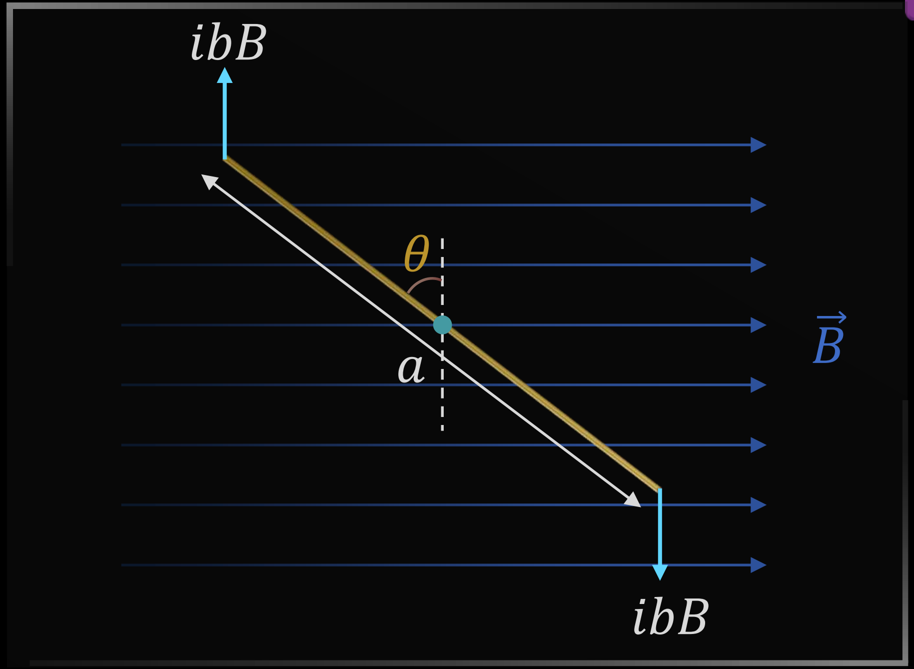
- Magnetic force on each side of the loop gets cancelled by the magnetic force acting on the opposite side of the loop. Hence, net magnetic force on the loop is zero.
- Since all the opposite pair of forces are acting on the same on line, the net torque on the loop is also zero.
- In this case, the direction of magnetic moment ($\vec{M}$) of the loop and the direction of magnetic field ($\vec{B}$) is same.

$$\vec{F}_{net} = \vec{0}$$

$$\vec{\tau}_{net} = \vec{0}$$

Torque on a Current Carrying Loop



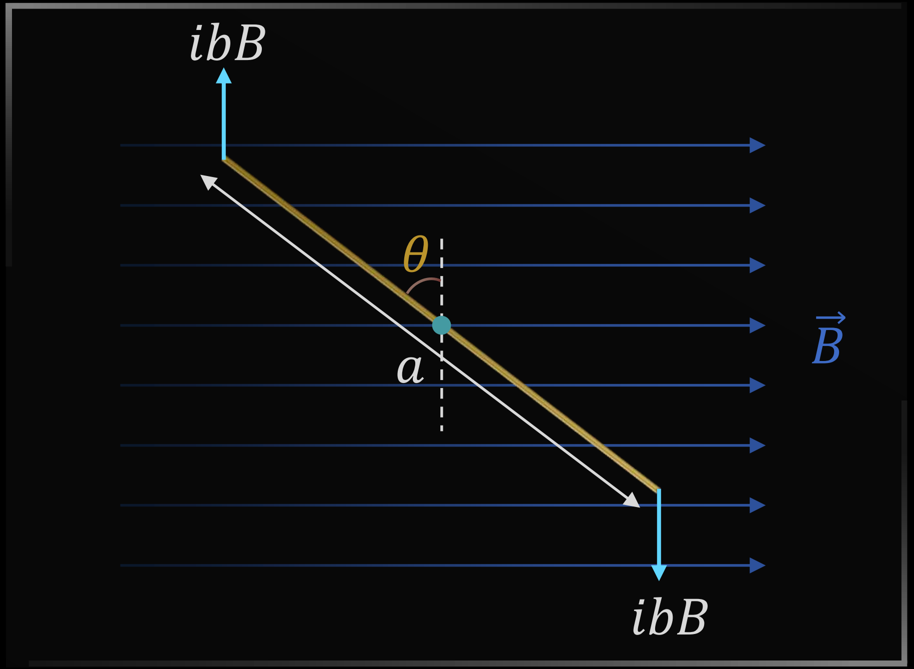


Side view of the loop

If we rotate the loop by an angle θ in a plane to perpendicular to its initial plane, then the force on each side of the loop remains same as previous and hence, the net force on the loop is still zero. But there is a separation between the line of action of opposite pair of force 'ibB' as shown by the side view of the loop and hence, they form a couple.

Torque on a Current Carrying Loop

Since the opposite pair of force 'ibB' form a couple, they will produce a non-zero torque in the system. Thus, the magnitude of the torque will be:



$$\tau = iaB\frac{b}{2}\sin\theta + iaB\frac{b}{2}\sin\theta$$

$$\tau = iabB\sin\theta$$

$$= iAB\sin\theta$$

$$= MB\sin\theta$$

$$\boxed{\vec{\tau} = \vec{M}\times\vec{B}}$$ Valid for Every loop.

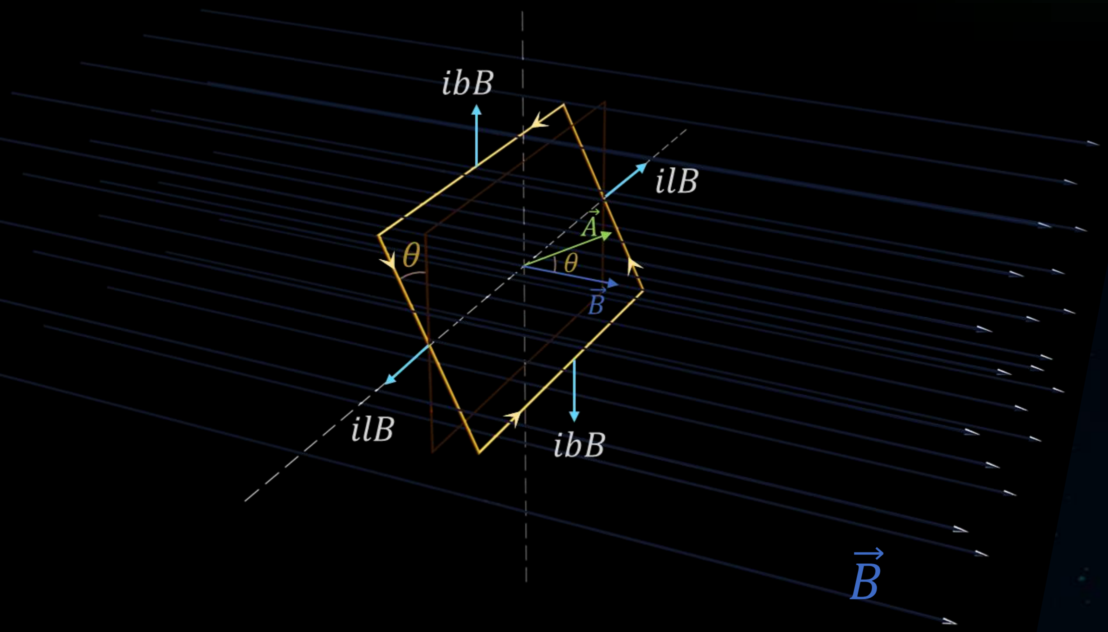
$F_{net} = 0$ {always in a loop in Constant M-f}

$\tau = 0$ {when $\vec{M}$ & $\vec{B}$ are $\parallel$ or Anti $\parallel$}
$\theta = 0°$ $\quad \theta = 180°$

$$\tau = iabB\sin\theta$$

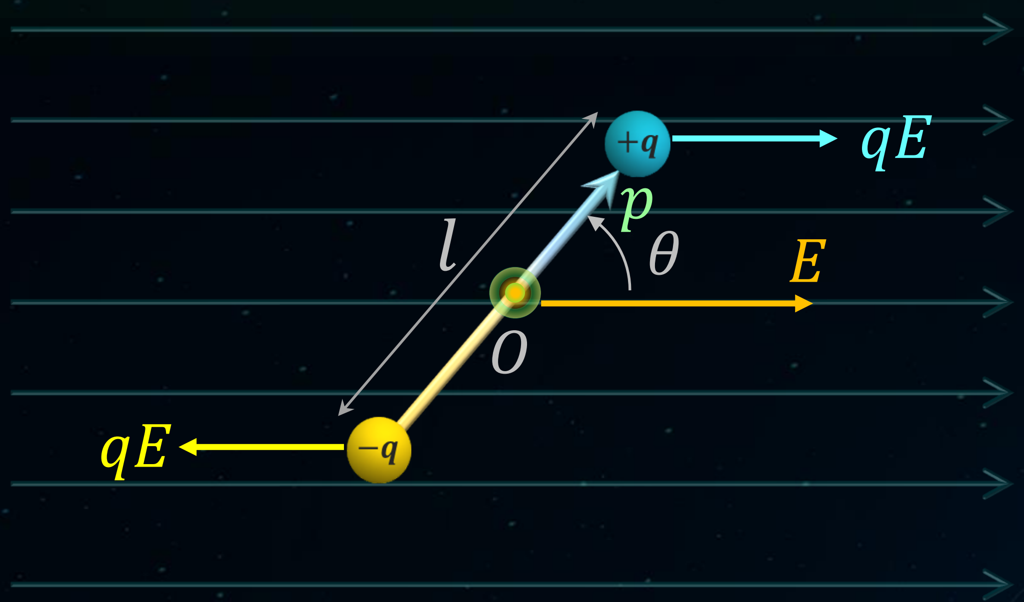
$$\vec{\tau} = \vec{M}\times\vec{B}$$

Torque on a Current Loop



$$\vec{\tau} = \vec{M} \times \vec{B}$$

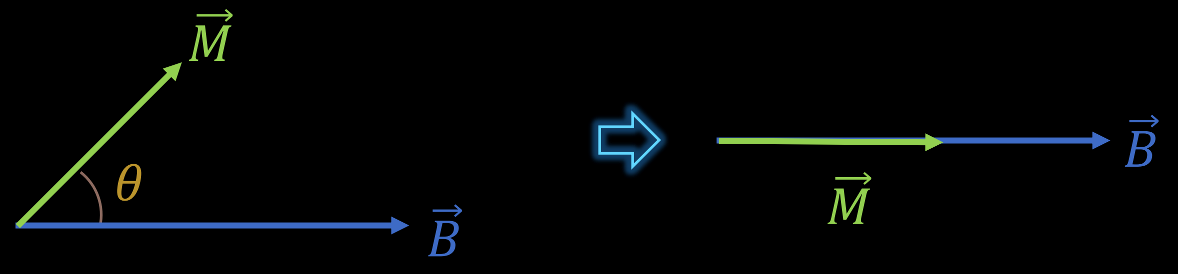
TORQUE ON AN ELECTRIC DIPOLE

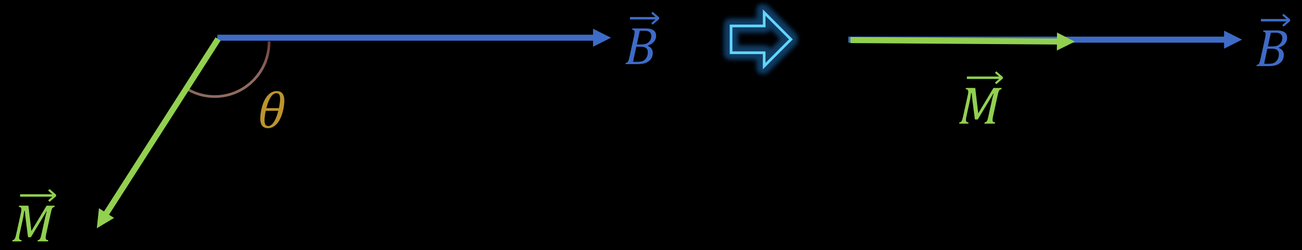


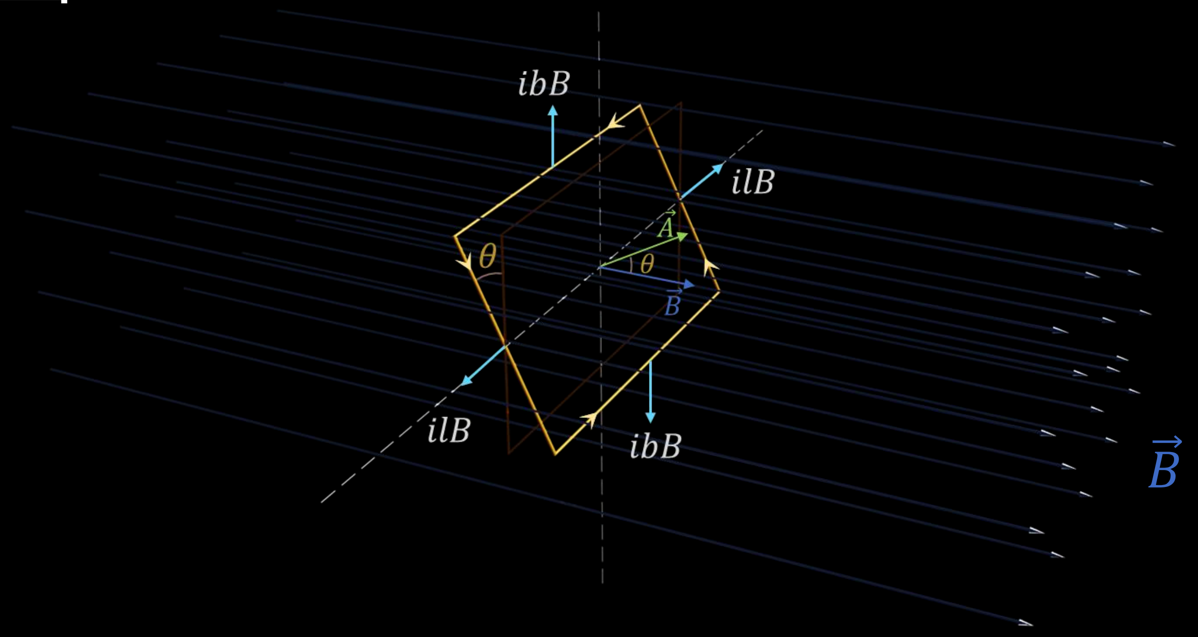
$$\vec{\tau}_{net} = \vec{p} \times \vec{E}$$

Torque on a Current Carrying Loop

$$\vec{\tau} = \vec{M} \times \vec{B}$$



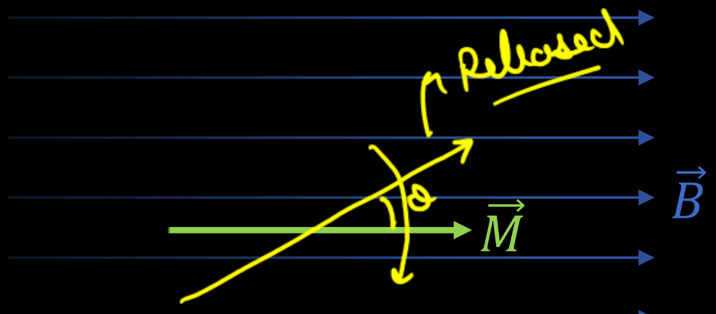


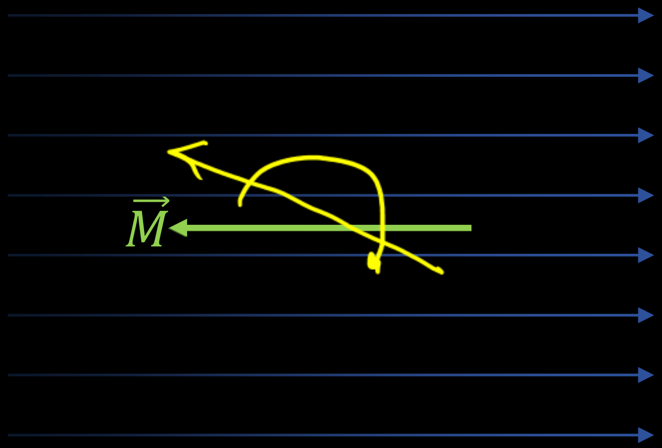


The effect of net torque is to align $\vec{M}$ with $\vec{B}$ from smaller angle side.

Torque on a Current Carrying Loop

$$\vec{\tau} = \vec{M} \times \vec{B}$$





$$\theta = 0°$$

Stable equilibrium

In this case, if we rotate the magnetic dipole from this equilibrium position, the dipole will align itself along the direction of the magnetic field i.e., the dipole will regain its initial alignment.

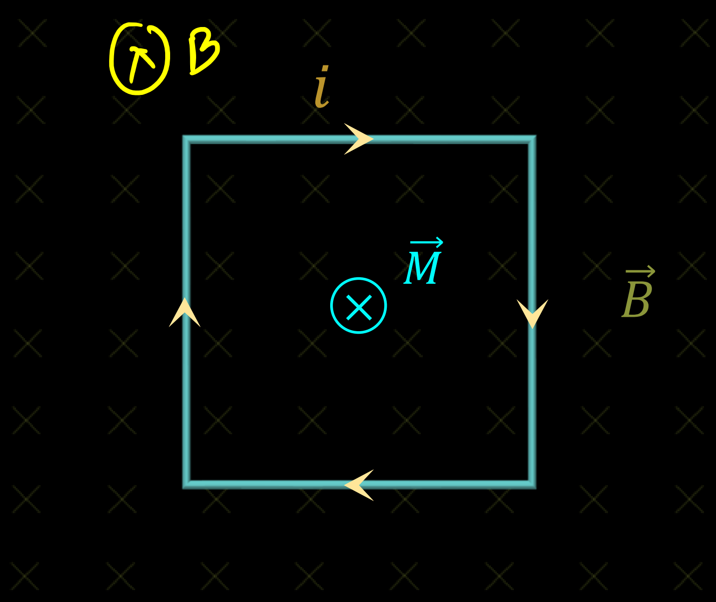
$$\theta = 180°$$

Unstable equilibrium

In this case, if we rotate the magnetic dipole from this equilibrium position, the dipole will flip and align itself along the direction of the magnetic field i.e., the dipole will not regain its initial alignment.

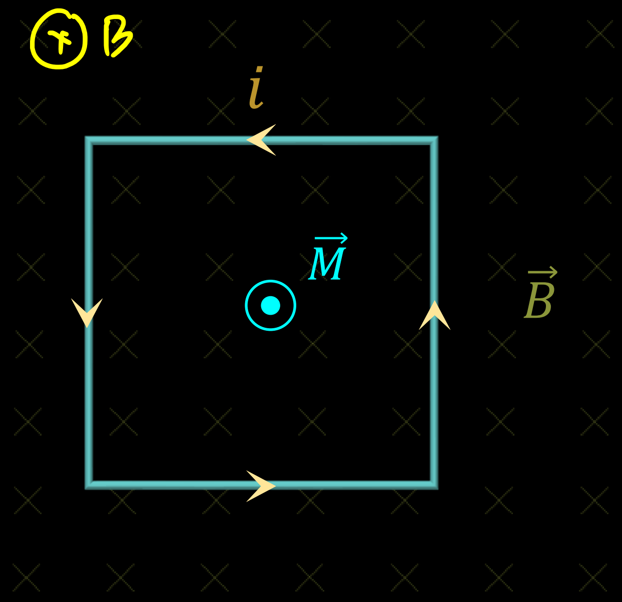
Torque on a Current Carrying Loop

$$\vec{\tau} = \vec{M} \times \vec{B}$$



$\theta = 0°$

Stable equilibrium

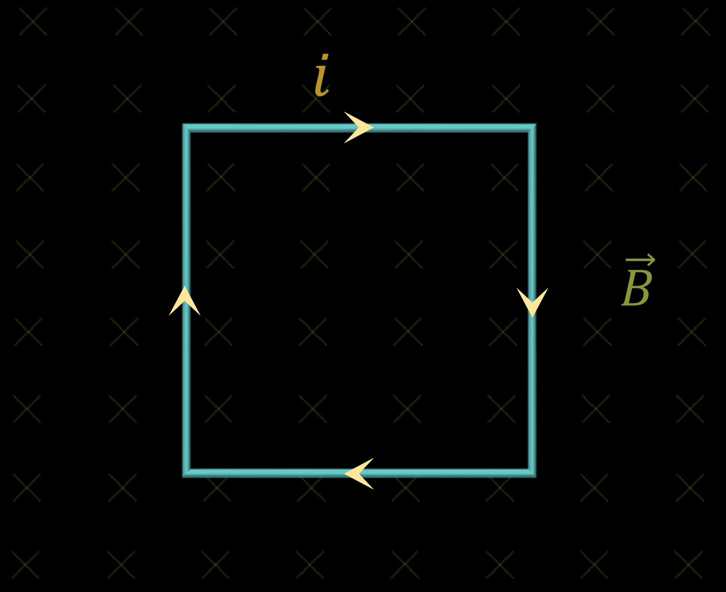


$\theta = 180°$

Unstable equilibrium

Torque on a Current Carrying Loop

Time period of Angular SHM



Stable equilibrium

If $\tau = -k\theta$

$$T = 2\pi\sqrt{\frac{I}{k}}$$

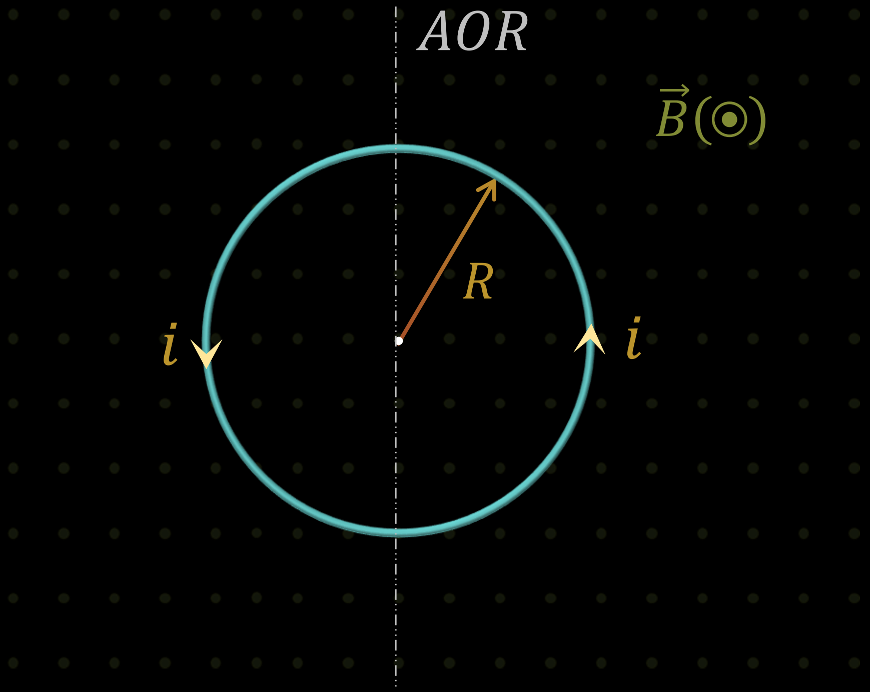
I is moment of Inertia

$$\vec{\tau} = \vec{M} \times \vec{B}$$

Question

A current carrying ring of mass m is placed inside a magnetic field B as shown. If the ring is slightly rotated and released, then find the time period of the oscillation.

Solution



The magnetic moment of the loop is $M = i(\pi R^2)$ and according to the direction of the current, the direction of the magnetic moment will be coming out of the plane of paper.

Thus, the direction of magnetic moment of the ring and the direction of magnetic field is same. Hence, the ring will possess stable equilibrium.

If we rotate the ring by a small angle θ, the ring will execute angular SHM and the torque on the ring :

$\tau = -MB\sin\theta$

$\tau = -MB\,\theta$ .....(1)

We know that if any rigid body execute angular SHM and the body has moment of inertia I about the axis of rotation, then, the expression of the torque on the body and the time period of oscillation are:

$\tau = -K\theta$ ....(2)

$T = 2\pi\sqrt{\dfrac{I}{K}}$

Question Solution

Comparing equation (1) and (2), we get,

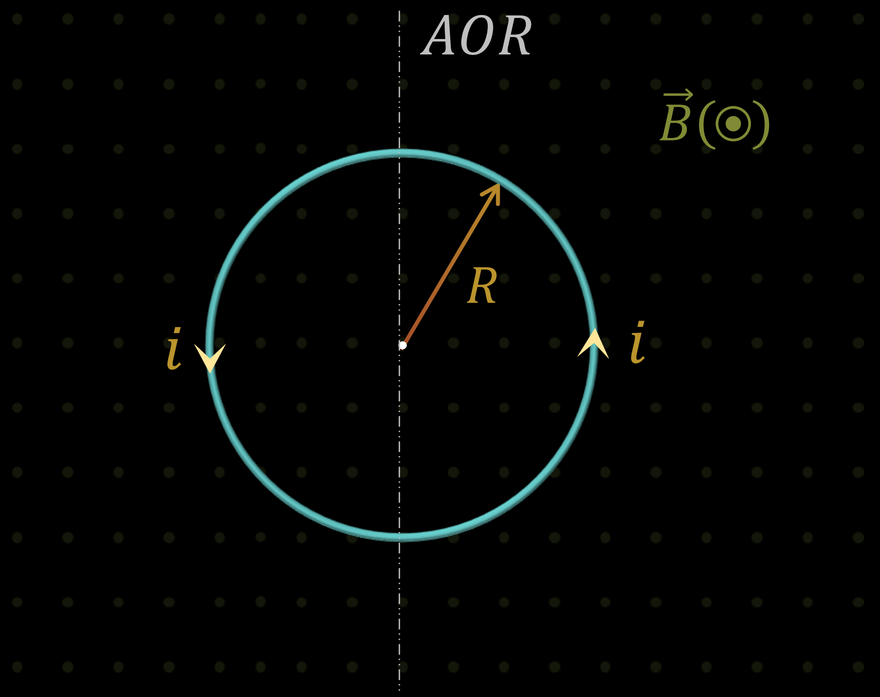
$K = MB$

The moment of inertia of the ring about the given axis of rotation (AOR) is given by,

$$I = \frac{mR^2}{2}$$

Therefore, the time period of oscillation becomes,

$$T = 2\pi\sqrt{\frac{I}{K}} = 2\pi\sqrt{\frac{mR^2}{2\cdot MB}} = 2\pi\sqrt{\frac{m\cancel{R^2}}{2\cdot i\cdot \pi \cancel{R^2}\cdot B}} = \sqrt{\frac{2\pi m}{iB}}$$

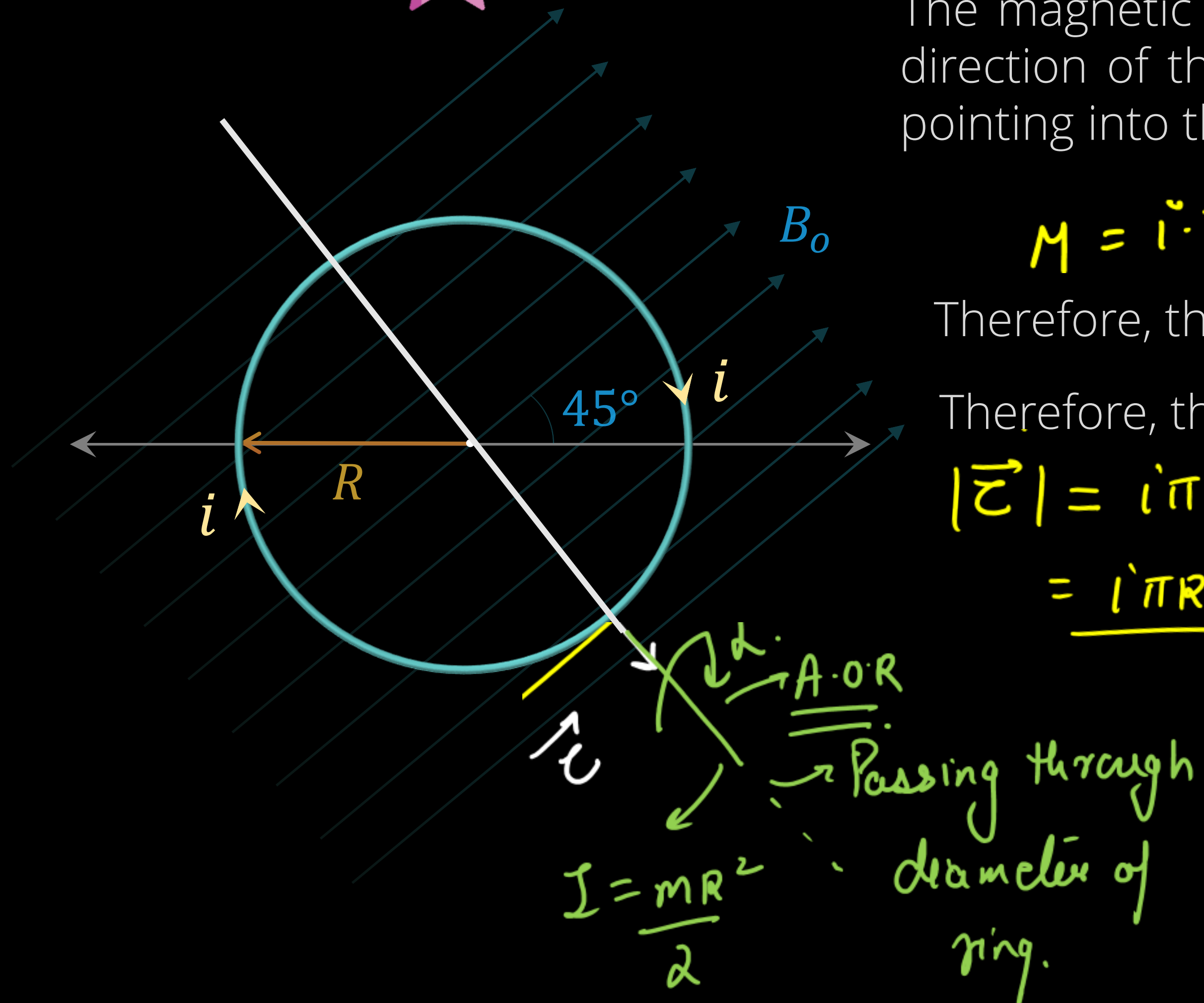


$$T = \sqrt{\frac{2\pi m}{iB}}$$

Question Solution

A current carrying ring of mass m is placed inside a magnetic field B_o as shown. Find the angular acceleration at this instant.

Solution



The magnetic moment of the loop is $M = i(\pi R^2)$ and according to the direction of the current, the direction of the magnetic moment will be pointing into the plane of paper.

$M = i \cdot \pi R^2 \otimes$

Therefore, the angle between $\vec{M}$ & $\vec{B} = 90°$

Therefore, the magnitude of the torque on the ring will be,

$|\vec{\tau}| = i\pi R^2 B \sin 90°$

$= i\pi R^2 B$

$\vec{\tau}$ direction

Axis of Rotation

Since the direction of $\vec{\tau}$ denotes the location of axis of rotation, the A.O.R for the given ring will as shown in the figure.

Question | Solution

The moment of inertia of the ring about the given axis of rotation ($A.O.R$) is given by,

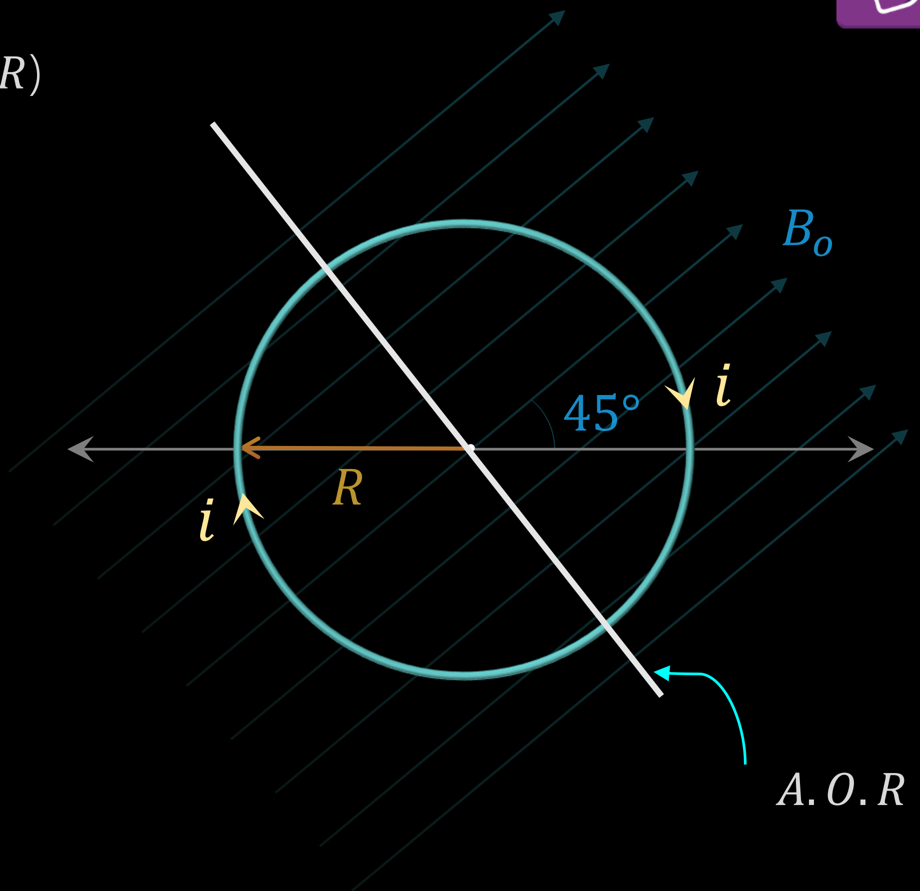
$$I = \frac{mR^2}{2}$$

If the angular acceleration of the ring is α, then,

$$\boxed{\tau = I\alpha}$$

$$i\pi \cancel{R^2} B = \frac{m\cancel{R^2}}{2} \cdot \alpha$$

$$\boxed{\frac{2i\pi B}{m} = \alpha}$$



$$\alpha = \frac{2\pi i B}{m}$$

If the body is free to rotate, then it will rotate about an axis passing through its Center of mass and parallel to the torque

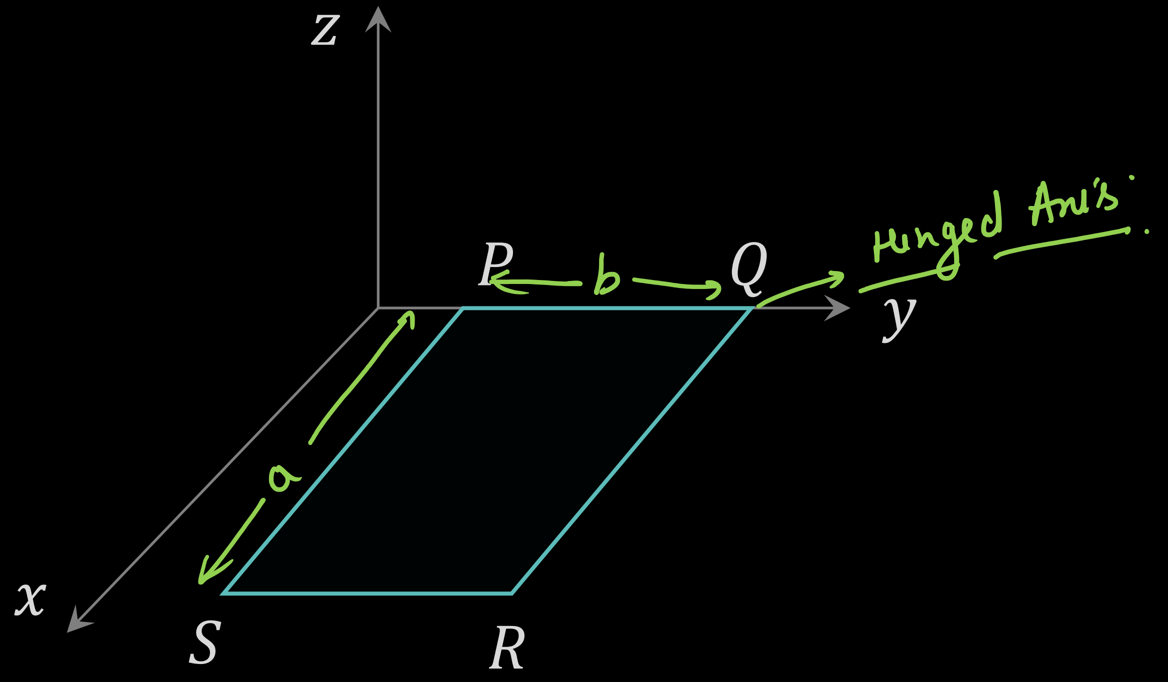
Question Solution

A rectangular loop $PQRS$ made from a uniform wire of length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along the horizontal line taken as y-axis. Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{\imath} + 4\hat{k})B_o$ exists in this region. The loop is held in the x-y plane and a current i is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium.

a. What is the direction of the current i in PQ?

b. Find the magnetic force on the arm RS.

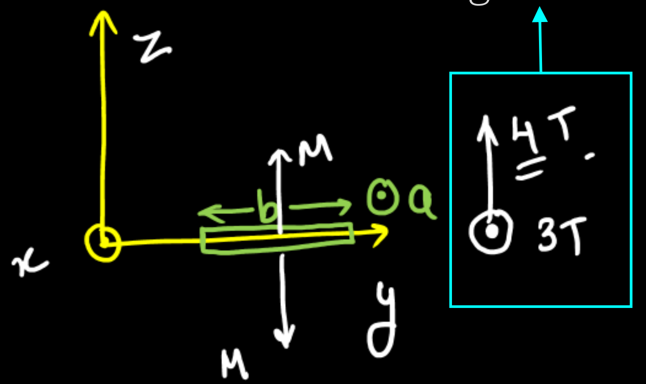
c. Find the expression for I in terms of B_o, a, b and m.



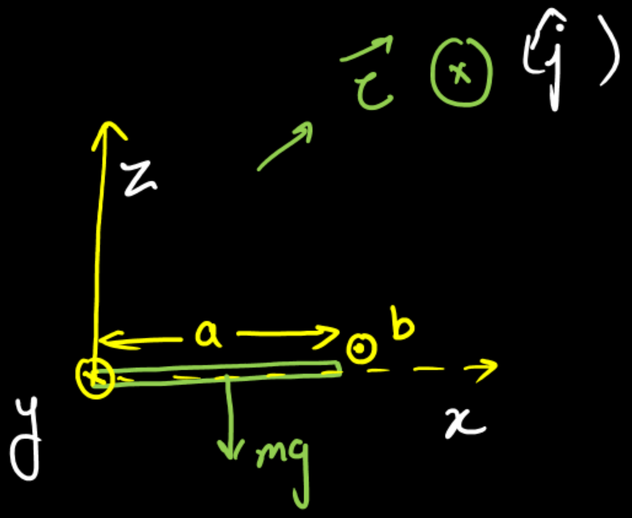
Question | Solution

(a)

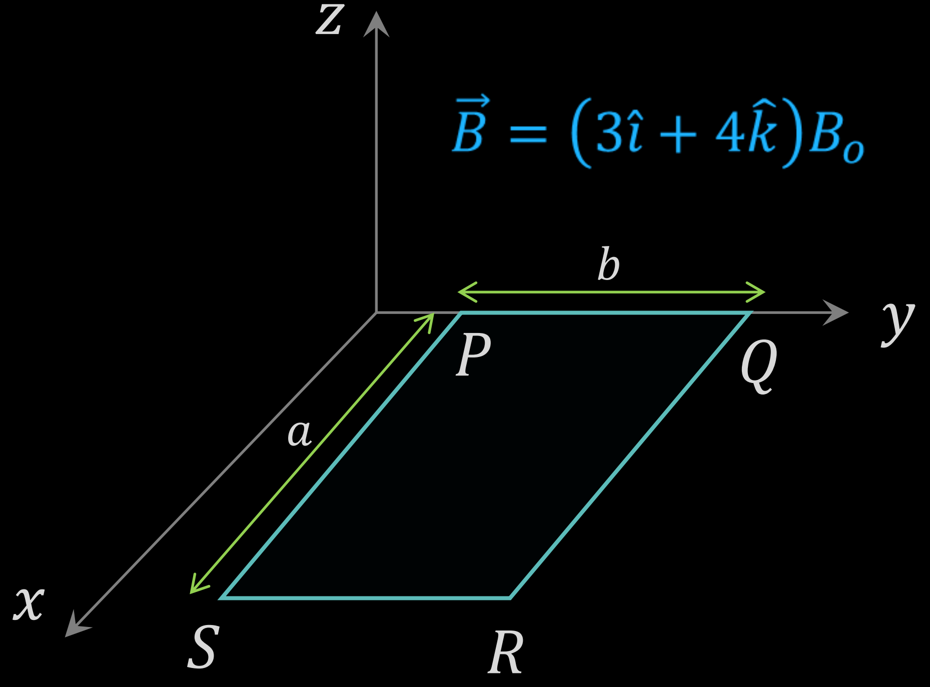
Component division of the magnetic field



View from *SR* side of the loop towards $-ve\ x$-axis



View from *PS* side of the loop towards $+ve\ y$-axis



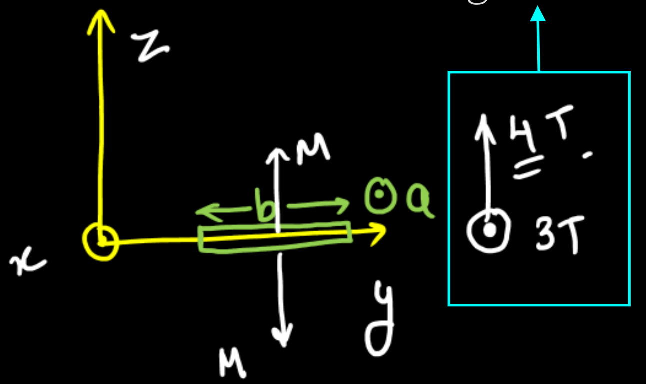
Assume that the current in the loop is i which is either clockwise or, anticlockwise. This means the magnetic moment of the loop $M = iab$ will either be parallel to z-axis (If current in anticlockwise from P to SRQ) or antiparallel to z-axis (If current in clockwise from P to QRS).

- Since the z-component of the magnetic field i.e., $4B_0\ Tesla$ is either parallel or antiparallel to the magnetic moment M, the torque on the loop produced by it is zero.
- Since the x-component of the magnetic field i.e., $3B_0\ Tesla$ is perpendicular to the magnetic moment M, the torque on the loop produced by it: $\tau_B = MB = (iab)3B_0$

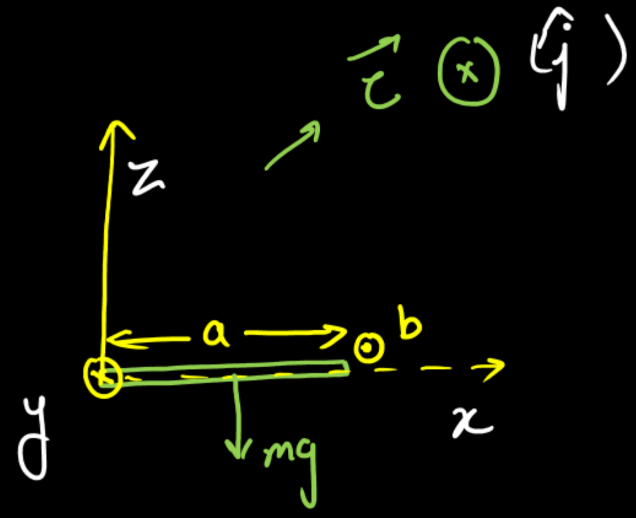
Question Solution

(a)

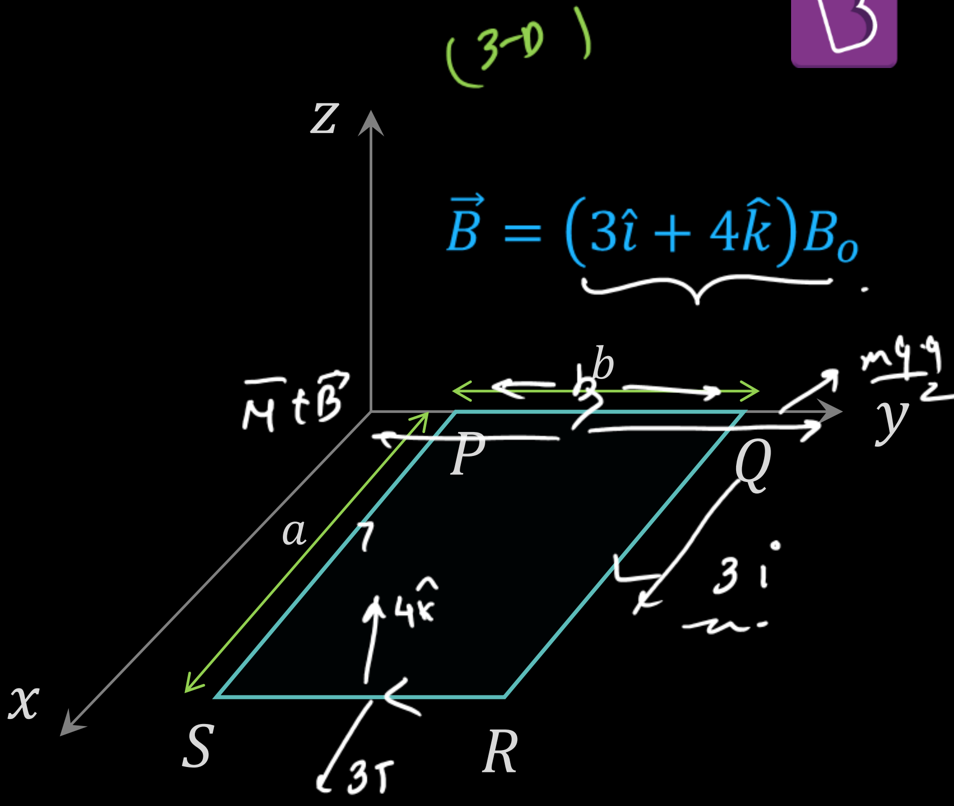
Component division of the magnetic field



View from SR side of the loop towards $-ve\ x$-axis



View from PS side of the loop towards $+ve\ y$-axis



The torque on the loop due to the gravitational force, $\tau_y = mg\left(\frac{a}{2}\right)$ and the direction of this torque is along $+ve\ y$-axis.

This torque τ_y should be balanced by the torque τ_B due to x-component of the magnetic field in order to stay the loop in the horizontal position in equilibrium. For this purpose, the direction of τ_B should be along $-ve\ y$-axis. It is possible only if magnetic moment M is antiparallel to z-axis (i.e., current in clockwise from P to QRS).

a. Current in loop $PQRS$ is clockwise from P to QRS

Question | Solution

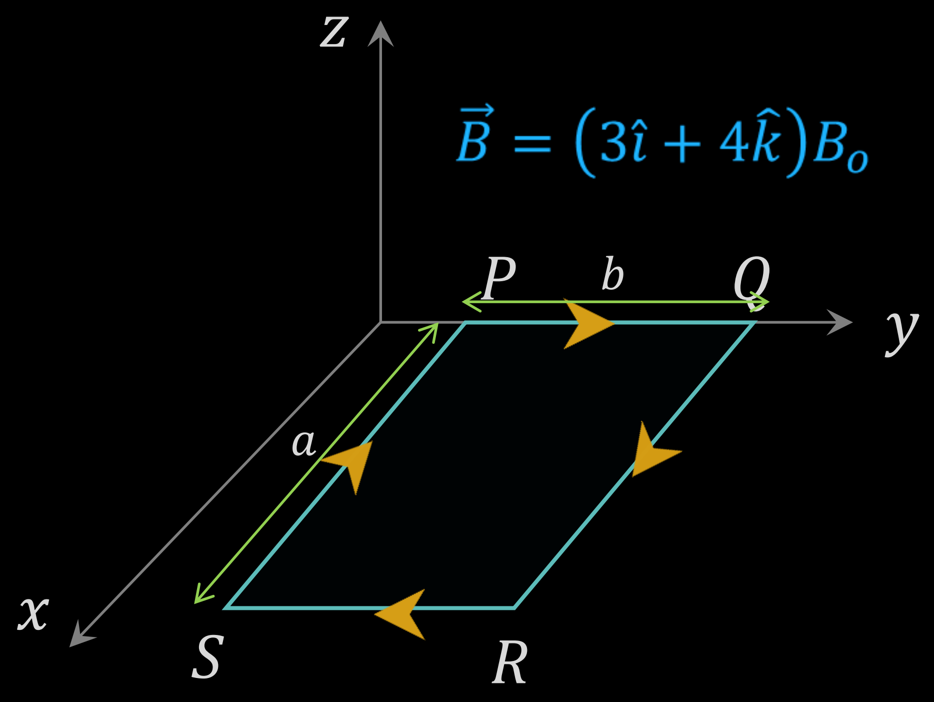
(b) According to the direction of the current in the arm RS, its length is denoted as:

$$\vec{l} = \underline{-b}\hat{j}$$

Therefore, the magnetic force on the arm RS is given by,

$$\vec{F}_{RS} = i\vec{l} \times \vec{B}$$

$$\vec{F}_{RS} = \underline{i(-b\hat{j})} \times \underline{(3\hat{i}+4\hat{k})B_o} = B_o ib(3\hat{k} - 4\hat{i})$$



(c) Equating the torque τ_B and τ_y, we get,

$$i\cancel{a}b \cdot 3B_o = mg \cdot \frac{\cancel{a}}{2} \Rightarrow i = \boxed{\frac{mg}{6bB_o}}$$

b. $\vec{F} = B_o ib(3\hat{k} - 4\hat{\imath})$

c. $I = \dfrac{mg}{6bB_o}$